Bethe-Salpeter dynamics and the constituent mass concept for heavy quark mesons

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The definition of a quark as heavy requires a comparison of its mass with the nonperturbative chiral symmetry breaking scale which is about 1 GeV ($\Lambda_\chi \sim 1$ GeV) or with the scale $\Lambda_{\mathrm{QCD}} \sim 0.2$ GeV that characterizes the distinction between perturbative and nonperturbative QCD. For quark masses significantly larger than these scales, nonperturbative dressing effects, or equivalently nonperturbative self-energy contributions, and relativistic effects are believed to be less important for physical observables. We explore the concept of a constituent mass for heavy quarks in the Dyson-Schwinger equations formalism, for light-heavy and heavy-heavy quark mesons by studying their masses and electroweak decay constants.

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I. INTRODUCTION

A large amount of research work on heavy quark systems has been done within a Hamiltonian approach containing a static potential interaction. The quark mass is a parameter to be fitted to experimental data. One refers to this mass as a model-dependent constituent mass (see, for example, [1,2]). In different potential models the range of the c quark mass is 1.30–1.84 GeV while for the b quark is 4.2–5.17 GeV [3,4]. Recent work has made a clear connection between a relativistic Hamiltonian and the QCD Lagrangian. However, there remains no more than a qualitative relation between the various mass definitions within the currently used approaches ([5–7], and references therein).

Another approach is through the framework of the so-called instantaneous Bethe-Salpeter model. In this case the instantaneous Bethe-Salpeter equation (BSE) kernel and the use of constituent mass quark propagators allow explicit integration over time in the rest frame of the mesons. Such a model can give pseudoscalar and vector quarkonia meson spectra and decay constants in reasonable agreement to experimental data; however the number of parameters is considerably greater than the present approach. The lack of retardation effects is also a drawback. As you go to higher radial or angular excited states, though, the masses and the decay constants are not described in a satisfying way (see, for example, [8,9], and references therein).

By using the Dyson-Schwinger equation formalism and a *heavy quark limit* approach, which essentially begins with a constituent or pole mass heavy quark propagator, Kalinovski *et al.* in a series of works examined heavy quark meson observables ([10,11], and references therein). For the light quark propagator, they employed a parametrization fitted to light quark meson experimental data. The solution of the BSE is simplified through a parametrization of the Bethe-Salpeter amplitude (BSA). In that way, the calculations of other physical observables, like decay constants, form factors, etc. are a relatively easy and straightforward task (for more details, see [12,13]).

Heavy quark effective field theories (HQET) have been developed by appropriately manipulating QCD Lagrangian with the ultimate goal of simplifying the description of nonperturbative physics. We stress that, unlike other effective theories, the purpose here is to describe heavy quark hadron observables, thus one can not completely remove the heavy quark degree of freedom. One can integrate out degrees of freedom that describe fluctuations around the mass shell. HQETs are also used in lattice QCD. In general, masses, decay constants, form factors, and other physical quantities for mesons and baryons with heavy quarks are described with reasonable agreement to experimental data. For more details and a pedagogical introduction in the HQETs, see for example [14,15] and for more applications, see [16–18], and references therein. A detailed overview of the present state of experimental and theoretical studies of quarkonia can be found in Ref. [19].

We will examine the constituent mass concept within the nonperturbative framework of Dyson-Schwinger equations, using a previously developed effective interaction.

II. DEFINITIONS OF A HEAVY QUARK MASS

In the case of quarkonia (heavy-heavy quark) $Q\bar{Q}$ mesons, the heavy quarks with mass m_Q have an effective coupling that is small; the strong interactions at the Compton scale, $\lambda_Q \sim 1/m_Q$, can be treated perturbatively. In this case the situation is quite simple, symmetric, and much like the positronium system. For the light-heavy mesons, the situation is not that simple since the heavy quark is essentially in a strongly interacting medium of light quarks, antiquarks, and gluons. The size of such mesons is typically 1 fm ($\sim 1/\Lambda_{\mathrm{QCD}}$) and the momentum exchange between the heavy quark and light quark is of the order of Λ_{QCD}. In order to couple to the heavy quark, one has to use a hard (short distance) probe, but the gluons

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exchanged between light and heavy degrees of freedom are soft. Therefore, the light quarks largely are insensitive to the flavor and the spin orientation of the heavy quark; they only feel its color field. Effective heavy quark field theories and techniques have been developed to emphasize these aspects [20,21].

An issue that has central significance for studies of heavy quark systems is the definition of the mass of a heavy quark. Since isolated quarks are never found, one has to introduce a specific mass definition for each different framework. The pole mass is the one we are going to use for our studies but there are others like the $\overline{\text{MS}}$ mass, the Georgi-Politzer mass, the potential model mass, and the heavy quark effective field theory (HQET) mass [22–24]. The idea behind the estimation of a heavy quark pole mass, which is infrared finite and independent on the renormalization scheme, is that, since the running coupling is small at this scale, one can use perturbative QCD to calculate the quark propagator. Therefore the pole mass is a perturbative quantity. Since the quarks are confined, the exact propagator can not have a true mass pole and that fact by itself reveals the importance of nonperturbative infrared effects in the mass function of the quark. Confinement also creates ambiguity for a nonperturbative definition of a quark pole mass. The light-heavy quark meson has a pole mass that is a physical quantity which can, in principle, be used to extract the heavy quark pole mass by subtracting the binding energy which must be of the order of Λ_{QCD}. But the definition of a binding energy for absolutely confined particles is itself ambiguous. Any so-determined heavy quark pole mass will have then an uncertainty in the order of Λ_{QCD}. The use of the heavy quark pole mass concept for the studies of heavy quark systems is limited [25–27].

The concept of a constituent mass means a constant mass, independent of momenta and used within a calculation of a hadron and its properties. In our case we will compare results from such an assumption with results from the dressed quarks of our model.

III. EQUATIONS OF STUDIES: GAP AND BSE

As a basis of our studies, we are going to use two equations. These are the equation for the quark propagator (gap equation) and the BSE for quarks bound states. In Euclidean metric [28] gap equation is

$$S(p)^{-1} = Z_2(i\not{p} + m_{\text{bm}}) + Z_1 \int_q^\Lambda g^2 D_{\mu\nu}(k)\gamma_\mu \frac{\lambda^i}{2} S(q)\Gamma^i_\nu(p,q), \quad (1)$$

where $D_{\mu\nu}(k)$ is the renormalized dressed gluon propagator and $\Gamma^i_\nu(p,q)$ is the renormalized dressed quark-gluon vertex. Z_1, Z_2 are the gluon and quark propagator renormalization constants correspondingly, Λ is the regularization mass scale, and finally m_{bm} is the bare quark mass.

The homogeneous BSE for the bound state of quarks has the form:

$$[\Gamma^{ab}_M(p,P)]_{tu} = \int^\Lambda \frac{d^4\tilde{q}}{(4\pi)^4} K^{rs}_{tu}(p,\tilde{q},P)[S^a(\tilde{q}+\eta P) \times \Gamma^{ab}_M(\tilde{q},P)S^b(\tilde{q}-\bar{\eta}P)]_{sr}. \quad (2)$$

$\Gamma^{ab}_M(p,P)$ is the meson amplitude (BSA) with a and b the quark flavors, P is the total momentum, η ($\bar{\eta}$) in the propagators argument is the momentum partitioning parameter for the quark (antiquark) with $\eta + \bar{\eta} = 1$, $\eta \in [0,1]$ while M signifies the type of the meson: scalar, pseudoscalar, vector, or axial vector. The indices r, s, t, and u are for the combined color and Dirac matrix indices; $K^{rs}_{tu}(p,\tilde{q},P)$ is the unknown renormalized amputated irreducible quark-antiquark scattering kernel.

For the psudoscalar mesons, the most general form of the BSA has four invariants, while for the vector mesons has eight (see, for example, [29,30]). The invariant amplitudes are Lorentz scalar functions of q^2, P^2, $q\cdot P$, *and* the very important for our studies, momentum partitioning parameter η. The parameter for qQ mesons is chosen so that propagator singularities are outside the BSE integration domain. A four Chebychev polynomial expansion is used for each invariant.

The normalization condition for the bound state of two quarks of flavor a and b is

$$1 = \frac{\partial}{\partial P^2}\left\{\int_q^\Lambda \text{Tr}_{CD}[\bar{\Gamma}^{ab}_M(q,-K)S^a(q_+)\Gamma^{ab}_M(q,K)S^b(q_-)] + \int_q^\Lambda \int_k^\Lambda [[\bar{\chi}^{ba}_M(k,K)]_{ut} K^{rs}_{tu}(k,q,P) \times [\chi^{ab}_M(q,K)]_{sr}]\right\}\Bigg|_{K^2=P^2=-m^2}, \quad (3)$$

where $\chi^{ab}_M(q,K) = S^a(q_+)\Gamma^{ab}_M(q,K)S^b(q_-)$ is the meson wave function with $q_+ := q + \eta P$, $q_- := q - \bar{\eta}P$ the quark momenta, $\bar{\Gamma}^{ab}_M(q,K) = [C^{-1}\Gamma^{ab}_M(-q,K)C]^t$ is the antimeson BSA in which $C = \gamma_2\gamma_4$ is the charge conjugation matrix, and A^t denotes the transpose of the matrix A.

The definition of the electroweak decay constant f_H of a charged pseudoscalar meson is the following [31]:

$$\langle 0|\bar{q}^b\gamma_\mu\gamma_5 q^a|H^{ab}(P)\rangle = i f^{PS}_H P_\mu, \quad (4)$$

where $|H^{ab}(P)>$ is the meson state with total momentum P_μ. The meson state is normalized according to relation (3), and additionally the phase has been chosen in such a way that the decay constant is real and positive. Working out the details in the definition Eq. (4), we end up with the following expression for the pseudoscalars decay constant expressed in terms of the meson normalized BSA and quark propagators:

$$f_H^{PS} = \frac{Z_2 N_C}{P^2} \times \left\{ \int^\Lambda \frac{d^4q}{(2\pi)^4} P_\mu \times \mathrm{Tr}_D[\Gamma_M^{ab}(q,P) S^b(q_-)\gamma_\mu\gamma_5 S^d(q_+)] \right\}, \quad (5)$$

where $P^2 = -m_H^2$ and $N_C = 3$ is the number of colors, from the trace over the color indexes. Z_2, the quark propagator renormalization constant, is quark mass independed, and numerically it was found that $Z_2 = 0.970$ for all quark masses up to about 4 GeV, with a variation only in the fourth decimal point. A similar expression can be obtained for vector mesons.

IV. RAINBOW-LADDER TRUNCATION, MARIS-TANDY (MT) EFFECTIVE KERNEL, AND HEAVY QUARK MESONS

With the Rainbow-Ladder truncation, we can isolate the equations of interest from the rest of the system of infinite coupled nonlinear integral equations of QCD. In rainbow truncation for the gap equation, we set

$$Z_1 g^2 D_{\mu\nu}(q)\Gamma^i_\nu(p,q) \to 4\pi\alpha(q^2) D^{\mathrm{free}}_{\mu\nu}(q)\gamma_\nu \frac{\lambda^i}{2}, \quad (6)$$

where $D^{\mathrm{free}}_{\mu\nu}(q)$ is the free gluon propagator and $\alpha(q^2)$ is an effective running coupling. The ladder truncation for the BSE consists of replacing the Kernel K with

$$[K(p,q,P)]^{rs}_{tu} \to -4\pi\alpha(q^2) D^{\mathrm{free}}_{\mu\nu}(q)\left[\frac{\lambda^i}{2}\gamma_\mu\right]^{ru} \otimes \left[\frac{\lambda^i}{2}\gamma_\nu\right]^{ts}. \quad (7)$$

For the running coupling $\alpha(k^2)$, we are going to use the so-called MT model [31] that has the form

$$\frac{4\pi\alpha(k^2)}{k^2} = \frac{(2\pi)^2 k^2 D}{\omega^6} e^{-(k^2/\omega^2)} + \frac{2(2\pi)^2 \gamma_m F(k^2)}{\ln[\tau + (1 + \frac{k^2}{\Lambda^2_{\mathrm{QCD}}})^2]}. \quad (8)$$

For our calculations, we choose $\omega = 0.4$ GeV, $D = 0.93\ \mathrm{GeV}^2$, and $m_t = 0.5$ GeV and the u/d- and s-current quark masses at the renormalization scale $\mu = 19$ GeV fitted to the masses of pion and kaon are $m_{u/d}(19\ \mathrm{GeV}) = 0.00374$ GeV and $m_s(19\ \mathrm{GeV}) = 0.083$ GeV.

The MT model was developed within the Dyson-Schwinger equations approach and the rainbow-ladder truncation to study the physics of dynamical chiral symmetry breaking and related phenomena, like the spectrum of light quark mesons e.g. pions, kaons, and light vector mesons [30–32]. Studies of other physical observables like form factors, charge radii, and hadronic decays, of light mesons can be found in [30,33–42]. All estimations appear to be in very good agreement with experimental data. The QCD mechanisms responsible for the agreement are not so obvious. A more detailed and deeper understanding of QCD dynamics responsible for the light quark results is required.

An extension to heavy quark meson properties can inform us on the physical content. Early applications could be extended, at most, to charmonium, due to numerical problems in the heavy quark gap equation [36,43–46]. Singularities that are near to, but not within, the domain of integration require careful numerical work, which has only been achieved recently [47–49]. The model gave us masses and decay constants that are in surprisingly good agreement with experimental data. The unequal heavy quark meson studies (i.e. the D, B mesons) reveal the limits of the model (see [46] for more details). Finally, it will be interesting to explore the effectiveness of a constituent mass approximation for the heavy quark propagators, especially for the meson masses and electroweak decay constants.

V. MESON'S AMPLITUDE AND MASSES WITHIN A CONSTITUENT MASS DYNAMICS

The solution of the gap Eq. (1) has the general form

$$S(p) = \frac{1}{A(p^2)} \frac{1}{(i\not{p} + M(p^2))} = -i\not{p}\sigma_v(p^2) + \sigma_s(p^2), \quad (9)$$

where the amplitudes σ_s and σ_v are

$$\sigma_s(p^2) = \frac{1}{A(p^2)} \frac{M(p^2)}{p^2 + M^2(p^2)} \quad (a),$$
$$\sigma_v(p^2) = \frac{1}{A(p^2)} \frac{1}{p^2 + M^2(p^2)} \quad (b). \quad (10)$$

For the solution of the BSE, propagator amplitudes momentum p^2 is in general a complex number and varies within a parabolic region in the complex plane determined by the total momentum P and the partitioning parameter η. A very small region, near the peak, is of special importance due to the weight provided by the infrared part of the effective kernel in the BSE. The mass amplitude also varies slowly in that small region. The peak point, on mass shell, is at $q_+^2 \sim -(\eta M_{\mathrm{meson}})^2$ with M_{meson} the meson mass. One may approximate the quark mass amplitude in the whole parabolic region, and for the mass-shell solution of the BSE, with the ‘‘representative’’ value of the mass amplitude at the peak. We call that point ’’mass-shell’’ point.

In the constituent mass approximation, the heavy (c or b) quark propagator simplifies to

$$S(p)^{-1} \sim i\not{p} + M_c, \quad (11)$$

where M_c is the constituent mass. The propagator amplitudes then reduce to the following expressions:

$$\sigma_s(p^2) \sim \frac{M_c}{(p^2 + M_c^2)}, \qquad \sigma_v(p^2) \sim \frac{1}{(p^2 + M_c^2)}. \quad (12)$$

In essence we have more than one approximations involved in order to get the above expressions from the complete propagator. One is the constituent or pole mass approximation in the real part of the mass function $[\mathrm{Real}(M(p^2)) \sim M_c]$ and at the same time we ignore the imaginary part of this function $[\mathrm{Im}(M(p^2)) \sim 0]$ which becomes more important at the peak of the parabolic region. So the complex quark mass function is replaced by a real constant and geometrically that means we replace the surface of the real part of the mass amplitude by a flat surface in the complex plane. Unlike the ambiguous and problematic physical definition of the approximation, a simple and precise mathematical description is possible by just referring to as a constant quark mass function approximation. The other approximation regards amplitude $A(=1/Z)$ where we set $\mathrm{Real}(A(p^2)) \sim 1$, and $\mathrm{Im}(A(p^2)) \sim 0$. By plotting that function for the c quark propagator [46], one can see that it is generally over one in the area near the peak of the parabolic region (mass-shell point) and again the imaginary part of this amplitude is quite important over there. The significance and the different role of the imaginary parts of these two functions (M, A) in the solution of the BSE and the sensitivity of physical observables for the light-heavy and heavy-heavy quark systems has been studied in detail [46].

A plot used for justifying a constituent mass approximation for the heavy quarks is the one in Fig. 1. From this plot, one can readily see that there are very strong infrared dressing effects for chiral quarks and the u/d and s quark making them much heavier in the infrared region. The dressing is relatively smaller for the much heavier c and b quarks, and their mass functions are almost flat lines; therefore, one can assume they can be well approximated by a constant. We should notice though that the plot is in logarithmic scale (both axes). The behavior of the mass amplitude and propagator amplitude $A(p^2)$ (real and imaginary parts) for the timelike or complex p^2, can be important influences upon the existence of a solution of the BSE.

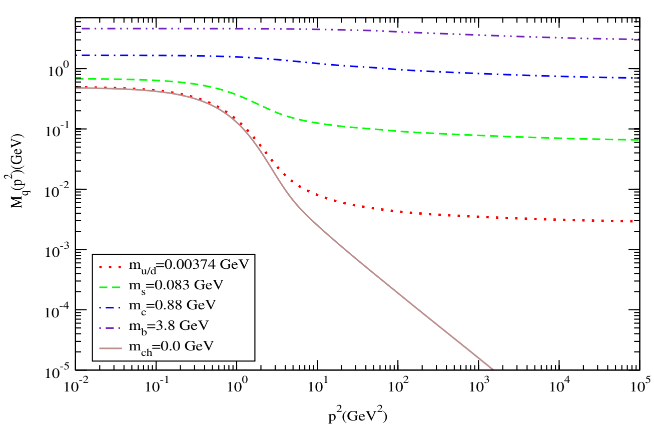

FIG. 1 (color online). Quark mass functions in the real spacelike axis, for the chiral limit, u/d, s, c, and b quarks. Current quark masses are at $\mu = 19$ GeV.

The fitted constituent mass for the c quark was found to be $M_c^c \sim 2.0$ GeV and for the b quark $M_c^b \sim 5.3$ GeV. For the light quarks, we use, for reasons of convenience, a three complex conjugate pole representation (3ccp) [Eq. (13)] that has been previously used [50] and studied in detail [46,51]:

$$S(q) = \sum_{k=1}^{3} \left(\frac{z_k}{i\not{q} + m_k} + \frac{z_k^*}{i\not{q} + m_k^*} \right). \tag{13}$$

m_k, z_k are complex parameters fitted to the propagator dynamical solution on the real axis. Besides the nonperturbative effects absorbed in the values of the constituent mass parameters, some error, because of the use of the light quark 3ccp representation, may be present, but considering the results of a previous analysis [46], we can safely assume that it is relatively very small (in the second decimal point in the fitted constituent masses) and hence insignificant for the purpose of the present studies. The results of studies where we actually solve the gap equation for the light quark propagator in the parabolic region (in the complex plane) needed for the solution of the BSE verify the last statement.

By varying parameter η, we can change the parabolic domain of the quark propagator sampled during the solution of the BSE. In that way we can move closer or further away from the propagator singularities and determine any influence in the calculated observables. The exact location and the type of the singularities are not known for the dynamical propagator. The knowledge of the exact location of the lowest pair of poles in the light quark propagator 3ccp representation makes it easier to determine the range of allowable values of η, so that these poles and the constituent mass pole of the heavy quark propagator are always outside of their integration domain. In earlier studies [31,38], it was found that if you do not have the complete Dirac structure for the meson amplitude (BSA) and/or if more Chebychev polynomials in the invariants' angle expansion are required for convergence, then the calculated observables will show a dependence on the value of η. Therefore, the variation of η can also serve as an indirect check for the convergence of the Chebychev polynomial expansion. In addition, we can check for any systematic uncertainties introduced by the light quark propagator 3ccp representations. For all these reasons, the calculations were repeated for two or three different values of η in the allowable range, even for equal quark mesons.

A direct comparison of the quark propagator amplitudes from the constituent mass approximation and the gap solution, in the real axis only is shown in Figs. 2 and 3. One can notice the stronger behavior of σ_s over that of σ_v near the mass-shell position because of the presence of the quark mass function in the numerator of σ_s. Far from the singularities, a better agreement is possible. As the current quark mass increases, there is comparatively less dressing,

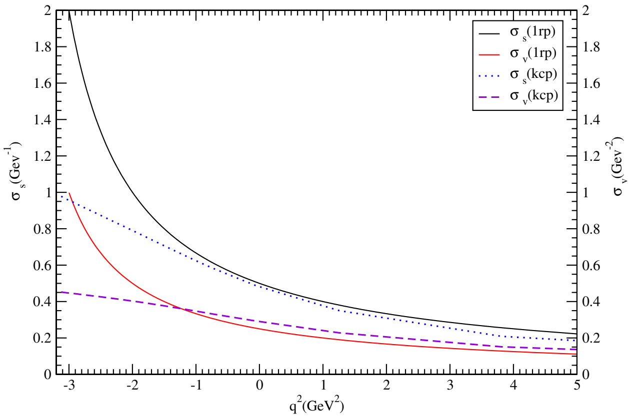

FIG. 2 (color online). c quark propagator amplitudes on the real axis and near the mass-shell point in the timelike region, from the constituent mass approximation ($M_c^c = 2.0$ GeV), noted as 1rp, compared to the DSE solution.

and the lower singularities of the propagator will be closer to the dominant region of the complex domain needed for the meson bound state calculations. That is $q_+^2 \sim (\eta P)^2 \sim -\mathrm{Re}\{M^2[(\eta P)^2]\} \sim -m_q^2(\mu)$ where $P^2 = -(M_{\mathrm{meson}})^2$ and $M_{\mathrm{meson}} \sim 2m_q$. The first pair of poles of the 3ccp representation for the b quark produce the real part of the pole location on q_+^2 plane to be at $m_r^2 - m_i^2 = 27.977\ \mathrm{GeV}^2$ which is very close to the pole location of the constituent mass approximation $(M_b^c)^2 = 28.09\ \mathrm{GeV}^2$. This is also very close to the real part of the quark mass function near the peak of the mass-shell parabolic region. We should notice here that for the heavy quarks studies we use the fitted current masses $m_c(19\ \mathrm{GeV}) = 0.88$ GeV, and $m_b(19\ \mathrm{GeV}) = 3.8$ GeV from [43].

The results for the c and b quark mesons masses are presented in Table I (see also [49]).

All calculated masses appear to be in excellent agreement with experiment with a deviation of no more than 3.1%. Variation of parameter η within the allowable region

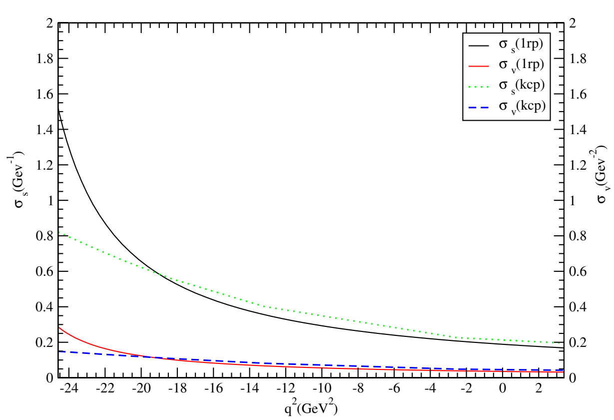

FIG. 3 (color online). b quark propagator amplitudes on the real axis and near the mass-shell point in the timelike region, from the constituent mass approximation ($M_c^b = 5.3$ GeV), noted as 1rp, compared to the DSE solution.

TABLE I. qQ and QQ pseudoscalar and vector meson masses using constituent mass approximations (noted as 1rp) for the c, b quark propagators. The pseudoscalar mesons $D(uc)$ and $B(ub)$ experimental masses were used to determine the values of the constituent c and b quark mass parameters. The last column has the percentage differences between experimental and calculated hadron masses $\Delta m/m^{\mathrm{exp}} = (m^{1\mathrm{rp}} - m^{\mathrm{exp}})/m^{\mathrm{exp}}$. No experimental data are available for the $B_c^*(cb)$ vector meson mass. The absolute relative percentage difference is from 0.13%–3.26% and no increasing difference between experimental and calculated masses with increasing quark mass is observed . Experimental data are from [52] and η_b mass from [53].

Meson	Exp. (GeV)	1rp (GeV)	$\Delta m/m^{\mathrm{exp}}\%$
$D(uc)$	1.864	1.852	−0.6
$D^*(uc)$	2.007	2.04	+1.6
$D_s(sc)$	1.969	1.975	+0.3
$D_s^*(sc)$	2.112	2.17	+2.8
$B(ub)$	5.279	5.254	−0.5
$B^*(ub)$	5.325	5.32	−0.1
$B_s(sb)$	5.370	5.38	+0.2
$B_s^*(sb)$	5.413	5.42	+0.1
$\eta_c(c\bar{c})$	2.980	3.025	+1.5
$J/\psi(c\bar{c})$	3.097	3.192	+3.1
$B_c(cb)$	6.286	6.36	+1.2
$B_c^*(cb)$		6.440	
$\eta_b(b\bar{b})$	9.389	9.603	+2.3
$\Upsilon(b\bar{b})$	9.460	9.645	+2.0

revealed that overall for pseudoscalar mesons a third decimal point (or more) in the masses can be trusted, while for the vector mesons, since the amplitude has eight invariants, there was a weak variation in the masses, usually less than 1% of the mass. This might be due to the influence of the singularities (BSA amplitude is now more sensitive and we also have to move closer to the singularities) and/or due to nonconvergence of the Chebychev polynomial expansion. The influence of these two can be distinguished only if we repeat the calculations with more polynomials in the expansion or use no expansion at all. Notice also that although the singularities are always outside of the integration domain, it is not possible to determine beforehand how far we need to be to reach a bound state solution. The influence of the constituent mass pole and the complex conjugate mass poles of the light quark representation is also different. For the present purposes, the achieved agreement between calculated and experimental D and B meson masses, for the estimation of the constituent c and b quark masses, and the accuracy in the rest of the calculated meson masses is deemed adequate to safely draw the desired conclusions.

The reason for this successful description of the heavy quark meson masses is due to the behavior of the mass function near the constituent mass shell point (peak of the parabolic region) and the infrared weight provided by the MT effective kernel. We next examine the deviation of the calculated BSA. For the quarkonia, we do this by compar-

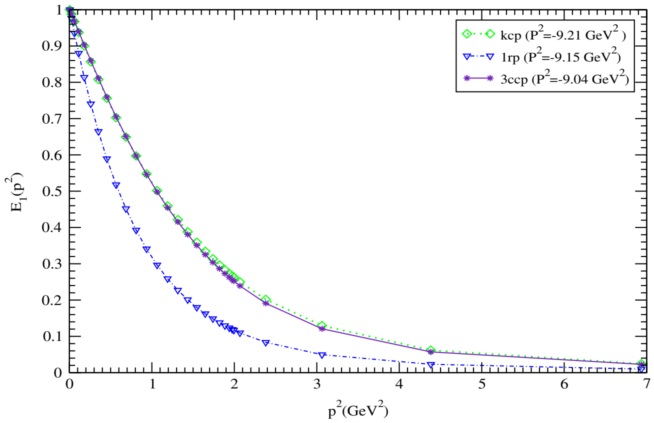

FIG. 4 (color online). First Chebychev moment of the dominant invariant amplitude of the $c\bar{c}$ pseudoscalar meson by using the DSE dynamical solution (kcp), the 3ccp propagator representation, and the constituent mass approximation (1rp). In the parenthesis for each line we indicate the meson total momentum squared.

ing the Chebychev moments of the amplitudes. In addition, we include results from using a three complex conjugate pole mass representation for the heavy quarks.

Figures 4 and 5 display the first Chebychev moment of the dominant invariant of the amplitude of the pseudoscalar quarkonia η_c, η_b. The effect of the constituent mass approximation for the heavy quark propagator is a faster fall off of the moments with momentum i.e. a larger meson in coordinate space. To see whether the difference in relative momentum dependence might be due to the different mass-shell locations, we calculate the amplitude relative momentum dependence of the 3ccp and constituent mass [one real pole(1rp)] approximation evaluated at the mass shell of the full dynamical solution [solution in the complex plane with the gluon momentum k as integration variable (kcp)]. We found there is no significant change and we can safely conclude that the amplitude differences are a true consequence of the different dynamics of the propagator approximations. The same pattern appears to hold true for the vector quarkonia. Heavy quark dressing effects appear to strengthen the binding of the quarks. For the qQ systems, a similar analysis is not possible since a meson mass-shell solution is not attained with the dynamical heavy quark propagator.

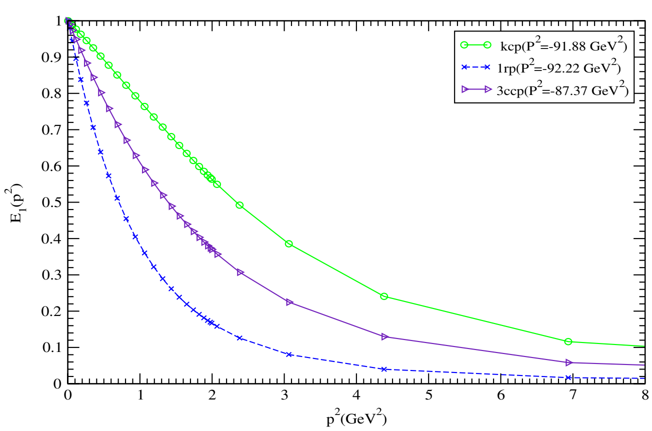

FIG. 5 (color online). First Chebychev moment of the dominant invariant amplitude of the $b\bar{b}$ pseudoscalar meson by using the DSE dynamical solution (kcp), the 3ccp propagator representation, and the constituent mass approximation (1rp). In the parenthesis for each line we include the meson total momentum squared.

VI. CONSTITUENT MASS DYNAMICS EFFECTS ON MESON ELECTROWEAK DECAY CONSTANTS

Using the constituent mass approximation, we calculate the mesons electroweak decay constants. Table II contains these results (see also [49]).

Unfortunately, there are not many experimental data for the electroweak decay constants of qQ vector mesons, but the available data for the other mesons are enough to get a clear picture of the pattern in the calculated decays. Overall it appears that the calculated decay constants are much smaller than the experimental data and the situation is rapidly deteriorating as we go to heavier quark systems. The relative difference for the $\Upsilon(b\bar{b})$ meson is -70.0%, so it appears that, even for the relatively simpler quarkonia systems, the agreement is poor. Intuitively, one would expect that the constituent mass approximation will get better for heavier quarks but the results for the electroweak decay constants indicate the opposite. To help understand how this comes about, we will use meson amplitudes obtained from constituent mass approximations and the dynamical propagator. Since we have no dynamical solution (kcp) for the qQ meson systems, we will use the 3ccp model for the heavy quarks in the N (normalization), f

TABLE II. qQ and QQ pseudoscalar and vector meson electroweak decay constants using constituent mass approximation for the c, b propagators. The last column has the percentage differences between experimental and calculated hadron decays $\Delta f/f^{\rm exp} = (f^{\rm 1rp} - f^{\rm exp})/f^{\rm exp}$. Experimental data are from [52].

Meson	Exp. (GeV)	1rp (GeV)	$\Delta f/f^{\rm exp}\%$
$D(uc)$	0.223	0.154	−31.1
$D^*(uc)$		0.164	
$D_s(sc)$	0.294	0.197	−33.0
$D_s^*(sc)$		0.180	
$B(ub)$	0.176	0.105	−40.3
$B^*(ub)$		0.182	
$B_s(sb)$		0.144	
$B_s^*(sb)$		0.20	
$\eta_c(c\bar{c})$	0.340	0.239	−29.7
$J/\psi(c\bar{c})$	0.416	0.198	−52.4
$B_c(cb)$		0.210	
$B_c^*(cb)$		0.18	
$\eta_b(b\bar{b})$		0.244	
$\Upsilon(b\bar{b})$	0.700	0.21	−70.0

(decay constant) loop integrals to contrast with the constituent mass approximation. For the quarkonia states, the results are in Table III.

The second column of Table III uses the constituent mass bound state amplitudes. We see that the decay constant differs by 0.088 GeV if the N, f integrals are computed using the dynamical (kcp) quark propagator. The corresponding difference for the full dynamical bound state (kcp) is 0.065 GeV. These differences are very close and reveal the effect of the different dynamics of the propagators in the decay constant value. On the other hand, the difference of the decays in the third row (0.322 GeV and 0.239 GeV) is 0.083 GeV and for the ones in the last row (0.327 GeV and 0.387 GeV) is 0.06 GeV. This shows the influence of the dressed bound state amplitudes on the decay constant. It appears that both 1rp propagators and 1rp BSA are about equally accountable for the very small value of the η_c decay constant. Also, we see that the 3ccp representation of propagators captures the full dynamical solution very well. For the corresponding vector meson, we reach the same conclusion, only now the differences (about 0.11 GeV) resulting from the 1rp BSA or 1rp c propagator are larger than before.

For the b quarkonia, we see, in general, a deteriorating situation. For the pseudoscalar meson, we have for both cases a 0.201 GeV difference due to the dynamics of the constituent mass b propagator (more than twice the difference in the pseudoscalar c quarkonium), and for the differences due to the constituent mass BSA we have 0.248 GeV and 0.248 GeV, so it appears now that the 1rp BSA is slightly more responsible for the very small decay. For the vector meson, we have 0.240 GeV, 0.258 GeV, 0.215 GeV, and 0.233 GeV for the corresponding cases. Therefore, we see there is a more balanced contribution form the 1rp propagator and BSA in the difference of the decay. The 3ccp model lacks the success it had in the calculation of the c quarkonia.

Overall we can conclude that the constituent quark mass approximation will give us smaller decay constants than the complete solution and the difference will increase as we go to heavier quarks. We found that the quark dressing indirectly through the BSA and directly in the N, f loop integrals, will have about the same responsibility for getting so small decay constants. It is very easy to understand the reason for the influence the constituent propagator has directly in these two integrals if we just notice that the total momentum derivative in the N integral in this case will act only in the momentum $q_{\pm}^2$, since it is the only P dependent quantity. With the full propagator though, we will get eight additional terms from the action of the operator in M and A amplitudes (they have real and imaginary parts). So at the end not only will augment the influence of the mass amplitude in the normalization integral by increasing the power of the denominator of the propagator amplitudes when it acts in the momenta $q_{\pm}^2$, but we will also have additional terms when it acts directly on the two functions (notice one of these terms will be $M \cdot \partial_P M$). All of these additional contributions from the extra terms are lost in the constituent mass approximation, making the decay constants very sensitive observables on quarks dressing.

For the light-heavy systems, the use of the heavy 3ccp model in the place of the 1rp propagator when calculating N, f gave us the results in Table IV. For reasons of completeness and comparison, we include the 3ccp decay constant results for quarkonia.

Before we start analyzing and discussing the data of the table, we can infer something about the approximation without inspecting the data. We found that the Dyson-

TABLE III. $\eta_c(c\bar{c})$, $\eta_b(b\bar{b})$ pseudoscalar, $J/\psi(c\bar{c})$, $\Upsilon(b\bar{b})$ vector meson decay constants by using a BSA(Γ) calculated with constituent mass approximation (1rp), representation (3ccp) or dynamical (kcp) c, $\bar{c}$ and b, $\bar{b}$ quark propagators and using 1rp, 3ccp, or kcp propagators in the calculations for N and f integrals for each case, indicated in the first column of the table. The index in Γ signifies the type of quark propagator used for the calculation of the meson amplitude. Here, $\eta = 0.50$ but calculations were also done for $\eta = 0.45$, 0.55. All calculated decay constants are in GeV.

c, $\bar{c}$	$f_{\eta_c}^{\exp} = 0.340$ GeV			$f_{J/\psi}^{\exp} = 0.416$ GeV		
N, f integ.	$\Gamma_{1\text{rp}}$	$\Gamma_{3\text{ccp}}$	Γ_{kcp}	$\Gamma_{1\text{rp}}$	$\Gamma_{3\text{ccp}}$	Γ_{kcp}
1rp, 1rp	0.239	0.319	0.322	0.198	0.296	0.308
3ccp, 3ccp	0.326	0.362	0.372	0.330	0.340	0.368
kcp, kcp	0.327	0.378	0.387	0.308	0.382	0.415
b, $\bar{b}$	$f_{\eta_b}^{\exp} = ?$			$f_{\Upsilon}^{\exp} = 0.700$ GeV		
N, f integ.	$\Gamma_{1\text{rp}}$	$\Gamma_{3\text{ccp}}$	Γ_{kcp}	$\Gamma_{1\text{rp}}$	$\Gamma_{3\text{ccp}}$	Γ_{kcp}
1rp, 1rp	0.243	0.398	0.491	0.209	0.342	0.449
3ccp, 3ccp	0.414	0.547	0.647	0.381	0.515	0.628
kcp, kcp	0.444	0.571	0.692	0.424	0.535	0.682

TABLE IV. Pseudoscalar and vector mesons decay constants with constituent mass approximation for the c or b quark propagators in the BSE and 1rp or 3ccp in f and N integrals. Experimental data are from [52].

Meson	Exp. (GeV)	$f^{1\text{rp}}$ (GeV)	$f^{3\text{ccp}}$ (GeV)
$D(uc)$	0.222	0.154	0.255 ± 0.010
$D^*(uc)$		0.164	0.288 ± 0.030
$D_s(sc)$	0.294	0.197	0.255 ± 0.005
$D_s^*(sc)$		0.180	0.326 ± 0.040
$B(ub)$	0.176	0.105	0.193 ± 0.005
$B^*(ub)$		0.182	0.549 ± 0.083
$B_s(sb)$		0.144	0.212 ± 0.002
$B_s^*(sb)$		0.20	0.425 ± 0.041
$\eta_c(c\bar{c})$	0.340	0.239	0.326
$J/\psi(c\bar{c})$	0.416	0.198	0.330
$B_c(cb))$		0.210	0.324 ± 0.004
$B_c^*(cb))$		0.18	0.328 ± 0.008
$\eta_b(b\bar{b})$		0.244	0.414
$\Upsilon(b\bar{b})$	0.700	0.210	0.381

Schwinger equations (DSE) solution for the heavy quark propagator failed to generate a bound state for the qQ mesons, but with the constituent mass approximation we did reach a physical state. Thus the constituent mass approximation does not reproduce the dynamics of the DSE dynamical propagator obtained with the effective MT interaction. This indicates that the dynamical propagator is deficient for heavy quarks because it departs from a constituent mass supporting behavior. On the other hand quarks are only found inside hadrons and cannot propagate macroscopically far from a hadron. Hence quarks cannot truly have a real pole mass. The fact that we obtain qQ bound states using a constituent mass approximation for the heavy quark, but not using a MT dynamical heavy quark propagator indicates that both are not very good representations of nature. It does not indicate that a constituent heavy quark mass approximation is generally better. It also indicates that the MT dynamical model has inadequacies that are minor for quarkonia but more serious for qQ mesons. From the data in Table IV we see that by use of the 3ccp representation for the heavy quark propagator gives us, decay constants that are much larger than the ones we have with the constituent mass propagator. The calculations for different values of η revealed a small dependence of the decay constants on that parameter, and for that we can blame the proximity of the singularities of the propagator. Nevertheless the decay constants now are closer to experiment, therefore the constituent mass qQ meson amplitudes (*unlike the QQ ones*) are of reasonable accuracy for their decay estimations. The explanation for this is rather simple. For QQ systems, we have essentially two propagators approximated by constituent-like ones while for the qQ mesons there is only one approximate propagator. The same reasoning can be applied for the calculations of the N, f loop integrals.

The variation of amplitudes M and A of the propagator along the real and imaginary p^2 axes near the mass-shell point are influenced, to a different degree as the quark mass increases, by the infrared behavior (infrared dressing effects) of the MT kernel. The derivatives of these functions will appear in the normalization integral and the latter is a more sensitive probe for a larger area in the mass-shell parabolic region in the complex plane. An approximation that was good for the meson mass is not necessarily good for a decay constant. The same is true for the approximations to the bound state amplitudes. The decay constant integrals show a sensitivity to self-energy corrections to the pole propagator and through them reveal their sensitivity to the MT model in the infrared region.

VII. THE EXTREME HEAVY QUARK LIMIT.

There are relations [12] for the masses and decay constants of heavy quark mesons that have been extracted by using the heavy quark limit behavior of the propagator and a dimensional analysis of the involved equations. The approximation for the behavior of the propagator is based on the fundamental assumption that *all* the meson momentum is carried by the heavy quark alone. In this case the momentum of the quark will be

$$P_\mu = M_H v_\mu = (m_Q + E_H) v_\mu, \tag{14}$$

where P_μ is the total momentum of the meson of mass M_H with a heavy quark of mass m_Q, and energy E_H. Then the heavy quark propagator can be written as [12]

$$S_Q(q+P) = \frac{1}{2}\frac{-i\not{v}+1}{q.v - E_H} + \mathcal{O}\left(\frac{|q|}{m_Q}, \frac{E_H}{m_Q}\right), \tag{15}$$

where q is the relative quark momentum and the dimensionless velocity satisfies $v^2 = -1$. This approximation is considered to work satisfactorily well for qQ mesons as far as the binding energy of the system and the momentum scale, defined as the width of the meson BSA, are much smaller than the heavy quark mass. The width ($\langle q \rangle$) of the BSA gives a measure of how fast the amplitude decreases with momentum and a typical definition, is the momentum where the amplitude falls in half of its value at the origin. The definition of a binding energy as we explained is ambiguous. We know that the difference between the meson mass and the heavy quark mass will be roughly of the order of $\Lambda_{\rm QCD} \sim 0.2$ GeV, and we assume this to be also the order of the binding energy. Therefore, since a typical constituent mass for the c quark was found to be about 1.3 GeV and for the b about 4.6 GeV, this was considered a reasonable approximation (see, for example, [11,12]). The consequences of the other approximation are more complicated ones and essentially require the beforehand knowledge of the behavior of the complete solution of the BSE. In the studies for the qQ systems using a pole mass heavy quark propagator, it was found that the Chebychev moment of the dominant invariant amplitude does not decrease faster with the momentum as the Q quark mass increases but the quality of the result is unknown, hence the reliability of the approximation as well. At the end it was found that in this case the decay constant of the light-heavy mesons should behave like [12]

$$f_H \sim 1/\sqrt{M_H}, \qquad 1/\hat{m}_Q \sim 0. \tag{16}$$

Using the same analysis it was found for the so-called in-hadron quark condensate:

$$-\langle q\bar{q}\rangle_\mu^H = \text{const} + \mathcal{O}(1/M_H), \quad \text{if } 1/M_H \sim 0, \tag{17}$$

and use of these in the pseudoscalar mass formula gives

$$M_H \sim \hat{m}_Q, \quad \text{if } 1/\hat{m}_Q \sim 0, \tag{18}$$

where $\hat{m}_Q$ is the renormalization-group-invariant current quark mass for the heavy quark. This last result is something one should intuitively expect, as well as the fact that the mass difference between pseudoscalar and vector mesons should get smaller as $\hat{m}_Q \to +\infty$, and they have been both numerically confirmed. Our studies of qQ mesons using a constituent mass approximation for the heavy quark propagator indicate the electroweak decay constants in the range of current mass up to the b quark (3.8 GeV) do not follow that behavior. Our calculated electroweak decay constants using a constituent mass approximation are much smaller than the experiment and probably not reliable conclusion can be drawn.

Going back to the initial step of the analysis for the light-heavy quark mesons, it was assumed that all the meson momentum is carried by the heavy quark and hence one can take $\eta = 1$. For our analysis, we will try to find an approximated form by keeping η general. Then, we have for the denominator of the propagator

$$\begin{aligned} D &= q_+^2 + M^2(q_+^2) \\ &= (1-\eta^2)M_H^2 + 2(i\eta|\tilde{q}|v + E)M_H + \tilde{q}^2 + E^2, \end{aligned} \tag{19}$$

where v is now the cosine of the angle between the four vectors $\tilde{q}$ (the quark relative momentum and BSE integration variable), and the total meson momentum P (with $P^2 = -M_H^2$). We also used the expression $M_H = M_Q + M_q - E \sim M_Q - E$ so $M_Q = M_H + E$, where E is the binding energy of the system, M_q and M_Q are the constituent masses for the light and heavy quarks correspondingly. We assume $M_q \ll M_Q$, so the light quark mass can be ignored, and finally made the approximation $M(q_+^2) \sim M_Q$ for the whole parabolic BSE integration region. Notice that the constant coefficient of M_H^2 is $(1-\eta^2)$ and this term does not exist if $\eta = 1$, dramatically changing the dependence of the denominator on M_H. So in this case if we write $P = iM_H u$ where u is a unit 4-vector, we get for the propagator:

$$S_Q(q_+) \sim \frac{(1+\eta \not{u})M_H + (E - i\tilde{\not{q}})}{D}, \tag{20}$$

and at the end, we can write

$$\begin{aligned} S_Q(q_+) \sim \frac{1}{M_H}\frac{(1+\eta\not{u})}{(1-\eta^2) + 2(i\eta|\tilde{q}|v + E)/M_H} \\ + \mathcal{O}\left(\frac{|\tilde{q}|}{M_H}, \frac{E}{M_H}\right). \end{aligned} \tag{21}$$

By replacing $E \to -E_H$, $u \to -iv$, $\tilde{q} \to q$ to match the notation in Ref. [12] and setting $M_H \sim M_Q$, we have

$$\begin{aligned} S_Q(q_+) \sim \frac{1}{M_Q}\frac{(1 - i\eta\not{v})}{(1-\eta^2) + 2(\eta q.v - E_H)/M_Q} \\ + \mathcal{O}\left(\frac{|q|}{M_Q}, \frac{E_H}{M_Q}\right), \end{aligned} \tag{22}$$

which gives [Eq. (15)] for $\eta = 1$

It is clear in this case that the heavy quark limit of the propagator has an η dependence that significantly modifies its dependence on the heavy quark mass. On the other hand, relations in (16)–(18), obtained by dimensional analysis arguments *for* $\eta = 1$, concern physical observables, so they should be independent of η [54]. Therefore, they should not hold true for the general case of (22) since that will introduce a different dependence on M_Q.

In Ref. [43] the calculated masses of q_1q_2 mesons up to about 0.8 GeV current quark masses were fit by the form

$$M_H^2 = M_0^2 + a_1(m_1 + m_2) + a_2(m_1 + m_2)^2, \tag{23}$$

where m_1, m_2 are the current quark masses at $\mu =$ 19 GeV, and the parameters of the relation fitted to the numerical data for the meson masses are $M_0^{PS} = 0$ GeV, $M_0^V = 0.75$ GeV, $a_1^{PS} = 2.96$ GeV, $a_1^V = 3.24$ GeV, while for both vector and psudoscalar mesons $a_2 =$ 1.12 GeV. We also notice that as $m_2 \to +\infty$, we have $M_H^2 \to a_2(m_2)^2$ (for both vector and pseudoscalar) and for the quarkonia $M_H^2 \to a_2(2m_Q)^2$ since $m_1 = m_2 = m_Q \to +\infty$. That relation was actually designed to have this behavior in the large quark limit. Expression (23) and the experimental masses of c and b pseudoscalar quarkonia were used to determine the c and b current quark masses in our studies.

VIII. CONCLUSIONS

In this work we explored the constituent mass concept for heavy quarks using a nonperturbative approach. For $c\bar{c}$ and $b\bar{b}$ mesons, a constituent mass approximation yields very good mass results but the electroweak decay constants are too low by some 30%–70%. Use of dynamically dressed propagators removes almost all of this deficiency and the decay constants are within 20% of the experiment. This improvement provided by dynamical dressing of c and b quarks is persistent and systematic in the sense that, when the dressing is progressively introduced into all three stages (bound state solution, normalization loop integral, and then the loop integral for evaluation of the decay constant), the final value always increases towards the experimental value. This indicates that a constituent mass approximation, even for b quarks is inadequate. Small departures from a strictly constant mass function and renormalization function Z for quarks in the relevant region of the complex plane are magnified due to the very weak binding of the mesons in question. These small deviations include confinement effects that appear to be, in general, important even for heavy quarks. The results of the MT model for heavy quark propagator dressing are not well reflected by a constituent mass approximation for the propagator.

However, our findings in the case of heavy-light mesons indicate that the role of self-energy dressing is a much more complicated and subtle topic. With fully dressed quark propagators, our model does not provide a physical bound state solution for qc, qb, $q = u/d$, s, or even cb

mesons. Such physical states are easily obtained with a constituent mass approximation, but again the indication is that the decay constants are too low. Therefore, it is evident that the constituent mass approximation is not reliable even for b quarks, and that our ladder-rainbow model kernel has deficiencies in the heavy quarks region that are masked in QQ mesons but are plainly evident in qQ mesons.

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