

Quantum correlations across the black hole horizon

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Inspired by the condensed-matter analogues of black holes, we study the quantum correlations across the event horizon reflecting the entanglement between the outgoing particles of the Hawking radiation and their in-falling partners. For a perfectly covariant theory, the total correlation is conserved in time and piles up arbitrarily close to the horizon in the past, where it merges into the singularity of the vacuum two-point function at the light cone. After modifying the dispersion relation (i.e., breaking Lorentz invariance) for large k , on the other hand, the light cone is smeared out and the entanglement is not conserved but actually created in a given rate per unit time.

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I. INTRODUCTION

One of the remaining mysteries of modern physics is the question of why black holes seem to behave as thermal objects, described by the Hawking temperature [1]. The thermal nature of black holes entails important concepts such as black hole entropy [2] and the information “paradox.” It is now widely believed that understanding the origin of thermality would be a major step towards unifying gravity and quantum theory.

Particularly puzzling is the fact that this thermal nature is apparently not caused by some sort of equilibration process, but by a dynamical quantum mechanism. In the semiclassical description, the state of the quantum fields propagating in the gravitational background is still a pure state. The thermal nature of the Hawking radiation is explained by the quantum correlations (across the horizon) between the Hawking particles and their in-falling partners [3].

For Hawking radiation itself, the origin of the created particles and its robustness against modifications of the microscopic structure have been studied in many publications—often inspired by the condensed-matter analogues of black holes (“dumb holes”); see, e.g., [4–7]. In contrast, the correlations across the horizon, their origin and dependence on the microscopic structure have been studied in far less detail [8–10].

II. TWO-POINT FUNCTION

Exploiting the fact that Hawking radiation is basically a one-dimensional effect and applies to all fields, we study a massless scalar field ϕ in a $1 + 1$ dimensional space-time described by the Painlevé-Gullstrand-Lemaître coordinates ($\hbar = c = G_N = k_B = 1$)

$$\begin{aligned} ds^2 &= dt^2 - [dx + v(x)dt]^2 \\ &= [1 - v^2(x)]dt^2 - 2v(x)dt dx - dx^2, \end{aligned} \quad (1)$$

where $v(x)$ can be visualized as the local velocity of freely falling frames as measured from infinity. The Schwarzschild metric is obtained by $v = \sqrt{2M/r}$, but we shall consider arbitrary profiles $v(x)$. In the standard manner, we introduce light cone variables u and v via

$$\begin{aligned} ds^2 &= [1 - v^2(x)] \left(dt - \frac{dx}{1 - v(x)} \right) \left(dt + \frac{dx}{1 + v(x)} \right) \\ &= [1 - v^2(x)] du dv, \end{aligned} \quad (2)$$

with u diverging at the (future) event horizon $v = 1$. The past horizon with $v = -1$ corresponds to $v \uparrow \infty$. For future convenience, we introduce the tortoise coordinate $dx_* = dx/[1 - v(x)]$ with $u = t - x_*$. After the standard transformation to regular coordinates for $v < 1$ (i.e., outside the horizon)

$$U = -\frac{1}{\kappa} e^{-\kappa u}, \quad V = \frac{1}{\kappa} e^{\kappa v}, \quad (3)$$

where $\kappa = (dv/dx)_{\text{horizon}}$ is the surface gravity, and analytic continuation beyond horizon $v > 1$, where $U > 0$, we obtain the line element with a regular conformal factor

$$ds^2 = \mathfrak{U}_{\text{regular}}^2 (UV) dU dV. \quad (4)$$

Because of the conformal invariance of the massless scalar field in $1 + 1$ dimensions, we may directly read off the two-point function(s) [11]. In the Boulware state [12], which is the ground state of the Hamiltonian generating the t -evolution (i.e., of all stationary observers), it behaves as $\ln(\Delta u \Delta v)$. However, this quantity is clearly divergent at both horizons, $u \uparrow \infty$ and $v \uparrow \infty$. Black hole evaporation is described by the Unruh state, which is regular across the black hole (future) horizon at $U = 0$ leading to the two-point function $\propto \ln(\Delta U \Delta v)$. In this case, the ingoing v -modes are in their ground state. The Israel-Hartle-

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Hawking state [13] is regular across both horizons $U = 0$ and $V = 0$ and is thermal in both directions. Its two-point function reads (up to an undetermined constant reflecting the IR-divergence in $1 + 1$ dimensions)

$$\langle \phi(U, V)\phi(U', V') \rangle = -\frac{1}{4\pi} \ln(\Delta U \Delta V), \quad (5)$$

where $\Delta U = U - U'$ and $\Delta V = V - V'$. Since the incoming (V, v) sector is decoupled from the outgoing (U, u) sector (in this $1 + 1$ dimensional setup), we focus on the relevant ΔU part $\langle \dots \rangle_U$ describing the Hawking radiation and the in-falling partner particles in the following. If U and U' lie on different sides of the horizon $UU' < 0$, we obtain

$$\langle \phi(U, V)\phi(U', V') \rangle_U = -\frac{1}{4\pi} \ln(e^{-\kappa u} + e^{-\kappa u'}). \quad (6)$$

In this form, correlations across the horizon do not become particularly apparent, but if we calculate, for example, the correlator

$$\begin{aligned} \langle \dot{\phi}(t, x)\dot{\phi}(t', x') \rangle_U &= -\frac{1}{4\pi} \partial_t \partial_{t'} \ln(e^{-\kappa u} + e^{-\kappa u'}) \\ &= \frac{\kappa^2}{16\pi} \frac{1}{\cosh^2(\kappa[u - u']/2)}, \end{aligned} \quad (7)$$

with $u - u' = t - t' - x_* + x'_*$, we see that it has a peak if we regard the Hawking particle at (x_*) at time t and its in-falling partner is at (x'_*) at time t' [8].

Because of $u \uparrow \infty$ at the horizon, this correlator vanishes where $v = 1$. This reflects the critical slow-down (of the u -modes) at the horizon in terms of the t -coordinate. However, for other quantities such as the momentum density $\Pi = (\partial_t - v\partial_x)\phi$, the correlator does not vanish when approaching the horizon

$$\begin{aligned} \langle \Pi(t, x)\Pi(t', x') \rangle_U &= -\frac{1}{4\pi} \partial_x \partial_{x'} \ln(e^{-\kappa u} + e^{-\kappa u'}) \\ &= \frac{\kappa^2 [1 - v(x)]^{-1} [1 - v(x')]^{-1}}{16\pi \cosh^2(\kappa[u - u']/2)}, \end{aligned} \quad (8)$$

in view of $\partial_x u = -1/(1 - v)$. We observe that the $\dot{\phi}$ correlator across the horizon is always positive whereas the Π correlator is negative. This can be explained by the fact that the particles of the Hawking radiation and their in-falling partners have the same conserved frequency ω measured with respect to the time t but their k -values have the opposite sign. Note that ω and k should not be confused with energy and momentum: The Hawking particle has positive energy and momentum whereas the in-falling partner has negative energy but positive momentum; see, e.g., [14].

III. INTEGRALS

Since the Π -correlator is total derivative, we may easily derive the total correlation integrated from the horizon at

$x = 0$ up to spatial infinity

$$\int_0^\infty dx \langle \Pi(t, x)\Pi(t', x' < 0) \rangle_U = \frac{\kappa}{4\pi} \frac{1}{1 - v(x')}, \quad (9)$$

and we find that it is independent of $t - t'$. Consequently, for each point $x' < 0$ inside the horizon (at $x = 0$), the integral of the correlations across the horizon is conserved, i.e., independent of the time slice. In the far future $t \uparrow \infty$, the Hawking particles carry the correlations to spatial infinity—as one would expect. In the past $t \downarrow -\infty$, however, the correlations pile up near the horizon where $u \approx t - \kappa^{-1} \ln(\kappa x)$

$$\begin{aligned} \langle \Pi(t \downarrow -\infty, x \downarrow 0)\Pi(t', x' < 0) \rangle_U \\ \approx \frac{\kappa}{16\pi} \frac{1}{\cosh^2(\kappa[u - u']/2)} \frac{1}{1 - v(x')} \frac{1}{x}, \end{aligned} \quad (10)$$

i.e., they are concentrated in a small spatial volume and have a large amplitude (in order to keep the integral constant). As we shall see in the next section, these piled-up correlations merge into the singularity of the two-point function at the light cone (which approaches the horizon) and become virtually indistinguishable from the quantum vacuum fluctuations. For a modified dispersion relation, this picture changes drastically; see Sec. V.

Similarly, we may evaluate the time integral

$$\int_{-\infty}^{+\infty} dt \langle \Pi(t, x)\Pi(t', x') \rangle_U = \frac{\kappa}{4\pi} \frac{1}{1 - v(x)} \frac{1}{1 - v(x')}. \quad (11)$$

In contrast to the x -integral above, this result will survive for a modified dispersion relation (apart from some corrections at short length scales; see Sec. V). Of course, we may also derive the time integral for

$$\int_{-\infty}^{+\infty} dt \langle \dot{\phi}(t, x)\dot{\phi}(t', x') \rangle_U = \frac{\kappa}{4\pi}. \quad (12)$$

The difference between the two expressions (11) and (12) can again be traced back to the fact that the Hawking particles and their in-falling partners have the same ω , but different k depending on their positions x and x' (gravitational red-shift $\omega = [1 - v]k$).

IV. ANALYTICALLY SOLVABLE EXAMPLE

It might be illustrative to apply the above formulas (which are valid for arbitrary profiles v) to some simple example which allows us to write down closed expressions. To this end, let us choose the velocity profile

$$v(x) = 1 - \frac{\kappa}{\gamma} \tanh(\gamma x), \quad (13)$$

where γ is some parameter and κ the surface gravity. In this case, the light cone coordinates read

$$u = t - \frac{1}{\kappa} \ln[\sinh(\gamma x)] \xrightarrow{U} -\frac{1}{\kappa} e^{-\kappa t} \sinh(\gamma x), \quad (14)$$

and the correlator for all x and x' becomes

$$\langle \dot{\phi}(t, x) \dot{\phi}(t, x') \rangle_U = -\frac{\kappa^2}{4\pi} \frac{\sinh(\gamma x) \sinh(\gamma x')}{[\sinh(\gamma x) - \sinh(\gamma x')]^2}. \quad (15)$$

This function is positive on opposite sides of the horizon $xx' < 0$, negative when x and x' lie on the same side $xx' > 0$, and goes to zero if one of the two points approaches the horizon at $x = 0$ (critical slow-down). It reproduces the usual $1/(x - x')^2$ singularity at $x = x'$ and vanishes asymptotically $x \rightarrow \pm\infty$ and $x' \rightarrow \pm\infty$. The correlations across the horizon manifest themselves in the global maximum at $x = -x'$ with $t = t'$.

Although one gets the same $1/(x - x')^2$ singularity at $x = x'$ and the same asymptotic behavior for $x \rightarrow \pm\infty$ and $x' \rightarrow \pm\infty$, the situation is a bit different for the canonical momentum density $\Pi = \dot{\phi} - v\phi'$

$$\langle \Pi(t, x) \Pi(t, x') \rangle_U = -\frac{\gamma^2}{4\pi} \frac{\cosh(\gamma x) \cosh(\gamma x')}{[\sinh(\gamma x) - \sinh(\gamma x')]^2}. \quad (16)$$

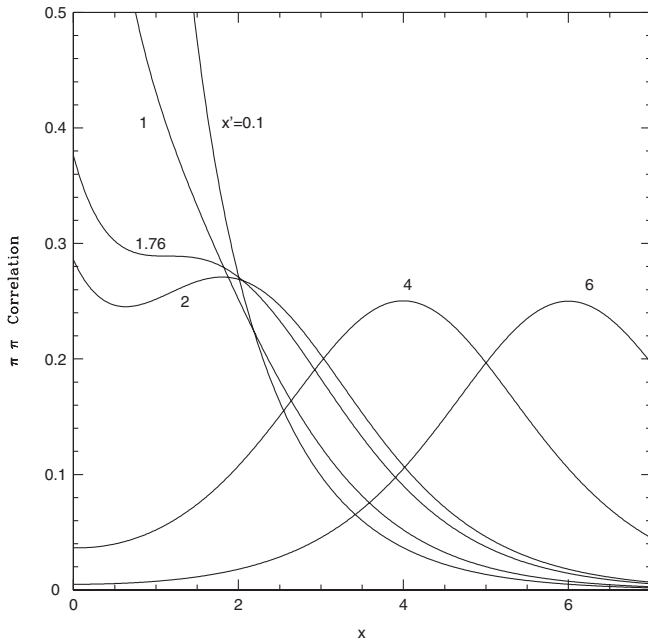


FIG. 1. Plot of the correlator (16) as a function of position x outside the horizon for various values of the partner position x' inside the horizon (both in units of γ). For large values of x' (i.e., far from the horizon) the correlation as a function of x is simply a bump of width about 2 and constant height which is located at $x = |x'|$. For small x' it merges into the general vacuum fluctuations near the horizon, i.e., the $1/(x - x')^2$ singularity. I.e., the correlation of the “Hawking” particles with their in-falling partners becomes visible only if the two points are well separated from the horizon, even though the integral (9) of the correlation over x is a constant.

First, this correlator is negative everywhere and does not vanish at the horizon. Second, we get a local minimum at $x = -x'$ only if $|x| > x_{\min} \approx 1.8/\gamma$, i.e., far enough away from the horizon. Close to the horizon, the local minimum merges into the light cone singularity and disappears; see Fig. 1. This absence of structure near the horizon can be interpreted as further confirmation of the picture that the Hawking radiation is not created very close to the horizon but rather in a region of finite spatial extent $\mathcal{O}(1/\gamma)$. Note that this length scale $1/\gamma$ could in principle be quite different from the scale set by the Hawking temperature, i.e., the surface gravity κ .

It should also be mentioned that the $\dot{\phi}$ -correlator *does* have a maximum arbitrarily close to the horizon, but the maximum value becomes very small. Consistent with this observation, the correlation conservation law (9) applies to the Π -correlator—for the $\dot{\phi}$ -correlator, one would need an additional integrating factor $1/[1 - v(x)]$.

V. MODIFIED DISPERSION

In the previous sections, we found the infinite pile-up of correlation close to the horizon in the past, such that the correlations constantly emerge out of the singularity at the light cone (which approaches the null surface of the horizon in the past). However, modifying the dispersion relation at large k (e.g., motivated by the condensed-matter analogues of black holes [4,6,7]), we expect this behavior to change: First, a modified dispersion relation smears out the light cone such that the two-point function typically becomes regular everywhere except at the space-time coincidence point $x = x'$ and $t = t'$. Second, tracing the particles of the Hawking radiation back in time, they do not originate from an small vicinity of the horizon in the presence of a modified dispersion relation [6,7].

In order to deal with a solvable example, let us switch to the Eddington-Finkelstein coordinates (v, r)

$$ds^2 = \left(1 - \frac{2M}{r}\right) dv^2 - 2dvdr, \quad (17)$$

and modify the dispersion relation by inserting a general function $f(\partial_r^2)$ containing higher-order spatial derivatives into the corresponding action for a scalar field ϕ

$$\mathcal{L} = -(\partial_v \phi) \partial_r \phi - \frac{(\partial_r \phi)}{2} \left[1 - \frac{2M}{r} + f(\partial_r^2) \right] \partial_r \phi. \quad (18)$$

This action allows us to define a conserved inner product

$$(\phi_1 | \phi_2) = i \int d\Sigma^\mu \phi_1^* \overleftrightarrow{\partial}_\mu \phi_2 = i \int dr \phi_1^* \overleftrightarrow{\partial}_r \phi_2, \quad (19)$$

where $\phi_1^* \overleftrightarrow{\partial}_r \phi_2 = \phi_1^* \partial_r \phi_2 - \phi_2 \partial_r \phi_1^*$. The momentum density $\Pi = -\partial_r \phi$ satisfies the wave equation

$$\left(2\partial_v + \partial_r \left[1 - \frac{2M}{r} + f(\partial_r^2)\right]\right)\Pi = 0, \quad (20)$$

which is of first order in ∂_v and hence automatically selects the outgoing u -sector only. In a stationary state, the two-point function can be Fourier expanded via

$$\langle \Pi(v, r)\Pi(v', r') \rangle = \int d\omega e^{-i\omega(v-v')} g_\omega(r, r'), \quad (21)$$

where $g_\omega(r, r')$ solves the ordinary differential equation

$$\left(-2i\omega + \partial_r \left[1 - \frac{2M}{r} + f(\partial_r^2)\right]\right)g_\omega(r, r') = 0 \quad (22)$$

for r and the same for r' with $+2i\omega$. Assuming that this differential equation together with the asymptotic conditions (freely falling ground state for large k at all positions r) uniquely determines the r -dependence of $g_\omega(r, r')$ (and thus the same for r'), we find that $g_\omega(r, r')$ factorizes

$$g_\omega(r, r') = h_\omega(r)h_\omega^*(r'). \quad (23)$$

Here, we are interested in the region near the horizon (where the pile-up of correlation occurred) and thus we employ the near-horizon approximation

$$1 - \frac{2M}{r} = 2\kappa x + \mathcal{O}(\kappa^2 x^2), \quad (24)$$

resulting in the (approximate) differential equation which can be solved via a Fourier-Laplace transformation

$$\begin{aligned} (-2i\omega + \partial_x[2\kappa x + f(\partial_x^2)])h_\omega(x) &= 0, \\ (-2i\omega + ik[2\kappa i\partial_k + f(-k^2)])\tilde{h}_\omega(k) &= 0. \end{aligned} \quad (25)$$

This is now a first-order ordinary differential equation in k and its general solution can be written as

$$h_\omega(x) = \int_{\mathfrak{C}} dk k^{-i\omega/\kappa} \exp\left\{ikx - \frac{i}{\kappa}F(k^2)\right\}, \quad (26)$$

where \mathfrak{C} is an appropriate contour in the complex plane

$$\langle \Pi(v, x)\Pi(v', x') \rangle = \int_{-\infty}^{+\infty} d\omega e^{-i\omega(v-v')} |\mathcal{N}_\omega^2| \int_0^\infty dk \int_0^\infty dk' \exp\left\{i\frac{\omega}{\kappa} \ln \frac{k'}{k} + ikx - ik'x' - \frac{i}{\kappa}F(k) + \frac{i}{\kappa}F(k')\right\}. \quad (30)$$

The ω -integral yields the Fourier transform of $|\mathcal{N}_\omega^2|$, which we denote by $\tilde{\mathcal{N}}$. Finally, introducing the new variable $\chi = e^{-\kappa(v-v')}k/k'$, the integrated correlation across the horizon in analogy to (9) yields

$$\int_0^\infty dx \langle \Pi(v, x)\Pi(v', x' < 0) \rangle = \int_0^\infty dk' \exp\left\{-ik'x' + \frac{i}{\kappa}F(k')\right\} \times \int_0^\infty \frac{d\chi}{\chi} \tilde{\mathcal{N}}(\ln \chi) \exp\left\{-\frac{i}{\kappa}F(\chi k' e^{-\kappa(v-v')})\right\}. \quad (31)$$

For a given point $x' < 0$ inside the horizon, Eq. (31) yields the total correlation between that point $x' < 0$ and all positions x outside the horizon up to spatial infinity. Setting $F = 0$, we rederive the result (9) for $\mathfrak{v}(x) = \kappa x$. In the far future $(v - v') \uparrow \infty$, the exponential prefactor $e^{-\kappa(v-v')}$ in Eq. (31) vanishes and thus the χ -integral becomes independent of k' . The remaining k' -integral then just yields $h_{\omega=0}^*(x')$. After coarse-graining over large length scales, this scales as $1/x'$. As a result, we get basically the same

and $dF/dk = f(-k^2)/2$ accounts for the modified dispersion relation. In order to determine the correct integration contour in the complex plane, we have to study different choices for the branch cut from $k^{-i\omega/\kappa}$. If the branch cut lies in the upper complex half-plane $\Im(k) > 0$, the solution connects the final Hawking mode to the initial positive/negative (pseudo) norm modes [7]

$$\alpha_\omega \phi_\omega^{k^+}(x) + \beta_\omega \phi_\omega^{k^-}(x) \rightarrow \phi_\omega^{\text{Hawking}}(x > 0), \quad (27)$$

where $\phi_\omega^{k^+}(x)$ has positive (pseudo) norm (19) and $\phi_\omega^{k^-}(x)$ negative (pseudo) norm. On the other hand, the branch cut in the lower complex half-plane $\Im(k) < 0$ connects the final mode of the in-falling partners to the initial positive/negative (pseudo) norm modes

$$\tilde{\alpha}_\omega \phi_\omega^{k^-}(x) + \tilde{\beta}_\omega \phi_\omega^{k^+}(x) \rightarrow \phi_\omega^{\text{partner}}(x < 0). \quad (28)$$

Assuming that the quantum state we have corresponds to the freely falling ground state for large k , the asymptotic condition for $h_\omega(x)$ implies that it has no contribution from $\phi_\omega^{k^-}(x)$. Thus, we take a suitable linear combination of the two solutions (27) and (28) with the branch cut in the upper and lower complex half-plane, respectively, which yields

$$h_\omega(x) = \mathcal{N}_\omega \int_0^\infty dk k^{-i\omega/\kappa} \exp\left\{ikx - \frac{i}{\kappa}F(k)\right\}, \quad (29)$$

where \mathcal{N}_ω is a normalization factor. This expression is quite natural since the boundary condition (freely falling ground state for large k) implies that $h_\omega(x)$ only contains positive k -values with a positive (pseudo) norm (19). Coarse-graining over large length scales, we do not see the impact of $F(k)$ and this function behaves as $h(x) \sim |x|^{i\omega/\kappa-1}$, but on short distances, it also contains the rapidly oscillating in-mode $\phi_\omega^{k^+}(x)$.

Now we are in the position to study the full correlator. Inserting (29) into (21) and (23), the total expression reads

conservation law as in (9). This is quite natural since it just reflects that fact that the Hawking particles carry the correlation away to spatial infinity.

In the far past $(v - v') \downarrow -\infty$, on the other hand, the exponential prefactor $e^{-\kappa(v-v')}$ diverges and thus the k' -integral in Eq. (31) is exponentially suppressed due to the rapidly oscillating phase $F(\chi k' e^{-\kappa(v-v')})$. This can most easily be seen by changing the integration variable to $k'' = k' e^{-\kappa(v-v')}$. In the resulting double integral over k''

and χ , the first exponent in Eq. (31) can be neglected and the integral scales as $e^{\kappa(v-v')}$. Consequently, in contrast to the perfectly covariant case (which implies an unbounded red-shift near the horizon), the total correlation is not conserved in this case (i.e., it does not pile up at the horizon) but created at finite times.

Note that the integral over the full x -interval (i.e, from $-\infty$ to $+\infty$) vanishes, since Π is a spatial derivative. Even with a modified dispersion relation, the time integral factorizes exactly

$$\int_{-\infty}^{+\infty} dv \langle \Pi(v, x) \Pi(v', x') \rangle = 2\pi h_{\omega=0}(x) h_{\omega=0}^*(x'), \quad (32)$$

and, after coarse-graining over large length scales, it behaves as $1/|xx'|$.

VI. CONCLUSIONS

We have studied the evolution of the quantum correlations across the black hole horizon. Since the quantum state under consideration is a pure state, all correlations imply entanglement and thus entanglement entropy, etc. Both, the ϕ and the Π correlator across the horizon (7) and (8), possess a peak at $u = u'$ if we are far enough away from the horizon. For black hole analogues (“dumb holes”) in Bose-Einstein condensates, the scalar field ϕ reflects the phase fluctuations while the momentum density Π corresponds to the density fluctuations $\delta\rho$, cf. [8].

For the Π correlator, we found a conservation law (9) for the total correlation across horizon in a perfectly covariant theory (up to arbitrarily small length scales). This means that the correlation to be carried away by the Hawking particles in the future must pile up arbitrarily close to the horizon in the past. However, below a minimum length scale γ set by the geometry (not necessarily the surface

gravity κ), this piled-up correlation becomes virtually indistinguishable from the vacuum singularity of the two-point function at the light cone; see Fig. 1 and Eqs. (13) and (16).

After modifying the microscopic structure via introducing a nonlinear dispersion relation at short distances, this picture changes drastically: In this case, the correlation carried away by Hawking radiation in the future cannot be traced back to arbitrarily early times and a small vicinity of the horizon. As a result, the entanglement is not conserved but actually created dynamically at a finite time. This complies with the intuitive picture for the origin of Hawking radiation in these systems: The initial wave packet with large k in its ground state approaches the horizon and is ripped apart by the gravitational red-shift into two parts—the outgoing Hawking radiation and the infalling partner particles. Initially, there is no entanglement (ground state), whereas the two final wave packets are in a squeezed state, i.e., entangled.

As a consequence, the total entanglement entropy grows with time due to the Hawking radiation emitted in the past (and hence would diverge for an eternal black hole; cf. [15]) whereas the radiation to be emitted in the future does not contribute (yet). These findings further support the view that Hawking radiation is not created at arbitrarily small length scales but at finite distances and could be relevant for the black hole information “paradox”, etc.

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