Analyses of decay constants and light-cone distribution amplitudes for *s*-wave heavy meson

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In this paper, a study of light-cone distribution amplitudes (LCDAs) for s-wave heavy meson are presented in both general and heavy quark frameworks. Within the light-front approach, the leading twist light-cone distribution amplitudes, $\phi_M(u)$, and their relevant decay constants of heavy pseudoscalar and vector mesons, f_M, have simple relations. These relations can be further simplified when the heavy quark limit is taken into consideration. After fixing the parameters that appear in both Gaussian and power-law wave functions, the corresponding decay constants are calculated and compared with those of other theoretical approaches. The curves and the first six ξ-moments of $\phi_M(u)$ are plotted and estimated. A conclusion is drawn from these results: Even though the values of the decay constants of the distinct mesons are almost equal, the curves of their LCDAs may have quite large differences, and vice versa. Additionally, in the heavy quark limit, the leading twist LCDAs, $\Phi_{Qq}(\omega)$ and $\Phi_{Qq}(\omega)$, are compared with the B-meson LCDAs, $\psi_+(\omega)$, suggested by the other theoretical groups.

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I. INTRODUCTION

The decay constants of heavy mesons with a c or a b quark are significant quantities and they play an important role in studies of CP violation, Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, the $D-\bar{D}$ or $B-\bar{B}$ mixing process, and leptonic or nonleptonic weak decay. Experimentally, new data on the decay constants of the pseudoscalar mesons f_D and f_{D_s} have been reported [1–4] which has provided a precise method for comparing different theoretical calculations and for checking their accuracy. During the last decade, the decay constants of both pseudoscalar and vector heavy mesons have been studied by lattice simulations [5], the relativistic quark model [6–10], and the field correlator method [11]. The light-cone distribution amplitudes (LCDAs) of hadrons are key ingredients in the description of the various exclusive processes of quantum chromodynamics (QCD), and their role is analogous to that of parton distributions in inclusive processes. In terms of the Bethe-Salpeter wave functions, $\varphi(u_i, k_{i\perp})$, the LCDAs, $\phi(u_i)$, are defined by retaining the momentum fractions, u_i, and integrating out the transverse momenta, $k_{i\perp}$ [12]. They provide essential information on the nonperturbative structure of the hadron for the QCD treatment of exclusive reactions and they play a central role in all known factorization formulas. Specifically, the leading twist LCDAs describe the probability amplitudes for finding the hadron in a Fock state with the minimum number of constituents. Experimentally, the fact that B-physics exclusive processes are under investigation in *BABAR* and Belle experiments also urges the detailed study of hadronic LCDAs. In the literature, the LCDAs of heavy quarkonia have been estimated by various nonperturbative approaches, such as QCD sum rules [13–16], NRQCD factorization [17], and the light-front quark model [18–20]. As for heavy-light mesons, the LCDAs of B-meson $\psi_\pm$ were first introduced within the heavy quark effective theory (HQET) [21], and the following studies were intensive [22–36], whereas the ones of other heavy-light mesons were discussed in a non-HQET framework [37].

In the past decade, the most significant progress made in the QCD description of hadronic physics was, perhaps, in the avenue of heavy quark dynamics. The analysis of heavy hadron structures has been tremendously simplified by the heavy quark symmetry (HQS) proposed by Isgur and Wise [38,39] and HQET developed from QCD in terms of $1/m_Q$ expansion [40–42]. HQET has provided a systematic framework for studying symmetry breaking $1/m_Q$ corrections (for a review, see [43]). Moreover, in terms of heavy quark expansion, HQET offered a new framework for the systematic study of the inclusive decays of heavy mesons [44–47]. However, the general properties of heavy hadrons, namely, their decay constants, transition form factors and structure functions, etc., are still incalculable within QCD, even in the infinite quark mass limit with the utilization of HQS and HQET. Hence, although HQS and HQET have simplified heavy quark dynamics, a complete first-principles QCD description of heavy hadrons has still been lacking due to the unknown nonperturbative QCD dynamics.

This paper has focused on the study of the decay constants and the leading twist LCDAs of pseudoscalar (D, D_s, B, B_s, B_c) and vector ($D^*, D_s^*, B^*, B_s^*, B_c^*$) mesons within both general and heavy quark frameworks. From the definitions of the decay constant and LCDA [or quark distribution amplitude (DA)] [12], these two properties seemed to be closely related. In terms of a detailed analysis, the purpose of this study is to transparently realize the relation between the decay constant and LCDA of the

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heavy meson. We believe that a thorough understanding of these universal nonperturbative objects would be of great benefit when analyzing the hard exclusive processes with heavy meson production or annihilation. Additionally, in this study, the s-wave heavy meson has been explored within the light-front quark model (LFQM), which is a promising analytic method for solving the nonperturbative problem of hadron physics [48], as well as offering many insights into the internal structures of bound states. The basic ingredient in LFQM is the relativistic hadron wave function which generalizes distribution amplitudes by including transverse momentum distributions, and which contains all the information of a hadron from its constituents. The hadronic quantities are represented by the overlap of wave functions and can be derived in principle. The light-front wave function is manifestly a Lorentz invariant, expressed in terms of internal momentum fraction variables which are independent of the total hadron momentum. Moreover, the fully relativistic treatment of quark spins and center-of-mass motion can be carried out using the so-called Melosh rotation [49]. This treatment has been successfully applied to calculate phenomenologically many important meson decay constants and hadronic form factors [50–55]. Therefore, the main purpose of this study was the calculation of the leading twist LCDAs of s-wave heavy mesons within LFQM.

The remainder of this paper is organized as follows. In Sec. II, the leading twist LCDAs of s-wave heavy meson states are derived within general and heavy quark frameworks. In Sec. III, the formulations of LFQM within the general and heavy quark frameworks are reviewed briefly, and the decay constants and the leading twist LCDAs then extracted. In Sec. IV, numerical results and discussions are presented. Finally, the conclusions are given in Sec. V.

II. LEADING TWIST LCDAS OF s-WAVE MESONS

A. General framework

The amplitudes of the hard processes involving s-wave mesons can be described by the matrix elements of gauge-invariant nonlocal operators, which are sandwiched between the vacuum and the meson states,

$$\langle 0|\bar{q}(x)\Gamma[x,-x]q(-x)|H(P,\epsilon)\rangle, \tag{2.1}$$

where P is the meson momentum, ϵ is the polarization vector (of course, ϵ does not exist in the case of pseudoscalar meson), Γ is a generic notation for the Dirac matrix structure, and the path-ordered gauge factor is

$$[x,y] = \mathrm{P}\exp\left[ig_s\int_0^1 dt(x-y)_\mu A^\mu(tx+(1-t)y)\right]. \tag{2.2}$$

This factor is equal to unity in the light-cone gauge which is equivalent to the fixed-point gauge, $(x-y)_\mu A^\mu(x-y)=0$, as the quark-antiquark pair is at the lightlike separation [56]. For simplicity, the gauge factor will not be shown below.

The asymptotic expansion of exclusive amplitudes in powers of large momentum transfer is governed by the expanding amplitude, Eq. (2.1), shown in powers of deviation from the light-cone $x^2=0$. The two lightlike vectors, p and z, can be introduced by

$$p^2=0, \qquad z^2=0, \tag{2.3}$$

so that $p\to P$ in the limit $M_H^2\to 0$ and $z\to x$ for $x^2=0$. From this it follows that [57]

$$\begin{aligned} z^\mu &= x^\mu - P^\mu\frac{1}{M_H^2}[Px-\sqrt{(Px)^2-x^2M_H^2}] \\ &= x^\mu - P^\mu\frac{x^2}{2Pz}+O(x^4), \\ p^\mu &= P^\mu - z^\mu\frac{M_H^2}{2Pz}, \end{aligned} \tag{2.4}$$

where $Px\equiv P\cdot x$ and $Pz=pz=\sqrt{(Px)^2-x^2M_H^2}$. In addition, if the meson is assumed to move in a positive $\hat{e}_3$ direction, then p^+ and z^- are the only nonzero components of p and z, respectively, in an infinite momentum frame. For the vector meson, the polarization vector ϵ^μ is decomposed into longitudinal and transverse projections as

$$\epsilon_\parallel^\mu = \frac{\epsilon z}{pz}\left(p^\mu - z^\mu\frac{M_H^2}{2pz}\right), \qquad \epsilon_\perp^\mu = \epsilon^\mu - \epsilon_\parallel^\mu, \tag{2.5}$$

respectively.

LCDAs are defined in terms of the matrix element of the nonlocal operator in Eq. (2.1). For the pseudoscalar (P) and vector (V) mesons, LCDAs can be defined as

$$\begin{aligned} &\langle 0|\bar{q}(z)\gamma^\mu\gamma_5 q(-z)|P(P)\rangle \\ &\quad = if_P\int_0^1 du e^{i\xi pz}\left[p^\mu\phi_P(u)+z^\mu\frac{M_P^2}{2pz}g_P(u)\right], \end{aligned} \tag{2.6}$$

$$\begin{aligned} &\langle 0|\bar{q}(z)\gamma^\mu q(-z)|V(P,\epsilon_{\lambda=0})\rangle \\ &= f_V M_V\int_0^1 du e^{i\xi pz}\left\{p^\mu\frac{\epsilon z}{pz}\phi_{V\parallel}(u)+\epsilon_\perp^\mu g_{V\perp}(u)\right. \\ &\quad \left. - z^\mu\frac{\epsilon z}{2(pz)^2}M_V^2 g_{V3}(u)\right\}, \end{aligned} \tag{2.7}$$

$$\begin{aligned} &\langle 0|\bar{q}(z)\sigma^{\mu\nu}q(-z)|V(P,\epsilon_{\lambda=\pm 1})\rangle \\ &\quad = f_V^\perp\int_0^1 du e^{i\xi pz}\left\{(\epsilon_\perp^\mu p^\nu-\epsilon_\perp^\nu p^\mu)\phi_{V_\perp}(u)\right. \\ &\qquad + (p^\mu z^\nu - p^\nu z^\mu)\frac{M_V^2\epsilon z}{(pz)^2}h_{V\parallel}(u) \\ &\qquad \left. + (\epsilon_\perp^\mu z^\nu-\epsilon_\perp^\nu z^\mu)\frac{M_V^2}{2pz}h_{V3}(u)\right\}, \end{aligned} \tag{2.8}$$

where u is the momentum fraction and $\xi \equiv (1-u) - u = 1-2u$. Here ϕ_P, $\phi_{V\parallel}$, and $\phi_{V\perp}$ are the leading twist-2 LCDAs, and the others contain contributions from higher-twist operators. The leading twist LCDAs are normalized as

$$\int_0^1 du\phi(u) = 1, \tag{2.9}$$

and can be expanded [58] in Gegenbauer polynomials $C_n^{3/2}(\xi)$ as

$$\phi(\xi,\mu) = \phi_{\rm as}(\xi)\bigg[\sum_{l=0}^{\infty} a_l(\mu)C_l^{3/2}(\xi)\bigg], \tag{2.10}$$

where $\phi_{\rm as}(\xi) = 3(1-\xi^2)/4$ is the asymptotic quark distribution amplitude and $a_l(\mu)$ are the Gegenbauer moments which describe to what degree the quark distribution amplitude deviates from the asymptotic one. $C_l^{3/2}(\xi)$'s have the orthogonality integrals

$$\int_{-1}^{1}(1-\xi^2)C_l^{3/2}(\xi)C_m^{3/2}(\xi)d\xi = \frac{2(l+1)(l+2)}{2l+3}\delta_{lm}. \tag{2.11}$$

Then a_l can be obtained by using the above orthogonality integrals as

$$a_l(\mu) = \frac{2(2l+3)}{3(l+1)(l+2)}\int_{-1}^{1} C_l^{3/2}(\xi)\phi(\xi,\mu)d\xi. \tag{2.12}$$

An alternative approach to parameterize the quark distribution amplitude is to calculate the so-called ξ-moments:

$$\langle \xi^n\rangle_\mu = \int_{-1}^{1} d\xi\xi^n\phi(\xi,\mu), \tag{2.13}$$

as calculated in this work.

To disentangle the twist-2 LCDAs from higher twists in Eqs. (2.6), (2.7), and (2.8), the twist-2 contribution of the relevant nonlocal operator $\bar{q}(z)\Gamma q(-z)$ must be derived. For the $\Gamma = \gamma^\mu(\gamma_5)$ case, the leading twist-2 contribution contains contributions of the operators which are fully symmetric in the Lorentz indices [59,60]:

$$[\bar{q}(-z)\gamma^\mu(\gamma_5)q(z)]_2 = \sum_{n=0}^{\infty}\frac{1}{n!}\bar{q}(0)\bigg\{\frac{(z\cdot\hat{D})^n}{n+1}\gamma^\mu + \frac{n(z\cdot\hat{D})^{n-1}}{n+1}\hat{D}^\mu \not{z}\bigg\}(\gamma_5)q(0), \tag{2.14}$$

where $\hat{D} = \vec{D} - \overleftarrow{D}$ and $\vec{D} = \vec{\partial} - igB^a(\lambda^a/2)$. The sum can be expressed in terms of a nonlocal operator,

$$[\bar{q}(-z)\gamma^\mu(\gamma_5)q(z)]_2 = \int_0^1 dt\frac{\partial}{\partial z_\mu}\bar{q}(-tz)\not{z}(\gamma_5)q(tz). \tag{2.15}$$

Taking the matrix element between the vacuum and the s-wave meson state, we obtained

$$\langle 0|[\bar{q}(-z)\gamma^\mu\gamma_5 q(z)]_2|P(P)\rangle = if_P\int_0^1 du\phi_P(u)\bigg\{p^\mu e^{i\xi pz} + (P^\mu - p^\mu)\int_0^1 dte^{i\xi tpz}\bigg\}, \tag{2.16}$$

$$\langle 0|[\bar{q}(-z)\gamma^\mu q(z)]_2|V(P,\epsilon_{\lambda=0})\rangle = f_V M_V\int_0^1 du\phi_{V\parallel}(u) \times\bigg\{p^\mu\frac{\epsilon z}{pz}e^{i\xi pz} + \bigg(\epsilon^\mu - p^\mu\frac{\epsilon z}{pz}\bigg)\int_0^1 dte^{i\xi tpz}\bigg\}. \tag{2.17}$$

The derivations of (2.17) as shown in Ref. [59], are applied to those of Eq. (2.16). We use Eq. (2.14), and then expand the right-hand sides of Eqs. (2.16) and (2.17) as

$$\sum_{n=0}^{\infty}\frac{1}{n!}\langle 0|\bar{q}(0)\bigg\{\frac{(z\cdot\hat{D})^n}{n+1}\gamma^\mu + \frac{n(z\cdot\hat{D})^{n-1}}{n+1}\hat{D}^\mu\not{z}\bigg\}\gamma_5 q(0)|P(P)\rangle = if_P\sum_{n=0}^{\infty}\frac{i^n}{n!}\int_0^1 du\phi_P(u)(\xi pz)^n\bigg\{p^\mu + (P^\mu - p^\mu)\int_0^1 dtt^n\bigg\}, \tag{2.18}$$

$$\sum_{n=0}^{\infty}\frac{1}{n!}\langle 0|\bar{q}(0)\bigg\{\frac{(z\cdot\hat{D})^n}{n+1}\gamma^\mu + \frac{n(z\cdot\hat{D})^{n-1}}{n+1}\hat{D}^\mu\not{z}\bigg\}q(0)|V(P,\epsilon)\rangle = f_V M_V\sum_{n=0}^{\infty}\frac{i^n}{n!}\int_0^1 du\phi_{V\parallel}(u)(\xi pz)^n \times\bigg\{p^\mu\frac{\epsilon z}{pz} + \bigg(\epsilon^\mu - p^\mu\frac{\epsilon z}{pz}\bigg)\int_0^1 dtt^n\bigg\}, \tag{2.19}$$

respectively. Picking $n=0$ in Eqs. (2.18) and (2.19), we obtain

$$\langle 0|\bar{q}(0)\gamma^\mu\gamma_5 q(0)|P(P)\rangle = if_P P^\mu\int_0^1 du\phi_P(u), \tag{2.20}$$

$$\langle 0|\bar{q}(0)\gamma^\mu q(0)|V(P),\epsilon_{\lambda=0}\rangle = f_V M_V\epsilon^\mu\int_0^1 du\phi_{V\parallel}(u). \tag{2.21}$$

From the normalization of Eq. (2.9), we have

$$\langle 0|\bar{q}\gamma^\mu\gamma_5 q|P(P)\rangle = if_P P^\mu, \quad \langle 0|\bar{q}\gamma^\mu q|V(P,\epsilon)\rangle = f_V M_V \epsilon^\mu \tag{2.22}$$

which are taken as the definitions of decay constants f_P and f_V in the literature.

Next, we consider the case of $\Gamma = \sigma_{\mu\nu}$, where the leading twist-2 contribution contains contributions of the operators:

$$[\bar{q}(-z)\sigma^{\mu\nu}q(z)]_2 = \sum_{n=0}^{\infty}\frac{1}{n!}\bar{q}(0)\bigg\{\frac{(z\cdot\hat{D})^n}{2n+1}\sigma^{\mu\nu} + \frac{n(z\cdot\hat{D})^{n-1}}{2n+1}\hat{D}^\mu\sigma^{\bullet\nu} + \frac{n(z\cdot\hat{D})^{n-1}}{2n+1}\hat{D}^\nu\sigma^{\mu\bullet}\bigg\}q(0). \tag{2.23}$$

The sum can also be represented in terms of nonlocal operators:

$$[\bar{q}(-z)\sigma^{\mu\nu}q(z)]_2 = \int_0^1 dt\bigg[\frac{\partial}{\partial z_\mu}\bar{q}(-t^2z)\sigma^{\bullet\nu}q(t^2z) + z_\alpha\frac{\partial}{\partial z_\nu}\bar{q}(-t^2z)\sigma^{\mu\alpha}q(t^2z)\bigg]. \tag{2.24}$$

Returning to Eq. (2.8), it can be rewritten as

$$\langle 0|\bar{q}(z)\sigma_{\mu\nu}q(-z)|V(P,\epsilon_{\lambda=\pm1})\rangle = f_V^\perp\int_0^1 du e^{i\xi pz}\bigg\{(\epsilon_\mu P_\nu - \epsilon_\nu P_\mu)\phi_{V\perp}(u) + (p_\mu z_\nu - p_\nu z_\mu)\frac{M_V^2\epsilon z}{(pz)^2}[h_{V\parallel}(u)-\phi_{V\perp}(u)] + (\epsilon_{\perp\mu}z_\nu - \epsilon_{\perp\nu}z_\mu)\frac{M_V^2}{2pz}[h_{V3}(u)-\phi_{V\perp}(u)]\bigg\}. \tag{2.25}$$

We sandwich both sides of Eq. (2.24) between the vacuum and the vector meson state as

$$\begin{aligned}\langle 0|[\bar{q}(z)\sigma_{\mu\nu}q(-z)]_2|V(P,\epsilon_{\lambda=\pm1})\rangle &= \int_0^1 dt\bigg[\frac{\partial}{\partial z^\mu}\langle 0|\bar{q}(-t^2z)\sigma_{\bullet\nu}q(t^2z)|V(P,\epsilon)\rangle + z^\alpha\frac{\partial}{\partial z^\nu}\langle 0|\bar{q}(-t^2z)\sigma_{\mu\alpha}q(t^2z)|V(P,\epsilon)\rangle\bigg]\\ &= f_V^\perp\int_0^1 du\bigg\{\phi_{V\perp}(u)\bigg[(\epsilon_\mu P_\nu - \epsilon_\nu P_\mu)\int_0^1 dt e^{i\xi t^2 pz} + 2pz\mathcal{S}_{\mu\nu}(i\xi)\int_0^1 dt t^2 e^{i\xi t^2 pz}\bigg]\\ &\quad + (h_{V\parallel}(u)-\phi_{V\perp}(u))\bigg[\mathcal{U}_{\mu\nu}\int_0^1 dt e^{i\xi t^2 pz} + 2pz\mathcal{T}_{\mu\nu}(i\xi)\int_0^1 dt t^2 e^{i\xi t^2 pz}\bigg]\bigg\}.\end{aligned} \tag{2.26}$$

The integral is performed as

$$i\xi\int_0^1 dt t^2 e^{i\xi t^2 pz} = \frac{1}{2pz}\int_0^1 dt t\frac{\partial}{\partial t}e^{i\xi t^2 pz} = \frac{1}{2pz}\bigg[e^{i\xi pz} - \int_0^1 dt e^{i\xi t^2 pz}\bigg], \tag{2.27}$$

and then we substitute Eq. (2.26) for Eq. (2.27) to obtain

$$\begin{aligned}\langle 0|[\bar{q}(-z)\sigma^{\mu\nu}q(z)]_2|V(P,\epsilon_{\lambda=\pm1})\rangle &= if_V^\perp\int_0^1 du\bigg\{\phi_{V\perp}(u)\bigg[\mathcal{S}^{\mu\nu}e^{i\xi pz} + ((\epsilon^\mu P^\nu - \epsilon^\nu P^\mu) - \mathcal{S}^{\mu\nu})\int_0^1 dt e^{i\xi t^2 pz}\bigg]\\ &\quad + (h_{V\parallel}(u)-\phi_{V\perp}(u))\bigg[\mathcal{T}^{\mu\nu}e^{i\xi pz} + (\mathcal{U}^{\mu\nu}-\mathcal{T}^{\mu\nu})\int_0^1 dt e^{i\xi t^2 pz}\bigg]\bigg\},\end{aligned} \tag{2.28}$$

where

$$\begin{aligned}\mathcal{S}^{\mu\nu} &= \frac{1}{2}\bigg[(\epsilon^\mu P^\nu - \epsilon^\nu P^\mu) - (\epsilon_\perp^\mu z^\nu - \epsilon_\perp^\nu z^\mu)\frac{M_V^2}{2pz}\bigg],\\ \mathcal{T}^{\mu\nu} &= \frac{\epsilon z M_V^2}{2(pz)^2}(p^\mu z^\nu - p^\nu z^\mu),\\ \mathcal{U}^{\mu\nu} &= \frac{M_V^2}{pz}(\epsilon^\mu z^\nu - \epsilon^\nu z^\mu).\end{aligned} \tag{2.29}$$

In contrast to Eqs. (2.16) and (2.17), the twist-2 LCDAs do not disentangle entirely from the higher twists in Eq. (2.28). Taking the product with $\epsilon_{\perp\mu}z_\nu$ in Eq. (2.28) to obtain

$$\langle 0|[\bar{q}(-z)\sigma^{\mu\bullet}\epsilon_{\perp\mu}\gamma_5 q(z)]_2|V(P,\epsilon_{\lambda=\pm1})\rangle = if_V^\perp\int_0^1 du\phi_{V\perp}(u)\frac{1}{2}(\epsilon\cdot\epsilon_\perp Pz)\bigg[e^{i\xi pz} + \int_0^1 dt e^{i\xi t^2 pz}\bigg], \tag{2.30}$$

we then use Eq. (2.23) and expand the right-hand side of Eq. (2.30) as

$$\sum_{n=0}^{\infty}\frac{1}{n!}\langle 0|\bar{q}(0)\frac{(n+1)(z\cdot\hat{D})^n}{2n+1}\sigma^{\mu\bullet}\epsilon_{\perp\mu}\gamma_5 q(0)|V(P,\epsilon_{\lambda=\pm1})\rangle = f_V^{\perp}\sum_{n=0}^{\infty}\frac{i^n}{n!}\int_0^1 du\phi_{V\perp}(u)\frac{1}{2}(\epsilon\cdot\epsilon_{\perp}Pz)(\xi pz)^n\left[1+\int_0^1 dt t^{2n}\right]. \tag{2.31}$$

Picking $n=0$ in Eq. (2.31), we obtain

$$\langle 0|\bar{q}(0)\sigma^{\mu\bullet}\epsilon_{\perp\mu}q(0)|V(P,\epsilon_{\lambda=\pm1})\rangle = f_V^{\perp}\int_0^1 du\phi_{V\perp}(u)(\epsilon\cdot\epsilon_{\perp}Pz). \tag{2.32}$$

From the normalization of Eq. (2.9), we have

$$\langle 0|\bar{q}(0)\sigma^{\mu\bullet}\epsilon_{\perp\mu}q(0)|V(P,\epsilon_{\lambda=\pm1})\rangle = f_V^{\perp}(\epsilon\cdot\epsilon_{\perp}Pz), \tag{2.33}$$

which is consistent with the usual definition of $f_V^{\perp}$ as

$$\langle 0|\bar{q}(0)\sigma^{\mu\nu}q(0)|V(P,\epsilon_{\lambda=\pm1})\rangle = f_V^{\perp}(\epsilon^{\mu}P^{\nu}-\epsilon^{\nu}P^{\mu}). \tag{2.34}$$

B. Heavy quark framework

In general, the theoretical description of meson properties relies on the bound state models with a relativistic normalization:

$$\langle M(P')|M(P)\rangle = 2P^0(2\pi)^3\delta^3(P'-P). \tag{2.35}$$

At low energies, however, these models have little connection to the fundamental theory of QCD. Then the reliable predictions are often made based on symmetries. A well-known example is HQS [43], which arises since the Compton wavelength, $1/m_Q$, of a heavy quark bound inside a hadron is much smaller than a typical hadronic distance (about 1 fm), and m_Q is unimportant for the low energy properties of the state. For a heavy-light meson system, it is more natural to use velocity v^{μ} instead of momentum variables. Then it is appropriate to work with a mass independent normalization of a heavy-light meson state:

$$\langle\hat{M}(v')|\hat{M}(v)\rangle = 2v^0(2\pi)^3\delta^3(\bar{\Lambda}v'-\bar{\Lambda}v), \tag{2.36}$$

where $\bar{\Lambda}=M-m_Q$ is the so-called residual center mass of a heavy-light meson. The relation between these two bound states is

$$|M(P)\rangle = \sqrt{M}|\hat{M}(v)\rangle. \tag{2.37}$$

In addition, the heavy quark field can be expanded as [43]

$$Q(x) = e^{-im_Q v\cdot x}\left[1+\frac{1}{iv\cdot D+2m_Q-i\varepsilon}i\not{D}_{\perp}\right]h_v(x), \tag{2.38}$$

where $h_v^*(x)$ is a field describing a heavy antiquark with velocity v. Then the current $\bar{q}\Gamma Q$ can be represented as

$$\bar{q}\Gamma Q = \bar{q}\Gamma\left(1+\frac{i\not{D}_{\perp}}{2m_Q}+\cdots\right)h_v. \tag{2.39}$$

Substituting Eqs. (2.37) and (2.39) into the definitions of LCDAs, Eqs. (2.6), (2.7), and (2.8) give

$$\langle 0|\bar{q}(z)\gamma^{\mu}\gamma_5 h_v(-z)|\hat{P}(v)\rangle = iF_P\int_0^{\infty}d\omega e^{i\omega vz}\left[v^{\mu}\Phi_P(\omega)+z^{\mu}\frac{1}{2vz}G_P(\omega)\right], \tag{2.40}$$

$$\begin{aligned}\langle 0|\bar{q}(z)\gamma^{\mu}h_v(-z)|\hat{V}(v,\epsilon_{\lambda=0})\rangle &= F_V\int_0^{\infty}d\omega e^{i\omega vz}\left\{v^{\mu}\frac{\epsilon z}{vz}\Phi_{V\parallel}(\omega)+\epsilon_{\perp}^{\mu}G_{V\perp}(\omega)\right.\\ &\quad\left.-z^{\mu}\frac{\epsilon z}{2(vz)^2}G_{V3}(\omega)\right\},\end{aligned} \tag{2.41}$$

$$\begin{aligned}\langle 0|\bar{q}(z)\sigma^{\mu\nu}h_v(-z)|\hat{V}(v,\epsilon_{\lambda=\pm1})\rangle &= F_V^{\perp}\int_0^{\infty}d\omega e^{i\omega vz}\left\{(\epsilon_{\perp}^{\mu}v^{\nu}-\epsilon_{\perp}^{\nu}v^{\mu})\Phi_{V\perp}(\omega)\right.\\ &\quad+(v^{\mu}z^{\nu}-v^{\nu}z^{\mu})\frac{\epsilon z}{(vz)^2}H_{V\parallel}(\omega)\\ &\quad\left.+(\epsilon_{\perp}^{\mu}z^{\nu}-\epsilon_{\perp}^{\nu}z^{\mu})\frac{1}{2vz}H_{V3}(\omega)\right\},\end{aligned} \tag{2.42}$$

where $F_M=\sqrt{M}f_M$, $\Phi_i(\omega)=\phi_i(u)/M$, and ω was first introduced in Ref. [61] as the product of longitudinal momentum fraction u of the light (anti)quark and the mass of heavy meson M, namely $\omega=uM$. Following a similar process, the leading twist LCDAs are obtained as

$$\langle 0|\bar{q}(0)\gamma^{\mu}\gamma_5 h_v(0)|\hat{P}(v)\rangle = iF_P v^{\mu}\int_0^{\infty}d\omega\Phi_P(\omega), \tag{2.43}$$

$$\langle 0|\bar{q}(0)\gamma^{\mu}h_v(0)|\hat{V}(v),\epsilon_{\lambda=0}\rangle = F_V\epsilon^{\mu}\int_0^{\infty}d\omega\Phi_{V\parallel}(\omega), \tag{2.44}$$

$$\langle 0|\bar{q}(0)\sigma^{\mu\bullet}\epsilon_{\perp\mu}h_v(0)|\hat{V}(v,\epsilon_{\lambda=\pm1})\rangle = F_V^{\perp}(\epsilon\cdot\epsilon_{\perp}vz)\int_0^{\infty}d\omega\Phi_{V\perp}(\omega). \tag{2.45}$$

The authors of Ref. [21] defined two quark-antiquark wave functions in momentum space $\psi_{\pm}(\omega)$ of a heavy-light meson in terms of the matrix element:

$$\langle 0|\bar{q}(z)\Gamma h_v(-z)|\hat{M}(v)\rangle$$
$$= f\int_0^\infty e^{i\omega vz}d\omega\,\mathrm{Tr}\Big\{\Big[\psi_+(\omega)+\frac{\not{z}}{2vz}[\psi_-(\omega)-\psi_+(\omega)]\Big]\times\mathcal{M}(v)\Gamma\Big\}, \quad (2.46)$$

where $f = F_M/2$ and

$$\mathcal{M}(v) = \frac{1+\not{v}}{2}\begin{cases}-i\gamma_5, & \text{for pseudoscalar meson } M(v),\\ \not{\epsilon}, & \text{for vector meson } M^*(v,\epsilon).\end{cases} \quad (2.47)$$

Evaluating the trace for various choices of Γ and taking the heavy quark limit, they obtained

$$\Phi_P(\omega) = \Phi_{V\|}(\omega) = \Phi_{V\perp}(\omega) = \psi_+(\omega), \quad (2.48)$$

and the normalization conditions

$$\int_0^\infty d\omega\,\psi_+(\omega) = 1. \quad (2.49)$$

In addition, the authors of Ref. [21] defined the moments of $\psi_+(\omega)$ as

$$\langle\omega^n\rangle_+ = \int_0^\infty d\omega\,\psi_+(\omega)\omega^n, \quad (2.50)$$

and used the equations of light and heavy quarks to obtain the relation between the first moment and the residual center mass:

$$\langle\omega\rangle_+ = \frac{4}{3}\bar{\Lambda}. \quad (2.51)$$

III. FORMULISM IN LIGHT-FRONT APPROACH

A. General framework

An s-wave meson bound state, consisting of a quark, q_1, and an antiquark, $\bar{q}_2$, with total momentum P and spin J, can be written as (see, for example [52])

$$|M(P,S,S_z)\rangle = \int\{d^3k_1\}\{d^3k_2\}2(2\pi)^3\delta^3(\tilde{P}-\tilde{k}_1-\tilde{k}_2)\times\sum_{\lambda_1,\lambda_2}\Psi^{SS_z}(\tilde{k}_1,\tilde{k}_2,\lambda_1,\lambda_2)|q_1(k_1,\lambda_1)\times\bar{q}_2(k_2,\lambda_2)\rangle, \quad (3.1)$$

where k_1 and k_2 are the on-mass-shell light-front momenta,

$$\tilde{k} = (k^+, k_\perp), \qquad k_\perp = (k^1, k^2), \qquad k^- = \frac{m_q^2 + k_\perp^2}{k^+}, \quad (3.2)$$

and

$$\{d^3k\} \equiv \frac{dk^+ d^2k_\perp}{2(2\pi)^3},$$
$$|q(k_1,\lambda_1)\bar{q}(k_2,\lambda_2)\rangle = b^\dagger(k_1,\lambda_1)d^\dagger(k_2,\lambda_2)|0\rangle,$$
$$\{b(k',\lambda'), b^\dagger(k,\lambda)\} = \{d(k',\lambda'), d^\dagger(k,\lambda)\} = 2(2\pi)^3\delta^3(\tilde{k}'-\tilde{k})\delta_{\lambda'\lambda}. \quad (3.3)$$

In terms of the light-front relative momentum variables $(u, \kappa_\perp)$ defined by

$$k_1^+ = (1-u)P^+, \qquad k_2^+ = uP^+,$$
$$k_{1\perp} = (1-u)P_\perp + \kappa_\perp, \qquad k_{2\perp} = uP_\perp - \kappa_\perp, \quad (3.4)$$

the momentum-space wave function Ψ^{SS_z} can be expressed as

$$\Psi^{SS_z}(\tilde{k}_1,\tilde{k}_2,\lambda_1,\lambda_2) = \frac{1}{\sqrt{Nc}}R^{SS_z}_{\lambda_1\lambda_2}(u,\kappa_\perp)\varphi(u,\kappa_\perp), \quad (3.5)$$

where $\varphi(u,\kappa_\perp)$ describes the momentum distribution of the constituent quarks in the bound state, and $R^{SS_z}_{\lambda_1\lambda_2}$ constructs a state of definite spin (S, S_z) out of the light-front helicity (λ_1, λ_2) eigenstates. Explicitly,

$$R^{SS_z}_{\lambda_1\lambda_2}(u,\kappa_\perp) = \sum_{s_1,s_2}\langle\lambda_1|\mathcal{R}_M^\dagger(1-u,\kappa_\perp,m_1)|s_1\rangle\times\langle\lambda_2|\mathcal{R}_M^\dagger(u,-\kappa_\perp,m_2)|s_2\rangle\times\Big\langle\frac{1}{2}\frac{1}{2};s_1s_2\Big|\frac{1}{2}\frac{1}{2};SS_z\Big\rangle, \quad (3.6)$$

where $|s_i\rangle$ are the usual Pauli spinors and $\mathcal{R}_M$ is the Melosh transformation operator [50]:

$$\langle s|\mathcal{R}_M(u_i,\kappa_\perp,m_i)|\lambda\rangle = \frac{m_i + u_iM_0 + i\vec{\sigma}_{s\lambda}\cdot\vec{\kappa}_\perp\times\vec{n}}{\sqrt{(m_i+u_iM_0)^2+\kappa_\perp^2}}, \quad (3.7)$$

with $u_1 = 1-u$, $u_2 = u$, and $\vec{n} = (0,0,1)$ a unit vector in the $\hat{z}$-direction. In addition,

$$M_0^2 = (e_1+e_2)^2 = \frac{m_1^2+\kappa_\perp^2}{1-u} + \frac{m_2^2+\kappa_\perp^2}{u},$$
$$e_i = \sqrt{m_i^2+\kappa_\perp^2+\kappa_z^2}. \quad (3.8)$$

where κ_z is the relative momentum in $\hat{z}$ direction and can be written as

$$\kappa_z = \frac{uM_0}{2} - \frac{m_2^2+\kappa_\perp^2}{2uM_0}. \quad (3.9)$$

M_0 is the invariant mass of $q\bar{q}$ and generally different from mass M of the meson which satisfies $M^2 = P^2$. This is due to the fact that the meson, quark and antiquark cannot be simultaneously on-shell. We normalize the meson state as

$$\langle M(P', S', S'_z)|M(P, S, S_z)\rangle = 2(2\pi)^3 P^+ \delta^3(\tilde{P}' - \tilde{P})\delta_{S'S}\delta_{S'_z S_z}, \qquad (3.10)$$

in order that

$$\int \frac{du d^2\kappa_\perp}{2(2\pi)^3} |\varphi(u, \kappa_\perp)|^2 = 1. \qquad (3.11)$$

In general, for any function $\mathcal{F}(|\vec{\kappa}|)$, $\varphi(u, \kappa_\perp)$ has the form of

$$\varphi(u, \kappa_\perp) = N\sqrt{\frac{d\kappa_z}{du}}\mathcal{F}(|\vec{\kappa}|), \qquad (3.12)$$

where normalization factor N is determined from Eq. (3.11).

In practice, it is more convenient to use the covariant form of $R^{SS_z}_{\lambda_1\lambda_2}$ [50,54,62]:

$$R^{SS_z}_{\lambda_1\lambda_2}(u, \kappa_\perp) = \frac{\sqrt{k_1^+ k_2^+}}{\sqrt{2}\tilde{M}_0(M_0 + m_1 + m_2)} \bar{u}(k_1, \lambda_1)(\bar{\not{P}} + M_0) \times \Gamma v(k_2, \lambda_2), \qquad (3.13)$$

where

$$\begin{aligned}
&\tilde{M}_0 \equiv \sqrt{M_0^2 - (m_1 - m_2)^2}, \qquad \bar{P} \equiv k_1 + k_2,\\
&\bar{u}(k, \lambda)u(k, \lambda') = \frac{2m}{k^+}\delta_{\lambda,\lambda'},\\
&\sum_\lambda u(k, \lambda)\bar{u}(k, \lambda) = \frac{\not{k} + m}{k^+},\\
&\bar{v}(k, \lambda)v(k, \lambda') = -\frac{2m}{k^+}\delta_{\lambda,\lambda'},\\
&\sum_\lambda v(k, \lambda)\bar{v}(k, \lambda) = \frac{\not{k} - m}{k^+}.
\end{aligned} \qquad (3.14)$$

For the pseudoscalar and vector mesons, we have:

$$\Gamma_P = \gamma_5, \qquad \Gamma_V = -\not{\epsilon}(\lambda), \qquad (3.15)$$

where

$$\begin{aligned}
&\epsilon^\mu_{\lambda=\pm1} = \left[\frac{2}{P^+}\vec{\epsilon}_\perp(\pm1)\cdot\vec{P}_\perp, 0, \vec{\epsilon}_\perp(\pm1)\right],\\
&\vec{\epsilon}_\perp(\pm1) = \mp(1, \pm i)/\sqrt{2},\\
&\epsilon^\mu_{\lambda=0} = \frac{1}{M_0}\left(\frac{-M_0^2 + P_\perp^2}{P^+}, P^+, P_\perp\right).
\end{aligned} \qquad (3.16)$$

Equations (3.13) and (3.15) can be further reduced by the applications of equations of motion on the spinors [54]:

$$R^{SS_z}_{\lambda_1\lambda_2}(u, \kappa_\perp) = \frac{\sqrt{k_1^+ k_2^+}}{\sqrt{2}\tilde{M}_0} \bar{u}(k_1, \lambda_1)\Gamma' v(k_2, \lambda_2), \qquad (3.17)$$

where

$$\Gamma'_P = \gamma_5, \qquad \Gamma'_V = -\not{\epsilon} + \frac{\epsilon \cdot (k_1 - k_2)}{M_0 + m_1 + m_2}. \qquad (3.18)$$

Next, the matrix elements of Eqs. (2.20), (2.21), and (2.32) are calculated within LFQM, and the relevant leading twist LCDAs are extracted. For the pseudoscalar meson state, we substitute Eqs. (3.1), (3.5), and (3.17) into Eq. (2.20) to obtain

$$\begin{aligned}
\langle 0|\bar{q}_2\gamma^\mu\gamma_5 q_1|P(P)\rangle &= N_c \int\{d^3k_1\}\sum_{\lambda_1,\lambda_2}\Psi^{SS_z}(k_1, k_2, \lambda_1, \lambda_2)\langle 0|\bar{q}_2\gamma^\mu\gamma_5 q_1|q_1\bar{q}_2\rangle\\
&= i\sqrt{N_c}\int\{d^3k_1\}\frac{\sqrt{k_1^+k_2^+}}{\sqrt{2}\tilde{M}_0}\varphi\,\mathrm{Tr}\left[\gamma^\mu\gamma_5\left(\frac{\not{k}_1 + m_1}{k_1^+}\right)\gamma_5\left(\frac{\not{k}_2 - m_2}{k_2^+}\right)\right] = if_P P^\mu\int du\phi(u).
\end{aligned} \qquad (3.19)$$

For the "good" component, $\mu = +$, the leading twist LCDA, ϕ_P, is extracted as

$$\phi_P(u) = \frac{2\sqrt{6}}{f_P}\int\frac{d^2\kappa_\perp}{2(2\pi)^3}\frac{[(1 - u)m_2 + um_1]}{\sqrt{u(1 - u)}\tilde{M}_0}\varphi(u, \kappa_\perp). \qquad (3.20)$$

A similar process is used for the vector meson which corresponded to Eqs. (2.21) and (2.32), and then the leading twist LCDAs are extracted as

$$\phi_{V\parallel}(u) = \frac{2\sqrt{6}}{f_V}\int\frac{d^2\kappa_\perp}{2(2\pi)^3}\frac{\varphi(u,\kappa_\perp)}{\sqrt{u(1-u)}\tilde{M}_0}\times\left\{um_1+(1-u)m_2+\frac{2\kappa_\perp^2}{M_0+m_1+m_2}\right\}, \tag{3.21}$$

$$\phi_{V\perp}(u) = \frac{2\sqrt{6}}{f_V^\perp}\int\frac{d^2\kappa_\perp}{2(2\pi)^3}\frac{\varphi(u,\kappa_\perp)}{\sqrt{u(1-u)}\tilde{M}_0}\times\left\{um_1+(1-u)m_2+\frac{\kappa_\perp^2}{M_0+m_1+m_2}\right\}. \tag{3.22}$$

From the normalization of Eq. (2.9), we found not only that the equations of f_P and f_V were consistent with that of [6], but also that the decay constants and the leading twist LCDAs has the simple relations

$$f_P+f_V=2f_V^\perp, \qquad \phi_P(u)+\phi_{V\parallel}(u)=2\phi_{V\perp}(u). \tag{3.23}$$

B. Heavy quark framework

If one takes $m_1=m_Q\to\infty$, that is, the heavy quark limit in the heavy-light meson, then two inequalities, $m_Q\simeq M_0\gg m_2,\kappa_\perp$ and $u\to 0$, are obtained. From Eqs. (3.20), (3.21), and (3.22), the decay constants and the leading twist LCDAs are simplified as

$$f_P\simeq f_V\simeq f_V^\perp\propto F_M, \qquad \phi_P\simeq\phi_{V\parallel}\simeq\phi_{V\perp}\propto\Phi_M, \tag{3.24}$$

which are independent of the form of $\mathcal{F}(|\vec{\kappa}|)$. These are consistent with HQS between the s-wave heavy-light mesons. The exact form of Φ_M, however, must be derived by the redefinition of the meson bound state. Let us consider the bound states of heavy mesons in the heavy quark limit:

$$|\hat{M}(v;S,S_z)\rangle = \int\{d^3q\}\{d^3k_2\}2(2\pi)^3\delta^3(\bar{\Lambda}\,\tilde{v}-\tilde{q}-\tilde{k}_2)\times\sum_{\lambda_1,\lambda_2}\hat{\Psi}^{SS_z}(\omega,\kappa_\perp,\lambda_1,\lambda_2)b_v^\dagger(q,\lambda_1)\times d^\dagger(k_2,\lambda_2)|0\rangle, \tag{3.25}$$

where $q=k_1-m_Qv$ is the residual momentum of heavy quark. The operators $b_v^\dagger(q,\lambda_1)$ create a heavy quark with

$$\{b_v(q,\lambda_1),b_{v'}^\dagger(q',\lambda_1')\}=2(2\pi)^3\delta_{vv'}\delta^3(\tilde{q}-\tilde{q}')\delta_{\lambda_1\lambda_1'}. \tag{3.26}$$

The relative transverse and longitudinal momenta, $\kappa_\perp$ and κ_z, are obtained by

$$\kappa_\perp=k_{2\perp}-\omega v_\perp, \qquad \kappa_z=\frac{\omega}{2}-\frac{m_2^2+\kappa_\perp^2}{2\omega}. \tag{3.27}$$

The momentum-space wave function $\hat{\Psi}^{SS_z}$ can be expressed as

$$\hat{\Psi}^{SS_z}(\omega,\kappa_\perp,\lambda_1,\lambda_2)=\frac{1}{\sqrt{Nc}}\hat{R}^{SS_z}_{\lambda_1\lambda_2}(\omega,\kappa_\perp)\hat{\varphi}^{SS_z}(\omega,\kappa_\perp), \tag{3.28}$$

where

$$\hat{R}^{SS_z}(\omega,\kappa_\perp,\lambda_1,\lambda_2)=\frac{k_2^+}{\sqrt{2}\sqrt{(\omega+m_2)^2+\kappa_\perp^2}}\times\bar{u}(v,\lambda_1)\Gamma v(k_2,\lambda_2) \tag{3.29}$$

with $\Gamma=\gamma_5(-\hat{\not\epsilon})$ for $S=0(1)$,

$$\hat{\epsilon}^\mu_{\lambda=\pm1}=\left[\frac{2}{v^+}\vec{\epsilon}_\perp(\pm1)\cdot\vec{v}_\perp,0,\vec{\epsilon}_\perp(\pm1)\right], \quad \hat{\epsilon}^\mu_{\lambda=0}=\left(\frac{-1+v_\perp^2}{v^+},v^+,v_\perp\right), \tag{3.30}$$

and $u(v,\lambda_1)$ is the spinor for the heavy quark,

$$\sum_\lambda u(v,\lambda)\bar{u}(v,\lambda)=\frac{\not v+1}{v^+}. \tag{3.31}$$

The normalization of the heavy meson bound states can then be given by

$$\langle\hat{M}(v',S',S_z')|\hat{M}(v,S,S_z)\rangle=2(2\pi)^3v^+\delta^3(\bar{\Lambda}v'-\bar{\Lambda}v)\times\delta_{SS'}\delta_{S_zS_z'}, \tag{3.32}$$

which not only leads to Eq. (2.37), but also to the space part $\hat{\varphi}^{SS_z}(\omega,\kappa_\perp^2)$ (called the light-front wave function) in Eq. (3.25) which has the following wave function normalization condition:

$$\int\frac{d\omega d^2\kappa_\perp}{2(2\pi)^3}|\hat{\varphi}^{SS_z}(\omega,\kappa_\perp^2)|^2=1. \tag{3.33}$$

In principle, the heavy quark dynamics are completely described by HQET, which is given by the $1/m_Q$ expansion of the heavy quark QCD Lagrangian:

$$\mathcal{L}=\bar{Q}(i\not D-m_Q)Q=\sum_{n=0}^\infty\left(\frac{1}{2m_Q}\right)^n\mathcal{L}_n. \tag{3.34}$$

Therefore, $|\hat{M}(v;S,S_z)\rangle$ and $\hat{\varphi}^{SS_z}(\omega,\kappa_\perp^2)$ are then determined by the leading Lagrangian $\mathcal{L}_0=\bar{h}_v iv\cdot Dh_v$. The authors of Ref. [63] have shown, from the light-front bound state equation, that $\hat{\varphi}^{SS_z}(U,\kappa_\perp^2)$ must be degenerate for $S=0$ and $S=1$. As a result, we can simply write

$$\hat{\varphi}^{SS_z}(\omega,\kappa_\perp^2)=\hat{\varphi}(\omega,\kappa_\perp^2) \tag{3.35}$$

in the heavy quark limit. Equation (3.25) together with Eqs. (3.29) and (3.35) are then the heavy meson light-front bound states in the heavy quark limit that obeyed HQS. From the normalization conditions of Eqs. (3.11) and (3.33), we obtain the relation between the wave functions

$\varphi(u, \kappa_\perp^2)$ and $\hat{\varphi}(\omega, \kappa_\perp^2)$:

$$\varphi(u, \kappa_\perp^2) = \sqrt{M}\hat{\varphi}(\omega, \kappa_\perp^2). \tag{3.36}$$

Next, the matrix elements of Eqs. (2.43), (2.44), and (2.45) can be calculated, and the relevant leading twist LCDAs extracted. For the pseudoscalar meson state, we substitute Eqs. (3.25), (3.28), and (3.29) into Eq. (2.43) to obtain

$$\begin{aligned}\langle 0|\bar{q}\gamma^\mu\gamma_5 h_v|P(v)\rangle &= N_c\int\{d^3k_2\}\sum_{\lambda_1,\lambda_2}\hat{\Psi}^{SS_z}(\omega, \kappa_\perp, \lambda_1, \lambda_2)\langle 0|\bar{q}_2\gamma^\mu\gamma_5 h_v|Q\bar{q}_2\rangle\\ &= i\sqrt{N_c}\int\{d^3k_2\}\frac{k_2^+}{\sqrt{2}\sqrt{(\omega+m_2)^2+\kappa_\perp^2}}\hat{\varphi}\,\mathrm{Tr}\bigg[\gamma^\mu\gamma_5\Big(\frac{\not{v}+1}{v^+}\Big)\gamma_5\Big(\frac{\not{k}_2-m_2}{k_2^+}\Big)\bigg] = iF_P v^\mu\int d\omega\Phi_P(\omega).\end{aligned} \tag{3.37}$$

For the "+" component, the leading twist LCDA Φ_P is extracted as

$$\Phi_P(\omega) = \frac{2\sqrt{6}}{F_P}\int\frac{d^2\kappa_\perp}{2(2\pi)^3}\frac{\omega+m_2}{\sqrt{(\omega+m_2)^2+\kappa_\perp^2}}\hat{\varphi}(\omega, \kappa_\perp). \tag{3.38}$$

In contrast with $\phi(u)$, $\Phi(\omega)$ represents the distribution of the longitudinal momentum carried by the light degree of freedom. A similar process is used for the vector meson which corresponds to Eqs. (2.44) and (2.45), and the results are

$$F_P = F_V = F_V^\perp, \qquad \Phi_P(\omega) = \Phi_{V\parallel}(\omega) = \Phi_{V\perp}(\omega), \tag{3.39}$$

which are consistent with Eqs. (2.48) and (2.49).

IV. NUMERICAL RESULTS AND DISCUSSIONS

In this section, the decay constants and LCDAs of $D^{(*)}$, $D_s^{(*)}$, $B^{(*)}$, $B_s^{(*)}$, and $B_c^{(*)}$ are studied. We consider two kinds of $\mathcal{F}(|\vec{\kappa}|)$, one is the Gaussian type, the other is the power-law type:

$$\mathcal{F}^g(|\vec{\kappa}|) = \exp\Big(-\frac{|\vec{\kappa}|^2}{2\beta^2}\Big), \tag{4.1}$$

$$\mathcal{F}^p(|\vec{\kappa}|) = \Big(\frac{1}{1+|\vec{\kappa}|^2/\beta^2}\Big)^2, \tag{4.2}$$

then the corresponding wave functions are

$$\varphi^g(u, \kappa_\perp) = 4\Big(\frac{\pi}{\beta^2}\Big)^{3/4}\sqrt{\frac{e_1e_2}{u(1-u)M_0}}\times\exp\bigg[-\frac{\kappa_\perp^2+(\frac{uM_0}{2}-\frac{m_2^2+\kappa_\perp^2}{2uM_0})^2}{2\beta^2}\bigg], \tag{4.3}$$

$$\varphi^p(u, \kappa_\perp) = 8\Big(\frac{2\pi}{\beta^3}\Big)^{1/2}\sqrt{\frac{e_1e_2}{u(1-u)M_0}}\times\bigg[\frac{\beta^2}{\kappa_\perp^2+(\frac{uM_0}{2}-\frac{m_2^2+\kappa_\perp^2}{2uM_0})^2+\beta^2}\bigg]^2, \tag{4.4}$$

and can be used to calculate decay constant f, the LCDAs $\phi(u)$, and the ξ-moments of $\phi(u)$. Prior to the numerical calculations, the parameters m_1, m_2 and β, which appeared in the wave function, have to first determined. For the light quark masses, we used the decay constants f_π, f_K and the mean square radii $\langle r_{\pi^+}^2\rangle$, $\langle r_{K^0}^2\rangle$ to fit $m_{u,d}(\equiv m_q)$ and m_s [64]. For the heavy quark masses, however, the relevant measurements are insufficient. We determined m_c and m_b by the mass of the spin-weighted average of the heavy quarkonium states and its variational principle for the relevant Hamiltonian [20].

As regards parameter β, it is determined by the decay constant of the heavy meson. Recently the CLEO collaboration updated their data concerning $\mathrm{Br}(D^+\to\mu^+\nu)$ and an average value was reported [65]: $f_{D^+} = 206.0 \pm 8.9$ MeV. In addition, the authors of Ref. [66] averaged $\mathrm{Br}(B^-\to\tau^-\bar{\nu})$ from the Belle [67] and *BABAR* [68,69] collaborations and extracted $f_B = 204 \pm 31$ MeV. The parameters β_{cq} and β_{bq} can then be determined. As mentioned in the previous work [64], the ratios, β_{cs}/β_{cq} and β_{bs}/β_{bq} can be related to the $SU(3)$ symmetry breaking, that is, m_s/m_q as follows:

$$\frac{\Delta M_{D_sD_s^*}}{\Delta M_{DD^*}} = \frac{m_q}{m_s}\Big(\frac{\beta_{cs}}{\beta_{cq}}\Big)^3, \qquad \frac{\Delta M_{B_sB_s^*}}{\Delta M_{BB^*}} = \frac{m_q}{m_s}\Big(\frac{\beta_{bs}}{\beta_{bq}}\Big)^3. \tag{4.5}$$

Therefore, β_{cs} and β_{bs} are not independent parameters. Concerning the decay constants of B_c, we quote the average result of QCD sum rules [70]: $f_{B_c} = 360$ MeV to extract the parameter β_{bc}. All the parameters are listed in Table I.

Next, we used the parameters in Table I as input to calculate the decay constants f_P, f_V, and $f_V^\perp$ of the relevant heavy mesons. The values of the ratios f_V/f_P, $f_{P'}/f_P$,

TABLE I. Input values of quark masses and β's (MeV).

	m_q	m_s	m_c	m_b	β_{cq}	β_{cs}	β_{bq}	β_{bs}	β_{bc}
φ^g	251	445 ± 36	1380	4780	465 ± 22	567 ± 42	587 ± 74	727 ± 120	815
φ^p	172	296 ± 12	1360	4770	505 ± 25	608 ± 40	575 ± 77	706 ± 113	815

$f_{V'}/f_V$ are also included. Tables II and III show a comparison of the results of this work with other theoretical calculations. In a previous work [64], we pointed out that ratios f_{D_s}/f_D and f_{B_s}/f_B were not only chiefly determined by the ratio of light quark masses, m_s/m_q, or the $SU(3)$ symmetry breaking, but also insensitive to the heavy quark masses $m_{c,b}$. This phenomenon also appears in the ratios $f_{D_s^*}/f_{D^*}$ and $f_{B_s^*}/f_{B^*}$ here for both Gaussian and power-

TABLE II. Decay constants of the pseudoscalar and vector heavy mesons (MeV). Linear and HO are the different potentials in Refs. [6,7], FC is the field correlators, BS is the Bethe-Salpeter equation, and RQM is the relativistic quark model.

	Experiment	This work[a]	Linear (HO)	FC [11]	BS [8,9]	Lattice [5]	RQM [10]
f_D	206.0 ± 8.9 [65]	$\underline{206.0 \pm 8.9}$ ($\underline{206.0 \pm 8.9}$)	211 (194)	210 ± 10	230 ± 25	$211 \pm 14^{+0}_{-12}$	234
f_{D^*}		259.6 ± 14.6 ($306.3^{+18.2}_{-17.7}$)	254 (228)	273 ± 13	340 ± 23	$245 \pm 20^{+0}_{-2}$	310
$f^{\perp}_{D^*}$		232.7 ± 11.7 ($256.2^{+13.6}_{-13.3}$)					
f_{D_s}	260.7 ± 6.5 [65]	267.4 ± 17.9 (259.7 ± 13.7)	248 (233)	260 ± 10	248 ± 27	$231 \pm 12^{+6}_{-0}$	268
$f_{D_s^*}$		338.7 ± 29.7 (391.0 ± 28.9)	290 (268)	307 ± 18	375 ± 24	$272 \pm 12^{+0}_{-20}$	315
$f^{\perp}_{D_s^*}$		303.1 ± 23.8 (325.3 ± 21.5)					
f_B	204 ± 31[b]	$\underline{204 \pm 31}$ ($\underline{204 \pm 31}$)	189 (180)	182 ± 8	196 ± 29	$179 \pm 18^{+26}_{-9}$	189
f_{B^*}		225 ± 38 (249^{+44}_{-42})	204 (193)	200 ± 10	238 ± 18	$196 \pm 24^{+31}_{-2}$	219
$f^{\perp}_{B^*}$		214 ± 34 (226 ± 37)					
f_{B_s}		281 ± 54 (270 ± 47)	234 (237)	216 ± 8	216 ± 32	$204 \pm 16^{+28}_{-0}$	218
$f_{B_s^*}$		313 ± 67 (335 ± 68)	250 (254)	230 ± 12	272 ± 20	$229 \pm 20^{+31}_{-16}$	251
$f^{\perp}_{B_s^*}$		297 ± 61 (302 ± 58)					
f_{B_c}		$\underline{360}$ ($\underline{360}$)	377 (508)	438 ± 10	322 ± 42		
$f_{B_c^*}$		387 (423)	398 (551)	453 ± 20	418 ± 24		
$f^{\perp}_{B_c^*}$		374 (392)					

[a]The value is obtained by $\varphi^g(\varphi^p)$.
[b]This value is extracted by the branching ratio: $\mathcal{B}(B^- \to \tau^- \bar{\nu}) = (1.42 \pm 0.43) \times 10^{-4}$ [66].

TABLE III. Ratio of the decay constants. In this work, $f^{\perp}_V/f_P = (1 + f_V/f_P)/2$.

	Experiment	This work[a]	Linear (HO)	FC [11]	BS [8,9]	Lattice [5]	RQM [10]
f_{D^*}/f_D		1.26 ± 0.02 (1.49 ± 0.02)	1.20 (1.18)	1.27 ± 0.05	1.48 ± 0.26		1.32
$f_{D_s^*}/f_{D_s}$		1.27 ± 0.03 (1.51 ± 0.03)	1.17 (1.15)	1.17 ± 0.04	1.51 ± 0.26		1.18
f_{D_s}/f_D	1.27 ± 0.06 [65]	1.30 ± 0.04 (1.26 ± 0.04)	1.18 (1.20)	1.24 ± 0.03	1.08 ± 0.01	1.10 ± 0.02	1.15
$f_{D_s^*}/f_{D^*}$		1.30 ± 0.05 (1.28 ± 0.05)	1.14 (1.18)		1.10 ± 0.06	1.11 ± 0.03	1.02
f_{B^*}/f_B		1.10 ± 0.02 (1.22 ± 0.03)	1.08 (1.07)	1.08 ± 0.04	1.21 ± 0.27		1.16
$f_{B_s^*}/f_{B_s}$		1.11 ± 0.03 (1.24 ± 0.05)	1.07 (1.07)	1.07 ± 0.04	1.26 ± 0.28		1.15
f_{B_s}/f_B		1.38 ± 0.07 (1.32 ± 0.08)	1.24 (1.32)	1.19 ± 0.03	1.10 ± 0.01	$1.14 \pm 0.03^{+0.00}_{-0.01}$	1.15
$f_{B_s^*}/f_{B^*}$		1.39 ± 0.08 (1.35 ± 0.08)	1.23 (1.32)		1.14 ± 0.08	$1.17 \pm 0.04^{+0.00}_{-0.03}$	1.15
$f_{B_c^*}/f_{B_c}$		1.08 (1.18)	1.06 (1.08)	1.03 ± 0.03	1.30 ± 0.24		

[a]The value is obtained by $\varphi^g(\varphi^p)$.

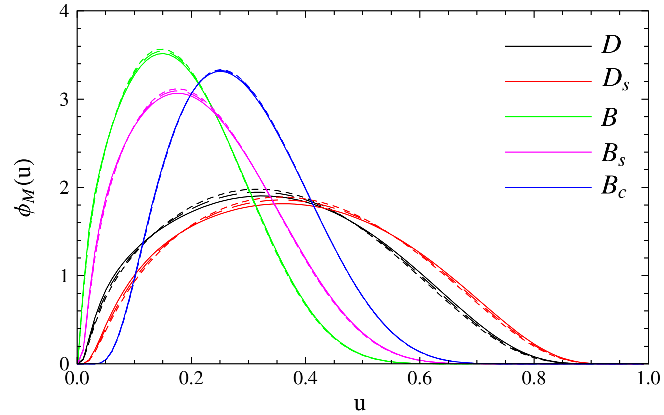

FIG. 1 (color online). Quark distribution amplitudes of the heavy meson for a Gaussian wave function. The solid, dotted, and dashed lines correspond to $\phi_P^g(u)$, $\phi_{V\|}^g(u)$, $\phi_{V\perp}^g(u)$, respectively.

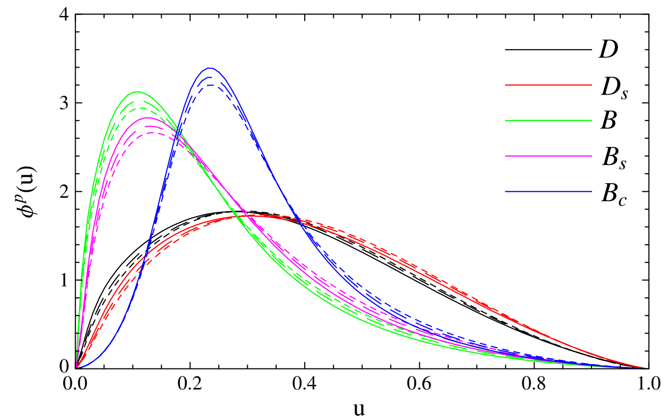

FIG. 2 (color online). Quark distribution amplitudes of the heavy meson for a power-law wave function. The solid, dotted, and dashed lines correspond to $\phi_P^p(u)$, $\phi_{V\|}^p(u)$, $\phi_{V\perp}^p(u)$, respectively.

law wave functions. On the contrary, as shown in Table III, the ratio f_V/f_P is not only dependent on the heavy quark mass, but also insensitive to the light quark mass. The reason is that, making a comparison between Eqs. (3.20) and (3.21), the difference between f_P and f_V is proportional to $2\kappa_\perp^2/(M_0 + m_1 + m2)$. In the Gaussian (power-law) wave function, the mean square value of the transverse momentum is equal (proportional) to the square value of the parameter β, or $\langle\kappa_\perp^2\rangle = \beta^2(\langle\kappa_\perp^2\rangle \propto \beta^2)$, so the ratio f_V/f_P is influenced by the parameter β and the quark mass. In the case of the different heavy quark, for example, f_{D^*}/f_D and f_{B^*}/f_B, as m_b is much greater than m_c, this effect is greater than that of $\beta_{bq} > \beta_{cq}$, so f_{B^*}/f_B is smaller than f_{D^*}/f_D. On the other hand, in the case of the different light quark, for example, f_{D^*}/f_D and $f_{D_s^*}/f_{D_s}$, as m_s is slightly greater than m_q, this effect is less than that of $\beta_{cs} > \beta_{cq}$, so $f_{D_s^*}/f_{D_s}$ is a little larger than f_{D^*}/f_D.

The quark distributions of the heavy meson, $\phi_P(u)$, $\phi_{V\|}(u)$, and $\phi_{V\perp}(u)$ are plotted in Fig. 1 and 2. Clearly the difference in the constituent quark masses is greater, the location where u peaked is closer to zero. This indicates, relatively, that the lighter the quark, the smaller its momentum fraction. We also find that, even though the difference between f_D and f_{D^*} was more than 25% (almost 50%) for the Gaussian (power-law) wave function, all curvilinear distinctions between $\phi_{V\|}(u)$ and $\phi_P(u)$ are quite small. The reason is that, after the $\kappa_\perp$ integration, the curve of $\phi_M(u)$ is influenced only by the quark mass, parameter β, and the total spin (that is, the pseudoscalar or the vector meson). As the quark mass and β are the same in $\phi_{V\|(\perp)}(u)$ and $\phi_P(u)$, the distinctions between them were slight. On the other hand, even though f_{D^*} is almost equal to f_{D_s}, as shown in Table II (or $f_{D^*}/f_D \simeq f_{D_s}/f_D$) for the Gaussian wave function, the curvilinear distinction between $\phi_{D^*\|}^g(u)$ and $\phi_D^g(u)$ is obviously smaller than that between $\phi_{D_s}^g(u)$ and $\phi_D^g(u)$. As for the power-law wave function, the situation is inverse. Therefore, we can infer that, even though the values of f_M's are almost the same between the distinct heavy mesons, the curves of $\phi_M(u)$ may have the quite large differences, and vice versa. Finally, the quark distribution function is displayed in terms of the ξ-moments, as in Eq. (2.13). The first six ξ-moments ($n > 0$) are listed in Table IV.

For the heavy quark framework, some models for B meson LCDAs have also been adopted in the literature. Inspired by the QCD sum rule analysis, the authors of Ref. [21] proposed a simple model:

$$\psi_{+I}(\omega) = \frac{\omega}{\lambda_I^2} e^{-\omega/\lambda_I}. \tag{4.6}$$

Additionally, the authors of Ref. [25] suggested a

TABLE IV. First six ξ-moments of the s-wave heavy meson.

	$\langle\xi^1\rangle$	$\langle\xi^2\rangle$	$\langle\xi^3\rangle$	$\langle\xi^4\rangle$	$\langle\xi^5\rangle$	$\langle\xi^6\rangle$
$\phi_D^{g(p)}$	−0.288 (−0.251)	0.210 (0.235)	−0.125 (−0.115)	0.0960 (0.111)	−0.0695 (−0.0673)	0.0558 (0.0664)
$\phi_{D_s}^{g(p)}$	−0.213 (−0.207)	0.183 (0.217)	−0.0890 (−0.0905)	0.0738 (0.0970)	−0.0468 (−0.0507)	0.0388 (0.0550)
$\phi_B^{g(p)}$	−0.617 (−0.531)	0.425 (0.398)	−0.312 (−0.288)	0.240 (0.234)	−0.191 (−0.185)	0.156 (0.157)
$\phi_{B_s}^{g(p)}$	−0.549 (−0.486)	0.359 (0.359)	−0.254 (−0.249)	0.189 (0.200)	−0.147 (−0.154)	0.117 (0.129)
$\phi_{B_c}^{g(p)}$	−0.536 (−0.368)	0.227 (0.230)	−0.133 (−0.123)	0.108 (0.0867)	−0.0553 (−0.0527)	0.0378 (0.0403)

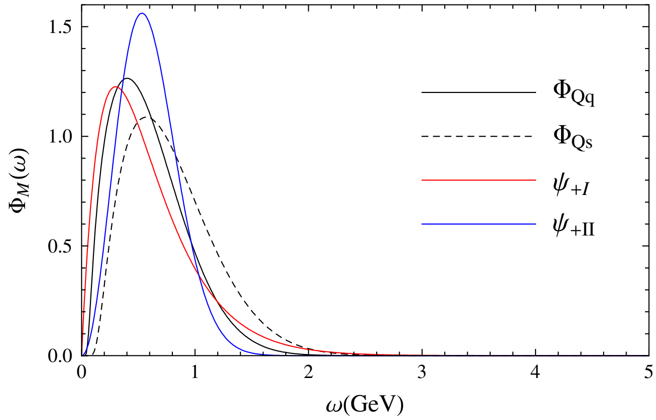

FIG. 3 (color online). Leading twist LCDAs in the heavy quark framework.

Gaussian-type model:

$$\psi_{+II}(\omega) = \sqrt{\frac{2}{\pi\lambda_{II}^2}}\frac{\omega^2}{\lambda_{II}^2}e^{-\omega^2/2\lambda_{II}^2}. \tag{4.7}$$

By applying Eqs. (2.50) and (2.51), the relation between the residual center mass and the parameter λ is

$$\bar{\Lambda}_q = \frac{3}{2}\lambda_I = \frac{3}{\sqrt{2\pi}}\lambda_{II}. \tag{4.8}$$

In Ref. [23], the value $\lambda_I = 0.3$ GeV corresponded to $\bar{\Lambda}_q = 0.45$ GeV. For a convenient comparison, we used this $\bar{\Lambda}_q$ and $\bar{\Lambda}_s = \bar{\Lambda}_q + m_s - m_q$ to fix parameters β_{Qq} and β_{Qs} in this work. Moreover, the Gaussian wave function $\hat{\varphi}$ is given by taking the heavy quark limit in Eq. (4.3) and using the relation Eq. (3.36):

$$\hat{\varphi}(\omega,\kappa_\perp) = 4\left(\frac{\pi}{\beta^2}\right)^{3/4}\sqrt{\frac{1}{2}+\frac{m_2^2+\kappa_\perp^2}{2\omega^2}} \times \exp\left(-\frac{\kappa_\perp^2+(\frac{\omega}{2}-\frac{m_2^2+\kappa_\perp^2}{2\omega})^2}{2\beta^2}\right). \tag{4.9}$$

The light quark masses $m_{q(s)} = 0.251(0.445)$ GeV are as in Table I, and we can then obtain the values $\beta_{Qq}^g = 0.279$ GeV and $\beta_{Qs}^g = 0.338$ GeV. In terms of these parameters, the leading twist LCDAs $\Phi_{Qq}(\omega)$, $\Phi_{Qs}(\omega)$, $\psi_{+I}(\omega)$, and $\psi_{+II}(\omega)$ are calculated and plotted as in Fig. 3. We find that the curve of $\Phi_{Qq}(\omega)$ is close to that of $\psi_{+I}(\omega)$.

V. CONCLUSIONS

This study has discussed the leading twist LCDAs of the s-wave heavy meson within the light-front approach in both general and heavy quark frameworks. These LCDAs are shown in terms of light-front variables and relevant decay constants. In the general frameworks, we find that the decay constants and LCDAs of the pseudoscalar and vector mesons have the following relations: $f_P + f_V = 2f_V^\perp$ and $\phi_P + \phi_{V\parallel} = 2\phi_{V\perp}$. The parameters m and β, which appear in both Gaussian and power-law wave functions, were determined as follows: (1) the light quark masses are fitted by the decay constants and the mean square radii of the light meson; (2) the heavy quark masses are determined by the mass of the spin-weighted average of the heavy quarkonium states and its variational principle for the relevant Hamiltonian; and (3) the hadronic parameter β's are evaluated by the decay constants of D^+, B^-, and B_c, with the former two and the latter one from the experimental data and the average result of QCD sum rules, respectively. We find that, for both Gaussian and power-law wave functions, the ratios $f_{D_s^*}/f_{D^*}$ and $f_{B_s^*}/f_{B^*}$, as well as f_{D_s}/f_D and f_{B_s}/f_B in the previous work, are chiefly determined by the ratio of light quark masses m_s/m_q, or the $SU(3)$ symmetry breaking. On the other hand, by making a comparison between f_{D^*}/f_D, $f_{D_s^*}/f_{D_s}$, f_{B^*}/f_B, and $f_{B_s^*}/f_{B_s}$, the ratio f_V/f_P is not only dependent on the heavy quark mass, but also insensitive to the light quark mass.

As shown in Figs. 1 and 2, we find that even though the difference between f_D and f_{D^*} is more than 25% (almost 50%) for the Gaussian (power-law) wave functions, all curvilinear distinctions between $\phi_{V\parallel}(u)$ and $\phi_P(u)$ are quite small because their main difference come from the variations of the quark mass and β. On the contrary, even though f_{D^*}/f_D is almost equal to f_{D_s}/f_D for the Gaussian wave function, the curvilinear distinction between $\phi_{D^*\parallel}^g(u)$ and $\phi_D^g(u)$ is obviously smaller than that between $\phi_{D_s}^g(u)$ and $\phi_D^g(u)$. As for the power-law wave function, the situation is inverse. Therefore, we conclude that even though the values of f_M's are almost equal among the distinct mesons, the curves of $\phi_M(u)$ may have quite large differences, and vice versa.

When the heavy quark framework is used, the above relations for the decay constant and the LCDAs can be further simplified as $F_P = F_V = F_V^\perp$ and $\Phi_P = \Phi_{V\parallel} = \Phi_{V\perp}$, as consistent with HQS. For a convenient comparison, the value $\bar{\Lambda}_q = 0.45$ GeV, as suggested in Ref. [23], is used to fix $\beta_{Qq(Qs)}$ and to plot the curves of Φ_{Qq}, Φ_{Qs}, ψ_{+I}, and ψ_{+II} in Fig. 3. We find that the curvilinear distinction between $\Phi_{Qq}(\omega)$ and $\psi_{+I}(\omega)$ is relatively small.

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