Quark model description of the tetraquark state $X(3872)$ in a relativistic constituent quark model with infrared confinement

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(Received 9 April 2010; published 4 June 2010)

We explore the consequences of treating the $X(3872)$ meson as a tetraquark bound state. As dynamical framework we employ a relativistic constituent quark model which includes infrared confinement in an effective way. We calculate the decay widths of the observed channels $X \to J/\psi + 2\pi(3\pi)$ and $X \to \bar{D}^0 + D^0 + \pi^0$ via the intermediate off-shell states $X \to J/\psi + \rho(\omega)$ and $X \to \bar{D} + D^*$. For reasonable values of the size parameter Λ_X of the $X(3872)$ we find consistency with the available experimental data. We also discuss the possible impact of the $X(3872)$ in a s-channel dominance description of the J/ψ dissociation cross section.

DOI: 10.1103/PhysRevD.81.114007 PACS numbers: 12.39.Ki, 13.25.Ft, 13.25.Jx, 14.40.Rt

I. INTRODUCTION

A narrow charmoniumlike state $X(3872)$ was observed in 2003 in the exclusive decay process $B^\pm \to K^\pm \pi^+ \pi^- J/\psi$ [1]. The $X(3872)$ decays into $\pi^+\pi^- J/\psi$ and has a mass of $m_X = 3872.0 \pm 0.6(\text{stat}) \pm 0.5(\text{syst})$ very close to the $M_{D^0} + M_{D^{*0}} = 3871.81 \pm 0.25$ mass threshold [2]. Its width was found to be less than 2.3 MeV at 90% confidence level. The state was confirmed in B-decays by the *BABAR* experiment [3] and in $p\bar{p}$ production by the Tevatron experiments CDF [4] and D0 [5]. The most precise measurement up to now was done in [6] with $M_X = 3871.61 \pm 0.16 \pm 0.19$. The new average mass given in [4] is

$$M_X = 3871.51 \pm 0.22 \text{ MeV}. \tag{1}$$

From the observation of the decay $X(3872) \to J/\psi\gamma$ reported by both the Belle [7] and *BABAR* [8] Collaborations and from the angular analysis performed by CDF [9], it was shown that the only quantum numbers compatible with the data are $J^{PC} = 1^{++}$ or 2^{-+}. However, the observation of the decays into $D^0\bar{D}^0\pi^0$ by the Belle and *BABAR* Collaborations [10,11] allows one to exclude the choice 2^{-+} because the near-threshold decay $X \to D^0\bar{D}^0\pi^0$ is expected to be strongly suppressed for $J = 2$.

The Belle Collaboration has reported evidence for the decay mode $X \to \pi^+\pi^-\pi^0 J/\psi$ with a strong three-pion peak between 750 MeV and the kinematic limit of 775 MeV [7], suggesting that the process is dominated by the subthreshold decay $X \to \omega J/\psi$. It was found that the branching ratio of this mode is almost the same as that of the mode $X \to \pi^+\pi^- J/\psi$:

$$\frac{\mathcal{B}(X \to J/\psi\pi^+\pi^-\pi^0)}{\mathcal{B}(X \to J/\psi\pi^+\pi^-)} = 1.0 \pm 0.4(\text{stat}) \pm 0.3(\text{syst}). \tag{2}$$

These observations imply strong isospin violation because the three-pion decay proceeds via an intermediate ω-meson with isospin 0 whereas the two-pion decay proceeds via the intermediate ρ-meson with isospin 1. Also the two-pion decay via the intermediate ρ-meson is very difficult to explain by using an interpretation of the $X(3872)$ as a simple $c\bar{c}$ charmonium state with isospin 0. There are several different interpretations of the $X(3872)$ in the literature:

(i) The $X(3872)$ is a molecule bound state ($D^0\bar{D}^{*0}$) with small binding energy (see, Refs. ([12–25]));
(ii) The $X(3872)$ is a tetraquark state composed of a diquark and antidiquark (see, Refs. ([26–31]));
(iii) The $X(3872)$ should be interpreted in terms of threshold cusps [32];
(iv) The $X(3872)$ should be interpreted in terms of hybrids [33] and glueballs [34].

A description of the current theoretical and experimental situation for the new charmonium states may be found in the reviews [35–43].

The most intriguing question at present is whether the $X(3872)$ is a loosely-bound charm-meson molecule with a binding energy of $M_X - (M_{D^{*0}} + M_{D^0}) = -0.30 \pm 0.40$ MeV, or a tetraquark composed of a color diquark and color antidiquark. Recently, the molecular nature of the $X(3872)$ has been carefully discussed in the literature. The authors of [44] assumed that the $X(3872)$ is a $D^0\bar{D}^{*0}$ molecule and, based on this assumption, estimated its prompt production cross section at the Fermilab Tevatron. They presented a theoretical upper limit on the prompt production cross section which is about 30 times smaller than the observed prompt production rate at the Tevatron. The conclusion was that S-wave resonant scattering is unlikely to allow the formation of a loosely bound $D^0\bar{D}^{*0}$ molecule in high-energy hadron collisions. In Ref. [45] it was argued that one needs to take into account charm-meson rescattering in the analysis of [44]. As a consequence the theoretical upper limit is increased by orders of magnitude and is then compatible with the observed production rate at the Tevatron. This conclusion was later on criticized in [46]. First, using the results of [45], the authors of [46] argue that a new unobserved $X_s(4080)$ molecule composed of a $D_s\bar{D}_s^*$ pair should have been observed at the Tevatron. Second, they cast some doubts on the applicability of the Watson theorem for final state interactions in the above calculation.

The tetraquark state interpretation of the $X(3872)$ was successfully applied to describe the available experimental data on the decays $X \to J/\psi\pi^+\pi^-$ and $X \to J/\psi\pi^+\pi^-\pi^0$ [26]. By using an effective three-meson Lagrangian with the coupling taken from a similar analysis of light scalar mesons, the authors found the width of the X-meson to be 1.6 MeV in accordance with the experimental bound. Contrary to this, the values of the widths calculated by using QCD sum rules were found to be too large—around 50 MeV [29].

In this paper we provide an independent analysis of the properties of the $X(3872)$ meson which we interpret as a tetraquark state as in [26]. We work in the framework of the relativistic constituent quark model which has recently been extended to include infrared confinement effects [47]. The improved model [47] is a successful generalization of the relativistic constituent quark model which some of us have developed over many years [48–52]. The relativistic constituent quark model can be viewed as an effective quantum field theory approach to hadronic interactions based on an interaction Lagrangian of hadrons interacting with their constituent quarks. Once the relevant interpolating quark current is written down one can evaluate the matrix elements of the physical processes in a self-consistent way. The nice feature of this approach is that multiquark systems as e.g. baryons and tetraquarks can be treated on the same footing as the simplest quark-antiquark states. The coupling strength of hadrons with their interpolating quark currents is determined by the compositeness condition $Z_H = 0$ [53] where Z_H is the wave function renormalization constant of the hadron. Matrix elements are generated by a set of quark loop diagrams according to a $1/N_c$ expansion. The ultraviolet divergences of the quark loops are regularized by including vertex form factors for the hadron-quark vertices which, in addition, describe finite size effects due to the nonpointlike structure of hadrons. The relativistic constituent quark model contains only a few model parameters: the light and heavy constituent quark masses, the confinement scale, and the size parameters that describe the size of the distribution of the constituent quarks inside the hadron.

The paper is organized as follows. In Sec. II we construct the nonlocal generalization of the four-quark interpolating current of the $X(3872)$ written down in [26]. This leads to a nonlocal effective Lagrangian describing the interaction of the $X(3872)$ meson with its constituent quarks. The coupling strength of the $X(3872)$ w.r.t. its constituent quarks is determined from the compositeness condition $Z_H = 0$. We also briefly discuss the implementation of infrared confinement in our scheme. In Sec. III we calculate the matrix elements of the transitions $X \to J/\psi + \rho(\omega)$ and $X \to D + \bar{D}^*$. The results are then used to evaluate the widths of the decays $X \to J/\psi + 2\pi(3\pi)$ and $X \to D^0 + \bar{D}^0 + \pi^0$. In Sec. IV we present the results of our numerical analysis and compare our results with the results of other approaches.

II. THEORETICAL FRAMEWORK

A. Effective Lagrangians

The authors of [26] suggested to consider the $X(3872)$ meson as a $J^{PC} = 1^{++}$ tetraquark state with a symmetric spin distribution: $[cq]_{S=0}[\bar{c}\,\bar{q}]_{S=1} + [cq]_{S=1}[\bar{c}\,\bar{q}]_{S=0}$, ($q = u, d$). The nonlocal version of the four-quark interpolating current reads

$$
\begin{aligned}
J^{\mu}_{X_q}(x) &= \int dx_1 \ldots \int dx_4 \delta\Big(x - \sum_{i=1}^{4} w_i x_i\Big)\Phi_X\Big(\sum_{i<j}(x_i - x_j)^2\Big) \\
&\quad \times \frac{1}{\sqrt{2}}\varepsilon_{abc}\varepsilon_{dec}\{[q_a(x_4)C\gamma^5 c_b(x_1)] \\
&\quad \times [\bar{q}_d(x_3)\gamma^{\mu}C\bar{c}_e(x_2)] + [q_a(x_4)C\gamma^{\mu}c_b(x_1)] \\
&\quad \times [\bar{q}_d(x_3)\gamma^5 C\bar{c}_e(x_2)]\}, \\
w_1 &= w_2 = w_c = \frac{m_c}{2(m_q + m_c)}, \\
w_3 &= w_4 = w_q = \frac{m_q}{2(m_q + m_c)}.
\end{aligned}
\tag{3}
$$

The matrix $C = \gamma^0\gamma^2$ is the charge conjugation matrix: $C = C^{\dagger} = C^{-1} = -C^T$, $C\Gamma^T C^{-1} = \pm\Gamma$ (" + " for $\Gamma = S, P, A$ and }−" for $\Gamma = V, T$). The numbering of the coordinates x_i is chosen such that one has a convenient arrangement of vertices and propagators in the Feynman diagrams. The effective interaction Lagrangian describing the coupling of the meson X_q to its constituent quarks is written in the form

$$
\mathcal{L}_{\rm int} = g_X X_{q\mu}(x) \cdot J^{\mu}_{X_q}(x), \qquad (q = u, d). \tag{4}
$$

The state X_u breaks isospin symmetry maximally:

$$
X_u = \frac{1}{\sqrt{2}}\Big\{\underbrace{\frac{X_u + X_d}{\sqrt{2}}}_{I=0} + \underbrace{\frac{X_u - X_d}{\sqrt{2}}}_{I=1}\Big\}. \tag{5}
$$

The authors of [26] take the physical states to be a linear superposition of the X_u and X_d states according to

$$
\begin{aligned}
X_l &\equiv X_{\rm low} = X_u \cos\theta + X_d \sin\theta, \\
X_h &\equiv X_{\rm high} = -X_u \sin\theta + X_d \cos\theta.
\end{aligned}
\tag{6}
$$

The mixing angle θ can be determined from fitting the ratio of branching ratios Eq. (2).

The coupling constant g_X in Eq. (4) will be determined from the compositeness condition $Z_H = 0$, see, e.g. Refs. [47,52,53]. The compositeness condition requires that the renormalization constant Z_X of the elementary meson X is set to zero, i.e.

$$Z_X = 1 - \Pi'_X(m_X^2) = 0, \tag{7}$$

where $\Pi_X(p^2)$ is the scalar part of the vector-meson mass operator

$$\begin{aligned}\Pi_X^{\mu\nu}(p) &= g^{\mu\nu}\Pi_X(p^2) + p^\mu p^\nu \Pi_X^{(1)}(p^2),\\ \Pi_X(p^2) &= \frac{1}{3}\Big(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\Big)\Pi_X^{\mu\nu}(p).\end{aligned} \tag{8}$$

The Fourier transform of the vertex function $\Phi_X(\sum_{i<j}(x_i - x_j)^2)$ can be calculated by using appropriately chosen Jacobi coordinates. One has

$$\begin{aligned}
x_1 &= x + \frac{2w_2 + w_3 + w_4}{2\sqrt{2}}\rho_1 - \frac{w_3 - w_4}{2\sqrt{2}}\rho_2 + \frac{w_3 + w_4}{2}\rho_3,\\
x_2 &= x - \frac{2w_1 + w_3 + w_4}{2\sqrt{2}}\rho_1 - \frac{w_3 - w_4}{2\sqrt{2}}\rho_2 + \frac{w_3 + w_4}{2}\rho_3,\\
x_3 &= x - \frac{w_1 - w_2}{2\sqrt{2}}\rho_1 + \frac{w_1 + w_2 + 2w_4}{2\sqrt{2}}\rho_2 - \frac{w_1 + w_2}{2}\rho_3,\\
x_4 &= x - \frac{w_1 - w_2}{2\sqrt{2}}\rho_1 - \frac{w_1 + w_2 + 2w_3}{2\sqrt{2}}\rho_2 - \frac{w_1 + w_2}{2}\rho_3,
\end{aligned}$$

where $x = \sum_{i=1}^4 x_i w_i$ and $\sum_{1\le i<j\le 4}(x_i - x_j)^2 = \sum_{i=1}^3 \rho_i^2$. One then has

$$\begin{aligned}
\Phi_X\Big(\sum_{i<j}(x_i - x_j)^2\Big) &= \prod_{i=1}^4 \int \frac{dp_i}{(2\pi)^4} e^{-i\sum_{i=1}^4 p_i x_i}\tilde{\Phi}_X(p_1,\ldots,p_4),\\
\tilde{\Phi}_X(p_1,\ldots,p_4) &= (2\pi)^4\delta\Big(\sum_{i=1}^4 p_i\Big)\bar{\Phi}_X(-\Omega^2),\\
\bar{\Phi}_X(-\Omega^2) &= \frac{1}{4}\prod_{i=1}^3\int d\rho_i e^{i\sum_{i=1}^3\omega_i\rho_i}\Phi_X(R^2),
\end{aligned} \tag{9}$$

where $\Omega^2 = \sum_{i=1}^3 \omega_i^2$ and $R^2 = \sum_{i=1}^3 \rho_i^2$. The Jacobi coordinates in momentum space read

$$\omega_1 = \frac{p_1 - p_2}{2\sqrt{2}}, \qquad \omega_2 = \frac{p_1 + p_2 + 2p_3}{2\sqrt{2}}, \qquad \omega_3 = \frac{p_1 + p_2}{2}. \tag{10}$$

For calculational convenience we will choose a simple Gaussian form for the vertex function $\bar{\Phi}_X(-\Omega^2)$. The minus sign in the argument of this function is chosen to emphasize that we are working in Minkowski space. One has

$$\bar{\Phi}_X(-\Omega^2) = \exp(\Omega^2/\Lambda_X^2) \tag{11}$$

where the parameter Λ_X characterizes the size of the X-meson. Since Ω^2 turns into $-\Omega^2$ in Euclidean space the form (11) has the appropriate fall-off behavior in the Euclidean region. We emphasize that any choice for Φ_X is appropriate as long as it falls off sufficiently fast in the ultraviolet region of Euclidean space to render the corresponding Feynman diagrams ultraviolet finite. As mentioned before we shall choose a Gaussian form for Φ_X for calculational convenience.

We are now in the position to write down an explicit expression for the derivative of the mass operator appearing in Eq. (7). The corresponding three-loop diagram describing the X-meson mass operator is shown in Fig. 1.

One has

$$\begin{aligned}
\Pi'_X(p^2) &= \frac{1}{2p^2}p^\alpha\frac{\partial}{\partial p^\alpha}\Pi_X(p^2)\\
&= \frac{2g_X^2}{3p^2}\Big(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\Big)\prod_{i=1}^3\int\frac{d^4k_i}{(2\pi)^4 i}\bar{\Phi}_X^2(-K^2)\\
&\quad\times\{-w_c\,\mathrm{tr}[S_c^{[12]}\not{p}S_c^{[12]}\gamma^5 S_q^{[2]}\gamma^5]\,\mathrm{tr}[S_c^{[3]}\gamma^\mu S_q^{[13]}\gamma^\nu]\\
&\quad + w_q\,\mathrm{tr}[S_c^{[12]}\gamma^5 S_q^{[2]}\not{p}S_q^{[2]}\gamma^5]\,\mathrm{tr}[S_c^{[3]}\gamma^\mu S_q^{[13]}\gamma^\nu]\\
&\quad - w_c\,\mathrm{tr}[S_c^{[12]}\gamma^5 S_q^{[2]}\gamma^5]\,\mathrm{tr}[S_c^{[3]}\not{p}S_c^{[3]}\gamma^\mu S_q^{[13]}\gamma^\nu]\\
&\quad + w_q\,\mathrm{tr}[S_c^{[12]}\gamma^5 S_q^{[2]}\gamma^5]\,\mathrm{tr}[S_c^{[3]}\gamma^\mu S_q^{[13]}\not{p}S_q^{[13]}\gamma^\nu]\}
\end{aligned} \tag{12}$$

where we have introduced the abbreviations

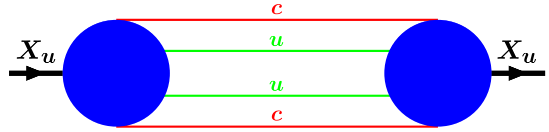

FIG. 1 (color online). Diagram describing the X_u-meson mass operator.

$$S_c^{[12]} = S_c(k_1 + k_2 - w_c p), \qquad S_c^{[3]} = S_c(k_3 - w_c p),$$
$$S_q^{[2]} = S_q(k_2 + w_q p), \qquad S_q^{[13]} = S_q(k_1 + k_3 + w_q p),$$
$$K^2 = \frac{1}{8}(k_1 + 2k_2)^2 + \frac{1}{8}(k_1 + 2k_3)^2 + \frac{1}{4}k_1^2.$$

In the next section we shall describe how to evaluate the integral (12).

B. Infrared confinement

In [47] we described how to integrate n-point one-loop diagrams and how to implement infrared confinement of quarks in this process. Since the present application involves also multiloop diagrams we need to extend our loop integration techniques to the case of an arbitrary number of loops. Let n, ℓ and m be the number of the propagators, loops and vertices, respectively. In Minkowski space the ℓ-loop diagram will be represented as

$$\Pi(p_1, \ldots, p_m) = \int [d^4k]^\ell \prod_{i_1=1}^{m} \Phi_{i_1+n}(-K^2_{i_1+n}) \times \prod_{i_3=1}^{n} S_{i_3}(\tilde{k}_{i_3} + v_{i_3}),$$
$$K^2_{i_1+n} = \sum_{i_2} (\tilde{k}^{(i_2)}_{i_1+n} + v^{(i_2)}_{i_1+n})^2 \tag{13}$$

where the vectors $\tilde{k}_i$ are linear combinations of the loop momenta k_i. The v_i are linear combinations of the external momenta p_i to be specified in the following. The strings of Dirac matrices appearing in the calculation need not concern us since they do not depend on the momenta. The external momenta p_i are all chosen to be ingoing such that one has $\sum_{i=1}^{m} p_i = 0$.

Using the Schwinger representation of the local quark propagator one has

$$S(k) = (m + \not{k}) \int_0^\infty d\beta e^{-\beta(m^2 - k^2)}. \tag{14}$$

For the vertex functions one takes the Gaussian form. One has

$$\Phi_{i+n}(-K^2) = \exp[\beta_{i+n} K^2] \qquad i = 1, \ldots, m, \tag{15}$$

where the parameters $\beta_{i+n} = s_i = 1/\Lambda_i^2$ are related to the size parameters. The integrand in Eq. (13) has a Gaussian form with the exponential $kak + 2kr + R$ where a is a $\ell \times \ell$ matrix depending on the parameter β_i, r is the ℓ-vector composed from the external momenta, and R is a quadratic form of the external momenta. Tensor loop integrals are calculated with the help of the differential representation

$$k_i^\mu e^{2kr} = \frac{1}{2} \frac{\partial}{\partial r_{i\mu}} e^{2kr}. \tag{16}$$

We have written a FORM [54] program that achieves the necessary commutations of the differential operators in a very efficient way. After doing the loop integrations one obtains

$$\Pi = \int_0^\infty d^n\beta F(\beta_1, \ldots, \beta_n), \tag{17}$$

where F stands for the whole structure of a given diagram. The set of Schwinger parameters β_i can be turned into a simplex by introducing an additional t–integration via the identity

$$1 = \int_0^\infty dt \delta\Big(t - \sum_{i=1}^{n} \beta_i\Big) \tag{18}$$

leading to

$$\Pi = \int_0^\infty dt t^{n-1} \int_0^1 d^n\alpha \delta\Big(1 - \sum_{i=1}^{n} \alpha_i\Big) F(t\alpha_1, \ldots, t\alpha_n). \tag{19}$$

There are altogether n numerical integrations: $(n-1)$ α-parameter integrations and the integration over the scale parameter t. The very large t-region corresponds to the region where the singularities of the diagram with its local quark propagators start appearing. However, as described in [47], if one introduces an infrared cutoff on the upper limit of the t-integration, all singularities vanish because the integral is now convergent for any value of the set of kinematic variables. We cut off the upper integration at $1/\lambda^2$ and obtain

$$\Pi^c = \int_0^{1/\lambda^2} dt t^{n-1} \int_0^1 d^n\alpha \delta\Big(1 - \sum_{i=1}^{n} \alpha_i\Big) F(t\alpha_1, \ldots, t\alpha_n). \tag{20}$$

By introducing the infrared cutoff one has removed all potential thresholds in the quark loop diagram, i.e. the quarks are never on-shell and are thus effectively confined. We take the cutoff parameter λ to be the same in all physical processes. The numerical evaluations have been done by a numerical program written in the FORTRAN code.

As a further illustration of the infrared confinement effect relevant to the applications in this paper we consider the case of a scalar one-loop two-point function. One has

$$\Pi_2(p^2) = \int \frac{d^4k_E}{\pi^2} \times \frac{e^{-sk_E^2}}{[m^2 + (k_E + \frac{1}{2}p_E)^2][m^2 + (k_E - \frac{1}{2}p_E)^2]}$$

where we have collected all the nonlocal Gaussian vertex form factors in the numerator factor $e^{-sk_E^2}$. Note that the momenta k_E, p_E are Euclidean momenta. Doing the loop integration one obtains

$$\Pi_2(p^2) = \int_0^\infty dt \frac{t}{(s+t)^2} \int_0^1 d\alpha \exp\left[-t z_{\rm loc} + \frac{st}{s+t} z_1\right],$$
$$z_{\rm loc} = m^2 - \alpha(1-\alpha)p^2, \qquad z_1 = \left(\alpha - \frac{1}{2}\right)^2 p^2. \quad (21)$$

The integral $\Pi_2(p^2)$ can be seen to have a branch point at $p^2 = 4m^2$ because $z_{\rm loc}$ is zero when $\alpha = 1/2$. By introducing a cutoff on the t-integration one obtains

$$\Pi_2^c(p^2) = \int_0^{1/\lambda^2} dt \frac{t}{(s+t)^2} \int_0^1 d\alpha \exp\left[-t z_{\rm loc} + \frac{st}{s+t} z_1\right]. \quad (22)$$

The one-loop two-point function $\Pi_2^c(p^2)$ Eq. (22) can be seen to have no branch point at $p^2 = 4m^2$.

III. THE TRANSITIONS $X \to J/\psi + \rho(\omega)$ AND $X \to D + \bar{D}^*$

In this section we evaluate the matrix elements of the transitions $X \to J/\psi + \rho(\omega)$ and $X \to D + \bar{D}^*$. The relevant Feynman diagrams are shown in Fig. 2. Since the $X(3872)$ is very close to the respective thresholds in both cases, cf.

$$m_X - (m_J/\psi + m_\rho) = -0.90 \pm 0.41\ \text{MeV},$$
$$m_X - (m_{D^0} + m_{D^{*0}}) = -0.30 \pm 0.34\ \text{MeV}$$

the intermediate ρ, ω and D^* mesons have to be treated as off-shell particles.

$$\begin{aligned} M^{\mu\nu\rho}(X_u(p,\mu) \to J/\psi(q_1,\nu) + v^0(q_2,\rho)) &= 6 g_X g_{J/\psi} g_{v^0} \int \frac{d^4k_1}{(2\pi)^4 i} \int \frac{d^4k_2}{(2\pi)^4 i} \bar{\Phi}_X(-K_1^2) \Phi_{J/\psi}\left(-\left(k_1 + \frac{1}{2}q_1\right)^2\right) \\ &\quad \times \Phi_{v^0}\left(-\left(k_2 + \frac{1}{2}q_2\right)^2\right) \mathrm{tr}[i\gamma^5 S_c(k_1)\gamma^\nu S_c(k_1+q_1)\gamma^\mu S_u(k_2)\gamma^\rho S_u(k_2+q_2)] \\ &= \varepsilon^{q_1 q_2 \mu\nu} q_1^\rho M_{XJv}^{(1)} + \varepsilon^{q_1 q_2 \mu\nu} q_2^\rho M_{XJv}^{(2)} + \varepsilon^{q_1 q_2 \mu\rho} q_2^\nu M_{XJv}^{(3)} + \varepsilon^{q_1 q_2 \nu\rho} q_1^\mu M_{XJv}^{(4)} \\ &\quad + \varepsilon^{q_1 \mu\nu\rho} M_{XJv}^{(5)} + \varepsilon^{q_2 \mu\nu\rho} M_{XJv}^{(6)} + \varepsilon^{q_1 q_2 \mu\rho} q_1^\nu M_{XJv}^{(7)} + \varepsilon^{q_1 q_2 \nu\rho} q_2^\mu M_{XJv}^{(8)}, \\ K_1^2 &= \frac{1}{2}\left(k_1 + \frac{1}{2}q_1\right)^2 + \frac{1}{2}\left(k_2 + \frac{1}{2}q_2\right)^2 + \frac{1}{4}(w_u q_1 - w_c q_2)^2, \end{aligned} \quad (23)$$

where $v^0 = \rho, \omega$. In the case where the X and J/ψ are on mass-shell, i.e. $\epsilon_\mu(q_1^\mu + q_2^\mu) = 0$ and $\epsilon_\nu q_1^\nu = 0$, the number of independent Lorentz structures reduces to 6. Because of the obvious relations:

$$M(X_d \to J/\psi + \rho) = -M(X_u \to J/\psi + \rho),$$
$$M(X_d \to J/\psi + \omega) = M(X_u \to J/\psi + \omega)$$

one can express the decay amplitudes of the physical states defined by Eq. (6) via the decay amplitudes of the X_u as

$$M(X_l \to J/\psi + \omega) = (\cos\theta + \sin\theta) M(X_u \to J/\psi + \omega),$$
$$M(X_h \to J/\psi + \omega) = (\cos\theta - \sin\theta) M(X_u \to J/\psi + \omega),$$
$$M(X_l \to J/\psi + \rho) = (\cos\theta - \sin\theta) M(X_u \to J/\psi + \rho),$$
$$M(X_h \to J/\psi + \rho) = -(\cos\theta + \sin\theta) M(X_u \to J/\psi + \rho).$$

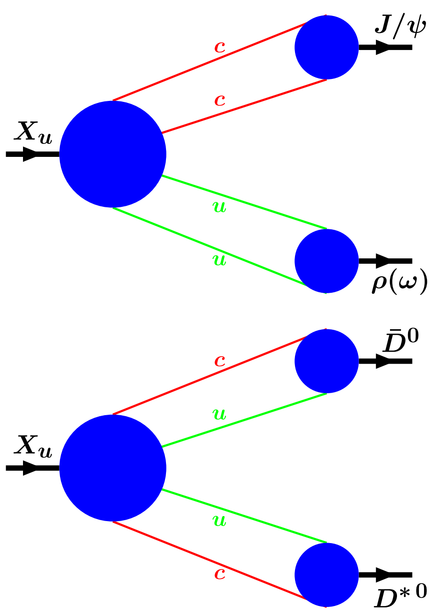

FIG. 2 (color online). Feynman diagrams describing the decays $X \to J/\psi + \rho(\omega)$ and $X \to D + \bar{D}^*$.

Next we turn to the decay $X \to \bar{D} + D^*$. The generic matrix element is written as

$$M^{\mu\nu}(X_q(p,\mu)\to\bar D(q_1)+D^*(q_2,\nu)) = 3\sqrt{2}g_X g_D g_{D^*}\int\frac{d^4k_1}{(2\pi)^4 i}\int\frac{d^4k_2}{(2\pi)^4 i}\bar\Phi_X(-K_2^2)\Phi_D(-(k_1+w_c q_1)^2$$
$$\times)\Phi_{D^*}(-(k_2+w_c q_2)^2)\,\mathrm{tr}[\gamma^5 S_c(k_1)\gamma^5 S_q(k_1+q_1)\gamma^\mu S_c(k_2)\gamma^\nu S_q(k_2+q_2)]$$
$$+(m_q\leftrightarrow m_c, w_q\leftrightarrow w_c)$$
$$= g^{\mu\nu}M^{(1)}_{XDD^*}+q_1^\mu q_1^\nu M^{(2)}_{XDD^*}+q_1^\mu q_2^\nu M^{(3)}_{XDD^*}+q_2^\mu q_1^\nu M^{(4)}_{XDD^*}+q_2^\mu q_2^\nu M^{(5)}_{XDD^*}.$$
$$K_2^2=\frac{1}{8}(k_1-k_2)^2+\frac{1}{8}(k_1-k_2+q_1-q_2)^2+\frac{1}{4}(k_1+k_2+w_c p)^2. \tag{24}$$

Note that the decay of the X-meson into $D\bar D$ is forbidden.

IV. NUMERICAL ANALYSIS

A. X-decays

Using the matrix elements (23) for the decay $X\to J/\psi+\rho(\omega)$ one can evaluate the decay widths $X\to J/\psi+2\pi(3\pi)$. We employ the narrow-width approximation which was extensively discussed in Ref. [55] and also used in Ref. [56]. One has

$$\frac{d\Gamma(X\to J/\psi+n\pi)}{dq^2}=\frac{1}{8m_X^2\pi}\cdot\frac{1}{3}|M_{XJv}|^2\frac{\Gamma_{v^0}m_{v^0}}{\pi}\times\frac{p^*(q^2)}{(m_{v^0}^2-q^2)^2+\Gamma_{v^0}^2m_{v^0}^2}\times\mathcal{B}(v^0\to n\pi),$$
$$\frac{1}{3}|M_{XJv}|^2=\frac{1}{3}\sum_{\mathrm{pol}}|\varepsilon_X^\mu\varepsilon_{J/\psi}^\nu\varepsilon_{v^0}^\rho M_{\mu\nu\rho}|^2. \tag{25}$$

Here $p^*(q^2)=\lambda^{1/2}(m_X^2,m_{J/\psi}^2,q^2)/2m_X$ is the momentum of the J/ψ or the $(n\pi)$ system in the center-of-mass frame and $(nm_\pi)^2\le q^2\le(m_X-m_{J/\psi})^2$ defines the kinematic region of the respective processes with $n=2$ for the ρ meson and $n=3$ for the ω meson. The essential point which allows one to derive Eq. (25) is the omission of polarization correlations. As was shown in Ref. [55] they produce no effects if the intermediate state is on-shell. Note that in our calculation we keep the q^2-dependence of the matrix elements as given by Eq. (23). We use the masses and widths of the $\rho(\omega)$-mesons from [2] (all in MeV): $m_\rho=775.49$, $\Gamma_\rho=146.2$, $\mathcal{B}(\rho\to2\pi)=1$, $m_\omega=782.65$, $\Gamma_\omega=8.49$, $\mathcal{B}(\omega\to3\pi)=0.892$.

The adjustable parameters of our model are the constituent quark masses m_q, the scale parameter λ characterizing the infrared confinement and the size parameters Λ_M. They were determined by using a least square fit to a number of physical observables, see [47]. Below we display the numerical values for the parameters which are relevant to the present paper.

$m_{u/d}$	m_s	m_c	λ	Λ_π	$\Lambda_{\rho/\omega}$	Λ_D	Λ_{D^*}	$\Lambda_{J/\psi}$	Λ_{η_c}	
0.217	0.360	1.6	0.181	0.711	0.295	1.4	2.3	3.3	3.0	GeV

(26)

Note that our fit values for the size parameters of the pion and the ρ-meson are in qualitative agreement with those found in the quark model based on the instanton vacuum Ref. [57].

There are two new free parameters: the mixing angle θ in Eq. (6) and the size parameter Λ_X. We have varied the parameter Λ_X in a large interval and found that the ratio

$$\frac{\Gamma(X_u\to J/\psi+3\pi)}{\Gamma(X_u\to J/\psi+2\pi)}\approx0.25 \tag{27}$$

is very stable under variations of Λ_X. Hence, by using this result and the central value of the experimental data given in Eq. (2), one finds

$$\frac{\Gamma(X_{l,h}\to J/\psi+3\pi)}{\Gamma(X_{l,h}\to J/\psi+2\pi)}\approx0.25\cdot\left(\frac{1\pm\tan\theta}{1\mp\tan\theta}\right)^2\approx1 \tag{28}$$

which gives $\theta\approx\pm18.4°$ for X_l ("+") and X_h ("−"), respectively. This is in agreement with the results obtained in both [26]: $\theta\approx\pm20°$ and [29]: $\theta\approx\pm23.5°$. The decay width is quite sensitive to the change of the size parameter Λ_X. A natural choice is to take a value close to $\Lambda_{J/\psi}$ and Λ_{η_c} which are both around 3 GeV. We have varied the size parameter Λ_X from 3 up to 4 GeV and display the dependence of the decay width in Fig. 3. One can see that the decay width $\Gamma(X\to J/\psi+n\pi)$ decreases from 0.30 up to 0.07 MeV, monotonously. This result is in accordance with the experimental bound $\Gamma(X(3872))\le2.3$ MeV and the result obtained in [26]: 1.6 MeV.

In a similar way we calculate the width of the decay $X\to D^0\bar D^0\pi^0$ which was observed by the Belle Collaboration and reported in [10]. Again using the narrow-width approximation the differential rate reads

$$\frac{d\Gamma(X_u\to\bar D^0D^0\pi^0)}{dq^2}=\frac{1}{2m_X^2\pi}\cdot\frac{1}{3}|M_{XDD^*}|^2\frac{\Gamma_{D^{*0}}m_{D^{*0}}}{\pi}\times\frac{p^*(q^2)\mathcal{B}(D^{*0}\to D^0\pi^0)}{(m_{D^{*0}}^2-q^2)^2+\Gamma_{D^{*0}}^2m_{D^{*0}}^2},$$
$$\frac{1}{3}|M_{XDD^*}|^2=\frac{1}{3}\sum_{\mathrm{pol}}|\varepsilon_X^\mu\varepsilon_{D^{*0}}^\nu M_{\mu\nu}|^2 \tag{29}$$

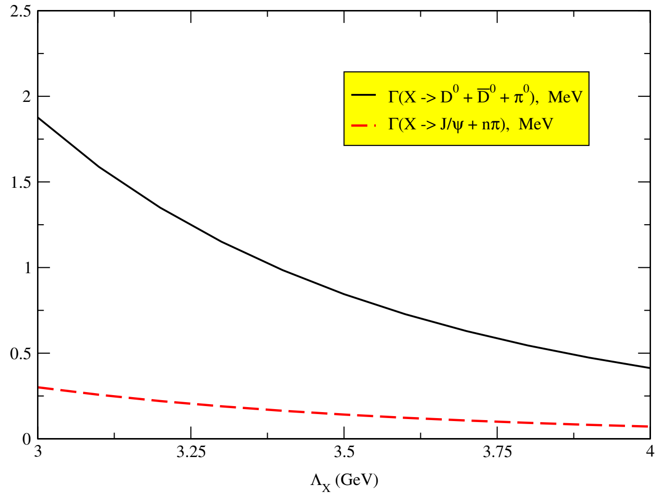

FIG. 3 (color online). The dependence of the decay widths $\Gamma(X_l \to \bar{D}^0 D^0 \pi^0)$ and $\Gamma(X \to J/\psi + n\pi)$ on the size parameter Λ_X.

where the matrix element $M_{\mu\nu}$ is defined by Eq. (24) and $p^*(q^2) = \lambda^{1/2}(m_X^2, m_{D^0}^2, q^2)/2m_X$ is the momentum in the center-of-mass system. The kinematical region defined by

$$(m_{D^0} + m_{\pi^0})^2 \le q^2 \le (m_X - m_{D^0})^2$$

is very narrow $3.99928 \le q^2 \le 4.02672$ GeV^2. Note that we have taken into account both channels with the intermediate D^{*0} and $\bar{D}^{*0}$ mesons. We use the masses and widths of the D^{*+} and D^{*0} mesons given in [2,16,27] (all dimensional quantities in MeV):

$$m_{D^{*+}} = 2010.27, \qquad \Gamma_{D^{*+}} = 0.096,$$
$$m_{D^{*0}} = 2006.97, \qquad \mathcal{B}(D^{*+} \to D^+\pi^0) = 0.307,$$
$$\Gamma_{D^{*0}} = 0.070, \qquad \mathcal{B}(D^{*0} \to D^0\pi^0) = 0.619.$$

Keeping in mind that

$$\Gamma(X_l \to \bar{D}^0 D^0 \pi^0) = \cos\theta^2 \Gamma(X_u \to \bar{D}^0 D^0 \pi^0),$$

we have varied Λ_X from 3 up to 4 GeV and found that the decay width $\Gamma(X_l \to \bar{D}^0 D^0 \pi^0)$ decreases from 1.88 up to 0.41 MeV, monotonously. In Fig. 3 we plot the dependence of the decay widths $\Gamma(X_l \to \bar{D}^0 D^0 \pi^0)$ and $\Gamma(X \to J/\psi + n\pi)$ on the size parameter Λ_X. Using the results of [2]

$$10^5 \mathcal{B}(B^\pm \to K^\pm X) \cdot \mathcal{B}(X \to J/\psi \pi^+\pi^-) = 0.95 \pm 0.19,$$
$$10^5 \mathcal{B}(B^\pm \to K^\pm X) \cdot \mathcal{B}(X \to D^0 \bar{D}^0 \pi^0) = 10.0 \pm 4.0$$

one calculates the rate ratio

$$\frac{\Gamma(X \to D^0 \bar{D}^0 \pi^0)}{\Gamma(X \to J/\psi \pi^+\pi^-)} = 10.5 \pm 4.7. \qquad (30)$$

The theoretical value for this rate ratio depends on the size parameter Λ_X as Fig. 3 shows. One has

$$\frac{\Gamma(X \to D^0 \bar{D}^0 \pi^0)}{\Gamma(X \to J/\psi \pi^+\pi^-)}\bigg|_{\rm theor} = 6.0 \pm 0.2, \qquad (31)$$

where the theoretical error reflects the Λ_X dependence of the ratio. The ratio lies within the experimental uncertainties given by Eq. (30).

B. J/ψ-dissociation

The last topic which we would like to discuss is the impact of the intermediate X-resonance on the value of the J/ψ-dissociation cross section, see [58–60]. The relevant s-channel diagram is shown in Fig. 4.

We evaluate the J/ψ-dissociation cross sections without using the narrow-width approximation. One has

$$\sigma(J/\psi + v^0 \to D + \bar{D}^*) + \sigma(J/\psi + v^0 \to \bar{D} + D^*) = 2(\cos\theta \mp \sin\theta)^2 \sigma(J/\psi + v^0 \to X_u \to \bar{D} + D^*),$$
$$\sigma(J/\psi + v^0 \to X_u \to \bar{D} + D^*) = \frac{1}{16\pi s} \frac{\lambda^{1/2}(s, m_D^2, m_{D^*}^2)}{\lambda^{1/2}(s, m_{J/\psi}^2, m_{v^0}^2)} \frac{1}{9} \sum_{\rm pol} \frac{|A|^2}{(s - m_X^2)^2 + \Gamma_X^2 m_X^2},$$
$$A = \varepsilon_{J/\psi}^{\nu} \varepsilon_{v^0}^{\rho} M_{\mu\nu\rho} \left(-g^{\mu\alpha} + \frac{p^\mu p^\alpha}{m_X^2}\right) \varepsilon_{D^*}^{\beta} M_{\alpha\beta} \qquad (32)$$

where $p = p_1 + p_2 = q_1 + q_2$. $v^0 = \rho$ (minus sign) or ω (plus sign). We also neglect isotopic spin breaking effects. Note that $E = \sqrt{s} \ge m_{D^+} + m_{D^{*+}}$ for charged D-mesons in the final states and $E \ge m_{J/\psi} + m_{v^0}$ for neutral D-mesons in the final states. In the first case the cross section is zero at the threshold of reaction $E = m_{D^+} + m_{D^{*+}}$. In the last case the cross section blows up at $E = m_{J/\psi} + m_{v^0}$ because the

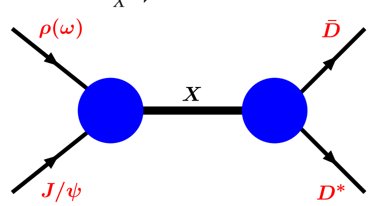

FIG. 4 (color online). Diagram describing the X-resonance contribution to the J/ψ-dissociation process.

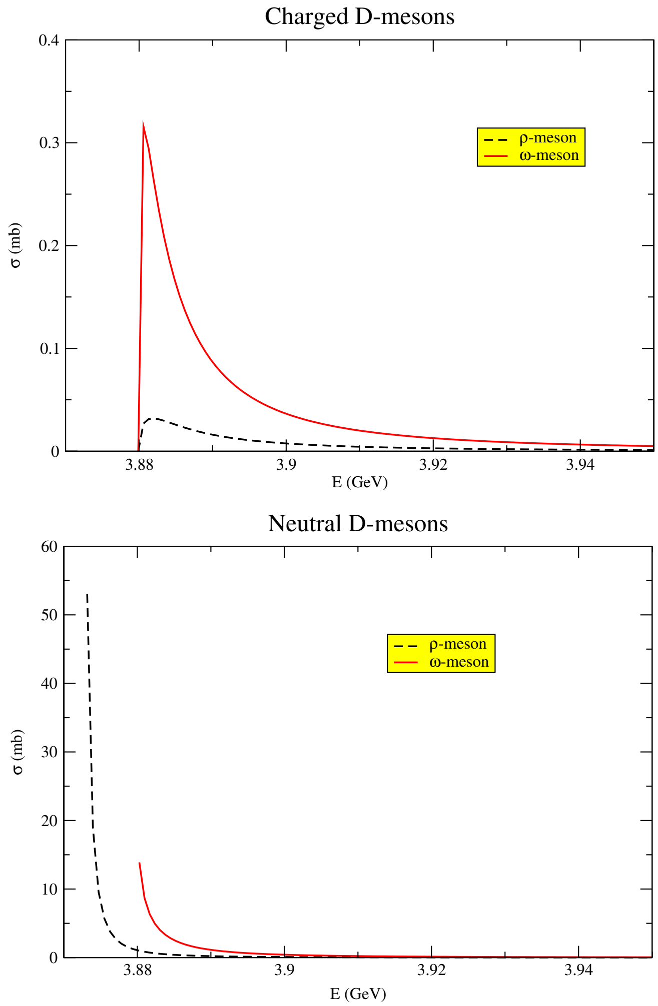

FIG. 5 (color online). The cross sections of the processes $J/\psi + v^0 \to X \to D + D^*$. Charged D-mesons-upper panel, neutral D-mesons-lower panel.

channel $J/\psi + v^0 \to D^0 + \bar{D}^{*0}$ is exothermic and the kinematical function $\lambda^{1/2}(s, m_{J/\psi}^2, m_{v^0}^2)$ in the denominator is equal to zero at this point. We take $\Gamma_X = 1$ MeV in the Breit-Wigner propagator and set $\Lambda_X = 3.5$ GeV when calculating the matrix elements. We plot the behavior of the relevant cross sections in Fig. 5. One can see that in the case of charged D-mesons (upper panel in Fig. 5) the maximum value of the cross section is about 0.32 mb at $E = 3.88$ GeV. This result should be compared with the result of the cross section $\sigma(J/\psi + \pi \to D + \bar{D}^*) \approx 0.9$ mb at $E = 4.0$ GeV, see, [59] and the result of the cross section $\sigma(J/\psi + \rho \to D + \bar{D}^*) \approx 2.9$ mb at $E = 3.9$ GeV, see, [58]. Thus the X-resonance gives a sizable contribution to the J/ψ-dissociation cross section. It would be interesting to do a complete analysis of the J/ψ dissociation cross section in view of our new results on the s-channel contribution of the $X(3872)$ tetraquark state. However, this certainly is beyond the scope of the present paper.

ACKNOWLEDGMENTS

This work was supported by the DFG grant KO 1069/13-1, the Heisenberg-Landau program, the Slovak aimed project at JINR and the grant VEGA No. 2/0009/10. M. A. I. also appreciates the partial support of the Russian Fund of Basic Research Grant No. 10-02-00368-a. We thank P. Santorelli for his collaboration in the early stages of this work. M. A. I. very much appreciates very interesting discussion remarks and useful comments by A. E. Dorokhov, G. V. Efimov, S. B. Gerasimov, N. I. Kochelev, D. V. Shirkov and O. V. Teryaev during a "Hadron Physics" seminar held in the Bogoliubov Laboratory of Theoretical Physics.

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