Unifiable supersymmetric dark left-right gauge model

Ernest Ma

Department of Physics and Astronomy, University of California, Riverside, California 92521, USA

The recently proposed dark left-right gauge model, with Z' and $W_R^\pm$ bosons at the TeV scale, is shown to have a simple supersymmetric extension which is unifiable. Its one-loop gauge-coupling renormalization-group equations are shown to have identical solutions to those of the minimal supersymmetric standard model. It also has a rich dark sector, with at least three stable particles.

I. INTRODUCTION

In the conventional left-right gauge extension of the standard model (SM) of particle interactions, the $SU(2)_R$ fermion doublet $(\nu, e)_R$ pairs up with the usual $SU(2)_L$ fermion doublet $(\nu, e)_L$ through a Higgs bidoublet, so that both the electron e and the neutrino ν obtain Dirac masses. Remarkably, this situation is not compulsory. It is in fact possible to have a symmetry such that ν_R is not the Dirac mass partner of ν_L. It becomes another particle entirely, call it n_R, and the same symmetry makes it a dark-matter fermion (scotino). This intriguing scenario has been elaborated in some recent papers [1–4]. The left-right structure itself was discussed already 23 years ago [5,6] in the context of superstring-inspired E_6 models. Called the alternative left-right model (ALRM), it has the important property of no tree-level flavor-changing neutral currents. This makes it possible for the $SU(2)_R$ breaking scale to be as low as a TeV, allowing both its charged $W_R^\pm$ and Z' gauge bosons to be observable at the large hadron collider (LHC). However, its relevance to dark matter was not considered until one year ago [1].

In this paper, the latest version [4] of the dark left-right model (DLRM) is shown to have a simple supersymmetric extension with gauge-coupling unification. The resulting one-loop renormalization-group equations turn out to have solutions identical to those of the minimal supersymmetric standard model (MSSM), as well as two previously proposed left-right extensions [7,8]. It also has a rich dark sector, with at least three stable particles [9].

II. MODEL

Consider the gauge group $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)$. Following Ref. [4], a new global U(1) symmetry S is imposed so that the spontaneous breaking of $SU(2)_R \times S$ will leave the combination $L = S + T_{3R}$ unbroken. Under $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1) \times S$, the superfields transform as shown in Table I. Because of supersymmetry, the Higgs sector is doubled, in analogy to the transition from the SM to MSSM. Another set of Higgs doublet superfields η and a new set of charged Higgs singlet superfields χ are added to obtain gauge-coupling unification [7,8].

The symmetry S is used here to distinguish ψ, Φ_{L1}, η_{L1} from one another, as well as ψ^c, Φ_{R2}, η_{R2}. The bilinear terms allowed by S are

$$\Delta_1 \Delta_2, \qquad \Phi_{L1}\Phi_{L2}, \qquad \Phi_{R1}\Phi_{R2},$$
$$\eta_{L1}\eta_{L2}, \qquad \eta_{R1}\eta_{R2}, \qquad \chi_1\chi_2. \tag{1}$$

The trilinear terms are

$$\psi\psi^c\Delta_1, \qquad QQ^c\Delta_2, \qquad Qd^c\Phi_{L1},$$
$$\psi\nu^c\Phi_{L2}, \qquad n\psi^c\Phi_{R1}, \qquad hQ^c\Phi_{R2}, \tag{2}$$

$$\Phi_{L1}\Phi_{R2}\Delta_2, \qquad \Phi_{L2}\Phi_{R1}\Delta_1,$$
$$\eta_{L1}\eta_{R2}\Delta_1, \qquad \eta_{L2}\eta_{R1}\Delta_2, \tag{3}$$

$$\Phi_{L1}\eta_{L1}\chi_2, \qquad \Phi_{R1}\eta_{R1}\chi_2,$$
$$\Phi_{L2}\eta_{L2}\chi_1, \qquad \Phi_{R2}\eta_{R2}\chi_1. \tag{4}$$

TABLE I. Particle content of proposed model.

Superfield	$SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)$	S
$\psi = (\nu, e)$	$(1, 2, 1, -1/2)$	1
$\psi^c = (e^c, n^c)$	$(1, 1, 2, 1/2)$	$-3/2$
ν^c	$(1, 1, 1, 0)$	-1
n	$(1, 1, 1, 0)$	2
$Q = (u, d)$	$(3, 2, 1, 1/6)$	0
$Q^c = (h^c, u^c)$	$(3^*, 1, 2, -1/6)$	$1/2$
d^c	$(3^*, 1, 1, 1/3)$	0
h	$(3, 1, 1, -1/3)$	-1
Δ_1	$(1, 2, 2, 0)$	$1/2$
Δ_2	$(1, 2, 2, 0)$	$-1/2$
Φ_{L1}	$(1, 2, 1, -1/2)$	0
Φ_{L2}	$(1, 2, 1, 1/2)$	0
Φ_{R1}	$(1, 1, 2, -1/2)$	$-1/2$
Φ_{R2}	$(1, 1, 2, 1/2)$	$1/2$
η_{L1}	$(1, 2, 1, -1/2)$	-2
η_{L2}	$(1, 2, 1, 1/2)$	2
η_{R1}	$(1, 1, 2, -1/2)$	$-3/2$
η_{R2}	$(1, 1, 2, 1/2)$	$3/2$
χ_1	$(1, 1, 1, -1)$	-2
χ_2	$(1, 1, 1, 1)$	2

Hence m_e comes from the $I_{3L} = 1/2$ and $I_{3R} = -1/2$ component of Δ_1 with $L = 1/2 - 1/2 = 0$, m_u from the $I_{3L} = -1/2$ and $I_{3R} = 1/2$ component of Δ_2 with $L = -1/2 + 1/2 = 0$, m_d from ϕ_{L1}^0, m_ν from ϕ_{L2}^0, m_n from ϕ_{R1}^0, and m_h from ϕ_{R2}^0. This structure guarantees the absence of tree-level flavor-changing neutral currents [10].

III. DARK MATTER

As it stands, both the neutrino ν ($L = 1$) and the scotino n ($L = 2$) are Dirac fermions, and lepton number L is conserved. If we now introduce a mass term $\nu^c \nu^c$ which breaks L by two units, then ν gets a small Majorana mass through the canonical seesaw mechanism, as is usually assumed. As for n, it remains a Dirac fermion, being still protected by a global U(1) symmetry. This can be understood by noting that with the addition of the $\nu^c \nu^c$ term, the same allowed bilinear and trilinear terms in Eqs. (1)–(4) are obtained, if an odd matter parity M is assumed for ψ, ψ^c, ν^c, n, Q, Q^c, d^c, h and the S assignments of ψ, ψ^c, ν^c, and n are changed to 0, $-1/2$, 0, and 1, respectively. There are thus two conserved quantities: the usual M (or its resulting R) parity and a global U(1) number $L' = S + T_{3R}$, with $L' = 0$ for the usual quarks and leptons and $L' = 1$ for the scotino n. Because of their S assignments, the η and χ superfields appear always in pairs, so there is another parity H which is conserved. Hence there are at least three stable particles. Note that η and χ communicate with the quarks and leptons only through Δ and Φ, i.e. the so-called Higgs "portals."

The various superfields of this model under L', M, and H are listed in Table II. The usual R parity is then defined as $R \equiv MH(-1)^{2j}$. A possible scenario for dark matter is to have the following three coexisting stable particles: the lightest neutralino ($L' = 0$, $H = +$, $R = -$), the scotino n ($L' = 1$, $H = +$, $R = +$), and the exotic η_{R2}^0 fermion ($L' = 1$, $H = -$, $R = +$). However, there may be additional stable particles due to kinematics. For example, if the scalar counterpart of n cannot decay into n plus the

lightest neutralino, then it will also be stable. There may even be several exotic stable η and χ particles. The dark sector may be far from just the one particle that is usually assumed, as in the MSSM.

As pointed out in Ref. [9], in the presence of several dark-matter candidates, the one with the largest annihilation cross section contributes the least, but may be the first to be discovered at the LHC. This means that in this model, the severe constraint due to dark-matter relic abundance on the one candidate particle of the MSSM, i.e. the lightest neutralino, may be relaxed, because it needs only to account for a fraction of the total dark-matter abundance. The allowed parameter space of the MSSM becomes much bigger and the opportunity for its discovery is enhanced at the LHC. As for the other two varieties of dark matter, both the scotino n and the η_{R2}^0 fermion will annihilate through the Z' into SM particles. This process results in both quarks and leptons, and will not explain the possible positron excess in recent astrophysical observations. A full analysis of all three (or more) dark-matter particles is difficult because of many unknown parameters.

IV. GAUGE-COUPLING UNIFICATION

The one-loop renormalization-group equations for the gauge couplings of $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_X$ are given by

$$\frac{1}{\alpha_i(M_1)} - \frac{1}{\alpha_i(M_2)} = \frac{b_i}{2\pi} \ln \frac{M_2}{M_1}, \quad (5)$$

where $\alpha_i = g_i^2/4\pi$ and the numbers b_i are determined by the particle content of the model between M_1 and M_2. In the SM with two Higgs scalar doublets, these are given by

$$SU(3)_C: b_C = -11 + (4/3)N_f = -7, \quad (6)$$

$$SU(2)_L: b_L = -22/3 + (4/3)N_f + 2(1/6) = -3, \quad (7)$$

$$U(1)_Y: b_Y = (20/9)N_f + 2(1/6) = 7, \quad (8)$$

where $N_f = 3$ is the number of quark and lepton families. As such, the gauge couplings do not unify at a common mass scale, i.e. they do not satisfy the condition

$$\alpha_C(M_U) = \alpha_L(M_U) = (5/3)\alpha_Y(M_U) = \alpha_U. \quad (9)$$

If the SM becomes the MSSM above M_S, the numbers b_i change, i.e.

$$SU(3)_C: b'_C = -11 + (2/3)(3) + (4/3 + 2/3)N_f = -3, \quad (10)$$

$$SU(2)_L: b'_L = -22/3 + (2/3)(2) + (4/3 + 2/3)N_f + (2/3 + 1/3)2(1/2) = 1, \quad (11)$$

$$U(1)_Y: (3/5)b'_Y = (4/3 + 2/3)N_f + (3/5)(2/3 + 1/3)(4)(1/4) = 33/5. \quad (12)$$

TABLE II. Superfields under $L' = S + T_{3R}$, M, and H.

L'	M	H	Superfields
0	$-$	$+$	u, d, ν, e
0	$+$	$+$	g, γ, $W_L^\pm$, Z, Z'
0	$+$	$+$	ϕ_{L1}^0, ϕ_{L1}^-, ϕ_{L2}^+, ϕ_{L2}^0, ϕ_{R1}^0, ϕ_{R2}^0
0	$+$	$+$	δ_{11}^0, δ_{11}^-, δ_{22}^+, δ_{22}^0
1	$-$	$+$	n, h^c
-1	$-$	$+$	n^c, h
1	$+$	$+$	W_R^+, ϕ_{R2}^+, δ_{12}^+, δ_{12}^0
-1	$+$	$+$	W_R^-, ϕ_{R1}^-, δ_{21}^0, δ_{21}^-
1	$+$	$-$	η_{R2}^0
-1	$+$	$-$	η_{R1}^0
2	$+$	$-$	η_{L2}^+, η_{L2}^0, η_{R2}^+, χ_2^+
-2	$+$	$-$	η_{L1}^-, η_{L1}^0, η_{R1}^-, χ_1^-

Therefore

$$\ln\frac{M_U}{M_Z} = \frac{\pi}{2}\left(\frac{1}{\alpha_L(M_Z)} - \frac{1}{\alpha_C(M_Z)}\right), \tag{13}$$

$$\ln\frac{M_S}{M_Z} = \frac{\pi}{4}\left(\frac{3}{\alpha_Y(M_Z)} - \frac{12}{\alpha_L(M_Z)} + \frac{7}{\alpha_C(M_Z)}\right). \tag{14}$$

Now [11]

$$\alpha(M_Z)^{-1} = 127.953 \pm 0.049, \tag{15}$$

$$\sin^2\theta_W(M_Z) = 0.231\,19 \pm 0.000\,14, \tag{16}$$

$$\alpha_L(M_Z) = \alpha(M_Z)/\sin^2\theta_W(M_Z) = 0.033\,81, \tag{17}$$

$$\alpha_Y(M_Z) = \alpha(M_Z)/\cos^2\theta_W(M_Z) = 0.010\,17, \tag{18}$$

$$\alpha_C(M_Z) = 0.1215 \pm 0.0017. \tag{19}$$

For $M_S > M_Z$, using Eq. (14),

$$\alpha_C < \frac{7\alpha_L\alpha_Y}{3(4\alpha_Y - \alpha_L)} = 0.1168, \tag{20}$$

in disagreement with Eq. (19). However, this problem is usually fixed by going to two loops and spreading out the SUSY particle thresholds.

Consider now the dark left-right model. At M_R, there is the boundary condition

$$\frac{1}{\alpha_Y(M_R)} = \frac{1}{\alpha_R(M_R)} + \frac{1}{\alpha_X(M_R)} = \frac{1}{\alpha_L(M_R)} + \frac{1}{\alpha_X(M_R)}. \tag{21}$$

Above M_R, assuming the minimal supersymmetric content, without the exotic η and χ superfields,

$$b_C = -11 + (2/3)(3) + (2+1)N_f = 0, \tag{22}$$

$$b_{L,R} = -22/3 + (2/3)(2) + (4/3 + 2/3)N_f$$
$$+ (2/3 + 1/3)(6)(1/2) = 3, \tag{23}$$

$$(3/2)b_X = (2+1)N_f + (1+1/2)(8)(1/4) = 12. \tag{24}$$

As such, again the gauge couplings do not unify, i.e. they do not satisfy

$$\alpha_C(M_U) = \alpha_L(M_U) = \alpha_R(M_U) = (2/3)\alpha_X(M_U) = \alpha_U. \tag{25}$$

However, this may easily be changed with the addition of new particles [12]. With the η superfields above M_R,

$$b'_C = -11 + (2/3)(3) + (2+1)N_f = 0, \tag{26}$$

$$b'_{L,R} = -22/3 + (2/3)(2) + (4/3 + 2/3)N_f$$
$$+ (2/3 + 1/3)(8)(1/2) = 4, \tag{27}$$

$$(3/2)b'_X = (2+1)N_f + (1+1/2)(16)(1/4) = 15, \tag{28}$$

and the χ superfields above M_X,

$$(3/2)b''_X = (2+1)N_f + (1+1/2)(16)(1/4)$$
$$+ (1+1/2)(2)(1) = 18. \tag{29}$$

The resulting solutions are

$$\ln\frac{M_U}{M_Z} = \frac{\pi}{2}\left(\frac{1}{\alpha_L(M_Z)} - \frac{1}{\alpha_C(M_Z)}\right), \tag{30}$$

$$\ln\frac{M_R^7}{M_X^3 M_Z^4} = \pi\left(\frac{3}{\alpha_Y(M_Z)} - \frac{12}{\alpha_L(M_Z)} + \frac{7}{\alpha_C(M_Z)}\right). \tag{31}$$

Note that Eqs. (30) and (31) are identical to Eqs. (13) and (14) of the MSSM, respectively, if we set $M_X = M_R = M_S$. Thus this model is no worse than the MSSM for gauge-coupling unification, and two-loop equations and particle thresholds may be invoked to fix it. The same one-loop solutions are obtained in two other previously proposed supersymmetric left-right models [7,8]. If the one-loop equations (30) and (31) are taken at face value, $M_U = 3.33 \times 10^{16}$ GeV and $M_R^{7/4}M_X^{-3/4} = 14.7$ GeV. Take for example $M_R = 500$ GeV, then $M_X = 55.2$ TeV. The additional singlets are thus much heavier, but since they do not affect either $SU(2)_R \times U(1)_X$ breaking or supersymmetry breaking, this is an acceptable scenario. In terms of $SO(10)$ multiplets, Q, Q^c, ψ, ψ^c belong to $\underline{16}$; h, d^c and Δ to $\underline{10}$; Φ and η to $\underline{16} + \underline{16}^*$; and χ to $\underline{120} + \underline{120}^*$. The chosen set of superfields is free of anomalies.

V. CONCLUSION

The dark left-right model, where the $SU(2)_R$ fermion doublet $(n, e)_R$ contains the dark-matter fermion (scotino) n which is distinguished from the usual lepton e by an U(1) global symmetry, is extended to include supersymmetry. New superfields are added, resulting in one-loop renormalization-group equations for the gauge couplings with solutions identical to those of the MSSM. The dark sector is automatically extended to include at least three stable particles.

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