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Chiral field theory of 0^{-+} glueball

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A chiral field theory of 0^{-+} glueball is presented. The Lagrangian of this theory is constructed by adding a 0^{-+} glueball field to a successful Lagrangian of the chiral field theory of pseudoscalar, vector, and axial-vector mesons. The couplings between the pseodoscalar glueball field and the mesons are revealed via a U(1) anomaly. Quantitative study of the physical processes of the 0^{-+} glueball of $m = 1.405$ GeV is presented. The theoretical predictions can be used to identify the 0^{-+} glueball.

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I. INTRODUCTION

It has been known for a very long time that the glueball is the solution of nonperturbative QCD and there are extensive study on pseudoscalar glueballs [1]. On the other hand, many candidates of 0^{++}, 0^{-+}, and 2^{++} glueballs have been discovered [2]. However, identification of a glueball is still in question. In order to identify a glueball quantitative study of the physical processes of a glueball is urgently needed. It is the attempt of this paper to present a chiral field theory which can be used to do systematic and quantitative study of the properties of the 0^{-+} glueball.

Both current algebra and lattice QCD successfully use quark operators to study nonperturbative hadron physics. Based on current algebra and QCD we have proposed a chiral field theory of pseudoscalar, vector, and axial-vector mesons [3], in which quark operators are used to study meson physics. The Lagrangian of quarks and mesons is constructed as

$$\mathcal{L}_1 = \bar{\psi}(x)(i\gamma\cdot\partial + \gamma\cdot v + \gamma\cdot a\gamma_5 - mu(x))\psi(x) - \bar{\psi}M\psi + \frac{1}{2}m_0^2(\rho_i^\mu\rho_{\mu i} + \omega^\mu\omega_\mu + a_i^\mu a_{\mu i} + f^\mu f_\mu + K_\mu^a\bar{K}^{a\mu} + K_1^\mu K_{1\mu} + \phi_\mu\phi^\mu + f_s^\mu f_{s\mu}), \tag{1}$$

where $a_\mu = \tau_i a_\mu^i + \lambda_a K_{1\mu}^a + (\frac{2}{3}+\frac{1}{\sqrt{3}}\lambda_8)f_\mu + (\frac{1}{3}-\frac{1}{\sqrt{3}}\lambda_8)f_{s\mu}$ ($i = 1, 2, 3$ and $a = 4, 5, 6, 7$); $v_\mu = \tau_i\rho_\mu^i + \lambda_a K_\mu^a + (\frac{2}{3}+\frac{1}{\sqrt{3}}\lambda_8)\omega_\mu + (\frac{1}{3}-\frac{1}{\sqrt{3}}\lambda_8)\phi_\mu$; $u = \exp\{i\gamma_5(\tau_i\pi_i + \lambda_a K^a + \lambda_8\eta_8 + \frac{1}{\sqrt{3}}\eta_0)\}$,; m is the constituent quark mass which originates in the quark condensate; M is the matrix of the current quark mass; m_0 is a parameter. In the limit, $m_q \to 0$, the theory (1) has global $U(3)_L \times U(3)_R$ symmetry. In this theory the meson fields are related to corresponding quark operators. For example, at the tree level the vector and the axial-vector mesons are expressed as the quark operators

$$\rho_\mu^i = -\frac{1}{m_0^2}\bar{\psi}\gamma_\mu\tau^i\psi, \qquad a_\mu^i = -\frac{1}{m_0^2}\bar{\psi}\gamma_\mu\gamma_5\tau^i\psi.$$

The pseudoscalar mesons are introduced via the mechanism of the nonlinear σ model. Under this mechanism the introduction of the constituent quark mass is natural and it plays an essential role in this theory. The mesons are bound states of quarks and they are not independent degrees of freedoms, and the kinetic terms of the meson fields are generated by the quark loop diagrams. Integrating out the quark fields, the Lagrangian of the meson fields is derived. N_c expansion is naturally embedded. The tree diagrams are at the leading order and the loop diagrams of the mesons are at the higher orders. Besides the N_C expansion there are current quark mass and momentum expansions in this theory. The major features of nonperturbative QCD—N_C expansion, quark condensate, and chiral symmetry—are all included in this meson theory. The masses of the pseudoscalar, the vector, and the axial-vector mesons are determined. The form factors of the pion and the kaons are calculated in both spacelike and timelike regions. The widths of strong, electromagnetic, and weak decays of the mesons are computed. $\pi - \pi$ and $\pi - K$ scatterings are studied. The Wess-Zumino-Witten anomaly is revealed. ChPT (chiral perturbation theory) is the low energy approximation of this theory. All the 10 coefficients of the ChPT are determined. Meson physics is systematically studied. The pion decay constant and a universal coupling constant are the two parameters in most cases. The third parameter, the quark condensate, only appears in the masses of the pseudoscalar mesons. Theory agrees with the data very well [3]. The meson physics are successfully studied by expressing the meson fields as the quark operators. The Lagrangian (1) is not complete. There are other degrees of freedoms, for instance, glueballs. It is known that lattice QCD has used the gluon operator to calculate glueball mass [4]. Following the manner of Eq. (1), using the gluon operator to construct an effective Lagrangian to study the physics of the 0^{-+} glueball is the attempt of this paper. This paper is organized as follows: (1) introduction; (2) chiral Lagrangian of 0^{-+} glueball and mesons; (3) mass mixing of the 0^{-+} glueball $\eta(1405)$ and the η, η'; (4) $\eta(1405) \to \gamma\gamma$ decay; (5) $\eta(1405) \to \gamma\rho$, $\gamma\omega$, $\gamma\phi$ decays; (6) kinetic mixing of χ and η_0 fields; (7) $J/\psi \to \gamma\eta(1405)$ decay; (8) $\eta(1405) \to \rho\pi\pi$ decay;

(9) $\eta(1405) \to a_0(980)\pi$ decay; (10) $\eta(1405) \to K^*K$ decay; (11) summary.

II. CHIRAL LAGRANGIAN OF 0^{-+} GLUEBALL AND MESONS

As mentioned above, lattice gauge theory has used the gluon operator, $F\tilde{F}$ (in the continuum limit), to calculate the mass of the pseudoscalar glueball by the quench approximation [4]. The meson theory is phenomenologically successful, in which the mesons are coupled to the quark operators. The same approach is used to construct an effective Lagrangian of the 0^{-+} glueball in this paper. This theory should be chiral symmetric in the limit, $m_q \to 0$ and the field of the 0^{-+} glueball χ can be expressed as the gluon operator $F\tilde{F}$. Under the least coupling principle the effective Lagrangian is constructed as

$$\mathcal{L} = -\frac{1}{4}F^{a\mu\nu}F^a_{\mu\nu} + F^a_{\mu\nu}\tilde{F}^{a\mu\nu}\chi + \frac{1}{2}G^2_\chi\chi\chi, \quad (2)$$

where $\tilde{F}^{\mu\nu} = \epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$, and G_χ is a mass-related parameter. In QCD the glueball is a bound state of gluons, not an independent degree of freedom, therefore, there is no kinetic term for the glueball field χ. Using Eq. (2), at the tree level the glueball field is expressed as the gluon operator

$$\chi = -\frac{1}{G^2_\chi}F_{\mu\nu}\tilde{F}^{\mu\nu}. \quad (3)$$

The relationship between the gluon operator $F\tilde{F}$ and the quark operators is found from the U(1) anomaly

$$\partial_\mu(\bar{\psi}\gamma_\mu\gamma_5\psi) = 2i\bar{\psi}M\gamma_5\psi + \frac{3g_s^2}{(4\pi)^2}F_{\mu\nu}\tilde{F}^{\mu\nu}. \quad (4)$$

Using Eq. (4), Eq. (2) is rewritten as

$$\mathcal{L} = -\frac{1}{4}F^{a\mu\nu}F^a_{\mu\nu} - \left(\frac{3g_s^2}{(4\pi)^2}\right)^{-1}\{\bar{\psi}\gamma_\mu\gamma_5\psi\partial_\mu\chi + 2i\bar{\psi}M\gamma_5\psi\chi\} + \frac{1}{2}G^2_\chi\chi\chi. \quad (5)$$

The constant $(\frac{3g_s^2}{(4\pi)^2})^{-1}$ can be absorbed by the χ field. By redefining the χ field and the parameter G_χ, Eq. (5) is rewritten as

$$\mathcal{L} = -\frac{1}{4}F^{a\mu\nu}F^a_{\mu\nu} - \{\bar{\psi}\gamma_\mu\gamma_5\psi\partial_\mu\chi + 2i\bar{\psi}M\gamma_5\psi\chi\} + \frac{1}{2}G^2_\chi\chi\chi. \quad (6)$$

The same symbols of χ and G_χ are used. Equation (6) is chiral symmetric in the limit, $m_q \to 0$. It is known that $g_s^2N_c \sim 1$ in the N_C expansion and the loop diagrams with gluon internal lines are at the higher orders in the N_C expansion. Therefore, at the leading order in N_C expansion the kinetic terms of gluon fields are decoupled from this theory.

Adding the two terms,

$$\mathcal{L}_2 = -\{\bar{\psi}\gamma_\mu\gamma_5\psi\partial_\mu\chi + 2i\bar{\psi}M\gamma_5\psi\chi\} + \frac{1}{2}G^2_\chi\chi\chi, \quad (7)$$

to the Lagrangian of mesons (1), the Lagrangian including the glueball field χ and the meson fields is found to be

$$\mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2. \quad (8)$$

As shown above, the ways of introducing the 0^{-+} mesons and the 0^{-+} glueball field to the theory (8) are very different. The couplings between the quark operators and the η_8, η_0 are different from the coupling of the 0^{-+} glueball. For η_8, η_0 there are two couplings [3]:

$$-\frac{c}{g}\frac{2}{f_\pi}\bar{\psi}\gamma_\mu\gamma_5\lambda\psi\partial_\mu(\eta, \eta'), \qquad -im\frac{2}{f_\pi}\bar{\psi}\gamma_5\lambda\psi(\eta, \eta'), \quad (9)$$

where $\lambda = \lambda_8$ for η_8 and $\lambda = \frac{\sqrt{2}}{\sqrt{3}}I$ for η_0, respectively, $c = \frac{f_\pi^2}{2gm_\rho^2}$, and g is a universal coupling constant and it is determined by the decay rate of $\rho \to e^+e$. In the chiral limit, the coupling for the 0^{-+} glueball is obtained from Eq. (6)

$$-\bar{\psi}\gamma_\mu\gamma_5\psi\partial_\mu\chi. \quad (10)$$

The differences between Eqs. (9) and (10) lead to different physical results for the 0^{-+} mesons and the 0^{-+} glueball. The different physical results which are presented in this paper should be able to distinguish a pseudoscalar glueball from the pseudoscalar mesons.

Integrating out the quark fields, the kinetic term of the χ field and the vertices between the χ field and other mesons are obtained from the Lagrangian (8). This procedure is equivalent to doing one quark loop calculation. In the chiral limit, using the coupling (10) the quark loop diagram

$$\langle\chi(p')|S|\chi(p)\rangle = -\frac{1}{2}\int d^4xd^4y\langle|T\{\{\bar{\psi}(x)\gamma_\mu\gamma_5\psi(x)\bar{\psi}(y)\times\gamma_\nu\gamma_5\psi(y)\rangle p'_\mu p_\nu e^{i(p'x-py)} \quad (11)$$

is calculated to $O(p^2)$ and the kinetic term of the χ field is found to be

$$\frac{3}{2}F^2\frac{1}{2}\partial_\mu\chi\partial_\mu\chi, \quad (12)$$

where $F^2(1-\frac{2c}{g}) = f_\pi^2$ [3]. The normalized χ field is determined as

$$\chi \to \sqrt{\frac{2}{3}}\frac{1}{F}\chi. \quad (13)$$

It is the same as what has been done in Ref. [3] the couplings between the mesons and the χ field can via the vertex (10) be derived from the Lagrangian (8). As a matter of fact, all the meson vertices obtained from the coupling $-\frac{c}{g}\frac{2\sqrt{2}}{f_\pi}\frac{1}{\sqrt{3}}\bar{\psi}\gamma_\mu\gamma_5\psi\partial_\mu\eta_0$ can be found in Ref. [3]. Replacing $\frac{c}{g}\frac{2\sqrt{2}}{f_\pi}\frac{1}{\sqrt{3}}\eta_0$ in these meson vertices by $\sqrt{\frac{2}{3}}\frac{1}{F}\chi$ all the vertices involving the χ field are obtained.

III. MASS MIXING OF THE 0^{-+} GLUEBALL $\eta(1405)$ AND THE η, η'

The matrix elements of Eq. (4) have been used in the studies of the mixing between η, η' and 0^{-+} glueball [5]. In Ref. [5], Cheng *et al.* present a solution for the pseudoscalar glueball mass around (1.4 ± 0.1) GeV. The mass of the χ field is taken as an input in this study. Besides η, η' there are other $I^G(J^{PC}) = 0^+(0^{-+})$ pseudoscalars listed in Ref. [2]: $\eta(1295)$, $\eta(1405)$, $\eta(1475)$, $\eta(1760)$. In Ref. [6] a systematic phenomenological analysis about these pseudoscalars is presented. The analysis concludes that the $\eta(1405)$ is a possible candidate of the 0^{-+} glueball. In this paper the theory (8) is applied to do a systematic and quantitative study of the physical processes of the possible glueball state $\eta(1405)$. The theoretical predictions can be used to decide whether the $\eta(1405)$ is indeed a 0^{-+} glueball. The same can be done to other possible candidates of the pseudoscalar glueball.

The chiral field theory (8) is applied to study the mixing of η, η' and $\eta(1405)$ in this section. In this chiral theory (1) the pion, kaon, and η are Goldstone bosons. In the leading order in the chiral expansion their masses are found to be [3]

$$\begin{aligned} m^2_{\pi^+} &= -\frac{4}{f^2_\pi}\frac{1}{3}\langle 0|\bar{\psi}\psi|0\rangle(m_u + m_d), \\ m^2_{K^+} &= -\frac{4}{f^2_\pi}\frac{1}{3}\langle 0|\bar{\psi}\psi|0\rangle(m_u + m_s), \\ m^2_{K^0} &= -\frac{4}{f^2_\pi}\frac{1}{3}\langle 0|\bar{\psi}\psi|0\rangle(m_d + m_s). \end{aligned} \tag{14}$$

To the first order in current quark masses, the following elements of the mass matrices are derived from Eqs. (1) and (14):

$$\begin{aligned} m^2_{\eta_8} &= -\frac{4}{f^2_\pi}\frac{1}{3}\langle 0|\bar{\psi}\psi|0\rangle\frac{1}{3}(m_u + m_d + 4m_s) \\ &= \frac{1}{3}\{2(m^2_{K^+} + m^2_{K^0}) - m^2_\pi\}, \\ m^2_{\eta_0} &= -\frac{4}{f^2_\pi}\frac{1}{3}\langle 0|\bar{\psi}\psi|0\rangle\frac{2}{3}(m_u + m_d + m_s) \\ &= \frac{1}{3}(m^2_{K^+} + m^2_{K^0} + m^2_\pi), \\ \Delta m^2_{\eta_8\eta_0} &= \frac{4\sqrt{2}}{9}\frac{1}{f^2_\pi}\frac{1}{3}\langle 0|\bar{\psi}\psi|0\rangle(m_u + m_d - 2m_s) \\ &= \frac{\sqrt{2}}{9}(m^2_{K^+} + m^2_{K^0} - 2m^2_\pi), \end{aligned} \tag{15}$$

$m^2_{\eta_8} = 0.3211\ \text{GeV}^2$ and $m^2_{\eta_0} = 0.1703\ \text{GeV}^2$. If there is no 0^{-+} glueball the mass of η' is determined to be

$$m^2_{\eta'} = m^2_{\eta_8} + m^2_{\eta_0} - m^2_\eta = 0.1911\ \text{GeV}^2 \tag{16}$$

which is much smaller than the physical value $0.9178\ \text{GeV}^2$. This problem is known as the U(1) anomaly [7]. The diagram of two gluon exchange leads to an additional mass term for $m^2_{\eta_0}$, which is proportional to $\frac{g_s^2}{(4\pi)^2} \times \langle 0|F\tilde{F}|\eta_0\rangle$ [7]. In this study $m^2_{\eta_0}$ is taken as a parameter. It is necessary to point out that the current quark mass expansion and the N_C expansion are two independent expansions in this theory. Using Eqs. (7), (9), and (14), the mixing between the η_8 and the χ is found to be

$$\begin{aligned} \Delta m^2_{\chi\eta_8} &= -\frac{4\sqrt{2}}{9}\frac{1}{f_\pi F}\frac{1}{3}\langle 0|\bar{\psi}\psi|0\rangle(m_u + m_d - 2m_s) \\ &= -\frac{\sqrt{2}}{9}\frac{f_\pi}{F}(m^2_{K^+} + m^2_{K^0} - 2m^2_\pi). \end{aligned} \tag{17}$$

Three of the elements of the mass matrix, $m^2_{\eta_8}$, $m^2_{\eta_8\eta_0}$, $m^2_{\chi\eta_8}$ are determined to the first order in the current quark masses. Both the current quark masses and two gluon exchange contribute to the matrix element $\Delta m^2_{\chi\eta_0} \equiv \Delta_3$, which is taken as a parameter. $m^2_{\eta_0}$, Δ_3, and m^2_χ are the three parameters of the mass matrix of η_8, η_0, χ. $m_{\eta(1405)}$, m_η, and η' are taken as inputs. The equation

$$m^2_{\eta_8} + m^2_{\eta_0} + m^2_\chi = m^2_\eta + m^2_{\eta'} + m^2_{\eta(1405)} \tag{18}$$

is one of the three eigenvalue equations of the mass matrix. The other two eigenvalue equations are derived as

$$\Delta_3^2 + m^4_{\eta_0} - 2.87m^2_{\eta_0} + 1.77 = 0, \tag{19}$$

$$\Delta_3^2 + m^4_{\eta_0} - 2.88m^2_{\eta_0} + 1.74 - 0.025\,27\Delta_3 = 0. \tag{20}$$

The difference between these two equations is very small. It is very interesting to notice that in the chiral limit, $m^2_{\eta_8}$, $\Delta m^2_{\eta_8\eta_0}$(15), and $\Delta m^2_{\chi\eta_8}(17) \to 0$. Therefore, in the limit, $m_q \to 0$, the η_8, η_0, χ mixing is reduced to the $\eta_0 - \chi$ mixing. The two eigenvalue equations of the mass matrix of η_0 and χ are found to be

$$\begin{aligned} m^2_{\eta_0} + m^2_\chi &= m^2_{\eta'} + m^2_{\eta(1405)}, \\ \Delta_3^2 + m^4_{\eta_0} - 2.89m^2_{\eta_0} + 1.81 &= 0. \end{aligned} \tag{21}$$

Because of $m^2_{\eta_8} \approx m^2_\eta$ the first equation of Eqs. (21) is very close to Eq. (18) and Eqs. (19) and (20) are reduced to Eq. (21) in the chiral limit. Therefore, the difference between Eqs. (19) and (20) is caused by the current quark masses. The masses of the current quarks listed in Ref. [2] spread in a wide range: $m_u = 1.5$ to 3.3 MeV, $m_d = 3.5$ to 6.0 MeV, $m_s = 104^{+26}_{-34}$ MeV. The numerical values of $m^2_{\eta_8}$, $\Delta m^2_{\eta_8\eta_0}$ (15), and $\Delta m^2_{\chi\eta_8}$ (17) are computed by using the mass formulas (14), (15), and (17) which are at the first order in the expansion of the current quark masses. The effect of the current quark masses at the second order can be seen from the pion masses. At the first order in the current quark masses $m^2_{\pi^+} = m^2_{\pi^0}$. The mass difference of π^+ and π^0 is about 3.5% of the average of the pion mass. Nonzero $m^2_{\pi^+} - m^2_{\pi^0}$ results in the second order of the current quark masses [8] (of course, the electromagnetic interactions too). There should be errors caused by the current quark masses at higher orders in those values (15) and (17). On the other hand, Eqs. (19)–(21) show that the effect of the current quark masses on the mixing of $\eta_0 - \chi$ is small. In the study of the physics of $\eta(1405)$ Eq. (19) is taken into account. Obviously, the values of the current quark masses affect the η meson more. The physics of the η meson will not be studied in this paper.

Solving the eigenvalue equations of the mass matrix of η_8, η_0, χ,

$$\begin{aligned}
&\eta = a_1\eta_8 + b_1\eta_0 + c_1\chi,\\
&a_1 = \frac{m_2 - 0.3 + 1.2453\Delta_3}{((m_2 - 0.3 + 1.2453\Delta_3)^2 + 0.0523\Delta_3 + 0.2422m_2 - 0.1944)^{1/2}},\\
&b_1 = \frac{0.071\,08 + 0.3679\Delta_3}{m_2 - 0.3 + 1.2453\Delta_3}a_1, \qquad c_1 = \frac{-0.3679m_2 + 0.199}{m_2 - 0.3 + 1.2453\Delta_3}a_1, \qquad \eta' = a_2\eta_8 + b_2\eta_0 + c_2\chi,\\
&a_2 = \frac{m_2 + 1.2453\Delta_3 - 0.9172}{0.071\,08\text{–}10.4432}b_2, \qquad c_2 = \frac{10.4432m_2 - 9.49}{0.071\,08\text{–}10.4432}b_2,\\
&b_2 = \frac{0.071\,08\text{–}10.4432}{((0.071\,08\text{–}10.4432)^2 + (10.4432m_2 - 9.49)^2 + (m_2 + 1.2453\Delta_3 - 0.9172)^2)},\\
&\eta(1405) = a_3\eta_8 + b_3\eta_0 + c_3\chi, \qquad a_3 = \frac{0.043}{1.9771 - m_2}(-1.5768 + \Delta_3 + 0.7988m_2)c_3, \qquad b_3 = \frac{0.002\,454 + \Delta_3}{1.9771 - m_2}c_3,\\
&c_3 = \left\{1 + \frac{1}{1.9771 - m_2}[(0.002\,454 + \Delta_3)^2 + 0.001\,849(-1.5768 + \Delta_3 + 0.7988m_2)^2]\right\}^{-1/2}
\end{aligned} \tag{22}$$

are obtained, where $m_2 = m^2_{\eta_0}$, a_1, b_2, c_3 are taken to be positive.

IV. $\eta(1405) \to \gamma\gamma$ DECAY

There is one independent parameter left in Eqs. (22), which can be determined by the decay rate of $\eta' \to \gamma\gamma$. The vector meson dominance (VMD) is a natural result of this chiral field theory [3]. The decay of pseudoscalar to two photons is an anomalous process. The couplings between η_8, η_0 and $\rho\rho$, $\omega\omega$, $\phi\phi$ are presented in Ref. [3]:

$$\begin{aligned}
\mathcal{L}_{\eta_8 vv} &= \frac{N_C}{(4\pi)^2}\frac{8}{\sqrt{3}g^2 f_\pi}\eta_8\epsilon^{\mu\nu\alpha\beta}\{\partial_\mu\rho^i_\nu\partial_\alpha\rho^i_\beta\\
&\quad + \partial_\mu\omega_\nu\partial_\alpha\omega_\beta - 2\partial_\mu\phi_\nu\partial_\alpha\phi_\beta\},\\
\mathcal{L}_{\eta_0 vv} &= \frac{N_C}{(4\pi)^2}\frac{8\sqrt{2}}{\sqrt{3}g^2 f_\pi}\eta_0\epsilon^{\mu\nu\alpha\beta}\{\partial_\mu\rho^i_\nu\partial_\alpha\rho^i_\beta\\
&\quad + \partial_\mu\omega_\nu\partial_\alpha\omega_\beta + \partial_\mu\phi_\nu\partial_\alpha\phi_\beta\}.
\end{aligned} \tag{23}$$

The VMD leads to following relationships:

$$\rho^0_\mu \to \frac{1}{2}egA_\mu, \qquad \omega_\mu \to \frac{1}{6}egA_\mu, \qquad \phi_\mu \to \frac{-1}{3\sqrt{2}}egA_\mu. \tag{24}$$

The couplings

$$\begin{aligned}
\mathcal{L}_{\eta_8\gamma\gamma} &= \frac{\alpha N_C}{4\pi}\frac{8}{\sqrt{3}f_\pi}\eta_8\frac{1}{6}\epsilon^{\mu\nu\alpha\beta}\partial_\mu A_\nu\partial_\alpha A_\beta,\\
\mathcal{L}_{\eta_0\gamma\gamma} &= \frac{\alpha N_C}{4\pi}\frac{8\sqrt{2}}{\sqrt{3}f_\pi}\eta_0\frac{1}{3}\epsilon^{\mu\nu\alpha\beta}\partial_\mu A_\nu\partial_\alpha A_\beta
\end{aligned} \tag{25}$$

are found from Eqs. (23) and (24). The vertex $\mathcal{L}_{\chi vv}$ is determined by the couplings (8)

$$\begin{aligned}
&-\sqrt{\frac{2}{3}}\frac{1}{F}\bar{\psi}\gamma_\mu\gamma_5\psi\partial_\mu\chi, \qquad \frac{1}{g}\bar{\psi}\tau^i\gamma_\mu\psi\rho^i_\mu,\\
&\frac{1}{g}\bar{\psi}\gamma_\mu\psi\omega_\mu, \qquad -\frac{\sqrt{2}}{g}\bar{s}\gamma_\mu s\phi_\mu,
\end{aligned} \tag{26}$$

where $\frac{1}{g}$ and $\frac{\sqrt{2}}{g}$ are the normalization factor of the fields ρ, ω and ϕ, respectively, and s is the strange quark field. The calculation (to the fourth orders in the covariant derivatives) shows that two terms are found from the triangle quark loop diagrams of the χvv vertex

$$\frac{3}{4}\frac{N_C}{g^2}\frac{\sqrt{2}}{\sqrt{3}}\frac{1}{F}\frac{1}{(4\pi)^2}\epsilon^{\mu\nu\alpha\beta}p_\mu(q_{1\nu} - q_{2\nu})e^{\lambda_1}_\alpha e^{\lambda_2}_\beta$$

and

$$-\frac{3}{4}\frac{N_C}{g^2}\frac{\sqrt{2}}{\sqrt{3}}\frac{1}{F}\frac{1}{(4\pi)^2}\epsilon^{\mu\nu\alpha\beta}p_\mu(q_{1\nu} - q_{2\nu})e^{\lambda_1}_\alpha e^{\lambda_2}_\beta,$$

where p, $q_{1,2}$ are the momenta of the χ and the two vectors, respectively; $e_\alpha^{\lambda_{1,2}}$ are the polarization vectors of the two vector fields, respectively. These two terms canceled each other. Therefore, in the chiral limit the glueball χ component is not coupled to the vector-vector meson pairs, the vertex χvv vanishes. The pure glueball field χ does not decay to two photons. Besides the axial-vector couplings [the first equation of Eqs. (9)] the pseudoscalars η_8, η_0 have the pseudoscalar couplings [the second equations of Eq. (9)]. The pseudoscalars η_8, η_0 are coupled to the vector-vector meson pairs and decay to two photons (23). This is a very important difference between the pure 0^{-+} glueball and the η_8, η_0 mesons.

In the chiral limit, only the $\eta_{8,0}$ components of the η' meson contribute to the $\eta' \to \gamma\gamma$ decay. Using Eqs. (23), the decay width is found to be

$$\Gamma(\eta' \to \gamma\gamma) = \frac{\alpha^2}{16\pi^3}\frac{m_{\eta'}^3}{f_\pi^2}\left(2\sqrt{\frac{2}{3}}b_2 + \frac{1}{\sqrt{3}}a_2\right)^2, \tag{27}$$

where $f_\pi = 0.182$ GeV is taken. The experimental data of $\Gamma(\eta' \to \gamma\gamma)$ is $4.31(1 \pm 0.13)$ keV. By inputting $\Gamma(\eta' \to \gamma\gamma)$ and using Eqs. (18), (19), and (22),

$$m_{\eta_0}^2 = 1.25\ \text{GeV}^2, \qquad \Delta_3 = 0.51\ \text{GeV}^2, \qquad m_\chi = 1.28\ \text{GeV}. \tag{28}$$

are determined. Substituting the values of $m_{\eta_0}^2$, Δ_3, and m_χ into Eqs. (22) the expressions of η, η', $\eta(1405)$ are found to be

$$\begin{aligned} \eta &= 0.9742\eta_8 + 0.1593\eta_0 - 0.16\chi, \\ \eta' &= -0.1513\eta_8 + 0.8208\eta_0 - 0.551\chi, \\ \eta(1405) &= -0.003\,522\eta_8 + 0.5724\eta_0 + 0.8199\chi. \end{aligned} \tag{29}$$

Equation (29) shows that the η' meson contains a large component of the glueball, the η_0 meson component in the $\eta(1405)$ is large, and the η_8 component in the $\eta(1405)$ is negligibly small. The mixing between the η_0 meson and the glueball χ is very strong. These results confirm that in the chiral limit the $\eta_8 - \eta_0 - \chi$ mixing is reduced to the $\eta_0 - \chi$ mixing. The effects of the current quark masses are small on the $\eta_0 - \chi$ mixing.

Assuming the η has zero gluonium content and by measuring the ratio $R_\phi = \text{BR}(\phi \to \eta'\gamma)/\text{BR}(\phi \to \eta\gamma)$, KLOE has determined the mixing matrix of $\eta - \eta' - G$ as (see Cheng *et al.* in [5], and [9])

$$U_{\text{KLOE}} = \begin{pmatrix} 0.766 & -0.6430 & 0 \\ 0.596 & 0.710 & 0.375 \\ -0.241 & -0.287 & 0.927 \end{pmatrix}.$$

In this paper a different approach has been applied to study the $\eta - \eta' - G$ mixing. There are differences between Eq. (29) and U_{KLOE}. For example, in U_{KLOE} the η has no glueball content and the glueball has a certain amount of the η_8 component. Equation (29) shows that there is glueball content in the η and the η_8 component of the glueball is very small.

The orthogonality between the expressions (29) show that the accuracy of the expression of the η is about 93% and for the η' and the $\eta(1405)$ the accuracies are about 98%. For the η the error is larger; for the η' and the $\eta(1405)$ the error is smaller. In the determination of Eq. (29) m_π^2, $m_{K^+}^2$, $m_{K^0}^2$ are taken as inputs to calculate the elements $m_{\eta_8}^2$, $\Delta m_{\eta_8\eta_0}^2$, $\Delta m_{\chi\eta_8}^2$ and the physical masses of the η, η', $\eta(1405)$ are inputs too. The errors of Eq. (29) are caused by the treatments of the current quark masses of the three elements $m_{\eta_8}^2$, $\Delta m_{\eta_8\eta_0}^2$, $\Delta m_{\chi\eta_8}^2$. As shown by Eq. (15) these quantities are expanded to the first order in the current quark masses and evaluated by inputting m_π^2, $m_{K^+}^2$, $m_{K^0}^2$ which are expressed to the first order in the current quark masses too (14). As mentioned in Sec. III these treatments are not accurate enough and errors result. The errors can be reduced greatly by expanding the elements to the second order of the current quark masses and by adjusting the values of the $m_{u,d,s}$ properly. The three elements $m_{\eta_8}^2$, $\Delta m_{\eta_8\eta_0}^2$, $\Delta m_{\chi\eta_8}^2$ affect the η meson most. As mentioned above in the chiral limit the $\eta - \eta' - G$ mixing is reduced to the $\eta - \eta'$ mixing. The effect of accurate treatments of the quantities related to the current quark masses on the 0^{-+} glueball is small. The study on accurate treatments of the current quark masses is beyond the scope of this paper.

The small error for $\eta(1405)$ indicates that the physical mass of the $\eta(1405)$ as one of the eigenvalues of the mass matrix is acceptable and the $\eta(1405)$ fits the room of the 0^{-+} glueball well. Equation (29) is applied to study the physics of the $\eta(1405)$ in this paper.

Only the η_0 component of $\eta(1405)$ contributes to the $\eta(1405) \to \gamma\gamma$ decay and the glueball component χ is suppressed. Using Eqs. (23) and (29),

$$\Gamma(\eta(1405) \to \gamma\gamma) = \frac{\alpha^2}{16\pi^3}\frac{m_{\eta(1405)}^3}{f_\pi^2}\left(2\sqrt{\frac{2}{3}}b_3\right)^2 = 4.55\ \text{keV} \tag{30}$$

is predicted. In Eq. (30) the mass of the $\eta(1405)$ contributes a factor of 3.5 in comparison with $\Gamma(\eta' \to \gamma\gamma)$. The total width of the $\eta(1405)$ is 51.5 ± 3.4 MeV [2] and

$$B(\eta(1405) \to 2\gamma) = 0.87(1 \pm 0.07) \times 10^{-4}$$

is obtained. This small branching ratio is consistent with $\eta(1405)$ having not been discovered in two photon collisions.

The physical processes of the η meson will not be studied in this paper. However, the $\eta \to 2\gamma$ decay is taken as an example of the effects of the current quark masses. Using Eqs. (23) and (29), $\Gamma(\eta \to \gamma\gamma) = 0.361$ keV is obtained. The data is $0.511(1 \pm 0.06)$ keV. The theoretical prediction is lower than the data by about 40%. The coefficients of the expression of the η (23) are sensitive to the values of the current quark masses. For instance, if the $m^2_{\eta_8}$ is changed by about 10%, which is allowed by the date of the current quark masses presented in Ref. [2], the agreement between the prediction of the $\Gamma(\eta \to \gamma\gamma)$ and the experimental value can be achieved.

V. $\eta(1405) \to \gamma\rho$, $\gamma\omega$, $\gamma\phi$ DECAYS

The η_8 component of $\eta(1405)$ is ignored and the χ component does not contribute to the coupling of the $\eta(1405)vv$. The vertex of $\eta(1405)vv$ is determined by the quark component η_0 only. Using the VMD and Eqs. (22) and (23),

$$\mathcal{L}_{\eta(1405)\rho\gamma} = \frac{eN_C}{(4\pi)^2}\frac{8\sqrt{2}}{\sqrt{3}f_\pi} b_3\eta_0\epsilon^{\mu\nu\alpha\beta}\partial_\mu\rho_\nu\partial\alpha A_\beta,$$
$$\mathcal{L}_{\eta(1405)\omega\gamma} = \frac{eN_C}{(4\pi)^2}\frac{8\sqrt{2}}{3\sqrt{3}f_\pi} b_3\eta_0\epsilon^{\mu\nu\alpha\beta}\partial_\mu\omega_\nu\partial\alpha A_\beta, \quad (31)$$
$$\mathcal{L}_{\eta(1405)\phi\gamma} = \frac{eN_C}{(4\pi)^2}\frac{16}{\sqrt{3}f_\pi} b_3\eta_0\epsilon^{\mu\nu\alpha\beta}\partial_\mu\phi_\nu\partial\alpha A_\beta$$

are obtained, where $b_3 = 0.5724$ (29). The universal coupling constant $g = 0.395$ is determined by the decay rate of $\rho \to e^+e$. The decay widths are found to be

$$\Gamma(\eta(1405) \to \rho\gamma) = (0.5724)^2\frac{3\alpha}{2\pi^4 g^2}\frac{1}{f_\pi^2}k_\rho^3,$$
$$k_\rho = \frac{m_{\eta(1405)}}{2}\left(1 - \frac{m_\rho^2}{m^2_{\eta(1405)}}\right),$$
$$\Gamma(\eta(1405) \to \omega\gamma) = \frac{1}{9}(0.5724)^2\frac{3\alpha}{2\pi^4 g^2}\frac{1}{f_\pi^2}k_\omega^3,$$
$$k_\omega = \frac{m_{\eta(1405)}}{2}\left(1 - \frac{m_\omega^2}{m^2_{\eta(1405)}}\right), \quad (32)$$
$$\Gamma(\eta(1405) \to \phi\gamma) = \left(\frac{2}{9}0.5724\right)^2\frac{3\alpha}{2\pi^4 g^2}\frac{1}{f_\pi^2}k_\phi^3,$$
$$k_\phi = \frac{m_{\eta(1405)}}{2}\left(1 - \frac{m_\phi^2}{m^2_{\eta(1405)}}\right).$$

The numerate results are

$$\Gamma(\eta(1405) \to \rho\gamma) = 0.84 \text{ MeV},$$
$$\Gamma(\eta(1405) \to \omega\gamma) = 90.3 \text{ kev}, \quad (33)$$
$$\Gamma(\eta(1405) \to \phi\gamma) = 58.2 \text{ kev}.$$

The branching ratios of these three decay modes are

$$1.63 \times 10^{-2}(1 \pm 0.07), \qquad 1.75 \times 10^{-3}(1 \pm 0.07),$$
$$1.13 \times 10^{-3}(1 \pm 0.07),$$

respectively.

VI. KINETIC MIXING OF χ AND η_0

Besides mass mixing between the pseudoscalar mesons and the 0^{-+} glueball studied in Sec. III, there is kinetic mixing between the η_0 meson and the χ glueball. While the mass matrix is diagonalized and the physical states are determined, however, the matrix of the kinetic terms might not be diagonalized by these new physical states. The $\rho - \omega$ system is a good example. Equation (1) shows that the mass matrix of the ρ and the ω mesons is diagonalized. The kinetic terms of the ρ and the ω fields are generated by the quark loop diagrams [3]. The ρ fields are non-Abelian gauge fields. The mixing between the kinetic terms of the ρ^0 and the ω fields is dynamically generated by the quark loops, which is determined by the mass difference of the current quark masses, $m_d - m_u$, and the electromagnetic interactions [3,10]

$$\mathcal{L}_{\rho-\omega} = \left\{-\frac{1}{4\pi^2 g^2}\frac{1}{m}(m_d - m_u) + \frac{1}{24}e^2g^2\right\}$$
$$\times (\partial_\mu\rho_\nu - \partial_\nu\rho_\mu)(\partial_\mu\omega_\nu - \partial_\nu\omega_\mu).$$

In this chiral field theory while the kinetic terms of the η_0 field [3] and the χ field are generated by the quark loop diagrams (12) the kinetic mixing, $\partial_\mu\eta_0\partial_\mu\chi$, is dynamically generated by the quark loops too. The coefficient of this mixing is determined by three vertices

$$-\sqrt{\frac{2}{3}}\frac{1}{F}\bar{\psi}\gamma_\mu\gamma_5\psi\partial_\mu\chi - \frac{1}{\sqrt{3}}\frac{c}{g}\frac{2\sqrt{2}}{f_\pi}\bar{\psi}\gamma_\mu\gamma_5\psi\partial_\mu\eta_0$$
$$- im\frac{1}{\sqrt{3}}\frac{2\sqrt{2}}{f_\pi}\bar{\psi}\gamma_5\psi\eta_0. \quad (34)$$

By calculating the S-matrix element $\langle\eta_0|S|\chi\rangle$, in the chiral limit the kinetic mixing is found to be

$$-\left(1 - \frac{2c}{g}\right)^{1/2}\partial_\mu\eta_0\partial_\mu\chi. \quad (35)$$

This kinetic mixing cannot be referred to the mass mixing. The amplitudes of $\eta(1405) \to \gamma\gamma$, γv have been calculated to the fourth orders in the covariant derivatives. At this order there is no contribution from the kinetic mixing term (35) to these processes.

VII. $J/\psi \to \gamma\eta(1405)$ DECAY

In pQCD the J/ψ radiative decay is described as $J/\psi \to \gamma gg$, $gg \to$ meson. Therefore, if the meson is strongly coupled to two gluons it should be produced in J/ψ radiative decay copiously. Both the η' and the $\eta(1405)$ contain large components of the pure glueball state χ. Therefore, large branching ratios of $J/\psi \to \gamma\eta'$, $\gamma\eta(1405)$ should be expected.

In Ref. [11] the decay width of the $J/\psi \to \gamma\chi$ is derived as

$$\Gamma(J/\psi \to \gamma\chi) = \frac{2^{11}}{81}\alpha\alpha_s^2(m_c)\psi_J^2(0)f_G^2\frac{1}{m_c^8} \times \frac{(1-\frac{m^2}{m_J^2})^3}{\{1-2\frac{m^2}{m_J^2}+\frac{4m_c^2}{m_J^2}\}^2} \times \left\{2m_J^2 - 3m^2\left(1+\frac{2m_c}{m_J}\right) - 16\frac{m_c^3}{m_J}\right\}^2, \quad (36)$$

where $\psi_J(0)$ is the wave function of the J/ψ at the origin, f_G is a parameter related to the glueball state χ, m is the mass of the physical state which contains the χ state and it will be specified. After replacing corresponding quantities in Eq. (36), $m_c \to m_b$, $m_J \to m_\Upsilon$, $Q_c = \frac{2}{3} \to Q_b = -\frac{1}{3}$, the Eq. (36) has been applied to study $B(\Upsilon(1S) \to \gamma\eta'(\eta))$ [12] and very strong suppression by the mass of the b quark has been found in these processes. The suppression leads to very small $B(\Upsilon(1S) \to \gamma\eta'(\eta))$, which are consistent with the experimental upper limits of $B(\Upsilon(1S) \to \gamma\eta'(\eta))$ [13].

The χ state of Eq. (36) is via both the mass mixing (29) and the kinetic mixing (35) related to the η' and the $\eta(1405)$, respectively

$$\langle\eta'|\chi(0)|0\rangle = -0.551 + 0.8208\left(1-\frac{2c}{g}\right)^{1/2} \times \frac{m_{\eta'}^2}{m_\chi^2 - m_{\eta'}^2} = 0.3044,$$
$$\langle\eta(1405)|\chi(0)|0\rangle = 0.8199 + 0.0.5724\left(1-\frac{2c}{g}\right)^{1/2} \times \frac{m_G^2}{m_\chi^2 - m_G^2} = -1.7788. \quad (37)$$

In Eqs. (37) the widths of η' and $\eta(1405)$ are ignored. Equations (37) show that the kinetic mixing [the second term of Eqs. (37)] (35) plays an essential role in those two matrix elements. Inputting $\Gamma(J/\psi \to \gamma\eta')$, the parameter f_G and $\psi_J^2(0)$ are canceled and the ratio

$$R = \frac{\Gamma(J/\psi \to \gamma\eta(1405))}{\Gamma(J/\psi \to \gamma\eta')} \quad (38)$$

is calculated. The ratio (38) is very sensitive to the value of the mass of the c quark and it increases with m_c dramatically. This sensitivity has already been found in the studies of $J/\psi \to \gamma(\eta, \eta')$, $\gamma f_2(1270)$ [11,14], $\Upsilon(1S) \to \gamma(\eta, \eta')$, $\gamma f_2(1270)$ [12]. In Ref. [2] $m_c = 1.27^{+0.07}_{-0.11}$ GeV is listed. Inputting $B(J/\psi \to \gamma\eta') = (4.71 \pm 0.27) \times 10^{-3}$, and using Eqs. (36) and (37), the dependence of $B(J/\psi \to \gamma\eta(1405))$ on the mass of the c quark is predicted. A few examples are presented:

$$\begin{aligned} m_c &= 1.22\ \text{GeV}, \\ B(J/\psi \to \gamma\eta(1405)) &= 3.67(1 \pm 0.06) \times 10^{-3}, \\ m_c &= 1.23\ \text{GeV}, \\ B(J/\psi \to \gamma\eta(1405)) &= 6.69(1 \pm 0.06) \times 10^{-3}, \\ m_c &= 1.24\ \text{GeV}, \\ B(J/\psi \to \gamma\eta(1405)) &= 1.13(1 \pm 0.06) \times 10^{-2}. \end{aligned} \quad (39)$$

A large branching ratio for $J/\psi \to \gamma\eta(1405)$ is predicted for $m_c > 1.2$ GeV. In Ref. [2] the measurements

$$\begin{aligned} &B(J/\psi \to \gamma\eta(1405/1475) \to \gamma K\bar{K}\pi) \\ &= (2.8 \pm 0.6) \times 10^{-3}, \\ &B(J/\psi \to \gamma\eta(1405/1475) \to \gamma\eta\pi^+\pi^-) \\ &= (3.0 \pm 0.5) \times 10^{-4}, \\ &B(J/\psi \to \gamma\eta(1405/1475) \to \gamma\rho^0\rho^0) \\ &= (1.7 \pm 0.4) \times 10^{-3}, \\ &B(J/\psi \to \gamma\eta(1405/1475) \to \gamma\gamma\rho^0) \\ &= (0.78 \pm 0.2) \times 10^{-4}, \\ &B(J/\psi \to \gamma\eta(1405/1475) \to \gamma\gamma\phi) < 8.2 \times 10^{-5} \end{aligned}$$

are listed. As discussed in Ref. [6] in the mass region of the $\eta(1440)$ there are two pseudoscalars $\eta(1405)$ and $\eta(1475)$. The $\eta(1475)$ could be the first radial excitation of the η' [15] which mainly decays to $K^*\bar{K}$. Although the $\eta(1405)$ is not separated from the $\eta(1405/1475)$ in these decays, the data shows those branching ratios are not small.

VIII. $\eta(1405) \to \rho\pi\pi$ DECAY

The decay $\eta(1405) \to \rho\pi\pi$ is an anomalous decay mode. There are two subprocesses: (1) $\eta(1405) \to \rho\rho$, $\rho \to \pi\pi$, (2) $\eta(1405) \to \rho\pi\pi$ directly. Because $\chi \to \rho\rho$ vanishes only the $\eta_0 \to \rho\rho$ (23) contributes to (1). The vertex $\rho\pi\pi$ can be found from Ref. [3]

$$\begin{aligned} \mathcal{L}_{\rho\pi\pi} &= \frac{2}{g}f_{\rho\pi\pi}\epsilon_{ijk}\rho_\mu^i\pi^j\partial_\mu\pi^k, \\ f_{\rho\pi\pi} &= 1 + \frac{q^2}{2\pi^2 f_\pi^2}\left\{\left(1-\frac{2c}{g}\right)^2 - 4\pi^2c^2\right\}, \end{aligned} \quad (40)$$

where q is the momenta of the ρ meson and the $f_{\rho\pi\pi}$ is the intrinsic form factor generated by the quark loop and it is a prediction of this chiral field theory of mesons [3]. This intrinsic form factor makes the theoretical results of the form factors of pion and kaons and the decay widths of ρ, K^*, ϕ mesons in excellent agreements with the data [3]. The amplitude of the subprocess (1) is derived as

$$T^{(1)} = -0.5724\frac{4\sqrt{6}}{\pi^2 g^3 f_\pi}\frac{f_{\rho\pi\pi}}{q^2 - m_\rho^2 + i\sqrt{q^2}\Gamma(q^2)} \times \epsilon^{\mu\nu\alpha\beta}k_\mu e_\nu^\lambda k_{1\alpha}k_{2\beta}, \tag{41}$$

where 0.5724 is the coefficient of the η_0 component of $\eta(1405)$ (29), $q = k_1 + k_2$, and $\Gamma(q^2)$ is the decay width of the ρ meson. When $q^2 > 4m_\pi^2$ [3]

$$\Gamma(q^2) = \frac{f_{\rho\pi\pi}^2(q^2)}{12\pi g^2}\sqrt{q^2}\left(1 - \frac{4m_\pi^2}{q^2}\right)^{3/2}. \tag{42}$$

The subprocess (2) is the decay mode without intermediate resonance. The vertex of this process is similar to $f_1 \to \rho\pi\pi$ presented in Ref. [3] (Eq. 111 of Ref. [3]) and it is found to be

$$\mathcal{L}_{\chi\rho\pi\pi} = \frac{2\sqrt{2}}{\sqrt{3}g\pi^2 f_\pi^2 F}\left(1 - \frac{4c}{g}\right)\epsilon_{ijk}\epsilon^{\mu\nu\alpha\beta}\partial_\mu\chi\partial_\nu\pi^i\partial_\alpha\pi^j\rho_\beta^k. \tag{43}$$

The amplitude of the subprocess (2) is derived from Eq. (43)

$$T^{(2)} = 0.8199\frac{4\sqrt{2}}{\sqrt{3}g\pi^2}\frac{1}{f_\pi^2}\frac{1}{F}\epsilon^{\mu\nu\alpha\beta}p_\mu e_\nu^\lambda k_{1\alpha}k_{2\beta}, \tag{44}$$

where 0.8199 is the coefficient of the χ component of $\eta(1405)$ (29). Only the glueball component χ contributes to $T^{(2)}$. Adding the two amplitudes (41) and (44) together the amplitude of the process $\eta(1405) \to \rho^0\pi^+\pi^-$ is found and the decay width is computed

$$\Gamma(\eta(1405) \to \rho^0\pi^+\pi^-) = 0.92\ \text{MeV}. \tag{45}$$

$T^{(1)}$ dominates the decay. The branching ratio of this channel is about 1.8%. The small branching ratio is the result of two factors: the invariant mass of $\pi\pi$ is less than m_ρ and the phase space of the three body decay is much smaller than the one of the two body decay. There are other two decay modes:

$$\Gamma(\eta(1405) \to \rho^+\pi^0\pi^-) = \Gamma(\eta(1405) \to \rho^-\pi^+\pi^0) = \Gamma(\eta(1405) \to \rho^0\pi^+\pi^-). \tag{46}$$

The total branching ratio of $\eta(1405) \to \rho\pi\pi$ is 5.4%.

IX. $\eta(1405) \to a_0(980)\pi$ DECAY

Two body decay, $\eta(1405) \to a_0(980)\pi$, should be the major decay mode of $\eta(1405)$. In the Lagrangian (1) the isovector scalar field $a_0(980)$ is not included and in order to study this decay mode the $a_0(980)$ field must be introduced to the Lagrangian (1). As mentioned in the section of introduction that a meson field is expressed as a quark operator in this theory. It is natural that

$$a_0(980) \sim \bar{\psi}\tau^i\psi a_0^i. \tag{47}$$

The quantum numbers of $a_0(980)$ are $J^{PC} = 0^{++}$ and in the Lagrangian (1) there is already a term $-m\bar{\psi}u\psi$. The parameter m is originated in the quark condensate whose $J^{PC} = 0^{++}$ too. It is proposed that the $a_0(980)$ field can be added to the Lagrangian by modifying $-m\bar{\psi}u\psi$ to

$$-\frac{1}{2}\bar{\psi}\{(m + \tau^i a_0^i)u + u(m + \tau^i a_0^i)\}\psi. \tag{48}$$

Of course a mass term

$$\frac{1}{2}m_{a_0}^2 a_0^i a_0^i \tag{49}$$

has to be introduced. At the tree level the combination of Eqs. (48) and (49) leads to

$$a_0^i = -\frac{1}{m_{a_0}^2}\bar{\psi}\tau^i\psi. \tag{50}$$

Therefore, Eq. (47) is revealed from this scheme.

The couplings between the mesons and the $a_0(980)$ can be derived from Eq. (48). Using the vertex

$$\mathcal{L} = -\bar{\psi}\tau^i\psi a_0^i \tag{51}$$

obtained from Eq. (48), the quark loop diagram of the S-matrix element $\langle a_0|S|a_0\rangle$ is calculated and the kinetic term of the a_0 field is found. The a_0 field is normalized to be

$$a_0 \to \sqrt{\frac{2}{3}}\frac{1}{g}\left(1 - \frac{1}{3\pi^2 g^2}\right)^{-1/2} a_0. \tag{52}$$

A mass term is generated from the quark loop diagram. The mass of the a_0 field has to be redefined, which is taken as a parameter.

In this paper we focus on the decay $\eta(1405) \to a_0(980)\pi$ and the decays of $a_0(980)$ will be studied in another paper. Ignoring the η_8 component of $\eta(1405)$, there are $\eta_0 \to a_0\pi$ and $\chi \to a_0\pi$ two processes. In this study the decay width of $\eta(1405) \to a_0\pi$ is calculated to the leading order in the momentum expansion. Because of the derivative coupling $-\sqrt{\frac{2}{3}}\frac{1}{F}\bar{\psi}\gamma_\mu\gamma_5\psi\partial_\mu\chi$ the $\chi \to a_0\pi$ channel is at the next leading order in the momentum expansion. Therefore, only the $\eta_0 \to a_0\pi$ channel is taken into account. The vertices related to this channel are found from the vertex (48)

$$\mathcal{L} = -i\frac{2\sqrt{2}}{\sqrt{3}f_\pi}m\bar{\psi}\gamma_5\psi\eta_0 - i\frac{2m}{f_\pi}\bar{\psi}\tau^i\gamma_5\psi\pi^i - \sqrt{\frac{2}{3}}\frac{1}{g}\left(1 - \frac{1}{3\pi^2 g^2}\right)^{-1/2}\bar{\psi}\tau^i\psi a_0^i - i\frac{2m}{f_\pi}\sqrt{\frac{2}{3}}\frac{1}{g}\left(1 - \frac{1}{3\pi^2 g^2}\right)^{-1/2}\bar{\psi}I\gamma_5\psi a_0^i\pi^i, \tag{53}$$

where I is a 2×2 unit matrix. To the leading order in the momentum expansion the amplitude obtained from these vertices (53) is found to be

$$T = -0.5724\frac{8\sqrt{2}}{\sqrt{3}f_\pi^2}\frac{1}{g}\left(1-\frac{1}{3\pi^2g^2}\right)^{-1/2}\times\left\{\frac{1}{3}\langle 0|\bar{\psi}\psi|0\rangle + 3m^3g^2\right\}, \quad (54)$$

where the coefficient 0.5724 is the component of the η_0 of the $\eta(1405)$ (29). In the amplitude (54) the quark condensate is obtained from the vertex $\bar{\psi}I\gamma_5\psi a_0^i\pi^i$ which is derived from

$$-\frac{1}{2}i\bar{\psi}\gamma_5\{a_0\pi + \pi a_0\}\psi \quad (55)$$

of Eq. (48). The vertices, $\bar{\psi}\tau^i\gamma_5\psi\pi^i$ and $\bar{\psi}\tau^i\psi a_0^i$, which are obtained from Eqs. (1) and (48), contribute to the term, $3m^3g^2$, of Eq. (54). It is known that the quark condensate is negative. Therefore, there is cancellation between the two terms of the amplitude (54). The cancellation makes the decay width narrower. The mechanism (48) introducing the a_0 field to this chiral field theory leads to the cancellation. The decay width of $\eta(1405)\to a_0\pi$ is sensitive to the value of the quark condensate.

$$\frac{1}{3}\langle 0|\bar{\psi}\psi|0\rangle = -(0.24)^3\ \text{GeV} \quad (56)$$

is taken and it is close to the value used in Ref. [16]. The constituent quark mass m is determined in Ref. [3]

$$m^2 = \frac{f_\pi^2}{6g^2(1-\frac{2c}{g})^2}. \quad (57)$$

The total decay width of the three modes, $a_0^+\pi^-$, $a_0^-\pi^+$, $a_0^0\pi^0$ of $\eta(1405)\to a_0\pi$ is calculated to be

$$\Gamma(\eta(1405)\to a_0\pi) = 44\ \text{MeV}. \quad (58)$$

The branching ratio $B(\eta(1405)\to a_0\pi) = 86(1\pm 0.07)\%$. Therefore, $\eta(1405)\to a_0\pi$ is the major decay mode of $\eta(1405)$.

The decays of $\eta(1405)\to K\bar{K}\pi$, $\eta\pi\pi$, $\eta'\pi\pi$ are more complicated, in which the couplings between a_0 and $K\bar{K}$, $\eta\pi$, $\eta'\pi$; $f_0(980)$ and $\pi\pi$, $K\bar{K}\ldots$ are involved. There are direct couplings (without intermediate state) too. The chiral field theory (8) can be applied to study these processes. The study will be presented in the near future.

X. $\eta(1405)\to K^*(890)K$ DECAY

The decay mode $\eta(1405)\to K\bar{K}\pi$ has been found [2]. $\eta(1405)\to K^*(890)K$ is a possible decay channel. This channel has normal parity. In order to study it the real part (with normal parity) of the Lagrangian (1) is quoted from Ref. [3]

$$\begin{aligned}\mathcal{L}_{\rm RE} = {} & \frac{N_c}{(4\pi)^2}m^2\frac{D}{4}\Gamma\left(2-\frac{D}{2}\right)\mathrm{Tr}D_\mu UD^\mu U^\dagger - \frac{1}{3}\frac{N_c}{(4\pi)^2}\frac{D}{4}\Gamma\left(2-\frac{D}{2}\right)\mathrm{Tr}\{v_{\mu\nu}v^{\mu\nu} + a_{\mu\nu}a^{\mu\nu}\} \\ & + \frac{i}{2}\frac{N_c}{(4\pi)^2}\mathrm{Tr}\{D_\mu UD_\nu U^\dagger + D_\mu U^\dagger D_\nu U\}v^{\nu\mu} + \frac{i}{2}\frac{N_c}{(4\pi)^2}\mathrm{Tr}\{D_\mu U^\dagger D_\nu U - D_\mu UD_\nu U^\dagger\}a^{\nu\mu} \\ & + \frac{N_c}{6(4\pi)^2}\mathrm{Tr}D_\mu D_\nu UD^\mu D^\nu U^\dagger - \frac{N_c}{12(4\pi)^2}\mathrm{Tr}\{D_\mu UD^\mu U^\dagger D_\nu UD^\nu U^\dagger + D_\mu U^\dagger D^\mu UD_\nu U^\dagger D^\nu U \\ & - D_\mu UD_\nu U^\dagger D^\mu UD^\nu U^\dagger\} + \frac{1}{2}m_0^2(\rho_i^\mu\rho_{\mu i} + \omega^\mu\omega_\mu + a_i^\mu a_{\mu i} + f^\mu f_\mu + K_\mu^a\bar{K}^{a\mu} + K_1^\mu K_{1\mu} + \phi_\mu\phi^\mu + f_s^\mu f_{s\mu}),\end{aligned} \quad (59)$$

where

$$\begin{aligned} D_\mu U &= \partial_\mu U - i[v_\mu, U] + i\{a_\mu, U\}, \\ D_\mu U^\dagger &= \partial_\mu U^\dagger - i[v_\mu, U^\dagger] - i\{a_\mu, U^\dagger\}, \\ v_{\mu\nu} &= \partial_\mu v_\nu - \partial_\nu v_\mu - i[v_\mu, v_\nu] - i[a_\mu, a_\nu], \\ a_{\mu\nu} &= \partial_\mu a_\nu - \partial_\nu a_\mu - i[a_\mu, v_\nu] - i[v_\mu, a_\nu], \\ D_\nu D_\mu U &= \partial_\nu(D_\mu U) - i[v_\nu, D_\mu U] + i\{a_\nu, D_\mu U\}, \\ D_\nu D_\mu U^\dagger &= \partial_\nu(D_\mu U^\dagger) - i[v_\nu, D_\mu U^\dagger] - i\{a_\nu, D_\mu U^\dagger\}. \end{aligned}$$

The K_μ field (K^*) is included in the v_μ and appears in either the commutators of $D_\mu U$, $D_\mu U^\dagger$, $D_\nu D_\mu U$, $D_\nu D_\mu U^\dagger$ or $v_{\mu\nu}$. The components of η_0 and χ are flavor singlets, therefore, only the component of η_8 which is associated with λ_8 appears in the commutator, $[\lambda_8, K_\mu]$. The vertex obtained from these commutators is

$$\mathcal{L}_{\eta(1405)K^*K} = a_3cf_{ab8}\partial_\mu\eta_8K_\mu^aK^b, \quad (60)$$

where c is a constant determined by Eq. (59) and $a_3 = 0.003\,52$ is from the component of the η_8 component of the $\eta(1405)$ (29). Obviously, the contribution of the vertex (60) to the decay $\eta(1405)\to K^*K$ is very small.

The field $\partial_\mu\chi$ can be included in the a_μ field. The term in Eq. (60),

$$\mathrm{Tr}\{D_\mu UD_\nu U^\dagger + D_\mu U^\dagger D_\nu U\}v^{\nu\mu},$$

needs a special attention. To the fourth order in derivatives

$$\begin{aligned}&\mathrm{Tr}\{D_\mu UD_\nu U^\dagger + D_\mu U^\dagger D_\nu U\}v^{\nu\mu} \\ &= -8\left(1-\frac{2c}{g}\right)\mathrm{Tr}\{\partial_\mu\chi\partial_\nu K + \partial_\mu K\partial_\nu\chi\}(\partial_\nu K_\mu - \partial_\mu K_\nu) \\ &= 0. \end{aligned} \quad (61)$$

This theory predicts that the decay width of $\eta(1405) \to K^*K$ is very small.

The decay rate of $\eta(1405) \to K^*K$ is determined by the η_8 component of the $\eta(1405)$ (29). It is shown in Sec. III that in the chiral limit the $\eta_8 - \eta_0 - \chi$ mixing is reduced to the $\eta_0 - \chi$ mixing and the η_8 component of the physical state $\eta(1405)$ vanishes. Therefore, in the chiral limit the chiral field theory (8) predicts that $\Gamma(\eta(1405) \to K^*K) = 0$. At the next leading order in the chiral expansion the $\eta(1405)$ contains the η_8 component and a small decay rate of $\eta(1405) \to K^*K$ is expected. As mentioned in Sec. III the component η_8 of $\eta(1405)$, a_3, determined in this paper (29) is not accurate. The accurate determination of $\Gamma(\eta(1405) \to K^*K)$ is beyond the scope of this paper.

A 0^{-+} resonance $\eta(1416)$ has been discovered in $\pi^- p \to K^+K^-\pi^0 n$ at 18 GeV [17]. The parameters of this state are determined as [17]

$$M = 1416 \pm 4 \pm 2 \text{ MeV}, \qquad \Gamma = 42 \pm 10 \pm 9 \text{ MeV}.$$

These values are close to $\eta(1405)'s$ [2]. The ratio of the branching ratios

$$R = \frac{B(\eta(1416) \to K^*\bar{K} + \text{c.c})}{B(\eta(1416) \to a_0\pi^0)} = 0.084 \pm 0.024 \qquad (62)$$

have been reported in Ref. [17]. The final state $a_0\pi$ has three states, therefore, the ratio (62) should be divided by 3 and

$$R = 0.028 \pm 0.008. \qquad (63)$$

Assuming the $\eta(1416)$ is the $\eta(1405)$, Eq. (63) shows that $\Gamma(\eta(1405) \to K^*K)$ is narrower than $\Gamma(\eta(1405) \to a_0\pi)$ by 2 orders of magnitude. This result supports the prediction made by this chiral field theory.

XI. SUMMARY

Based on a phenomenologically successful chiral meson theory and the U(1) anomaly a chiral field theory of 0^{-+} glueball has been constructed. Systematic and quantitative study of the properties of the candidate of the 0^{-+} glueball $\eta(1405)$ have been done by this theory. The study of η_8, η_0, χ mixing shows that the mass of $\eta(1405)$ fits the room of the pseudoscalar glueball well. The prediction of the small branching ratio of $\eta(1405) \to 2\gamma$ is consistent with the fact that $\eta(1405)$ has not been found in two photon collisions. The theory predicts that $\eta(1405) \to a_0(980)\pi$ is the major decay mode of $\eta(1405)$. A very small branching ratio of $\eta(1405) \to K^*K$ is predicted and the theory is consistent with the data. The glueball component χ of the $\eta(1405)$ is the dominant contributor of the $J/\psi \to \gamma\eta(1405)$ decay. According to Refs. [6,15], the $K^*\bar{K}$ is the dominant decay mode of the $\eta(1475)$. Large $B(J/\psi \to \gamma\eta(1405))$ is via the kinetic mixing predicted. This is a very important channel to identify the $\eta(1405)$ as a 0^{-+} glueball. As indicated in Sec. VII, there are two states $\eta(1405/1475)$ in the $\eta(1440)$ region. It is suggested that measuring of the branching ratio of $J/\psi \to \gamma\eta(1405)$, $\eta(1405) \to \delta\pi$ and $J/\psi \to \gamma\eta(1475)$, $\eta(1475) \to K^*\bar{K}$ separately will be able to determine $B(J/\psi \to \gamma\eta(1405))$. Then the decay modes of the $\eta(1405)$ can be measured. The comparison between the experimental results and theoretical predictions should be able to determine whether the $\eta(1405)$ is, indeed, a 0^{-+} glueball. The quark component η_0 of the $\eta(1405)$ is the dominant contributor of the decay $\eta(1405) \to \gamma\gamma$, γV, $\rho\pi\pi$, $a_0\pi$. The glueball component χ of the $\eta(1405)$ is suppressed in these processes. This chiral field theory can be applied to study other possible candidates of the 0^{-+} glueball by input their masses into the theory to make quantitative predictions.

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