Spectator stresses and CMB observables

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The large-scale curvature perturbations induced by spectator anisotropic stresses are analyzed across the matter-radiation transition. It is assumed that the anisotropic stress is associated with a plasma component whose energy density is subdominant both today and prior to photon decoupling. The enforcement of the momentum constraint and the interplay with the neutrino anisotropic stress determine the regular initial conditions of the Einstein-Boltzmann hierarchy. The cosmic microwave background observables have shapes and phases which differ both from the ones of the conventional adiabatic mode as well as from their nonadiabatic counterparts.

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Supplementary anisotropic stresses may arise before matter-radiation equality for diverse physical reasons. For instance, on a phenomenological ground, one is often led to consider a putative anisotropic stress associated with the dark-matter component [1]. Along a similar line it is also legitimate to consider the possibility that the dark-energy component is endowed with an appropriate anisotropic stress [2]. The two possibilities mentioned so far are characterized by a common feature: in both cases the anisotropic stress is attributed to species which are dominant (or just slightly subdominant) today. This is the case for the dark-matter and for the dark-energy components in the ΛCDM paradigm whose basic parameters will be taken to coincide, for the purposes of the present analysis, with the ones determined by analyzing the WMAP 5 yr data alone[1] [3,4]. The purpose of the present paper is to scrutinize the situation where the source of large-scale inhomogeneity resides in a spectator anisotropic stress whose associated energy density is subdominant today. It will be argued that the "stressed" initial conditions for the CMB anisotropies constitute an intermediate case between the purely adiabatic and the purely nonadiabatic initial conditions (see, e.g., [5]). The energy-momentum tensor of the spectator fluid can be parametrized as

$$\mathcal{T}_{0}^{0} = \delta_s \rho_x, \qquad \mathcal{T}_{i}^{j} = -\delta_s p_x + \Pi_i^j, \qquad \mathcal{T}_{0}^{i} = 0, \quad (1)$$

where $\delta_s p_x = w_x \delta_s \rho_x$ and $\partial_i \partial_j \Pi^{ij} = (p_\gamma + \rho_\gamma)\nabla^2 \sigma_x$; p_γ and ρ_γ are the pressure and energy density of the photons; the $(0i)$ components of the energy-momentum tensor of the spectator field are set to zero since it is of higher order when the energy density and pressure of the same field are fully inhomogeneous (i.e. they do not have homogeneous background contribution). The barotropic index w_x will be taken to be constant for the purposes of the present investigation. In the present paper, only the scalar fluctuations of the metric will be considered: the symbol δ_s appearing in Eq. (1) denotes the scalar fluctuations of the corresponding quantity. The time evolution of the system[2] will be parametrized in terms of the scale factor $a(\tau)$ normalized at equality, i.e. $\alpha = a/a_{\rm eq}$; within this parametrization the evolution equations for the fluctuations of the geometry and of the sources shall be written in Fourier space and in terms of the rescaled wave number $\kappa = k\tau_1$; if $\kappa < 1$ the given wavelength exceeds the Hubble radius right before the equality time. In the opposite case (i.e. $\kappa > 1$) the given wavelength is smaller than the Hubble radius at the same time $\tau_1 = (\sqrt{2}+1)\tau_{\rm eq}$. In the synchronous coordinate system the perturbed entry of the geometry reads[3]

$$\delta_s g_{ij}(\kappa, \alpha) = a_{\rm eq}^2 \alpha^2(\tau)\left[\hat{\kappa}_i\hat{\kappa}_j h + 6\xi\left(\hat{\kappa}_i\hat{\kappa}_j - \frac{\delta_{ij}}{3}\right)\right], \quad (2)$$

where $h(\kappa, \alpha)$ and $\xi(\kappa, \alpha)$ describe the scalar fluctuations of the metric and enter also the contravariant components of the energy-momentum tensor, i.e.

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[1]For sake of concreteness the values of the cosmological parameters used in the present paper will be taken to coincide with the parameters arising from the 5 yr best fit to the WMAP data alone, i.e., using standard notations, $(\Omega_{b0}, \Omega_{c0}, \Omega_\Lambda, h_0, n_s, \epsilon_{re}) = (0.0441,\ 0.214,\ 0.742,\ 0.719,\ 0.963, 0.087)$, where ϵ_{re} denotes the optical depth to reionization and n_s the spectral index of (adiabatic) curvature perturbations.

[2]Without loss of generality the space-time geometry will be taken to be conformally flat implying that the background geometry can be written as $\bar{g}_{\mu\nu} = a^2(\tau)\eta_{\mu\nu}$ where $\eta_{\mu\nu} \equiv \mathrm{diag}(1, -1, -1, -1)$ is the Minkowski metric with signature mostly minus.

[3]The conventions employed in Refs. [6,7] differ from the ones employed here (and match the ones of [8] and references therein).

$$\delta_s T^{00} = \frac{\delta_s \rho_t}{a_{\rm eq}^2 \alpha^2}, \qquad \delta_s T^{0i} = \frac{1}{a_{\rm eq}^2\alpha^2}(p_t + \rho_t)v_t^i,$$
$$\delta_s T^{ij} = \frac{1}{a_{\rm eq}^2\alpha^2}\left\{\delta_s p_t \delta^{ij} + 2p_t\left[-\xi\delta^{ij} + \frac{\hat{\kappa}^i\hat{\kappa}^j}{2}(h + 6\xi)\right] - \Pi_t^{ij}\right\}, \tag{3}$$

where Π_t^{ij} denotes the *total* anisotropic stress. Introducing the density contrasts for neutrinos, photons, CDM particle, and baryons (i.e., respectively, δ_ν, δ_γ, δ_c, and δ_b) as well as $\Omega_x = \delta_s\rho_x/\rho_\gamma$, the fluctuation of the total energy density and the total anisotropic stress are[4]

$$\delta_s\rho_t = \rho_t\left[\Omega_R(R_\gamma\delta_\gamma + R_\nu\delta_\nu) + \Omega_M\left(\frac{\omega_{c0}}{\omega_{M0}}\delta_c + \frac{\omega_{b0}}{\omega_{M0}}\delta_b\right) + R_\gamma\Omega_R\Omega_x\right], \tag{4}$$

$$\kappa_i\kappa_j\Pi_t^{ij} = \kappa^2\left[(p_\nu + \rho_\nu)\sigma_\nu + \sum_a(p_a + \rho_a)\sigma_a + (p_\gamma + \rho_\gamma)\sigma_x\right], \tag{5}$$

where $\Omega_R(\alpha) = 1/(\alpha + 1)$ and $\Omega_M(\alpha) = \alpha/(\alpha + 1)$; $\sigma_\nu(\kappa, \alpha)$ denotes the neutrino anisotropic stress; $R_\nu = 0.4052$, $R_\gamma = (1 - R_\nu)$, and $R_b(\alpha) = (3/4)\rho_b/\rho_\gamma \simeq 0.215\alpha$ denote, respectively, the neutrino fraction, the photon fraction, and the ratio of the baryon and photon energy densities (weighted by a factor 3/4 which arises in the photon-baryon sound speed). In Eq. (5) a sum over other potential components has been added: these components could account for the possible stresses arising either in the dark-matter or in the dark-energy sectors (see, e.g., [1]) but will not be explicitly considered hereunder. The timelike and the spacelike components of the covariant conservation of the energy-momentum tensor of Eq. (1) demand, respectively, that

$$\frac{\partial\Omega_x}{\partial\alpha} + \frac{3w_x - 1}{\alpha}\Omega_x = 0, \qquad \Omega_x(\kappa, \alpha) = \frac{4}{3w_x}\sigma_x(\kappa, \alpha). \tag{6}$$

In the α parametrization the Hamiltonian and the momentum constraints read, respectively,

$$\frac{\partial h}{\partial\alpha} = \frac{\kappa^2\alpha}{2(\alpha + 1)}\xi - \frac{3}{\alpha}\left[\Omega_R(R_\nu\delta_\nu + R_\gamma\delta_\gamma) + \Omega_M\left(\frac{\omega_{c0}}{\omega_{M0}}\delta_c + \frac{\omega_{B0}}{\omega_{M0}}\delta_b\right) + R_\gamma\Omega_R\Omega_x\right], \tag{7}$$

$$\kappa^2\alpha^2\frac{\partial\xi}{\partial\alpha} = -\frac{4}{\sqrt{\alpha + 1}}\left\{R_\nu\theta_\nu + R_\gamma[1 + R_b(\alpha)]\theta_{\gamma b} + \frac{3}{4}\frac{\omega_{c0}}{\omega_{M0}}\alpha\theta_c\right\}, \tag{8}$$

where $\theta_{\gamma b} = \theta_\gamma \simeq \theta_{\gamma b}$ represents the common value of the photon and baryon velocities; the same identification will be made hereunder [see Eqs. (14) and (15)] and it is fully justified to lowest order in the tight-coupling approximation. Defining the variable $\mathcal{Q} = (h + 6\xi)$, the remaining two (perturbed) Einstein equations can be written as

$$\frac{\partial^2 h}{\partial\alpha^2} + \frac{5\alpha + 4}{2\alpha(\alpha + 1)}\frac{\partial h}{\partial\alpha} - \frac{\kappa^2\xi}{2(\alpha + 1)} = \frac{3}{\alpha^2(\alpha + 1)}[R_\gamma\delta_\gamma + R_\nu\delta_\nu + 3w_xR_\gamma\Omega_x], \tag{9}$$

$$\frac{\partial^2\mathcal{Q}}{\partial\alpha^2} + \frac{5\alpha + 4}{2\alpha(\alpha + 1)}\frac{\partial\mathcal{Q}}{\partial\alpha} = \frac{\kappa^2\xi}{2(\alpha + 1)} + \frac{12}{\alpha^2(\alpha + 1)}(R_\nu\sigma_\nu + R_\gamma\sigma_x). \tag{10}$$

The evolution equations of the neutrinos obey

$$\frac{\partial\delta_\nu}{\partial\alpha} = -\frac{2\theta_\nu}{3\sqrt{\alpha + 1}} + \frac{2}{3}\frac{\partial h}{\partial\alpha}, \qquad \frac{\partial\theta_\nu}{\partial\alpha} = \frac{\kappa^2}{8\sqrt{\alpha + 1}}\delta_\nu - \frac{\kappa^2}{2\sqrt{\alpha + 1}}\sigma_\nu, \tag{11}$$

$$\frac{\partial\sigma_\nu}{\partial\alpha} = \frac{2\theta_\nu}{15\sqrt{\alpha + 1}} - \frac{2}{15}\frac{\partial\mathcal{Q}}{\partial\alpha} - \frac{3}{20}\frac{\kappa\mathcal{F}_{\nu3}}{\sqrt{\alpha + 1}}. \tag{12}$$

The evolution equations of the dark-matter sector obey instead

$$\frac{\partial\delta_c}{\partial\alpha} = -\frac{\theta_c}{2\sqrt{\alpha + 1}} + \frac{1}{2}\frac{\partial h}{\partial\alpha}, \qquad \frac{\partial\theta_c}{\partial\alpha} + \frac{\theta_c}{\alpha} = 0. \tag{13}$$

The system of photons and baryons can be treated, within an excellent approximation, to lowest order in the tight-coupling expansion where the quadrupole of the photons vanishes and the governing equations are given by

$$\frac{\partial\theta_{\gamma b}}{\partial\alpha} + \frac{R_b\theta_{\gamma b}}{\alpha(R_b + 1)} = \frac{\kappa^2\delta_\gamma}{8\sqrt{\alpha + 1}(R_b + 1)}, \tag{14}$$

$$\frac{\partial\delta_\gamma}{\partial\alpha} = -\frac{2}{3}\frac{\theta_{\gamma b}}{\sqrt{\alpha + 1}} + \frac{2}{3}\frac{\partial h}{\partial\alpha}, \qquad \frac{\partial\delta_b}{\partial\alpha} = -\frac{\theta_{\gamma b}}{2\sqrt{\alpha + 1}} + \frac{1}{2}\frac{\partial h}{\partial\alpha}. \tag{15}$$

From Eq. (6), it follows that $\mathcal{P}_\Omega(\kappa, \alpha) = [4/(3w_x)]\mathcal{P}_\sigma(\kappa)(\alpha/\alpha_i)^{1-3w_x}$, where $\mathcal{P}_\Omega(\kappa, \alpha)$ denotes the power spectrum of Ω_x while $\mathcal{P}_\sigma(k)$ denotes the power spectrum of σ_x whose explicit expression is assigned as $\mathcal{P}_\sigma(k) = \mathcal{B}(k/k_p)^{n_x-1}$; $k_p = 0.002\ \mathrm{Mpc}^{-1}$ is the pivot

[4]Using standard notations $\omega_{c0} = h_0^2\Omega_{c0}$, $\omega_{b0} = h_0^2\Omega_{b0}$, and $\omega_{M0} = h_0^2\Omega_{M0}$.

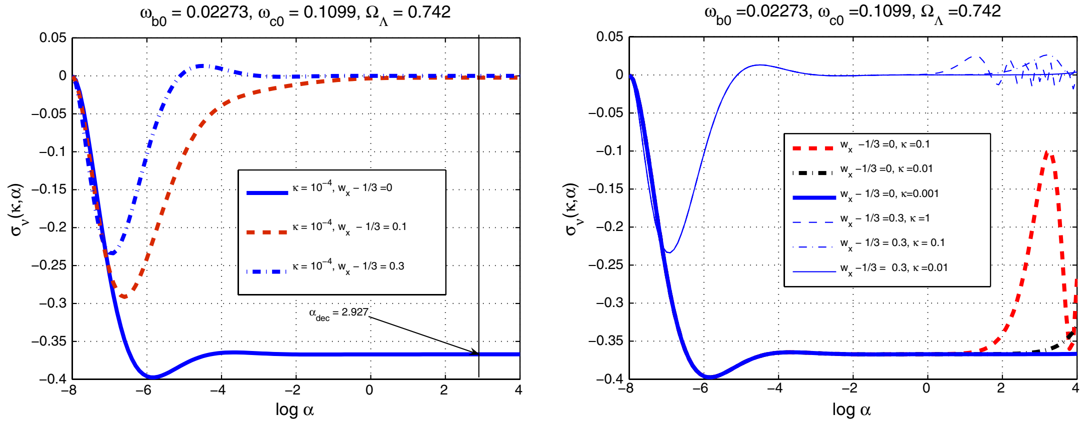

FIG. 1 (color online). The relaxation of the neutrino anisotropic stress for different values of w_x and κ.

scale at which the power spectrum coincides with its amplitude $\mathcal{B}$. Diverse initial conditions can be imposed to the whole system of equations. They are customarily divided into adiabatic and nonadiabatic [5,9] (see also [6,7]). The adiabaticity condition together with the constraints on the evolution of the fluctuations implies the following set of initial conditions:

$$\delta_\nu(\kappa,\alpha_i)\simeq\delta_\gamma(\kappa,\alpha_i)\simeq\tfrac{3}{4}\delta_b(\kappa,\alpha_i)\simeq\tfrac{3}{4}\delta_c=-R_\gamma\Omega_x(\kappa,\alpha_i),$$
$$\sigma_\nu(\kappa,\alpha_i)=0, \qquad (16)$$

$$\theta_\nu(\kappa,\alpha_i)\simeq\theta_{\gamma b}(\kappa,\alpha_i)\simeq\theta_c(\kappa,\alpha_i)\simeq 0,$$
$$\sigma_x(\kappa,\alpha_i)=\frac{3w_x}{4}\Omega_x(\kappa,\alpha_i), \qquad (17)$$

where α_i denotes the initial integration time when the neutrino anisotropic stress vanishes exactly. We are now interested to see what happens to the neutrino anisotropic stress when the initial conditions of the system obey Eqs. (16) and (17). The results of the numerical integration are illustrated in Fig. 1 for different values of w_x and different values of κ. The initial conditions are set in the limit $\alpha \ll \alpha_{\rm dec}$, where $\alpha_{\rm dec}=a_{\rm dec}/a_{\rm eq}$ denotes the value of α at photon decoupling (for the best fit to the 5 yr WMAP data alone $\alpha_{\rm dec}\simeq 2.92$ as indicated in Fig. 1 with the vertical line in the plot at the left). If $w_x > 1/3$ the anisotropic stress is driven to zero; if $\kappa \ll 1$, then $\sigma_\nu(\kappa,\alpha)$ will not oscillate for $\alpha \geq 1$ while in the opposite case (i.e. $\kappa > 1$) the neutrino anisotropic stress will be oscillating for the same range of α. If $w_x = 1/3$ the asymptotic value of $\sigma_\nu(\kappa,\alpha)$ will be, approximately, $-R_\gamma/R_\nu\sigma_x(k,\alpha_{\rm dec})$ for $\kappa < 1$; in the opposite case (i.e. $\kappa > 1$) σ_ν will oscillate around the same asymptote reached in the $\kappa < 1$ case. A similar phenomenon has been discussed, in the past, in the case of large-scale magnetic fields which affect the anisotropic stress but which also interact with the charged particles [10]. Finally notice that, if $w_x < 1/3$, $\Omega_x(\kappa,\alpha)$ grows with α and might even get dominant. This situation is more similar to the one treated in [1] and will not be specifically addressed here. From the numerical solution of the system in terms of $h(\kappa,\alpha)$ and $\xi(\kappa,\alpha)$ one can also compute other fluctuations with relevant gauge-invariant interpretation such as the curvature perturbations on comoving orthogonal hypersurfaces (i.e. $\mathcal{R}$), the density contrast on comoving orthogonal hypersurfaces (i.e. ϵ_m), and also ζ the curvature perturbations on constant density hypersurfaces (see, e.g. [11]). Denoting with $\alpha_i \ll 1$ the value of α at the onset of the numerical integration, $\xi(\kappa,\alpha)\propto \ln(\alpha/\alpha_i)$ provided $\sigma_\nu(\kappa,\alpha_i)\to 0$ since θ_ν and $\theta_{\gamma b}$ are both proportional to $\kappa^2\alpha$.

It is interesting to assume now that the whole source of large-scale inhomogeneity resides in the spectator anisotropic stresses. Could we get reasonable shapes of the CMB observables in the absence of the conventional adiabatic mode? The answer is negative and it is contained in Fig. 2 where the temperature autocorrelations (i.e. TT power spectra) and the temperature-polarization cross correlations (i.e. TE power spectra) are computed in the most favorable case, i.e. $w_x = 1/3$. In both plots of Fig. 2 the full line illustrates the WMAP 5 yr best fit (in terms of the WMAP data alone and in the case of the conventional adiabatic mode) while the dashed and dot-dashed lines refer to the case of the spectator stresses with different spectral indices (one of them coinciding with the one of the standard adiabatic mode). For the purposes of Fig. 2, the amplitude of the power spectrum of the anisotropic stress has been taken to be $\mathcal{B}=2.41\times 10^{-9}$. In spite of possible adjustments in the amplitude as well as in the pivot scale, the shapes of the TT and TE correlations cannot be made to coincide either with the ones of the conventional adiabatic mode nor with the typical patterns of the isocurvature modes.

The numerical integration has been carried on by using, as initial conditions of the Boltzmann solver, the result of the numerical integration of the explicit system discussed above. The matching regime between the two regimes coincides with $\alpha \simeq 10^{-4}$ when the tight coupling between baryons and photons is valid. The first regime of evolution dictated by the equations derived here avoids a potentially stiff problem if α is initially very small. The Boltzmann solvers is a modified version of [8,12] which, in turn, is

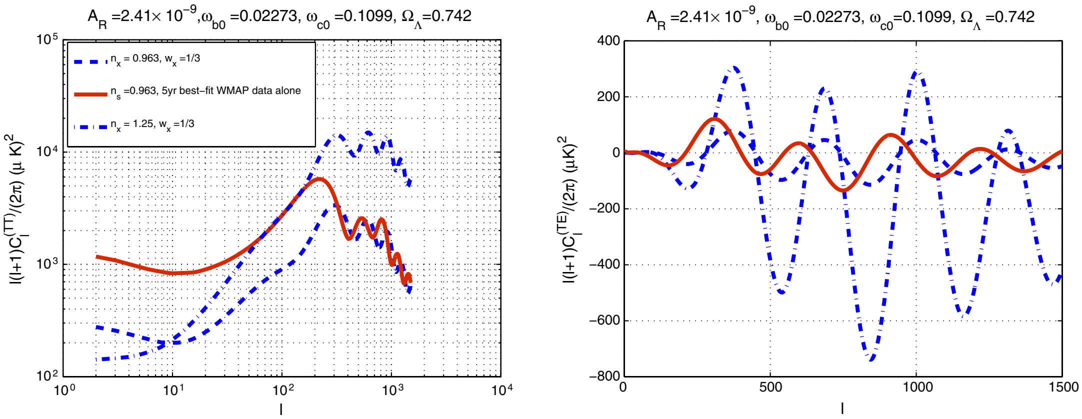

FIG. 2 (color online). The TT and TE power spectra induced by the spectator anisotropic stress in the absence of the conventional adiabatic mode (dashed and dot-dashed lines in both plots). The WMAP 5 yr best fit is reported, for comparison, with the full lines in both plots. Double logarithmic scale is used in the plot at the left while a linear scale is employed in the plot at the right.

based on [6,7]. From Fig. 2 it appears that, as expected, the shapes and phases of the TT and TE correlations obtained in the case of the adiabatic mode (full lines in both plots) differ from the ones induced by the spectator stresses (dashed and dot-dashed lines in both plots of Fig. 2). Spectator stresses can be treated and discussed in conjunction with a dominant adiabatic mode; absent of the adiabatic mode the shapes of the TT correlations have intermediate features between the isocurvature humps of the CDM-radiation mode [5]. The situation is also different from the case of the magnetized CMB observables when the adiabatic mode is absent [8] (see also [12]). In [13] (first reference) it is claimed that large-scale magnetic fields provide the same shapes for the TT correlations obtainable in the case of an adiabatic mode with appropriate amplitude. The initial conditions derived in [10] have been used in [13] (with supplementary typos) and then subsequently criticized by a subset of the authors (see, respectively, third and second references of [13]). Since we find the references [13] self-contradictory we do not want to comment on the possibility that a fully inhomogeneous magnetic field can seed structure formation in the absence of an adiabatic mode (as, suggested in the first reference of [13]). The present paper shows more modestly that spectator stresses alone [as introduced in Eq. (1)] do not reproduce the observed patterns of the TT and TE correlations assuming the initial conditions given in (16) and (17).

[1] W. Hu, Astrophys. J. **506**, 485 (1998); W. Hu and D. J. Eisenstein, Phys. Rev. D **59**, 083509 (1999); R. Maartens, J. Triginer, and D. Matravers, Phys. Rev. D **60**, 103503 (1999).

[2] T. Koivisto and D. F. Mota, Phys. Rev. D **73**, 083502 (2006).

[3] G. Hinshaw *et al.* (WMAP Collaboration), Astrophys. J. Suppl. Ser. **180**, 225 (2009); E. Komatsu *et al.* (WMAP Collaboration), Astrophys. J. Suppl. Ser. **180**, 330 (2009).

[4] J. Dunkley *et al.* (WMAP Collaboration), Astrophys. J. Suppl. Ser. **180**, 306 (2009); M. R. Nolta *et al.* (WMAP Collaboration), Astrophys. J. Suppl. Ser. **180**, 296 (2009).

[5] H. Kurki-Suonio, V. Muhonen, and J. Valiviita, Phys. Rev. D **71**, 063005 (2005); R. Keskitalo, H. Kurki-Suonio, V. Muhonen, and J. Valiviita, J. Cosmol. Astropart. Phys. 09 (2007) 008; M. Giovannini and K. E. Kunze, Phys. Rev. D **77**, 123001 (2008).

[6] C. P. Ma and E. Bertschinger, Astrophys. J. **455**, 7 (1995); E. Bertschinger, arXiv:astro-ph/9506070.

[7] U. Seljak and M. Zaldarriaga, Astrophys. J. **469**, 437 (1996); M. Zaldarriaga, D. N. Spergel, and U. Seljak, Astrophys. J. **488**, 1 (1997).

[8] M. Giovannini, Phys. Rev. D **79**, 121302 (2009); **79**, 103007 (2009).

[9] H. Kodama and M. Sasaki, Prog. Theor. Phys. Suppl. **78**, 1 (1984).

[10] M. Giovannini, Phys. Rev. D **74**, 063002 (2006); Classical Quantum Gravity **23**, 4991 (2006); Phys. Rev. D **70**, 123507 (2004).

[11] J. Hwang, Astrophys. J. **375**, 443 (1991); J. Hwang and H. Noh, Phys. Rev. D **64**, 103509 (2001); Classical Quantum Gravity **19**, 527 (2002).

[12] M. Giovannini, PMC Phys. A **1**, 5 (2007); M. Giovannini and K. E. Kunze, Phys. Rev. D **77**, 063003 (2008); **77**, 123001 (2008).

[13] K. Kojima, T. Kajino, and G. J. Mathews, J. Cosmol. Astropart. Phys. 02 (2010) 018; K. Kojima and K. Ichiki, arXiv:0902.1367; D. G. Yamazaki *et al.*, Phys. Rev. D **77**, 043005 (2008).