Inverse neutrinoless double beta decay revisited: Neutrinos, Higgs triplets, and a muon collider

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(Received 1 April 2010; published 1 June 2010)

We revisit the process of inverse neutrinoless double beta decay ($e^-e^- \to W^-W^-$) at future linear colliders. The cases of Majorana neutrino and Higgs triplet exchange are considered. We also discuss the processes $e^-\mu^- \to W^-W^-$ and $\mu^-\mu^- \to W^-W^-$, which are motivated by the possibility of muon colliders. For heavy neutrino exchange, we show that masses up to 10^6 (10^5) GeV could be probed for ee and $e\mu$ machines, respectively. The stringent limits for mixing of heavy neutrinos with muons render $\mu^-\mu^- \to W^-W^-$ less promising, even though this process is not constrained by limits from neutrinoless double beta decay. If Higgs triplets are responsible for inverse neutrinoless double beta decay, observable signals are only possible if a very narrow resonance is met. We also consider unitarity aspects of the process in case both Higgs triplets and neutrinos are exchanged. An exact seesaw relation connecting low energy data with heavy neutrino and triplet parameters is found.

DOI: 10.1103/PhysRevD.81.114001 PACS numbers: 13.66.Lm, 14.60.St, 14.80.Fd

I. INTRODUCTION

Observation of lepton number violation (LNV) would show that neutrinos are Majorana particles [1] and would add most interesting information on the origin of neutrino masses. In particular, the process

$$e^-e^- \to W^-W^-, \qquad (1)$$

called "inverse neutrinoless double beta decay," has frequently been proposed as a probe of LNV and new physics in general [2–7]. Running a future linear collider in an e^-e^- mode would allow looking for this and other lepton number violating and conserving processes [8]. We study inverse neutrinoless double beta decay here in the presence of Majorana neutrinos and a Higgs triplet. We also point out that the currently discussed muon colliders [9,10] would allow one to search for lepton number (and flavor) violating processes like

$$\mu^-\mu^- \to W^-W^- \quad (\text{and } e^-\mu^- \to W^-W^-), \qquad (2)$$

if the machines are run in a like-sign lepton mode. Physics potential of like-sign muon collisions has also been discussed in Ref. [11], and is mentioned as a possibility in Ref. [9], where the prospects and technology of muon colliders are outlined. See [12] for a recent review on the current status of muon collider research. We summarize the model-independent limits on heavy neutrino and Higgs triplet parameters which are relevant to these processes and give the corresponding values for the cross sections. We show in which situations the processes are observable. For heavy neutrino exchange, we show that in electron-electron collisions masses up to 10^6 GeV could be probed, while like-sign $e\mu$ machines can reach 10^5 GeV. The process $\mu^-\mu^- \to W^-W^-$ is less promising, due to strong constraints on mixing of heavy neutrinos with muons (note that this process is not constrained by limits from neutrinoless double beta decay). If Higgs triplets are exchanged in inverse neutrinoless double beta decay, observable signals are unlikely unless a very narrow resonance is met. We stress already here that we do not consider situations in which there are cancellations. This means we assume in the processes only the exchange of one heavy neutrino or of one Higgs triplet, and not the cases in which several neutrinos are present or both a triplet and a heavy neutrino are present. However, we note that if both terms are present (the type I + II seesaw mechanism) there is an exact seesaw relation, which uses low energy data coming from neutrinoless double beta decay and other neutrino data to constrain a particular combination of high-energy (i.e., heavy neutrino and Higgs triplet) parameters. It generalizes a previously discussed formula for the type I seesaw [13,14]. We also comment on unitarity of the cross section $e^-e^- \to W^-W^-$ in case of heavy neutrinos and triplets being simultaneously present.

The paper is organized as follows: in Sec. II we discuss the present limits on parameters relevant for inverse neutrinoless double beta decay. Those come from neutrinoless double beta decay, global fits, and other data, in particular, lepton flavor violation. Section III discusses the process of inverse neutrinoless double beta decay for heavy Majorana neutrinos, while Sec. IV discusses the situation with a Higgs triplet. In Sec. V we argue that unitarity of the cross section is automatically fulfilled in case the type I + II seesaw is present. We conclude in Sec. VI.

II. NEUTRINOLESS DOUBLE BETA DECAY, ITS INVERSE, AND LIMITS ON NEUTRINO AND TRIPLET PARAMETERS

Figure 1 shows the three different possible diagrams for inverse neutrinoless double beta decay in the presence of Majorana neutrinos and a Higgs triplet. Figures 1(a) and

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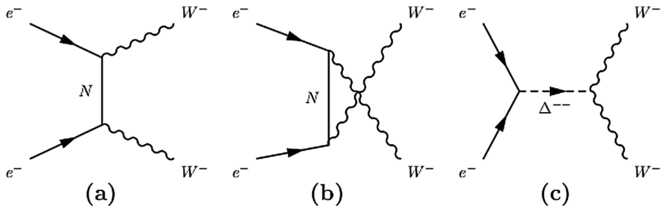

FIG. 1. Diagrams for $e^-e^- \to W^-W^-$ with Majorana neutrinos N and a doubly charged Higgs scalar Δ^{--}. (a) is the t channel, (b) the u channel, and (c) the s channel.

1(b) are connected to the diagram of neutrinoless double beta decay ($0\nu\beta\beta$). This is the process $(A, Z) \to (A, Z + 2) + 2e^-$, for which the electrons are outgoing and the W^- couple to incoming u and outgoing d quarks. Indeed, as each vertex receives a factor U_{ei} and the propagator of the neutrino introduces a term $m_i/(q^2 - m_i^2)$, the dependence on neutrino parameters is the same as for neutrinoless double beta decay.

Because a hypercharge $Y = 2$ Higgs triplet contains a doubly charged member (Δ^{--}), Fig. 1(c) is possible. The Δ^{--} can also lead to $0\nu\beta\beta$ [15]. One should note that the Higgs triplet contains also a singly charged scalar Δ^-, which can contribute to $0\nu\beta\beta$ as well (these are diagrams in which one W^- is replaced by a Δ^-). However, its coupling to quarks is suppressed by v_L/v, where v_L is the vacuum expectation value (VEV) of the neutral component of the triplet, and v the VEV of the standard model (SM) doublet. Moreover, the triplets are presumably heavier than the W. Hence, the diagrams for $0\nu\beta\beta$ containing Δ^- are suppressed with respect to the diagram containing Δ^{--} and consequently there is a direct connection between inverse neutrinoless double beta decay and neutrinoless double beta decay also in scenarios with Higgs triplets.

We begin by studying constraints on light and heavy neutrinos, as well as on Higgs triplet parameters from lepton number and flavor violation.

The most commonly assumed mechanism of $0\nu\beta\beta$ is light neutrino exchange, for which the "effective mass" $|m_{ee}|$ is constrained as follows [16]:

$$|m_{ee}| \equiv \sum N_{ei}^2 (m_\nu)_i \lesssim 1 \text{ eV}. \quad (3)$$

We have introduced here the notation that light neutrino masses are called $(m_\nu)_i$ and their mixing matrix is N. The above limit generously takes nuclear matrix element uncertainties into account. The most model-independent neutrino mass limit is 2.3 eV from the Mainz and Troitsk experiments [17], and $|m_{ee}|$ cannot exceed this value. Hence, the above upper value of 1 eV is of the same order as the "theoretical" upper value of 2.3 eV, which is valid in the case of quasidegenerate light neutrinos [i.e., $(m_\nu)_1 \simeq (m_\nu)_2 \simeq (m_\nu)_3 \equiv m_0$].

In case heavy neutrinos are exchanged in neutrinoless double beta decay, we have [18]

$$\left|\frac{1}{M_{ee}}\right| = \left|\sum S_{ei}^2 \frac{1}{M_i}\right| \lesssim 5 \times 10^{-8} \text{ GeV}^{-1}. \quad (4)$$

Here heavy neutrino masses are called M_i and the matrix describing their mixing with leptons is called S. Note that at the current stage we have not discussed any seesaw mechanism connected to light and heavy neutrino masses, which would link $|m_{ee}|$ and $|\frac{1}{M_{ee}}|$.

With regards to the mixing of electrons and muons with heavy neutral fermions, there are upper limits of [19]

$$\sum |S_{ei}|^2 \leq 0.0052, \qquad \sum |S_{\mu i}|^2 \leq 0.0001, \quad (5)$$

respectively, obtained from global fits, in particular, of LEP data. Note that the limit on $|S_{\mu i}|^2$ is more stringent. Comparing with the $0\nu\beta\beta$ limit in Eq. (4), the global one on $|S_{ei}|^2$ is stronger for masses $M_i \gtrsim 10^5$ GeV.

The origin of the difference between $|m_{ee}|$ and $|\frac{1}{M_{ee}}|$ is nothing but the two extreme limits of the fermion propagator of the Majorana neutrinos, which is central to the Feynman diagram of $0\nu\beta\beta$:

$$\frac{\not{q} + m}{q^2 - m^2} \propto \begin{cases} m & \text{for } q^2 \gg m^2, \\ \frac{1}{m} & \text{for } q^2 \ll m^2. \end{cases} \quad (6)$$

Here q denotes the momentum transfer in the process, which is around 100 MeV and corresponds to $1/r$, where r is the average distance of the two decaying nuclei. This helps us to understand roughly the numerical value of the limit on $|m_{ee}|$ and $|\frac{1}{M_{ee}}|$: the amplitude for light neutrino exchange is proportional to

$$\mathcal{A}_{\text{light}} \simeq G_F^2 \frac{|m_{ee}|}{q^2}, \quad (7)$$

while for heavy neutrinos it is proportional to

$$\mathcal{A}_{\text{heavy}} \simeq G_F^2 \left|\frac{1}{M_{ee}}\right|. \quad (8)$$

Therefore, a limit of 1 eV on $|m_{ee}|$ corresponds to a limit on $|\frac{1}{M_{ee}}|$ of 10^{-7} GeV^{-1}. This rather crude estimate is surprisingly close to the actual limit in Eq. (4), which takes the complicated nuclear physics into account. In the same approximation, we can estimate that the contribution of the doubly charged Higgs triplet to $0\nu\beta\beta$ has an amplitude proportional to

$$\mathcal{A}_{\text{triplet}} \simeq G_F^2 \frac{h_{ee} v_L}{m_\Delta^2}, \quad (9)$$

where the factor h_{ee} stems from the coupling of the triplet with the electrons, v_L from the coupling of the triplet to the two W, and $1/m_\Delta^2$ is its propagator for $m_\Delta^2 \gg q^2$. Hence, we estimate the following limit on the triplet parameters from $0\nu\beta\beta$:

$$\left|\frac{h_{ee}v_L}{m_\Delta^2}\right| \lesssim 10^{-(7\cdots 8)}\ \mathrm{GeV}^{-1}. \tag{10}$$

The triplet may be connected to the neutrino mass because of the following term in the Lagrangian:

$$\mathcal{L} = h_{\alpha\beta}\bar{L}_\alpha i\tau_2 \Delta L^c_\beta + \mathrm{H.c.} \tag{11}$$

Here h is a symmetric matrix, τ_2 is a Pauli matrix, L_α a lepton doublet of flavor $\alpha = e, \mu, \tau$, and

$$\Delta = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix} \tag{12}$$

contains the neutral, singly, and doubly charged members of the Higgs triplet. After the SM Higgs and the neutral component of the triplet obtain a VEV [$\langle\Phi\rangle = (0, v/\sqrt{2})^T$ and $\langle\Delta^0\rangle = v_L/\sqrt{2}$], a direct contribution to the neutrino mass $m_L = \sqrt{2}v_L h$ arises. The electroweak ρ parameter is modified to $\rho \simeq 1 - 2v_L^2/v^2$, which leads to the constraint $v_L \lesssim 8$ GeV. Direct and model-independent collider limits on the mass of the doubly charged triplet are $m_\Delta \gtrsim 100$ GeV [20]. It is interesting to compare this limit to limits stemming from searches for lepton flavor violation (see e.g. [21,22]):

$$\begin{aligned}
|h_{ee}|^2|h_{e\mu}|^2\left(\frac{250\ \mathrm{GeV}}{m_\Delta}\right)^4 &< 2.1\times 10^{-12},\\
|h_{ee}|^2|h_{\tau\mu}|^2\left(\frac{250\ \mathrm{GeV}}{m_\Delta}\right)^4 &< 2.5\times 10^{-7},\\
|h_{e\mu}|^2|h_{\tau\mu}|^2\left(\frac{250\ \mathrm{GeV}}{m_\Delta}\right)^4 &< 1.3\times 10^{-7},\\
|h_{\mu\mu}|^2|h_{\tau\mu}|^2\left(\frac{250\ \mathrm{GeV}}{m_\Delta}\right)^4 &< 4.0\times 10^{-7},\\
|(hh^\dagger)_{e\mu}|\left(\frac{250\ \mathrm{GeV}}{m_\Delta}\right)^4 &< 6.5\times 10^{-9},\\
|(hh^\dagger)_{e\tau}|\left(\frac{250\ \mathrm{GeV}}{m_\Delta}\right)^4 &< 1.1\times 10^{-4},\\
|(hh^\dagger)_{\mu\tau}|\left(\frac{250\ \mathrm{GeV}}{m_\Delta}\right)^4 &< 1.4\times 10^{-4}.
\end{aligned} \tag{13}$$

Here we have used the current limits on the processes $\mu \to 3e$, $\tau \to \mu 2e$, $\tau \to e 2\mu$, $\tau \to 3\mu$, $\mu \to e\gamma$, $\tau \to e\gamma$, and $\tau \to \mu\gamma$ [20]. Constraints from $(g-2)_\mu$ (the anomalous magnetic moment of the muon, constraining $|h_{\mu\mu}|^2/m_\Delta^4$) and muonium-antimuonium conversion (constraining $|h_{\mu\mu}|^2|h_{ee}|^2/m_\Delta^4$) are very weak.

III. INVERSE NEUTRINOLESS DOUBLE BETA DECAY WITH MAJORANA NEUTRINOS

A. e^-e^- collider

For the process of inverse $0\nu\beta\beta$ with Majorana neutrinos, Figs. 1(a) and 1(b) apply. In the Appendix the lengthy formulas for the cross section including the mass of the W can be found. In the useful and appropriate limit of negligible mass of the W one has

$$\frac{d\sigma}{d\cos\theta} = \frac{G_F^2}{32\pi}\left\{\sum m_i U_{ei}^2\left(\frac{t}{t - m_i^2} + \frac{u}{u - m_i^2}\right)\right\}^2. \tag{14}$$

Here $U_{\alpha i} = \{N_{\alpha 1}, N_{\alpha 2}, N_{\alpha 3}, S_{\alpha 1}, S_{\alpha 2}, \ldots, S_{\alpha n}\}$ is in our notation the general mixing matrix for the coupling of charged leptons with light and heavy neutrinos, which are given by $m_i = \{(m_\nu)_1, (m_\nu)_2, (m_\nu)_3, M_1, M_2, \ldots, M_n\}$. The extreme limits of the cross section are

$$\sigma(e^-e^- \to W^-W^-) = \begin{cases} \frac{G_F^2}{4\pi}(U_{ei}^2 m_i)^2 & \text{for } s \gg m_i^2, \\ \frac{G_F^2}{16\pi}s^2\left(\frac{U_{ei}^2}{m_i}\right)^2 & \text{for } s \ll m_i^2. \end{cases} \tag{15}$$

We will comment in Sec. V on the apparent violation of unitarity in the limit of $s \to \infty$. There are two interesting special cases for the cross section [6]:

- If only light active Majorana neutrinos contribute to the process, then we can bound the cross section as

$$\sigma(e^-e^- \to W^-W^-) = \frac{G_F^2}{4\pi}|m_{ee}|^2 \leq 4.2\times 10^{-18}\left(\frac{|m_{ee}|}{1\mathrm{eV}}\right)^2 \mathrm{fb}. \tag{16}$$

- If only heavy Majorana neutrinos contribute to the process, then we can bound the cross section using the $0\nu\beta\beta$ limit from Eq. (4) as

$$\sigma(e^-e^- \to W^-W^-) = \frac{G_F^2}{16\pi}s^2\left|\frac{1}{M_{ee}}\right|^2 \leq 2.6 \times 10^{-3}\left(\frac{\sqrt{s}}{\mathrm{TeV}}\right)^4\left(\frac{|\frac{1}{M_{ee}}|}{5\times 10^{-8}\ \mathrm{GeV}^{-1}}\right)^2 \mathrm{fb}. \tag{17}$$

Both numbers are far too small to be observable. In order to calculate the cross section for arbitrary neutrino masses, we have two limits to take into account: first, the global limit on $|S_{ei}|^2$ from (5), and the limit on S_{ei}^2/M_i from neutrinoless double beta decay given in Eq. (4). Assuming the exchange of only one heavy neutrino results in Fig. 2, where we plot the cross section for $e^-e^- \to W^-W^-$ as a function of M_i for $\sqrt{s} = 1$ TeV and $\sqrt{s} = 4$ TeV. We give the curves for applying no limit, only the global one, and finally the $0\nu\beta\beta$ limit in addition to the global one. We indicate in the plot the cross section where five events for a luminosity of 80 (s/TeV^2) fb^{-1} [6] would arise. From the plot one can see that the limit from $0\nu\beta\beta$ renders the process unobservable for $\sqrt{s} = 1$ TeV, while for $\sqrt{s} = 4$ TeV up to several 10^4 events are possible. The masses for which events are observable range from TeV to 10^3 TeV. This has to be compared with the situation at the

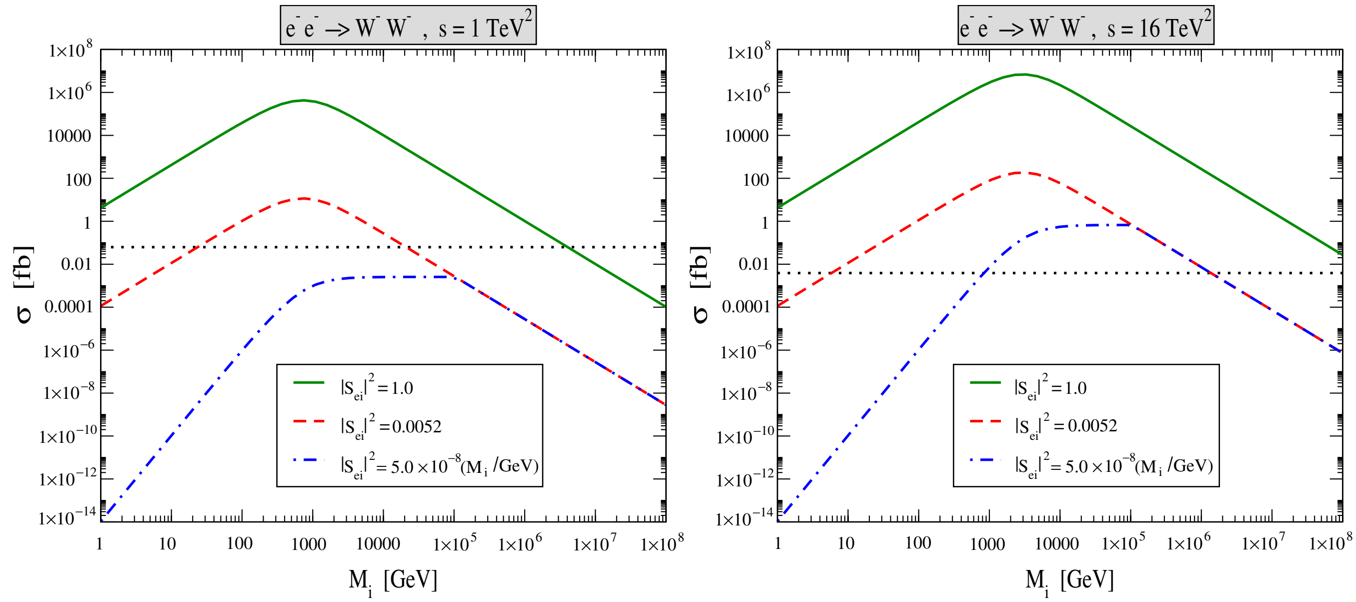

FIG. 2 (color online). Cross section for $e^-e^- \to W^-W^-$ with $\sqrt{s} = 1$ TeV (left panel) and $\sqrt{s} = 4$ TeV (right panel) and three limits for the mixing parameter $|S_{ei}|^2$. The dotted line corresponds to five events for an assumed luminosity of 80 (s/TeV^2) fb^{-1}.

LHC, where heavy Majorana neutrinos are observable in the range of 10–400 GeV for 100 fb^{-1} [23] (see [24] for a review on neutrino production at colliders).

In Fig. 3 we show the differential cross section $d\sigma/d\cos\theta$ for three different values of the neutrino mass and the mixing $|U_{ei}|^2$ chosen such that the total cross sections are the same. We see that once $m_i \gg \sqrt{s}$ the differential cross section is essentially flat, which is also obvious from Eq. (14).

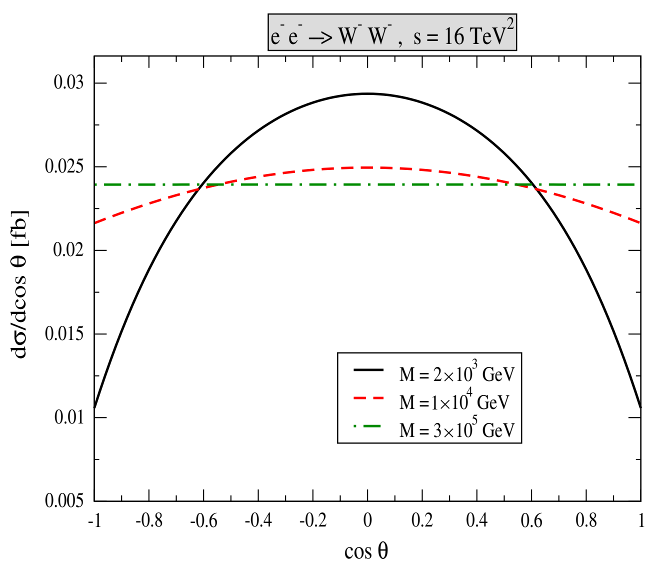

FIG. 3 (color online). Differential cross section for $e^-e^- \to W^-W^-$ with $\sqrt{s} = 4$ TeV and three different values of the neutrino mass, with the mixing chosen such that the total cross sections are identical.

B. $e^-\mu^-$ collider

The plans of building a muon collider open up the possibility of studying the lepton number and flavor violating mode $e^-\mu^- \to W^-W^-$. The differential cross section is

$$\frac{d\sigma}{d\cos\theta} = \frac{G_F^2}{32\pi}\left\{\sum m_i U_{ei} U_{\mu i}\left(\frac{t}{t - m_i^2} + \frac{u}{u - m_i^2}\right)\right\}^2. \tag{18}$$

If there are only light active neutrinos, then the cross section is proportional to $|m_{e\mu}|^2$, which is the $e\mu$ element of the mass matrix [25,26]. As this element cannot be larger than 2.3 eV either, there is no hope of seeing the process in this case. In the case of heavy neutrinos contributing to $e^-\mu^- \to W^-W^-$, the limit on $|\frac{1}{M_{ee}}|$ influences this process as well. One needs to compare its effect with the global limit of $|S_{ei}S_{\mu i}| \lesssim 0.000\,72$. The cross section is given in Fig. 4. One can note that the global limit can be stronger than the $0\nu\beta\beta$ limit for a large part of the parameter space.

With regards to the luminosity of like-sign $e\mu$ or $\mu\mu$ machines, it is currently not clear what numbers can be achieved. Let us use the numbers of $\mu^+\mu^-$ muon colliders as examples. According to Ref. [9], integrated luminosities of 45 (s/TeV^2) fb^{-1}, where we have assumed a year of 10^7 s, are possible. We will take this value in the following for both $e\mu$ and $\mu\mu$ like-sign collisions. While large uncertainty is presumably associated with this value, our results are easy to modify once more realistic estimates are present.

From the plots one can see that for $\sqrt{s} = 1$ TeV there is only a tiny window around (400–600) GeV where a few

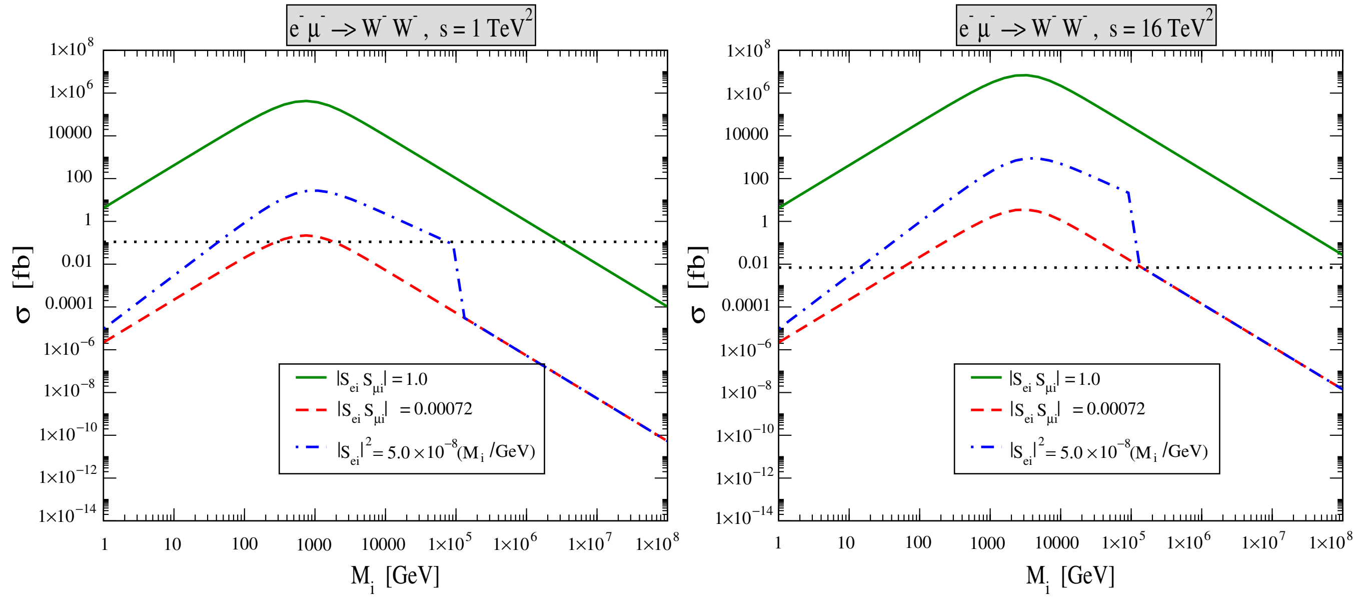

FIG. 4 (color online). Cross section for $e^-\mu^- \to W^-W^-$ with $\sqrt{s} = 1$ TeV (left panel) and $\sqrt{s} = 4$ TeV (right panel) and three limits for the mixing parameter. The dotted lines correspond to five events for an assumed luminosity of 45 (s/TeV^2) fb^{-1}.

events may happen, but for $\sqrt{s} = 4$ TeV up to a few 100 events between 100 and 10^5 GeV are possible. The situation is thus slightly worse than for *ee* collisions, even though there are strong constraints from neutrinoless double beta decay on S_{ei}^2/M_i. The reason is that the global limits on $|S_{\mu i}|^2$ are significantly stronger than on $|S_{ei}|^2$.

C. $\mu^-\mu^-$ collider

Finally, let us discuss the possibility of a $\mu^-\mu^-$ mode. The cross section is

$$\frac{d\sigma}{d\cos\theta} = \frac{G_F^2}{32\pi}\left\{\sum m_i U_{\mu i}^2\left(\frac{t}{t-m_i^2} + \frac{u}{u-m_i^2}\right)\right\}^2. \quad (19)$$

The only constraint comes from the global limit in Eq. (5), which however is rather strong. Figure 5 shows the cross section. From the plots one can see that for $\sqrt{s} = 4$ TeV there is only a smallish window between 400 and 10^4 GeV in which up to a few 10 events are possible.

We conclude from this section that like-sign *ee* lepton collisions are most promising to search for heavy Majorana

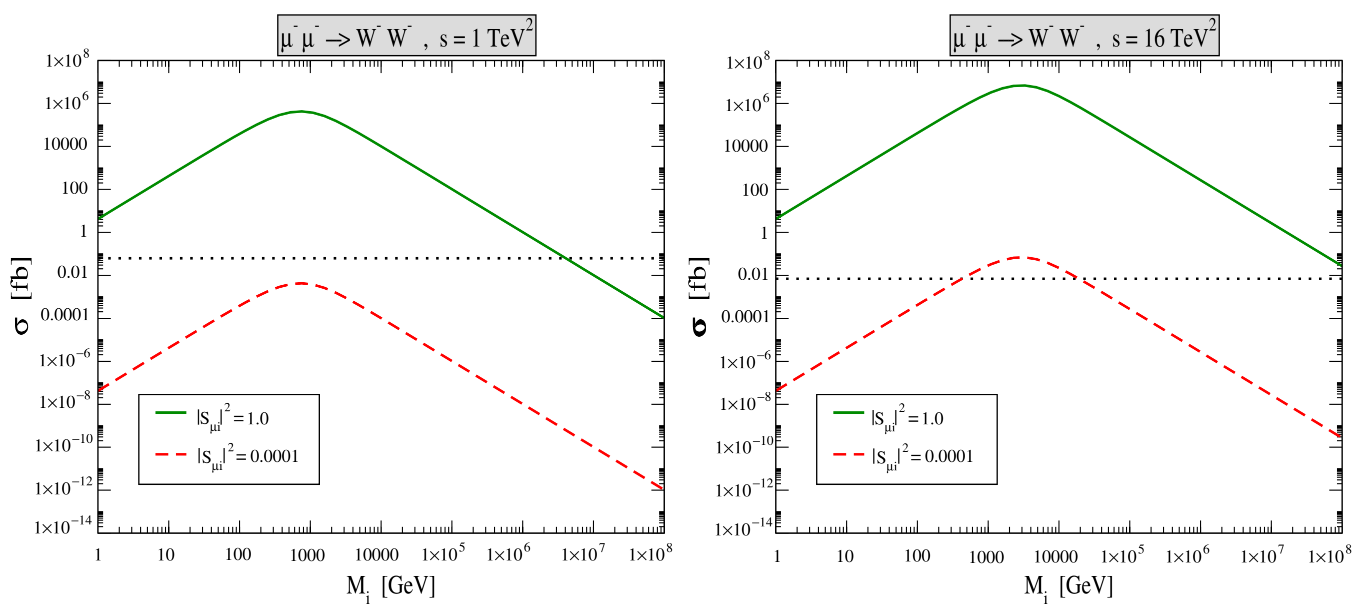

FIG. 5 (color online). Cross section for $\mu^-\mu^- \to W^-W^-$ with $\sqrt{s} = 1$ TeV (left panel) and $\sqrt{s} = 4$ TeV (right panel) and two limits for the mixing parameter $|S_{\mu i}|^2$. The dotted lines correspond to five events for an assumed luminosity of 45 (s/TeV^2) fb^{-1}.

neutrinos, and to constrain the parameter space of mixing matrix elements and mass. However, the center-of-mass energies should exceed 1 TeV. The already rather stringent limit on the mixing of heavy neutrinos with muons renders like-sign $e\mu$ collisions a bit less promising, and $\mu\mu$ facilities show little prospects to determine LNV due to Majorana neutrinos. As a numerical example, for $M_i = 1.5$ TeV and $\sqrt{s} = 4$ TeV, a like-sign $e\mu$ or ee collider would generate 5 events even for $|S|^2 = 3 \times 10^{-5}$, and improvement on the present bound by 1 to 2 orders of magnitude would be possible. Here $|S|^2$ denotes the respective combination of mixing parameters. For $M_i = 2 \times 10^5$ GeV, 5 events are possible even for $|S|^2 = 7 \times 10^{-4}$, resulting in an improvement of the bound on the mixing by 1 order of magnitude.

Note that such limits and considerations apply most probably not for heavy neutrinos of the type I seesaw mechanism. In its natural form there is a clash between production of colliders and TeV-scale masses of the heavy neutrinos: the mixing of the heavy neutrinos with the SM fermions is of order $|S| \sim m_D/M_R$, and the contribution to neutrino mass is $m_\nu \simeq m_D^2/M_R$. Since $m_\nu \lesssim$ eV, TeV-scale M_R implies MeV-scale m_D, and hence $|S|$ is of order 10^{-6}. However, the seesaw mechanism involves matrices, and highly fine-tuned scenarios in which the contributions of several heavy neutrinos compensate each other are possible, though they seem rather unnatural and, in particular, unstable. We continue by studying inverse neutrinoless double beta decay in an often studied extension of the Higgs sector.

IV. INVERSE NEUTRINOLESS DOUBLE BETA DECAY WITH A HIGGS TRIPLET

The production of Higgs triplets in like-sign lepton collisions has been discussed also in Refs. [2,3]. The cross section for $\alpha^-\beta^- \to W^-W^-$ is

$$\sigma = 2\frac{d\sigma}{d\cos\theta} = \frac{G_F^2}{2\pi} v_L^2 |h_{\alpha\beta}|^2 \frac{(s - 2m_W^2)^2 + 8m_W^4}{(s - m_\Delta^2)^2 + m_\Delta^2\Gamma_\Delta^2}\sqrt{1 - 4\frac{m_W^4}{s}} \simeq \frac{G_F^2}{2\pi} v_L^2 |h_{\alpha\beta}|^2 \frac{s^2}{(s - m_\Delta^2)^2 + m_\Delta^2\Gamma_\Delta^2}, \tag{20}$$

where Γ_Δ is the width of the Δ^{--}. We also note that, in case only a Higgs triplet contributes to neutrino mass, the process $\alpha^-\beta^- \to W^-W^-$ cannot take place if the entry $(m_\nu)_{\alpha\beta}$ vanishes. Recall that $v_L h = m_L/\sqrt{2}$, where m_L is the triplet contribution to neutrino mass. Hence $|v_L h_{\alpha\beta}|$ cannot exceed 1 eV, unless there are cancellations between the triplet and another contribution to neutrino mass, e.g., a type I seesaw term. Neglecting this unnatural possibility, $v_L h$ can be at most $m_\nu/\sqrt{2}$, and the order of $\frac{G_F^2}{4\pi}|(m_\nu)_{\alpha\beta}|^2$ is 10^{-18} fb for $m_\nu \simeq$ eV. It is therefore clear that the resonance needs to be met in order to see an observable signal. On resonance ($s = m_\Delta^2$) we find

$$\sigma^{\rm res} = \frac{G_F^2}{2\pi} v_L^2 |h_{\alpha\beta}|^2 \frac{m_\Delta^2}{\Gamma_\Delta^2}. \tag{21}$$

Assuming 40 inverse femtobarn of luminosity and asking for more than 5 events gives the requirement $m_\Delta/\Gamma_\Delta \gtrsim 10^8$.

We need to discuss the width of the triplet. Since the mass of Δ^{--} is very close to the mass of Δ^- and Δ^0, decays in final states containing the other members of the triplet are very much suppressed.[1] The other decays of interest are into like-sign lepton pairs

$$\Gamma_\ell^{\alpha\beta} \equiv \Gamma(\Delta^{--} \to \alpha^-\beta^-) = \frac{|h_{\alpha\beta}|^2}{4\pi(1 + \delta_{\alpha\beta})} m_\Delta \simeq 19.9\frac{|h_{\alpha\beta}|^2}{(1 + \delta_{\alpha\beta})}\left(\frac{m_\Delta}{250\ \text{GeV}}\right)\ \text{GeV}, \tag{22}$$

and into a pair of W:

$$\Gamma_W \equiv \Gamma(\Delta^{--} \to W^-W^-) = \frac{v_L^2 g^4}{16\pi m_\Delta^2}\sqrt{m_\Delta^2 - 4m_W^2}\left(2 + \frac{(m_\Delta^2 - 2m_W^2)^2}{4m_W^4}\right) \simeq \frac{G_F^2}{2\pi} v_L^2 m_\Delta^3 \simeq 3.4 \times 10^{-10}\left(\frac{v_L}{\text{MeV}}\right)^2\left(\frac{m_\Delta}{250\ \text{GeV}}\right)^3\ \text{GeV}. \tag{23}$$

We have neglected the mass of the W in the last row. Summing $\Gamma_\ell^{\alpha\beta}$ over all leptons and taking for simplicity $h_{\alpha\beta} = h$ gives $m_\Delta/\sum\Gamma_\ell^{\alpha\beta} \simeq 1/|h|^2$. Thus, for $|h| \simeq 10^{-4}$ the condition $m_\Delta/\Gamma_\Delta \simeq 10^8$ can be met. These order of magnitude estimates imply $v_L \simeq 10$ keV if $m_\nu \simeq 1$ eV. Indeed, choosing for instance $m_\Delta = 500$ GeV, for such values of the triplet VEV the width in a W pair is of order 10^{-13} GeV, while $\sum\Gamma_\ell^{\alpha\beta} \simeq 10^{-6}$ GeV. We have therefore found a consistent scenario. Hence, the resonance condition is obtainable in cases in which the decay into charged lepton pairs is favored. Interestingly, these cases are the ones frequently studied in the literature [28,29]. Pairs of Δ^{--} and Δ^{++} are produced mainly in Drell-Yan processes, and the cross section [29] between 250 and 800 GeV can approximately be written as $\sigma \simeq 30$ $(250\ \text{GeV}/m_\Delta)^4$ fb, so that 100 fb^{-1} of luminosity can generate a sizable amount of triplet pairs. This in turn would motivate the study of $\alpha^-\beta^- \to W^-W^-$ at a lepton collider, and to scan the center-of-mass energy to make precision tests at resonance.

[1]The mass splitting due to electroweak corrections between the doubly and singly charged members (if they have initially the same mass) is of order $G_F m_W^3$ [27] and therefore too small to allow for decays such as $\Delta^{--} \to \Delta^- W^-$.

One may wonder about another process in which a triplet is exchanged in the s channel, namely $\alpha^-\beta^- \to \gamma^-\delta^-$, i.e., production of two like-sign leptons γ and δ by collisions of two like-sign leptons α and β. The cross section is

$$\sigma(\alpha^-\beta^- \to \gamma^-\delta^-) = \frac{|h_{\alpha\beta}|^2|h_{\gamma\delta}|^2}{4\pi(1+\delta_{\gamma\delta})}\frac{s}{(s-m_\Delta^2)^2+m_\Delta^2\Gamma_\Delta^2}. \tag{24}$$

The ratio of the cross sections is

$$\frac{\sigma(\alpha^-\beta^- \to W^-W^-)}{\sigma(\alpha^-\beta^- \to \gamma^-\delta^-)} \equiv \frac{\sigma_{WW}}{\sigma_{\rm lep}} \simeq 2\frac{G_F^2 v_L^2 s}{|h_{\gamma\delta}|^2/(1+\delta_{\gamma\delta})} \overset{\rm res}{\longrightarrow} \frac{\Gamma_W}{\Gamma_\ell^{\gamma\delta}}. \tag{25}$$

At resonance, the ratio of cross sections equals the ratio of decay widths. In Fig. 6 we show for two values of m_Δ the ratio of decay widths Γ_W and Γ_ℓ, as well as the ratio of cross sections (at $\sqrt{s} = 1$ TeV) as a function of v_L. The ratio m_Δ to Γ_Δ is also plotted, where Γ_Δ is the total width of the triplet. We demanded the neutrino mass matrix $m_L = \sqrt{2}v_L h$ to be of order 0.1 eV with $h_{\alpha\beta} = h$.

The simultaneous requirement of $\frac{\sigma(\alpha^-\beta^-\to W^-W^-)}{\sigma(\alpha^-\beta^-\to\gamma^-\delta^-)} \gg 1$ and $m_\Delta/\Gamma_\Delta \gtrsim 10^8$ implies a certain region in $m_\Delta - v_L$ space; see Fig. 6. A typical point is $v_L = 0.002$ GeV, leading to $h \simeq 3\times 10^{-8}$. For such small couplings the limits from lepton flavor violation given above are obeyed.

The width of the Δ^{--} is extremely small, much smaller than the beam spread, which has been estimated to be about $R = 10^{-2}\sqrt{s}$ for ee colliders [8] and $R = 4\times 10^{-4}\sqrt{s}$ for muon colliders [9]. For instance, if $m_\Delta = 600$ GeV than for $\sqrt{s} = 600$ GeV the cross section is 50.1 fb, while for $\sqrt{s} = 599.995$ GeV the cross section is only 1.4×10^{-10} fb. Picturing the spread as a box of width R and convoluting the cross section over this box [3] will smear out the resonance and give a $1/(R\Gamma)$ instead of a $1/\Gamma^2$ dependence of the cross section, thus reducing the cross section by several orders of magnitude. For instance, with $m_\Delta = \sqrt{s} = 600$ GeV and a spread R the result is $1.3/R \times 10^{-6}$ pb. We conclude that observing triplet induced inverse $0\nu\beta\beta$ at like-sign lepton colliders is very unlikely.

The situation is better for $\alpha^-\beta^- \to \gamma^-\delta^-$, where with $h \simeq 0.1$ one estimates the cross section far away from resonance to be of order $\simeq h^4/(4\pi s) \simeq 3({\rm TeV}/\sqrt{s})^2$ fb, which could lead to sizable event numbers. With maximal Yukawa couplings of 4π the cross section would be $\sigma \simeq (4\pi)^3/s \simeq 0.8\ ({\rm GeV}/\Gamma_\Delta)^2$ mb. At resonance one has again with $h \simeq 0.1$ for the cross section $\sigma \simeq 3\ ({\rm TeV}/\sqrt{s})^2$ nb. We will study the prospects of this process elsewhere.

V. ON UNITARITY OF $e^-e^- \to W^-W^-$ AND THE TYPE I + II SEESAW MECHANISM

It is a useful exercise to consider the cross section of $e^-e^- \to W^-W^-$ in the presence of both the triplet and heavy neutrinos, and study the unitarity behavior of the process.

Toward this, consider scenarios with fermion singlets and Higgs triplets. Such a scenario is called type I + II seesaw, while the presence of only a triplet may be called type II seesaw (sometimes denoted triplet seesaw). The presence of only fermion singlets is called type I seesaw.

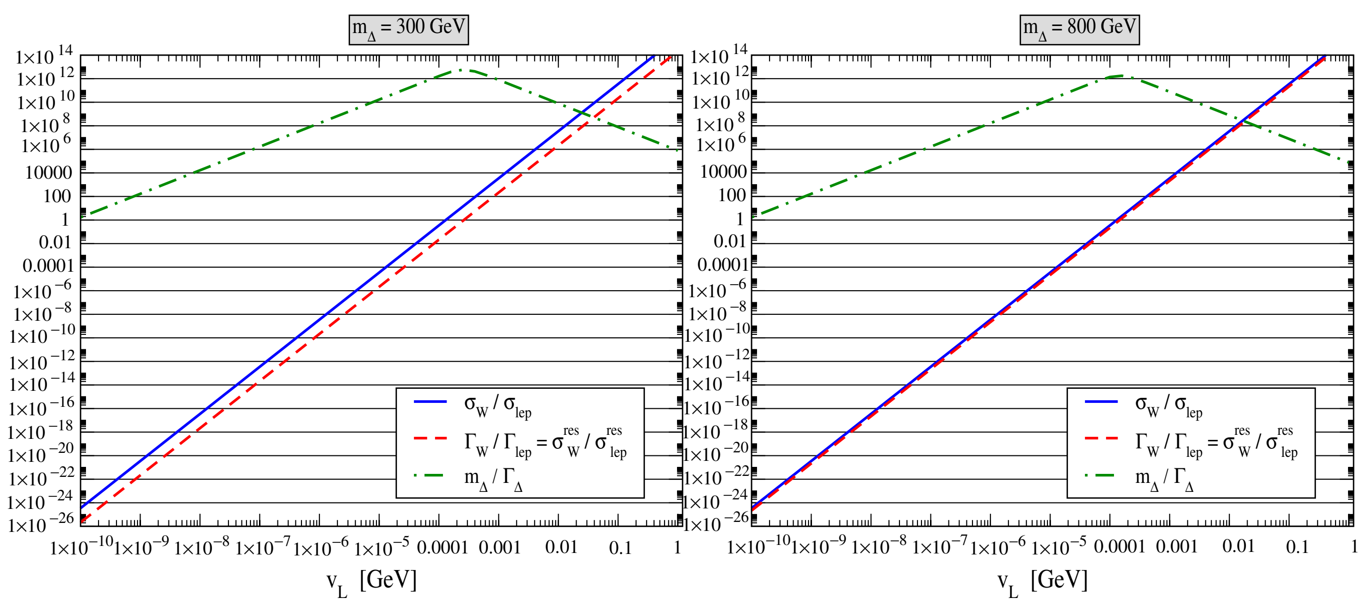

FIG. 6 (color online). The ratio of decay widths Γ_W and Γ_ℓ, as well as the ratio of cross sections (at $\sqrt{s} = 1$ TeV) as a function of v_L. The ratio m_Δ to Γ_Δ is also plotted, where Γ_Δ is the total width of the triplet.

We can write down a coupling of lepton doublets with the triplet, a Yukawa mass term for the coupling of lepton doublets with fermion singlets, and a direct bare mass term for the singlets:

$$\mathcal{L} = h_{\alpha\beta}\bar{L}_\alpha i\tau_2 \Delta L^c_\beta + \bar{L}_\alpha (Y_D)_{\alpha i}\Phi N_{R,j} + \tfrac{1}{2}\bar{N}^c_{R,i}(M_R)_{ij}N_{R,j} + \text{H.c.} \quad (26)$$

Here $\Phi = (\phi^+, \phi^0)^T$ is the SM Higgs doublet and M_R is a symmetric matrix. After the SM Higgs and the neutral component of the triplet obtain a VEV, the complete mass term containing the Dirac and Majorana masses can be written as

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}\bar{\nu}_L m_L \nu^c_L + \bar{\nu}_L m_D N_R + \frac{1}{2}\bar{N}^c_R M_R N_R + \text{H.c.}\\ &= \frac{1}{2}(\bar{\nu}_L, \bar{N}^c_R)\begin{pmatrix} m_L & m_D \\ m_D^T & M_R\end{pmatrix}\begin{pmatrix}\nu^c_L \\ N_R\end{pmatrix} + \text{H.c.}\\ &\equiv \frac{1}{2}(\bar{\nu}_L, \bar{N}^c_R)\mathcal{M}\begin{pmatrix}\nu^c_L \\ N_R\end{pmatrix} + \text{H.c.},\end{aligned} \quad (27)$$

with $m_D = Y_D v/\sqrt{2}$ and $m_L = \sqrt{2}v_L h$. There are in general six eigenvalues,

$$m^{\rm diag} = \text{diag}(m_1, m_2, m_3, m_4, m_5, m_6) \quad (28)$$

arising from diagonalizing the full 6×6 mass matrix $\mathcal{M}$ by a unitary 6×6 matrix

$$U = \begin{pmatrix} N & S \\ T & V\end{pmatrix} \quad \text{with} \quad \mathcal{M} = U\begin{pmatrix} m_\nu^{\rm diag} & 0 \\ 0 & M_R^{\rm diag}\end{pmatrix}U^T. \quad (29)$$

Here $m_\nu^{\rm diag} = \text{diag}((m_\nu)_1, (m_\nu)_2, (m_\nu)_3)$ contains the light "active" neutrino masses and $M_R^{\rm diag} = \text{diag}(M_1, M_2, M_3)$ the heavy ones. This difference between light and heavy neutrinos is valid if m_L is much smaller than m_D and M_R is much bigger than m_D. The entries of S and T are in this case of order m_D/M_R, and hence one can obtain the expression

$$N^\dagger(m_L - m_D M_R^{-1} m_D^T)N^* \simeq m_\nu^{\rm diag}. \quad (30)$$

Therefore, the mixing matrix in type I seesaw scenarios is strictly speaking not unitary, since $NN^\dagger = \mathbb{1} - SS^\dagger \neq \mathbb{1}$. The other set of heavy eigenvalues of $\mathcal{M}$ is obtained from $V^\dagger M_R V^* \simeq M_R^{\rm diag}$. We have illustrated the approximate nature of these expressions with the symbol $\simeq$, but for the usual magnitude of m_L, m_D, and M_R the implied nonunitarity of N is completely negligible. The matrix S characterizes the mixing of the light neutrinos with the heavy ones:

$$\nu_\alpha = N_{\alpha i}\nu_i + S_{\alpha i}N_i, \quad (31)$$

where ν_i (N_i) are the light (heavy) neutrinos with $i = 1, 2, 3$ and $\alpha = e, \mu, \tau$. The masses $(m_\nu)_i$ and M_i, and the associated mixing matrix elements N and S can be constrained by neutrinoless double beta decay; see Sec. II.

Note that the 11-element of Eq. (29) together with Eq. (27) reads

$$U m^{\rm diag} U^T = N m_\nu^{\rm diag} N^T + S M_R^{\rm diag} S^T = m_L. \quad (32)$$

We stress that this is an exact relation. It generalizes the relation $N m_\nu^{\rm diag} N^T + S M_R^{\rm diag} S^T = 0$, which is valid in the absence of a triplet contribution to neutrino mass and which has been discussed in Ref. [13] and further studied in [14]. The relation links, in type I + II seesaw scenarios, the measurable light neutrino parameters (the Pontecorvo-Maki-Nakagata-Sakata matrix N and the light neutrino masses) with the heavy Majorana neutrinos and the Higgs triplet couplings and VEV. In particular, Eq. (32) implies for the effective mass that

$$|m_{ee}| = |(m_L)_{ee} - \sum S^2_{ei}M_i|. \quad (33)$$

Consequently, the experimental limits on $|m_{ee}|$ apply directly to this combination of parameters:

$$|(m_L)_{ee} - \sum S^2_{ei}M_i| = |\sqrt{2}h_{ee}v_L - \sum S^2_{ei}M_i| \lesssim 1\ \text{eV}. \quad (34)$$

Obviously, in type I + II seesaw scenarios there is the interesting potential of cancellations between terms involving neutrino and triplet parameters. The individual limits on them can thus be evaded, and interesting phenomenology can arise. In this paper, however, we have discussed only the cases in which the triplets and neutrinos dominate in $e^-e^- \to W^-W^-$, $e^-\mu^- \to W^-W^-$, and $\mu^-\mu^- \to W^-W^-$, respectively, and will treat the effect of cancellations elsewhere. However, an interesting aspect regarding unitarity of the cross sections in case neutrinos and triplets contribute to inverse neutrinoless double beta decay is worth discussing: the full expression for the cross section is given in the Appendix. In the high-energy limit $\sqrt{s} \to \infty$, setting m_W to zero gives

$$\begin{aligned}\sigma &= \frac{G_F^2}{4\pi}((U^2_{ei}m_i)^2 + 2v_L^2 h_{ee}^2 - 2\sqrt{2}v_L h_{ee}U^2_{ei}m_i)\\ &= \frac{G_F^2}{4\pi}((U^2_{ei}m_i)^2 + (m_L)^2_{ee} - 2(m_L)_{ee}U^2_{ei}m_i)\\ &= \frac{G_F^2}{4\pi}((U^2_{ei}m_i) - (m_L)_{ee})^2 = 0.\end{aligned} \quad (35)$$

In the last line we have used the exact type I + II seesaw relation Eq. (32). Thus, the cross section becomes exactly zero in the high-energy limit. Recall that in case of no cancellation the cross section would be a constant, i.e., the amplitude would grow with $\sqrt{s}$, thus violating unitarity. The exact seesaw relation cures this. This observation generalizes the findings in [5,6], in which it was shown that in case of type I seesaw the cross section is $G_F^2/(4\pi)\times (U^2_{ei}m_i)^2$ which is equal to zero in type I seesaw scenarios [see Eq. (32) for $m_L = 0$].[2] Note that the requirement of vanishing $U_{ei}^2 m_i$ means that there cannot be only one neutrino: there must be necessarily two or more in order to make the cross section vanish in the high-energy limit. However, if a Higgs triplet is present then one neutrino is enough.

VI. CONCLUSIONS

Future lepton colliders may be run in a like-sign lepton mode, thereby probing lepton number violation. Here we have studied inverse neutrinoless double beta decay, $\alpha^-\beta^- \to W^-W^-$, with $(\alpha, \beta) = (e, e)$, (e, μ), and (μ, μ). We have discussed two sources of lepton number violation, namely, heavy Majorana neutrinos and Higgs triplets. The former possibility is shown (for ee and $e\mu$ collisions and center-of-mass energies larger than 1 TeV) to be observable for masses up to 10^6 GeV, which has to be compared with an LHC reach not exceeding 400 GeV. Triplet effects are unlikely to be seen, as a very narrow resonance has to be met. Surprisingly, even though no limits from neutrinoless double beta decay apply, like-sign muon colliders are a less promising option, because of strong constraints on heavy neutrino mixing with muons.

ACKNOWLEDGMENTS

This work was supported by the ERC under the Starting Grant MANITOP and by the Deutsche Forschungsgemeinschaft in the Transregio 27 "Neutrinos and beyond—weakly interacting particles in physics, astrophysics and cosmology."

APPENDIX: CROSS SECTION INCLUDING m_W

The three possibilities for $e^-(p_1)e^-(p_2) \to W^-(k_1, \mu)W^-(k_2, \nu)$ are shown in Fig. 1. Here $p_{1,2}$ and $k_{1,2}$ are the momenta of the particles and μ, ν the Lorentz indices of the W polarization vectors. The matrix element is

$$-i\mathcal{M} = -i(\mathcal{M}_t + \mathcal{M}_u + \mathcal{M}_s), \tag{A1}$$

where the subscript denotes whether it is the t, u, or s channel. The vertex for ΔWW is $i\sqrt{2}g^2 v_L g_{\mu\nu}$. In order to evaluate the cross section

$$\frac{d\sigma}{d\Omega} = \frac{1}{2}\frac{1}{64\pi^2 s}\frac{1}{4}|\bar{M}|^2\sqrt{\frac{\lambda(s, m_W^2, m_W^2)}{\lambda(s, 0, 0)}}, \tag{A2}$$

where the first $\frac{1}{2}$ is due to two identical particles in the final state and $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2(ab + ac + bc)$, we need

$$|\bar{\mathcal{M}}|^2 = |\bar{\mathcal{M}}_t|^2 + |\bar{\mathcal{M}}_u|^2 + |\bar{\mathcal{M}}_s|^2 + 2\,\mathrm{Re}(\bar{\mathcal{M}}_t^*\bar{\mathcal{M}}_u + \bar{\mathcal{M}}_t^*\bar{\mathcal{M}}_s + \bar{\mathcal{M}}_u^*\bar{\mathcal{M}}_s). \tag{A3}$$

The result is

$$\begin{aligned}
|\bar{\mathcal{M}}_t|^2 &= \frac{g^4}{4m_W^4}U_{ei}^2U_{ej}^2 m_i m_j\left[\frac{4m_W^6 - 4m_W^4(t+u) - t^2(t+u) + 2m_W^2(t+2u)}{(t-m_i^2)(t-m_j^2)}\right],\\
|\bar{\mathcal{M}}_u|^2 &= |\bar{\mathcal{M}}_t|^2(t \leftrightarrow u),\\
\bar{\mathcal{M}}_t^*\bar{\mathcal{M}}_u &= \frac{g^4}{4m_W^4}U_{ei}^2U_{ej}^2 m_i m_j\left[\frac{4m_W^6 - 2m_W^2 tu - tu(t+u)}{(u-m_i^2)(t-m_j^2)}\right],\\
|\bar{\mathcal{M}}_s|^2 &= 2\frac{g^4}{m_W^4}v_L^2 h_{ee}^2\frac{s(8m_W^4 + (s-2m_W^2)^2)}{(s-m_\Delta^2)^2},\\
\bar{\mathcal{M}}_t^*\bar{\mathcal{M}}_s &= \sqrt{2}\frac{g^4}{m_W^4}v_L h_{ee}U_{ei}^2 m_i\frac{(2m_W^2 - t - u)(4m_W^4 + t(t+u))}{(t-m_i^2)(s-m_\Delta^2)},\\
\bar{\mathcal{M}}_u^*\bar{\mathcal{M}}_s &= \bar{\mathcal{M}}_t^*\bar{\mathcal{M}}_s(t \leftrightarrow u).
\end{aligned}$$

[2]That the Higgs triplet restores unitarity of the process has been noted also in [5,6].

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