Nucleation of quark matter in protoneutron star matter

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The phase transition from hadronic to quark matter may take place already during the early post-bounce stage of core collapse supernovae when matter is still hot and lepton rich. If the phase transition is of first order and exhibits a barrier, the formation of the new phase occurs via the nucleation of droplets. We investigate the thermal nucleation of a quark phase in supernova matter and calculate its rate for a wide range of physical parameters. We show that the formation of the first droplet of a quark phase might be very fast and therefore the phase transition to quark matter could play an important role in the mechanism and dynamics of supernova explosions.

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I. INTRODUCTION

The possibility that a first-order phase transition in dense matter has implications for the explosions of supernovae was first proposed by Migdal *et al.* about 30 years ago [1]. Since then, a large number of papers addressed this issue with different hypotheses on the nature of the phase transition: pion or kaon condensate matter or quark matter, different models to compute the equation of state (EoS), different simplifications for the complex hydrodynamical evolution of collapsing massive stars, and different approaches for the neutrino Boltzmann transport [2–4]. However, only very recently was it possible to perform simulations using general relativistic Boltzmann neutrino transport equations and adopting realistic equations of state for quark matter [5,6]. In Ref. [6], in particular, it was shown that the phase transition to quark matter can occur already during the early post-bounce phase of a core collapse supernova event and that it produces a second shock wave (the first being the usual shock wave after the bounce) which triggers a delayed supernova explosion, even within spherical symmetry, for masses of the progenitor star up to $15M_{\odot}$. The formation of quark matter is also responsible for the emission of a neutrino burst, typically a few hundred milliseconds after the first neutronization burst, which could be detected by currently available neutrino detectors, representing a spectacular possible signature of quark matter formation in compact stars (see also [7]).

In the studies mentioned above, an important physical phenomenon is neglected for the sake of simplicity: the process of phase conversion in a first-order transition is actually driven by the nucleation of finite-size structures, such as droplets or bubbles, of the new phase within the old phase. The surface tension, σ, is the physical quantity that determines the nature of the process of phase conversion. If σ is sufficiently small, nucleation can be very fast and the new phase is produced almost in mechanical, thermal, and chemical equilibrium with the nuclear matter phase. An intermediate value of σ might render nucleation very difficult and the nuclear phase can be metastable for a significant amount of time. Then, the process of formation of the new phase, once triggered, would be a genuine nonequilibrium process, in which different mechanisms can take place: deflagration, detonation, convective instabilities (see, e.g., Refs. [8,9] and references therein). Finally, nucleation is highly suppressed for large values of σ, and the formation of the new phase may only proceed via spinodal decomposition if the density achieved is high enough to flatten out the activation barrier. (See Ref. [10] for a recent detailed discussion of the phase conversion process in a first-order phase transition.)

The nucleation of quark matter in neutron stars has been explored mainly within a scenario, proposed in Ref. [11], in which the formation of quark matter occurs only when the protoneutron star is almost completely deleptonized and the temperature has already dropped to, say, 1 MeV. Under these conditions, quantum nucleation has been shown to be the most important mechanism for the formation of a quark phase [12–16], also when color superconducting quark phases are present [17,18]. A less explored scenario is the nucleation of quark matter in hot and lepton-rich protoneutron stars. Refs. [19,20] present the first estimates and calculations showing thermal nucleation to be very efficient for temperatures of roughly 10 MeV and practically negligible for temperatures below 2 MeV. The simplifying assumption adopted in those papers (no leptons are included in the quark equation of state) might be, however, not realistic considering that for the large temperatures needed to nucleate quark matter the neutrino mean free path is small, and therefore neutrinos are trapped. In the presence of neutrinos, the critical densities for the phase transition are shifted toward larger values compared to a deleptonized neutron star. Moreover, as we will discuss in the following, the assumption of flavor conservation during the phase transition (also adopted in Ref. [21]) might be too conservative in light of the quark density fluctuations that are evidently present.

The main goal of this paper is to compare the time scale associated with the phase conversion with the dynamical time scale of core collapse supernovae during which quark matter might be eventually formed. In particular, since the explosion process occurs within a time scale of few hundred ms, the nucleation time must be of the same order of magnitude if quark matter plays indeed a role in the explosion mechanism.

For this purpose, we reconsider thermal nucleation of quark matter within the scenario proposed in Ref. [6] of a phase transition occurring already during the early post-bounce stage of a core collapse supernova. This implies a value for the critical density $n_c \lesssim 2n_0$ (where $n_0 = 0.16\ \mathrm{fm}^{-3}$ is the nuclear matter saturation density). We systematically investigate the windows of free parameters of the model adopted to compute the equations of state and the corresponding nucleation time scale. Our strategy is to present *underestimates* of this time scale, so that a successful phase conversion could somehow constrain the equation of state parameter space. We also discuss the important issue of flavor number conservation during the phase transition. We argue that thermal nucleation might indeed be efficient under the conditions realized in a star soon after bounce. In such case, the phase transition would proceed very fast, thus confirming the results found in Ref. [6].

The paper is organized as follows. In Sec. II we present the model we adopt for the equation of state, as well as a brief self-contained description of homogeneous nucleation and the role of statistical fluctuations. In Sec. III we present our results for the nucleation of nonstrange and strange quark matter. There, we also discuss quantum nucleation and the spinodal instability. Besides, we verify our equations of state by computing the stellar structure that emerge form the Tolman-Oppenheimer-Volkov (TOV) equations. Finally, Sec. IV contains our conclusions.

II. PHENOMENOLOGICAL FRAMEWORK

A. Equations of state

The phase transition from nuclear to quark matter is implemented, as customary, by matching the equations of state for each phase. In order to do that, one has to choose appropriate models to compute the equation of state for each of the two phases and impose the conditions for mechanical, thermal, and chemical equilibrium to determine the transition point. For nuclear matter, we adopt the relativistic mean field model with the TM1 parametrization [22], often used in supernovae simulations. For quark matter, we choose the MIT bag model [23,24] including perturbative QCD corrections [25,26].

Moreover, we consider two types of EoS for quark matter, both including the pressure from electrons and neutrinos, which are still present at this point of the stellar evolution. The first EoS contains only *up* and *down* quarks, while in the second we include a massive s quark as well. The quark model has as free parameters the bag constant B, the mass of the strange quark m_s (when present), and the value of the coefficient c that accounts for the perturbative QCD corrections to the free gas pressure as follows:

$$p(\{\mu\}) = (1-c)\left[\sum_{i=u,d}\frac{\mu_i^4}{4\pi^2}\right] + p_s + \frac{\mu_e^4}{12\pi^2} + \frac{\mu_\nu^4}{12\pi^2} - B, \tag{1}$$

where B is the bag constant, p_s is the contribution from the (massive) strange quark

$$p_s = (1-c)\frac{\mu_s^4}{4\pi^2} - \frac{3}{4\pi^2}m_s^2\mu_s^2, \tag{2}$$

and terms $\mathcal{O}(m_s^4/\mu_s^4) \sim 1\%$ in Eq. (2) have been neglected. Notice that, for the u-d equation of state, we neglect the terms related to the *strange* quark.

The free parameters are fixed by requiring a critical density for the phase transition below 2 times the nuclear saturation density $n_0 = 0.16\ \mathrm{fm}^{-3}$ for the typical conditions of matter in the core of a star during a supernova collapse, i.e. temperatures $T = 10$–20 MeV and lepton fractions $Y_L = 0.3$–0.4. In this way, we fulfill our initial hypothesis of formation of quark matter in the early post-bounce stage. Another important criterion to fix our free parameters comes from the computation of the maximum mass of cold hybrid stars. Taking into account the recent measurement of the mass of PSR J1903+0327, $M = (1.671 \pm 0.008)M_\odot$ [27], we require a maximum mass in agreement with this value. Finally, we investigate two possible scenarios for the appearance of strange quarks in the system. Since the nuclear EoS does not contain strangeness, a phase transition directly to strange quark matter might be difficult if we consider the slowness of weak reactions producing strange quarks with respect to the fast deconfinement/chiral phase transition process driven by the strong interaction. Therefore, we discuss a first case in which the phase transition involves two-flavor quark matter (strange quarks will be produced only later, via weak interaction, as suggested in Ref. [28]). In the second scenario, we consider a fast production of strange quarks: since we assume critical densities of the order of 2 times the saturation density and temperatures of a few tens of MeV, it is possible that a small seed of strange matter appears in the system through hyperons or kaons [29]. Once strangeness is produced in the hadronic matter, this would trigger the phase transition directly to strange quark matter.

The equations of state for nuclear matter and quark matter are calculated under conditions of local charge neutrality, local lepton fraction conservation (i.e., the two phases have the same Y_L), and weak equilibrium. Under these assumptions, the conditions of phase coexistence, as found in Ref. [30], are the equality of the total pressure of the two phases $P^H = P^Q$ and the following condition of chemical equilibrium:

$$\mu_n + Y_L \mu_\nu^H = \mu_u + 2\mu_d + Y_L \mu_\nu^Q \equiv \mu_{\rm eff}, \qquad (3)$$

where μ_n and μ_ν^H are the chemical potentials of neutron and neutrinos within the nuclear phase, and μ_u, μ_d, and μ_ν^Q are the chemical potentials of up and down quarks and of neutrinos within the quark phase, respectively. The quantity $\mu_{\rm eff}$ is an effective chemical potential, which is always the same in both phases. Notice that the condition (3) is always valid, although the condition $P^H = P^Q$ is valid only in the transition point. Here we use the zero-temperature equations of state, since a temperature of the order of a few tens of MeV does not alter considerably the equation of state.[1]

Moreover, we assume that the different degrees of freedom in both phases are in chemical equilibrium with respect to weak reactions (see Sec. II C):

$$\mu_n + \mu_\nu^H = \mu_p + \mu_e^H, \qquad (4)$$

$$\mu_d + \mu_\nu^Q = \mu_u + \mu_e^Q, \qquad (5)$$

$$\mu_d = \mu_s. \qquad (6)$$

Finally, the conditions of local charge neutrality and a local lepton fraction within the two phases allow us to compute all chemical potentials in terms of only one independent chemical potential:

$$n_p = n_e^H, \qquad (7)$$

$$\tfrac{2}{3}n_u - \tfrac{1}{3}n_d - \tfrac{1}{3}n_s = n_e^Q, \qquad (8)$$

$$\frac{n_e^H + n_\nu^H}{n_B^H} = \frac{n_e^Q + n_\nu^Q}{n_B^Q} = Y_L, \qquad (9)$$

where n_i ($i = n, p, u, d, s, e, \nu$) are the densities of the different species of particles.

Notice that fixing Y_L locally results in a jump of the chemical potential of neutrinos at the interface of the phase transition. This might be not very realistic, as discussed in Ref. [31] where it has been shown that a mixed phase should instead be considered due to the global conservation of the lepton number. We leave the discussion of nucleation in mixed phase for a future study.

B. Thermal homogeneous nucleation

First-order phase transitions are very well known even from simple everyday examples, such as in the melting of ice. In such a case, the conversion from one phase to the other usually occurs slowly and very close to the thermodynamical equilibrium, following the so-called Maxwell construction. However, when some relevant external control parameter (such as the temperature or the density) changes abruptly when a system is near the transition, the system finds itself in an unstable situation. For definiteness, consider a system initially homogeneous in a low-density phase (a "gas"), and close to the transition line to a high-density ("liquid") phase. Now, let it suffer a sudden compression. Although the system was prepared at the gas phase, the free energy at the new, higher density disfavors the gas phase and the liquid phase now becomes the stable one: phase conversion is about to begin.

The ever-present thermal and quantum fluctuations will not be suppressed, as expected in equilibrium, due to the instability of the system. Such fluctuations will drive the system to another point of stability of the phase diagram. The evolution in time of those fluctuations are at the heart of our discussion.

For first-order phase transitions, there can be two kinds of instabilities that dominate the dynamics of the phase conversion [32]. If a homogeneous system is brought into instability close enough to the coexistence line of the phase diagram, its dynamics will be dominated by large-amplitude, small-ranged fluctuations. In these cases, large amplitudes are necessary for the development of the phase transition once the system is in a metastable equilibrium. Those are usually referred to as bubbles (or droplets) and the process that creates them is called nucleation. In the other case, if the external perturbation is big enough and the system finds itself far from the coexistence line, the dominant fluctuations will have small amplitudes and large wavelengths, the process of phase conversion is called spinodal decomposition. In this work, we focus on thermal nucleation of quark matter as nuclear matter is compressed in a stellar collapse, leaving a discussion on a possible role for spinodal decomposition and quantum nucleation to the final section.

A standard, field-theoretical approach for thermal nucleation in one-component metastable systems was developed by Langer in the late sixties [33]. In this formalism, a key quantity for the calculation of the rate of nucleation is the coarse-grained free energy functional

$$F[\phi] = \int d^3r \left\{ \frac{1}{2}[\nabla\phi(\mathbf{r})]^2 + V[\phi(\mathbf{r})] \right\}, \qquad (10)$$

where $\phi(\mathbf{r})$ is the order parameter of the phase transition at a given point $\mathbf{r}$ of space. By assumption, the potential $V(\phi)$ has a global (true) minimum at ϕ_t and a local (false) one at ϕ_f. At a given baryon chemical potential μ of the metastable phase, the difference $\Delta V \equiv V(\phi_t) - V(\phi_f)$ is identified with the pressure difference between the stable and the metastable phases, with opposite sign: $\Delta V = -\Delta p(\mu) = p_t - p_f$, where p_t (p_f) is the pressure for the true (false) phase at baryon chemical potential μ.

[1]For a free massless gas, the corrections would be $\mathcal{O}(T^2/\mu^2) \sim 1\%$.

The field equation for $\phi(\mathbf{r})$ is given by a minimum of the functional F. One can easily think of three static solutions. Two of them are the trivial ones given by homogeneous field configurations with $\phi(\mathbf{r}) = \phi_t$ or $\phi(\mathbf{r}) = \phi_f$. The third is a spherically symmetric bubblelike solution that has as boundary conditions

$$\phi(r = 0) = \phi_t, \qquad \phi(r \to \infty) = \phi_f. \tag{11}$$

Roughly speaking, this means that the stable phase is found deep in the bubble and the metastable one is found away from it. Somewhere in-between, the order parameter must change from its central value ϕ_t to ϕ_f at $r \to \infty$. The relatively thin region which marks the border between "inside" ($\phi = \phi_t$) and "outside" ($\phi = \phi_f$) the bubble is called the bubble wall.

Exactly at the coexistence line, one can prepare one (infinite) system with the two homogeneous phases in equal proportions divided by a plane wall with a small width. This configuration is static, once no phase is favored. Further, each phase occupies a semi-infinite volume. If the system is slightly pushed into metastability, the static solution for $\phi(\mathbf{r})$ is a bubble with a very large radius and still a small wall width. This is the starting point for the *thin-wall approximation*: the free energy (10) of the system of volume $(4\pi/3)L^3$ ($L \to \infty$) is determined by the outcome of a competition between a surface energy term, which is positive and comes from $|\nabla\phi|^2$ in (10), and a bulk term, which is negative and corresponds to the potential V, or to the pressure difference between the phases. Notice that, within this approximation, $\phi(r)$ is constant, except over the (thin) wall of the bubble, and so $V(\phi)$ is also essentially constant both inside and outside the bubble. This means that the free energy for the bubble configuration of radius R in the thin-wall approximation of Eq. (10) is given by

$$F_{\text{bubble}}(R) = 4\pi R^2\sigma - \frac{4\pi}{3}(L^3 - R^3)p_f - \frac{4\pi}{3}R^3 p_t, \tag{12}$$

whereas the homogeneous metastable configuration has $|\nabla\phi|^2 = 0$ and

$$F_{\text{metastable}} = -\frac{4\pi}{3}L^3 p_f. \tag{13}$$

In Eq. (12), we introduced the surface tension σ, which is merely the energy per unit area of the bubble wall. As will be clear below, it is a key physical quantity in our analysis.

According to the standard theory [33], the nucleation rate has as its main ingredient the free energy shift when a bubble is created from fluctuations in the homogeneous metastable phase. From to Eqs. (12) and (13) we have

$$\Delta F(R) \equiv F_{\text{bubble}}(R) - F_{\text{metastable}} = 4\pi R^2\sigma - \frac{4\pi}{3}R^3(\Delta p), \tag{14}$$

where $\Delta p = p_t - p_f > 0$. Here, the pressures in each of the phases are calculated for the same value of μ_{eff}. Notice that this implies different baryon chemical potentials and densities for each phase, due to the conditions (3)–(9).

Bubble configurations of given radii R arise from the homogeneous metastable phase due to thermal fluctuations, and each of those has an associated value of $\Delta F(R)$. From Eq. (14), we can see that $\Delta F(R)$ has a maximum at the critical radius $R_c \equiv 2\sigma/\Delta p$. The equations of motion show that any bubble with $R < R_c$ will shrink and disappear whereas any bubble with $R > R_c$ will grow as a consequence of the competition between the positive surface energy and the negative bulk energy. Hence, the critical bubbles are the smallest bubbles that can start to drive the phase conversion dynamics. To give a quantitative meaning to the process of nucleation, one can calculate the rate Γ of critical bubbles created by fluctuations per unit volume, per unit time. In Langer's formalism [33]:

$$\Gamma = \frac{\mathcal{P}_0}{2\pi}\exp\left[-\frac{\Delta F(R_c)}{T}\right], \tag{15}$$

where the prefactor $\mathcal{P}_0$ is usually factorized into two parts: a statistical prefactor, which measures the rate of successful creation of a critical bubble by thermal fluctuations, and a dynamical prefactor, which measures the early growth rate of the bubble. As customary, we adopt $\mathcal{P}_0/2\pi = T^4$, which corresponds to an overestimate of the actual prefactor. (For an exact calculation of $\mathcal{P}_0$ see, e.g., Ref. [34].) This constitutes one of our main reasons to interpret our results as providing an overestimate for the nucleation rate. It goes in line with the thin-wall approximation, which is also known to overestimate Γ when compared to the exact (numerical) result [35]. Although this overestimate can lead to an overall factor of $\sim 10^2$ or even higher [34], the qualitative aspects of the results shown in the next section can be barely changed. And, since we are concerned with providing *underestimates* for thermal nucleation time scales under core collapse supernovae typical conditions, these details are not relevant.

Our final formula for the nucleation rate reads

$$\Gamma = T^4\exp\left[-\frac{16\pi}{3}\frac{\sigma^3}{(\Delta p)^2 T}\right], \tag{16}$$

where we used Eqs. (14) and $R_c = 2\sigma/\Delta p$. Notice that the influence of the equation of state is present through Δp. Also, there is a remarkably strong dependence of Γ on the surface tension σ, which will be determinant for the nucleation time scale.

It is convenient to introduce the *nucleation time* τ, defined as the time it takes for the nucleation of one single critical bubble inside a volume of 1 km^3 inside the core of the protoneutron star:

$$\tau \equiv \left(\frac{1}{1\ \text{km}^3}\right)\frac{1}{\Gamma}. \tag{17}$$

This is the time scale to be compared with the duration of the early post-bounce phase of a supernova event, few hundreds of milliseconds, during which it has been shown that quark matter formation could trigger the explosion [6]. With this definition, we assume that temperature and density are constant within this central volume of 1 km^3. Of course, this also goes in the direction of underestimating this time scale. In a more realistic calculation, one should first compute the pressure and density profiles using the TOV equations, then calculate the *local* value of Γ, and finally integrate over the region containing metastable matter. However, the density profiles are almost flat within the central kilometers of the star, thus making our assumption quite reasonable.

Finally, we note that we calculate the time of production of one single critical bubble, which has a typical size of some fermi. We do not study, in this paper, the growth regime of the *quark front*. Once again, this leads to an underestimate: in comparing τ with the bounce time scale as a criterion for the formation of a quark core, we tacitly assume that the quark matter bubble becomes macroscopic almost instantaneously, an obviously artificial simplification.

C. The role of statistical fluctuations

As discussed in the Introduction, a possible alternative approach to model the phase transition to quark matter assumes isospin (and strangeness) conservation during the phase transition [15,20,21]. This implies a transition from hadronic matter in chemical equilibrium to an intermediate quark matter phase Q^* in which quarks are not in chemical equilibrium. The chemical potentials of quarks are calculated, indeed, by assuming that the fractions of different quark flavors are the same, both in the hadronic and in the quark phase. This assumption is based on the argument that the time scale of the phase transition is regulated by QCD, thus typically of the order of 10^{-23} s, and much faster than the weak interaction time scale (weak interactions would produce quark matter in chemical equilibrium Q^{eq} only after the phase transition is completed).

However, as noticed in Refs. [29,36], statistical fluctuations of the number densities of quarks can have important effects on nucleation. To estimate these effects one can calculate the phase transition point by considering the two equations of state in chemical equilibrium. At the critical density one compares the average numbers $N^{\text{eq}}_{u,d}$ of up and down quarks within Q^{eq} and within the nuclear phase $N^*_{u,d}$ in a fixed volume V. If these numbers are different only by $\sim 3\sqrt{N^*_{u,d}}$, it means that fluctuations would most probably drive the transition directly to Q^{eq}.

The volume V in which we consider fluctuations corresponds to the volume of a drop of the new phase with critical radius $R_{\text{crit}} = 2\sigma/\Delta P$. Again, the surface tension σ is the crucial quantity that determines whether the phase transition occurs via the intermediate phase Q^* or directly to Q^{eq}. Taking into account the uncertainties on the value of σ we estimate the critical radii to be of order ~ 6 fm. Using the simple calculation explained above, we obtain that statistical fluctuations are efficient for radii of the order of 2–4 fm and thus of the same order of magnitude as the critical radii. So, we assume, as in Ref. [14], that the first drop is nucleated already in the Q^{eq} phase.

Now that we have all the ingredients for the calculation of the nucleation times, we can proceed to our results in the next section.

III. RESULTS AND DISCUSSIONS

In order to evaluate if the time scale τ for the nucleation of quark matter is compatible with the bounce time scale τ_B, we underestimate the time τ for the formation of a critical bubble as a function of density under various conditions of temperature, as well as for different equations of state and values of surface tension, given the scenario described in the previous section.

Recent supernova simulations [6] indicate that the central density of a protoneutron star can be as high as $2n_0$ during the bounce, and this value will serve as a cutoff density in our analysis. Still in the spirit of underestimating τ (or, equivalently, overestimating the nucleation rate Γ), we consider that nucleation is effective if $\tau < \tau_B \lesssim 100$ ms for some $n < 2n_0$.

A. Nucleation times for nonstrange matter

As our first case, we consider the transition from beta-stable nuclear matter to beta-stable quark matter composed of u and d quarks, plus electrons and electron neutrinos, with a fixed lepton fraction Y_L. Later on, we will discuss the case of a transition from nuclear matter to u-d-s quark matter (both lepton-rich and in beta equilibrium).

We start our analysis by the case of a low-density transition: $n_c = 1.5n_0$. We assume a lepton fraction $Y_L = 0.4$ and consider two values for the temperature, representing a "minimum" and a "maximum" value that can be expected during the bounce, and two values for the surface tension.

In Fig. 1 one can see the behavior of the nucleation time of a single critical bubble (as defined in the previous section) versus the density, in units of n_0.

As expected, the nucleation time τ has an extremely strong dependence on both density (notice the logarithmic scale for τ) and on the surface tension, a feature that can also be seen in Fig. 2. For low values of σ nucleation becomes feasible at relatively low densities, although such densities increase steadily as the surface tension rises. However, if the (basically unknown) surface tension is larger, the density for $\tau \sim 100$ ms may be higher than our $2n_0$ cutoff, and nucleation should not be an efficient

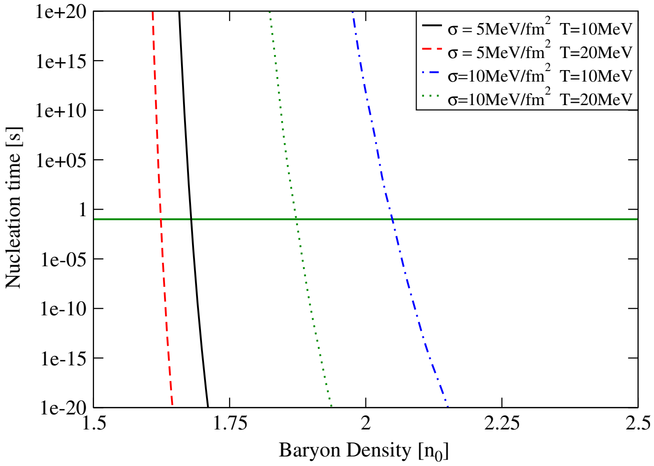

FIG. 1 (color online). Nucleation time as a function of baryon density for u-d quark matter ($n_c = 1.5n_0$). The horizontal line corresponds to $\tau = 100$ ms.

mechanism for phase conversion. In this sense, we can expect that if the nuclear-quark matter transition occurs in this scenario the surface tension must be quite small.

We can also investigate how the choice of the critical density can affect the nucleation time (Fig. 3). In any case, a surface tension larger than roughly 15 $\mathrm{MeV/fm^2}$ seems to be sufficient to prevent thermal nucleation (for $T = 20$ MeV or lower).

In Fig. 4, we show the role played by the temperature in the process of nucleation. We may notice that the precise value of the temperature does not affect the nucleation times as strongly as the surface tension does, as long as it is kept in the range expected during the early post-bounce phase at the protoneutron star core, i.e., roughly from 10 to 25 MeV.

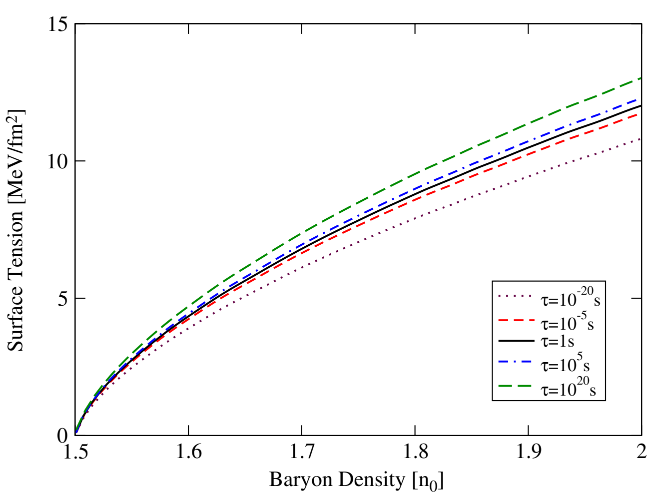

FIG. 2 (color online). Contour lines of constant nucleation time (contour lines) as a function of density and surface tension for u-d quark matter. Here, $n_c = 1.5n_0$, $Y_L = 0.4$, and $T = 20$ MeV.

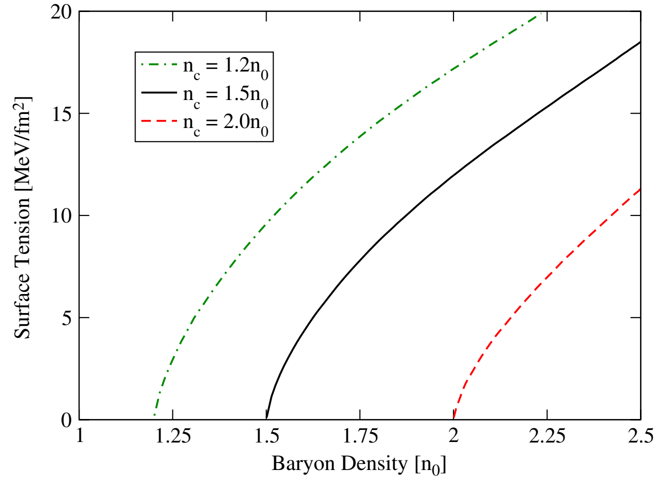

FIG. 3 (color online). Lines of constant nucleation time ($\tau = 100$ ms) for $n_c/n_0 = 1.2, 1.5, 2.0$ with $T = 20$ MeV, $c = 0$, and $Y_L = 0.4$.

Although the exact numbers should not be taken at face value, given the uncertainties involved, we believe that the order of magnitude of the actual limiting value of the surface tension for $\tau = 100$ ms is correct.

B. Nucleation of strange matter

Although the core of a supernova progenitor star before its collapse does not contain any strangeness, the energy density achieved during and right after bounce allows for the presence of a small amount of hyperons in the hadronic phase [37]. Such particles do not contribute significantly to the pressure or to the energy density, but density fluctuations of such hadrons may induce the formation of bubbles of strange quark matter.

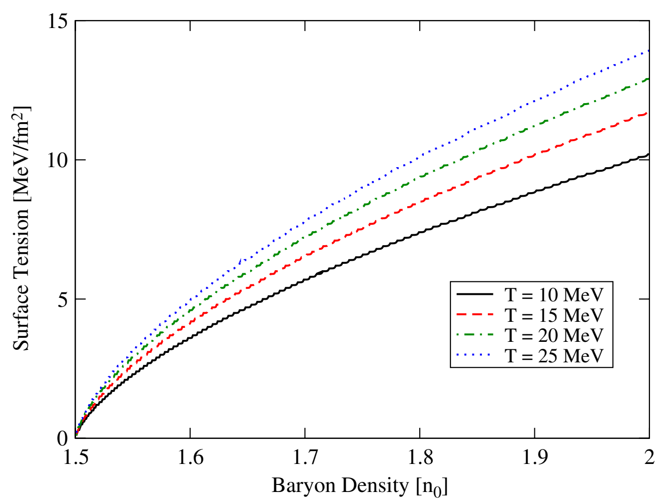

FIG. 4 (color online). Lines of constant nucleation time ($\tau = 100$ ms) for noninteracting u-d quarks, for temperatures between 10 and 25 MeV ($n_c = 1.5n_0$, $Y_L = 0.4$).

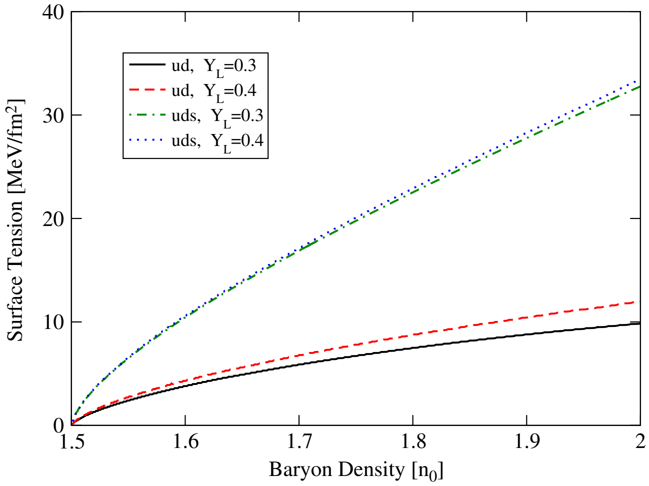

FIG. 5 (color online). Lines of constant nucleation time ($\tau = 100$ ms) for the transition from nuclear matter to *u-d* or *u-d-s* quark matter, with $Y_L = 0.3$, 0.4 ($n_c = 1.5 n_0$, $T = 20$ MeV, $c = 0$, and $m_s = 0$).

The introduction of strange quarks makes the EoS stiffer, i.e. for a given baryon chemical potential μ the corresponding pressure becomes higher. Once the nuclear EoS is the same, Δp will be higher for a given value of μ, and therefore the nucleation rate will also be higher. Figure 5 shows a comparison between the transition from nuclear matter to either *u-d* or *u-d-s* quark matter for two values of the lepton fraction Y_L. We can notice that a decrease in Y_L increases the efficiency of thermal nucleation. This is expected because deleptonization not only renders nuclear matter less stable but it also heats up the stellar core (the former effect, however, is not accounted for in this work).

Up to now, we have only considered noninteracting quarks. Results from two-loop perturbative three-flavor

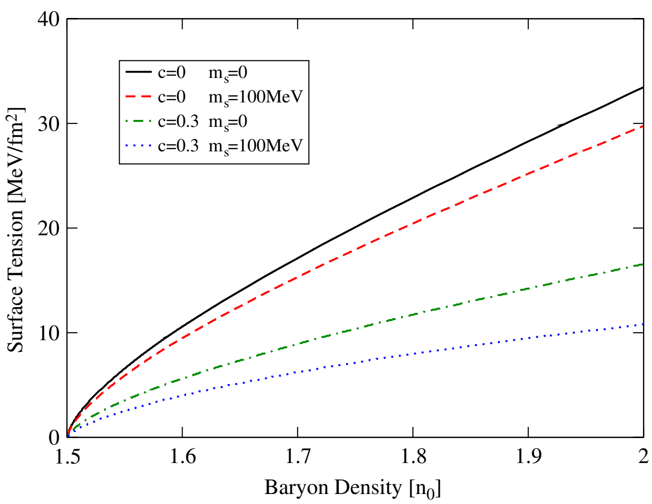

FIG. 6 (color online). Lines of constant nucleation time ($\tau = 100$ ms) for $c = 0$ and $c = 0.3$, and for $m_s = 0$ and $m_s = 100$ MeV ($n_c = 1.5 n_0$, $T = 20$ MeV, and $Y_L = 0.4$).

TABLE I. Maximum masses of cold deleptonized compact stars for some of the EoS used (corresponding to the cases $Y_L = 0.4$ and $m_s = 100$ MeV considered for nucleation).

n_c/n_0	c	$B^{1/4}$ (MeV)	$M_{max}/M_\odot$
1.2	0	159.22	1.60
	0.3	144.65	1.90
1.5	0	161.77	1.55
	0.3	145.89	1.87
2.0	0	166.64	1.48
	0.3	147.56	1.83

QCD at finite density [25] show that strong interactions can be effectively accounted for in the equation of state by introducing a factor $c < 1$ [26], as in Eq. (1). This factor makes the quark EoS softer in the pressure-chemical potential plane [see Eq. (1)] and, therefore, Δp should be smaller, making the nucleation time τ larger for a given density, according to Eqs. (16) and (17). As an explicit example, we compare the $c = 0$ case with $c = 0.3$, in the case of $n_c = 1.5 n_0$, as displayed in Fig. 6, where we also show the influence of strange quark mass m_s. Notice that the introduction of interactions via the parameter c drastically increases the nucleation time, so that only for a low value of the surface tension, e.g. $\sigma \sim 10$ MeV/fm^2 (for $Y_L = 0.4$), nucleation can be efficient if the density reaches a value close to $2n_0$.[2]

C. Stellar structure of selected equations of state

In order to check if the equations of state we used are compatible with observed pulsar data, we calculate their associated mass-radius diagram using the TOV equations. In Table I, we report the quark model parameters, for strange matter, $Y_L = 0.4$ and different choices of the critical density and the resulting maximum mass for cold and beta-stable stars.

Notice that only by including the effect of QCD perturbative interactions is it possible to obtain masses for hybrid stars compatible with the recent observation of PSR J1903 +0327, $M = (1.671 \pm 0.008) M_\odot$ [27].

D. Quantum nucleation and spinodal instability

There are other mechanisms which can compete with thermal nucleation in driving the phase transition: quantum nucleation and spinodal decomposition. Unfortunately, our treatment is blind to the spinodal instability: an effective potential is indeed needed to take it into account effectively (see Ref. [10]). In any case, this is a process that will be relevant only if the system is taken into values of density

[2]Of course, the surface tension should also be affected by loop corrections, and that could eventually reduce or even balance out this effect on the final nucleation dynamics. As becomes clear from all this analysis, a reliable estimate of the surface tension for cold dense matter is called for.

that are high enough to flatten out the activation barrier, so that there is no more extra cost to create a region of the true vacuum inside the initially homogeneous false vacuum configuration by thermal fluctuations. If that is possible, the phase conversion process will be rather explosive, a possibility we will consider in a forthcoming publication.

On the other hand, we can easily estimate the contribution of quantum nucleation by using the formalism considered in Ref. [12], i.e. a WKB treatment of the tunneling through a barrier of an effective potential similar in form to Eq. (14). We find that only for temperatures smaller than $\sim$5 MeV the quantum nucleation rate is comparable or larger than the thermal nucleation rate. Since we are considering supernova matter, with $T \gtrsim 10$ MeV, thermal nucleation is by far the dominant mechanism for the production of the first drop of quark matter.

IV. CONCLUSIONS

We investigated the possibility of the formation of quark matter in supernova matter, i.e. for temperatures of the order of a few tens of MeV and in the presence of trapped neutrinos, assuming that the corresponding critical density does not exceed $2n_0$. We argued that thermal nucleation of quark phase droplets is eventually the dominant mechanism for the formation of the new phase and that, due to fluctuations in the number densities of quarks, the phase transition involves directly the beta equilibrated quark phase. We have calculated the nucleation rate for different values of the free parameters. The surface tension, as expected, is the physical quantity which mainly controls the nucleation process.

Among the different equations of state and conditions at bounce we have tested, the choice $T = 20$ MeV and $Y_L = 0.4$ is the most likely to occur. Within this choice of physical conditions, if the phase transition involves only noninteracting up and down quarks, a value of σ smaller than $\sim$15 MeV/fm^2 should be required for nucleation to be efficient. Such a low value of σ is compatible with lattice QCD calculations, although they are obtained at large temperatures and small densities [38]. On the other hand, phenomenological estimates at large density and zero temperature indicate larger values of σ [39]. Therefore, we consider this scenario to be unlikely.

If strange quarks are produced during the phase transition, we conclude that, if σ is smaller than $\sim$10 MeV/fm^2 (for $m_s = 100$ MeV and $c = 0.3$), the nucleation time for the first drop of quark matter is sufficiently small and the appearance of quark matter can indeed strongly affect the supernova evolution as shown in [6]. The cold hybrid stars obtained after deleptonization and cooling have, if perturbative QCD corrections are included in the equation of state, maximum masses compatible with recent pulsar observations.

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Note added—After finishing this work a paper has been published which discusses similar issues [40]. In that work, the authors also conclude that nucleation is possible in protoneutron star matter. The main difference between our work and Ref. [40] is that in the latter the thermal nucleation of quark matter is studied within hot and deleptonized hadronic matter.

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