

**Dynamic transition to spontaneous scalarization in boson stars**Miguel Alcubierre,<sup>1,\*</sup> Juan C. Degollado,<sup>1,†</sup> Darío Núñez,<sup>1,‡</sup> Milton Ruiz,<sup>2,§</sup> and Marcelo Salgado<sup>1,||</sup><sup>1</sup>*Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México, A. P. 70-543, México D. F. 04510, México*<sup>2</sup>*Theoretical Physics Institute, University of Jena, 07743 Jena, Germany*

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We show that the phenomenon of spontaneous scalarization predicted in neutron stars within the framework of scalar-tensor theories of gravity, also takes place in boson stars without including a self-interaction term for the boson field (other than the mass term), contrary to what was claimed before. The analysis is performed in the physical (Jordan) frame and is based on a  $3 + 1$  decomposition of spacetime assuming spherical symmetry.

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**I. INTRODUCTION**

Scalar-tensor theories of gravity (STT) are alternative metric theories of gravitation where a spin-0 gravitational degree of freedom  $\phi$  accompanies the usual tensor spin-2 modes (see Ref. [1] for a review). In the so-called Jordan frame, the scalar field  $\phi$  couples nonminimally to the curvature while in the Einstein frame, it couples nonminimally to the ordinary matter fields. Scalar-tensor theories are perhaps the most analyzed and competitive theories of gravitation after general relativity, the most prominent example being the well-known Brans-Dicke theory [2]. Nevertheless, it is only recently that many issues and theoretical discoveries concerning STT have been settled. In the cosmological context, STT have been proposed as alternatives to dark energy in order to explain the accelerated expansion of the Universe while avoiding the so-called *coincidence problem*, which is associated with a cosmological constant [3–8]. In the astrophysical context, Damour and Esposito-Farèse [9,10] discovered that neutron star models (polytropes) within STT may undergo a *phase transition* that consists in the appearance of a non-trivial configuration of the scalar field  $\phi$  in the absence of sources and with vanishing asymptotic value. Such configurations are endowed with a new global quantity termed *scalar charge*. Because of the similarities of this scalarization process with the spontaneous magnetization in ferromagnets at low temperatures, these authors coined the term *spontaneous scalarization* (SC) to describe this phenomenon. Further analysis [11–13] confirmed that this phenomenon also takes place in realistic neutron star models and occurs independently of the details of the equation of state used to describe the nuclear matter.

The stability analysis for the transition to SC was first performed by Harada [14,15]. It is now understood that SC arises under certain conditions where the appearance of the

scalar-field gives rise to a configuration that minimizes the star's energy (the Arnowitt-Deser-Misner [ADM] mass) with fixed baryon number. This interpretation can in fact accommodate the Newtonian expectations despite the fact that the effective gravitational constant decreases during the transition [13]. Following the analogy with ferromagnetism, the ADM mass plays the same role as the (free) energy of the ferromagnet, while the baryon mass is the analogue of the inverse of the temperature. The order parameter is the scalar charge, which mirrors the magnetization. One important aspect of this phenomenon is that it occurs even when the parameters of the theory satisfy the stringent bounds put by the solar system experiments. The important point is that SC appears precisely when the asymptotic (cosmological) value  $\phi_0$  of the scalar field vanishes. Therefore, the phenomenon arises even when the associated effective Brans-Dicke parameter of the theory is arbitrarily large (see Sec. V). On the other hand, the binary pulsar does put limits on the magnitude of SC. In some classes of STT, these bounds restrict the nonminimal coupling to the curvature. However, the bounds are not so stringent since the couplings can still be of order unity [10].

More recently, studies of neutron star oscillations within STT reveal that, in addition to the emission of scalar gravitational waves, SC can also disturb the quadrupolar gravitational radiation as compared to the corresponding signals in general relativity (GR). Therefore, even if the detection channels of scalar gravitational waves are “switched off,” the detection of gravitational waves of spin-2 coming from these sources might validate STT or put even more stringent bounds on their parameters. Of course, the direct detection (or lack) of scalar gravitational waves would also help to discriminate between several alternative theories. In this regard it is important to emphasize another striking feature of STT. While GR predicts only quadrupolar gravitational radiation in the “far zone,” STT predicts monopolar gravitational waves [16], so that even in spherical symmetry scalar waves can be emitted. This is in part because Birkhoff's theorem does not apply in this case. The new polarization scalar mode is of the *breathing* type since it affects all the directions isotropi-

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cally [16]. In particular, during the SC process in spherical neutron stars this kind of radiation might be emitted. In fact, using a fully relativistic spherically symmetric (SS) code, Novak [12] not only confirmed the dynamical transition to the scalarization state but also the emission of such scalar waves.

In a more recent analysis, Whinnett [17] corroborated that SC can also occur in boson stars (see Ref. [18] for a thorough introduction to the subject), but only if these are endowed with a self-interaction. In that work the spacetime was assumed to be static and spherically symmetric, although the boson stars were only stationary.

In this paper we want to report that by performing a dynamical transition to SC in boson stars where the real scalar field  $\phi$  is excited to a nontrivial configuration, we find that the self-interaction term for the boson field (other than the mass term) is not required, contrary to what was found by Whinnett. This result is not the only novelty. In order to perform our analysis we have developed several new tools. Unlike Novak's work, we always work in the Jordan frame where the interpretation of results has a more direct physical meaning. To achieve this goal we have constructed a new code based on a 3 + 1 approach for STT developed in [19] and adapted to a Baumgarte-Shapiro-Shibata-Nakamura formulation [20,21], in which we have implemented constraint preserving boundary conditions. In this regard, it is important to mention that STT have been proved to possess a well-posed Cauchy problem in the Jordan frame [19,22]. In particular, we shall show elsewhere [23], that the SS equations used in this paper are strongly hyperbolic and therefore the Cauchy problem is well posed as well for this particular case.

The paper is organized as follows: In Sec. II, we introduce the scalar-tensor theories and discuss briefly some properties associated with the Jordan frame. In Sec. III, we describe the boson-star model. Section IV describes the numerical results. Finally, Sec. V contains the conclusions.

## II. SCALAR-TENSOR THEORIES OF GRAVITY

The general action for STT with a single scalar field is given by

$$S[g_{ab}, \phi, \psi] = \int \left\{ \frac{F(\phi)}{16\pi} R - \frac{1}{2} (\nabla\phi)^2 - V(\phi) \right\} \sqrt{-g} d^4x + S_{\text{matt}}[g_{ab}, \psi], \quad (2.1)$$

with  $\phi$  the nonminimally coupled scalar field, and where  $\psi$  represents collectively the ordinary matter fields (i.e. fields other than  $\phi$ ; we use units such that  $c = 1 = G_0$ ). For the problem at hand,  $\psi$  will represent the complex scalar field that we use to model the boson star (see Sec. III).

The representation of the STT given by Eq. (2.1) is called the *Jordan frame* representation. The field equations obtained from the action (2.1) are given by

$$G_{ab} = 8\pi T_{ab}, \quad (2.2)$$

$$\square\phi + \frac{1}{2}f'R = V', \quad (2.3)$$

where a prime indicates  $\partial_\phi$ ,  $\square := g^{ab}\nabla_a\nabla_b$  is the covariant d'Alambertian operator,  $G_{ab} = R_{ab} - \frac{1}{2}g_{ab}R$ , and

$$T_{ab} := G_{\text{eff}}(T_{ab}^f + T_{ab}^\phi + T_{ab}^{\text{matt}}), \quad (2.4)$$

$$T_{ab}^f := \nabla_a(f'\nabla_b\phi) - g_{ab}\nabla_c(f'\nabla^c\phi), \quad (2.5)$$

$$T_{ab}^\phi := (\nabla_a\phi)(\nabla_b\phi) - g_{ab}[\frac{1}{2}(\nabla\phi)^2 + V(\phi)], \quad (2.6)$$

$$G_{\text{eff}} := \frac{1}{8\pi f}, \quad f := \frac{F}{8\pi}, \quad (2.7)$$

where  $T_{ab}^{\text{matt}}$  is the stress-energy tensor for all matter fields other than  $\phi$ .

Using Eq. (2.2), the Ricci scalar can be expressed in terms of the energy-momentum tensor Eq. (2.4). Equation (2.3) then takes the final form

$$\square\phi = \frac{fV' - 2f'V - \frac{1}{2}f'(1 + 3f'')(\nabla\phi)^2 + \frac{1}{2}f'T_{\text{matt}}}{f(1 + 3f'^2/2f)}, \quad (2.8)$$

where  $T_{\text{matt}}$  stands for the trace of  $T_{ab}^{\text{matt}}$ . It is important to stress the fact that, although we have included here a potential  $V(\phi)$  for the STT, for the actual analysis of SC in boson stars that we shall present below it was not taken into account.

Using a modified harmonic gauge [19], it has been proved that the previous equations (in vacuum) can be put in a quasilinear diagonal hyperbolic form. Moreover, such equations can also be put in a full first order 3 + 1 form, from which one can prove directly their strong hyperbolic character [22]. Therefore, the Cauchy problem is well posed in the Jordan frame.

The Bianchi identities directly imply

$$\nabla_c T^{ca} = 0. \quad (2.9)$$

However, the use of the field equations leads to the conservation of the energy-momentum tensor of the matter alone

$$\nabla_c T_{\text{matt}}^{ca} = 0, \quad (2.10)$$

which implies the fulfillment of the (weak) equivalence principle (i.e. test particles follow geodesics of the metric  $g_{ab}$ .)

## III. BOSON STARS

As we mentioned in the introduction, the phenomenon of SC was first discovered in static neutron star models. Afterwards, the dynamic transition from the unscalarized to the scalarized state was also analyzed [12]. In this paper,

we propose to study a similar transition but using boson stars instead of neutron stars. In some way this matter model is even more fundamental than neutron stars in that one does not have to assume a perfect fluid, but rather a field theory. It is also simpler since one does not have to deal with the equation of state for the nuclear matter (where uncertainties in the model are always an issue), as well as the technical difficulties associated with the numerical simulation of shock fronts.

In a previous investigation, Whinnett [17] constructed stationary configurations of boson stars within a STT, and showed that SC was only possible if one included a self-interaction potential for the boson field. He carried out the analysis for three different classes of STT, one of which in fact is almost identical to the one we use in this paper. In our case, instead of constructing stationary configurations of scalarized boson stars, we analyze the dynamical transition from the unscalarized state to the one with scalarization. The final state of this process corresponds in principle to one of the stationary states that one would find using the method by Whinnett. Nevertheless, unlike Whinnett's results, we find SC without the need of a self-interaction. A possible explanation for this will be elucidated in Sec. V. We point out that boson-star models have been constructed in the past within STT [24–27], but apart from Whinnett's work none other study has looked at the spontaneous scalarization of such objects.

We shall consider boson stars described by the following energy-momentum tensor

$$T_{ab}^{\text{matt}} = \frac{1}{2}[(\nabla_a \psi^*)(\nabla_b \psi) + (\nabla_b \psi^*)(\nabla_a \psi)] - g_{ab}[\frac{1}{2}|\nabla \psi|^2 + V_\psi(|\psi|^2)], \quad (3.1)$$

where  $\psi$  is a complex-valued scalar field that represents the bosons,  $|\nabla \psi|^2 := g^{ab}(\nabla_a \psi)(\nabla_b \psi^*)$  and  $|\psi|^2 := \psi^* \psi$ . This model arises from the Lagrangian  $\mathcal{L}_{\text{matt}} = -\sqrt{-g}[\frac{1}{2}|\nabla \psi|^2 + V_\psi(|\psi|^2)]$ . The potential  $V_\psi(\psi^* \psi)$  will be taken to be of the form

$$V_\psi(\psi^* \psi) = \frac{1}{2}m_\psi^2 \psi^* \psi, \quad (3.2)$$

which includes a mass term but no self-interaction potential (which is typically associated with a term  $\frac{1}{4}\lambda(\psi^* \psi)^2$ ) [28].

The boson field obeys the Klein-Gordon equation:

$$\square \psi - m_\psi^2 \psi = 0. \quad (3.3)$$

Notice that the problem we wish to study involves two *different* scalar fields, a real-valued field  $\phi$  coupled non-minimally to the curvature, and a complex-valued field  $\psi$  (the boson field) coupled only minimally.

The matter Lagrangian is invariant with respect to a global phase transformation  $\psi \rightarrow e^{iq} \psi$  (with  $q$  a real constant). Noether's theorem then implies the local conservation of the boson number  $\nabla_a \mathcal{J}^a = 0$ , where the number-density current is given by  $\mathcal{J}^a = \frac{1}{2}g^{ab}[\psi \nabla_b \psi^* -$

$\psi^* \nabla_b \psi]$ . This means that the total boson number

$$\mathcal{N} = - \int_{\Sigma_t} n_a \mathcal{J}^a \sqrt{h} d^3x \quad (3.4)$$

is conserved, where  $n^a$  is the normal vector to the spatial Cauchy hypersurfaces  $\Sigma_t$  and  $h$  is the determinant of the induced metric on  $\Sigma_t$ . The total boson mass is then given by  $M_{\text{bos}} = m_b \mathcal{N}$ , with  $m_b := \hbar m_\psi / c$  the mass of a single boson (where we have restored the speed of light  $c$  and the Planck's constant  $\hbar$ ).

In fact, when the scalar field  $\phi$  acquires a nontrivial value one can define another global quantity associated to it, which for asymptotically flat spacetimes is given by

$$Q_{\text{scalar}} = \lim_{r \rightarrow \infty} \frac{1}{4\pi} \int_S s_a \nabla^a \phi ds, \quad (3.5)$$

where  $s^a$  is the unit outward normal to a topological 2-sphere  $S$  embedded in  $\Sigma_t$ , and  $r$  is a radial coordinate that provides the area of  $S$  asymptotically.

#### IV. NUMERICAL RESULTS

For the analysis at hand, we shall consider a SS space-time with a metric given by

$$ds^2 = -N^2 dt^2 + A^2 dr^2 + r^2 B^2 (d\theta^2 + \sin^2 \theta d\varphi^2), \quad (4.1)$$

where the metric coefficients ( $N, A, B$ ) are all functions of the coordinates  $t$  and  $r$ . The scalar-field variables will be functions of  $t$  and  $r$  as well. As can be seen from the form of (4.1), we consider a null shift vector. Also, the area of 2-spheres is given by  $\mathcal{A} = 4\pi r^2 B^2$ , which only coincides with the area coordinates when  $B \equiv 1$ .

We have constructed a spherically symmetric evolution code based on the BSSN system of equations, together with a 3 + 1 formulation of the STT equations developed in Ref. [19]. The details of this system, including its hyperbolicity properties, will be reported elsewhere [23].

For the evolution we have also used a generalization of the Bona-Masso slicing condition [29] that led to a well-behaved hyperbolic system [22].

We have constructed initial data for stationary boson stars like in GR, by assuming  $\psi(t, r) = \Psi(r)e^{i\omega t}$  and  $\phi(t, r) \equiv 0$ , and solving the eigenvalue problem resulting from Eq. (3.3) using a shooting method to find  $\omega$ . Notice that in the absence of a potential and for the class of STT we use (see below),  $\phi(t, r) \equiv 0$ , solves exactly Eq. (2.8). The resulting configuration corresponds to a strictly static spacetime. In order to study the dynamic transition to SC, we then considered a small Gaussian perturbation for the scalar field  $\phi$ . We then solved the new Hamiltonian constraint for this perturbed initial data. The momentum constraints are trivially satisfied initially by assuming a *moment of time symmetry* for which the extrinsic curvature vanishes, as well as the momenta associated with the real scalar field  $\phi$  and the boson field  $\psi$ .

For the dynamical simulation presented below, we have taken the nonminimal coupling function to be of the form  $F(\phi) = 8\pi f(\phi) = 1 + 8\pi\xi\phi^2$ , with  $\xi$  a positive constant.

Several scenarios can happen depending on the initial configuration. Figure 1 (top panel) depicts a curve that represents the boundary (i.e. the critical values) of the transition to SC for different values of the constant  $\xi$  and the central value of the norm of the complex scalar field  $\Psi(0)$ . Initial configurations below the critical line are stable with respect to the perturbations and do not lead to a SC transition. This means that for such configurations the scalar field  $\phi$  simply radiates away during the evolution and leaves behind a stable stationary configuration with a globally null  $\phi$ . On the other hand, initial configurations above the critical line (but below the line marked “maximum mass”) are unstable, and when perturbed lead to a dynamical transition to SC where the final configuration is a stationary boson star endowed with a nontrivial scalar field  $\phi$ .

The horizontal line marked “maximum mass” in Fig. 1 denotes the maximum mass for stable boson-star configurations. For boson stars with no perturbation from the scalar field  $\phi$  the mass increases with increasing  $\Psi(0)$  until a threshold value  $\Psi(0)_{\text{crit}}$  is reached, after which the masses start to decrease for larger  $\Psi(0)$ . This value separates two regions, the values with  $\Psi(0) < \Psi(0)_{\text{crit}}$  represent stable boson stars, while the configurations with  $\Psi(0) > \Psi(0)_{\text{crit}}$  are unstable and either collapse to a black hole or migrate to a configuration on the stable branch [30]. The maximum mass is  $0.633 m_{\psi}^{-1}$  and corresponds to  $\Psi(0)_{\text{crit}} = 0.076$  or  $\sigma(0)_{\text{crit}} = 0.27$  with the usual normal-

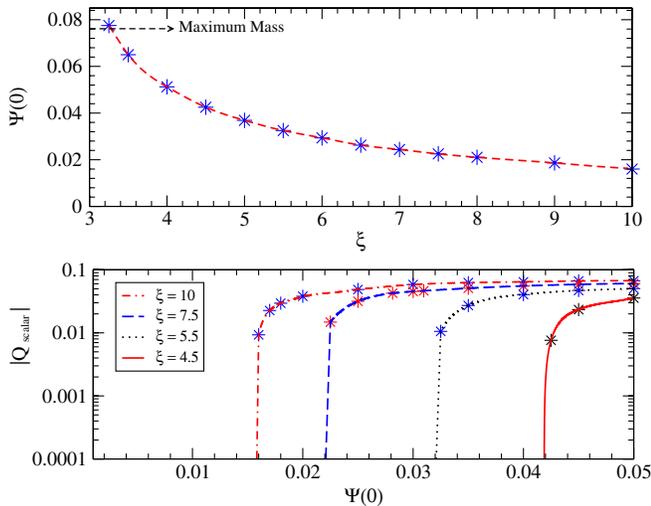


FIG. 1 (color online). (Top) Critical values of central boson field  $\Psi_{\text{central}}$  and nonminimal coupling parameter  $\xi$  for spontaneous scalarization; (Bottom) Scalar charge at the end of the transition to spontaneous scalarization for different values of the parameters.

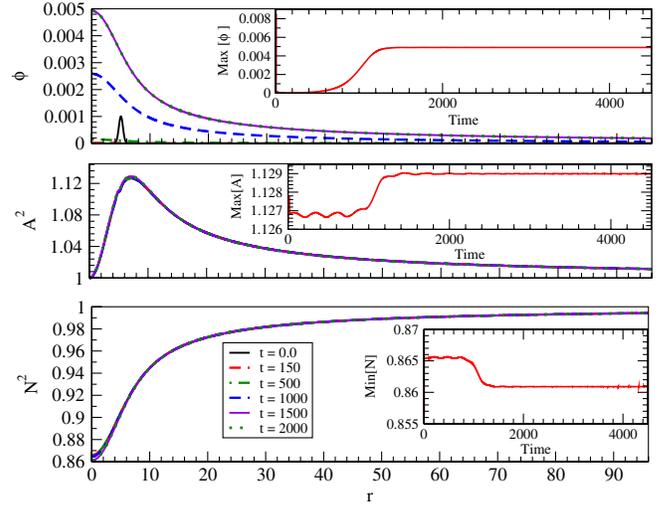


FIG. 2 (color online). Nonminimally coupling (NMC) scalar field  $\phi$  and metric components at different times for the simulation with  $\xi = 6$  and  $\Psi(0) = 0.03$ .

ization ( $\sigma = \sqrt{4\pi}\Psi$ ). Similar behavior is expected to occur in STT [23].

Figure 2 depicts a series of snapshots taken at different times during the evolution for the unstable case with  $\xi = 6$  and  $\Psi(0) = 0.03$ . Note from this figure that initially the scalar field  $\phi$  almost vanishes, whereas at the end of the evolution it settles down into a stationary configuration which results in a nontrivial profile that interpolates between a finite value  $\phi(t_{\text{final}}, r = 0)$  at the center of the boson star and a vanishing value  $\phi(t_{\text{final}}, r_{\infty})$  asymptotically. The final stationary configuration is also characterized by the appearance of a global *scalar charge* (see Fig. 1 bottom panel).

## V. DISCUSSION

Boson stars are stable self-gravitating configurations of a complex scalar field, and as such they could in principle exist in nature. If the bosons are light ( $m_b := \hbar m_{\psi}/c \sim \text{eV}/c^2$ ), one expects the mass of the boson star to be of order  $M \sim 10^{20}$  kg, which is in the mass range of some asteroids. On the other hand, for heavy bosons ( $m_b \sim 100 \text{ GeV}/c^2$ ) the boson-star mass turns out to be much lower ( $M \sim 10^9$  kg). However, by including self-interactions it is possible to increase the mass of the boson star to the order of the solar mass [31]. It has also been speculated that supermassive boson stars instead of black holes could be at the center of galaxies [32]. If boson stars actually exist in nature, they could serve as natural “laboratories” to test different alternative theories of gravity.

In this work we have used a scalar-tensor theory of gravity to study dynamical simulations of boson stars. We have found that, just like in the case of neutron stars, boson stars can also undergo a spontaneous scalarization process. We have analyzed this transition in a dynamical

fashion using a fully relativistic code in spherical symmetry. Unlike previous studies of stationary boson-star configurations, we have found that self-interactions are in fact not required to produce scalarization. We have implemented a STT which is very similar to the one studied by Whinnett, where scalarization was not found without self-interaction. However, he used a value  $\xi_W = 1$  [33] (corresponding to our  $\xi = 2$ ) which is actually in the region of Fig. 1 where scalarization does not ensue. He also used  $\xi_W = 2$  (ours  $\xi = 4$ ), where our results of Fig. 1 show that there is a small region where scalarization is found. Actually, from Whinnett's Fig. 4 is not clear that scalarization is not found for  $\xi_W = 2$  since his coupling parameter is not null. By increasing the value of the nonminimal coupling parameter  $\xi$ , one can reach a threshold where the energetically favorable configurations are those with the presence of a nontrivial scalar field  $\phi$ . Note from Fig. 1 that the larger the value of the parameter  $\xi$ , the lower the central energy density required to produce the transition to scalarization.

At this point it is perhaps appropriate to mention that the effective Brans-Dicke parameter given by  $\omega_{\text{BD}} = f/(f')^2|_{\phi_0}$  (where  $\phi_0$  is the asymptotic [cosmological] value) takes the explicit form  $\omega_{\text{BD}} = (1 + 8\pi\xi\phi_0^2)/[32\pi(\xi\phi_0)^2]$ . Therefore, as the spontaneous sca-

larization ensues with  $\phi_0 \rightarrow 0$  with a finite value of  $\xi$ , one has  $\omega_{\text{BD}} \rightarrow \infty$ , which obviously passes the constraints imposed by the Cassini probe  $|\gamma - 1| \lesssim 2.3 \times 10^{-5}$  [34], where  $\gamma$  is the post-Newtonian parameter, which in terms of  $\omega_{\text{BD}}$  is given by  $\gamma = (\omega_{\text{BD}} + 1)/(\omega_{\text{BD}} + 2)$ , implying  $\omega_{\text{BD}} \gtrsim 4.3 \times 10^4$ . As was already mentioned in the introduction, the phenomenon of spontaneous scalarization can therefore appear independently of the bounds imposed on  $\omega_{\text{BD}}$  by the solar system experiments.

Though here we have focused mainly on such a phenomenon and presented only our main results, in [23] we will present a more detailed explanation of our code and methods, together with a systematic study of the phenomenon. Also, in a future work we will analyze the collapse of a scalarized boson star to a black hole, and the corresponding emission of scalar gravitational waves.

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density is measured in units  $\rho_c := m_\psi^2 c^4 / G_0 = \rho_p (m_b / M_p)^2$  where  $\rho_p := c^7 / (\hbar G_0^2)$  is the Planck energy density. Note that  $m_\psi^{-1} = l_p (m_b / M_p)$  where  $l_p = \sqrt{\hbar G_0} / c^3$  is the Planck length.

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