Oscillation of neutrinos produced by the annihilation of dark matter inside the Sun

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(Received 28 February 2010; published 29 June 2010)

The annihilation of dark matter particles captured by the Sun can lead to a neutrino flux observable in neutrino detectors. Considering the fact that these dark matter particles are nonrelativistic, if a pair of dark matter annihilates to a neutrino pair, the spectrum of neutrinos will be monochromatic. We show that in this case, even after averaging over the production point inside the Sun, the oscillatory terms of the oscillation probability do not average to zero. This leads to interesting observable features in the annual variation of the number of muon track events. We show that smearing of the spectrum due to thermal distribution of dark matter inside the Sun is too small to wash out this variation. We point out the possibility of studying the initial flavor composition of neutrinos produced by the annihilation of dark matter particles via measuring the annual variation of the number of μ-track events in neutrino telescopes.

DOI: 10.1103/PhysRevD.81.113010 PACS numbers: 14.60.Pq, 95.35.+d

I. INTRODUCTION

Growing evidence from a wide range of cosmological and astrophysical observations shows that about 82% of the matter content of the universe is composed of dark matter (DM) whose exact identity is yet unknown (i.e., $\rho_{\rm DM}/(\rho_{\rm DM}+\rho_{\rm baryon}) = 82\%$ [1]). In the literature [2], various candidates for DM have been suggested, among which weakly interacting massive particles (WIMPs) are arguably the most popular class of candidates [3]. The WIMPs are expected to propagate in the space between the stars and planets just like an asteroid that is subject to gravitational force from the various astrophysical bodies. In time, a considerable number of WIMPs will interact with the nuclei inside the Sun and lose energy. If the velocity drops below the escape velocity, the particle will be captured by the gravitational potential of the Sun [4]. The DM particles will eventually thermalize inside the Sun. Having a relatively large density inside the core, the DM particles will annihilate with each other. Depending on the annihilation modes, two classes of the WIMP models can be identified: (1) DM pair directly annihilate to neutrino pairs (that is $\nu\bar{\nu}$ or $\nu\nu+\bar{\nu}\bar{\nu}$). (2) DM pair particles annihilate to various particles whose subsequent decay produce neutrinos alongside other particles. In both cases, the neutrinos can be detected in the neutrino telescopes such as IceCube, provided that the cross section of the WIMPs with nuclei is of the order of $\sim 10^{-6}$ pb or larger. Indirect detection of DM through registering the neutrinos has been thoroughly studied in the literature [5].

The oscillation length of the neutrinos due to 12-mixing can be estimated as

$$L_{\rm osc} = \frac{4\pi E_\nu}{\Delta m_{12}^2} \sim 3\times 10^{11}\ {\rm cm}\left(\frac{E_\nu}{100\ {\rm GeV}}\right)\left(\frac{8\times 10^{-5}\ {\rm eV}^2}{\Delta m_{12}^2}\right), \quad (1)$$

which is of the order of a few percent of the distance between the Sun and the Earth. If the energy resolution of the detector ($\delta E/E$) is worse than 1% and the width of the spectrum is larger than $\delta E/E$, averaging the oscillatory terms is justified. However, for a monochromatic spectrum, dropping the oscillatory terms may lead to an error. Investigating the observable effects of these terms and information that they carry is the subject of the present paper. We shall demonstrate this effect by diagrams based on explicit calculation. We also demonstrate that integrating over the production point does not justify dropping the oscillatory terms.

The DM particles captured inside the Sun are nonrelativistic and have an average velocity of $(3T_\odot/m_{\rm DM})^{1/2} \simeq 60\ {\rm km/sec}$. As a result, in the case that the DM pairs directly annihilate to neutrino pairs, the spectrum of neutrinos will be monochromatic. Considering the fact that $L_{\rm osc}$ is of the order of the variation of Earth and Sun distance over a year, we expect these oscillatory terms lead to a significant variation of the number of events during a year. This expectation is similar to the prediction of seasonal variation of the beryllium line in the seminal paper by Gribov and Pontecorvo [6].

In the papers of Ref. [7], dropping the oscillatory terms, the oscillation of the neutrinos on the way to the detectors has been studied. In papers [8–10], propagation of the monochromatic neutrinos has been numerically studied within the 3-ν oscillation scenario, but no emphasis has been put on the potential effects of the oscillatory terms. In [10] the potential effects of the oscillatory terms and the seasonal variations have been pointed out, but since the emphasis was on the flavor blind initial composition, the effects had not been explored.

In this paper we focus on the effects of oscillatory terms on the flux of monochromatic neutrinos from the Sun without assuming a democratic initial flavor content. Monochromatic flavor dependent neutrino flux can emerge within various models. A particular scenario leading to

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monochromatic neutrino flux has been recently worked out in [11]. The scenario is based on an effective coupling of form $g_\alpha N \phi \nu_\alpha$ where N is a Majorana neutrino and ϕ is the scalar playing the role of the dark matter. Within this scenario, the main annihilation mode is $\mathrm{DM}+\mathrm{DM}\to \nu_\alpha+\nu_\beta,\ \bar\nu_\alpha+\bar\nu_\beta$ and the flavor composition can be determined by the flavor structure of the coupling g_α. The scenario can be embedded within various models that respect the electroweak symmetry [12]. Another example of a model that can give rise to monochromatic neutrino spectrum with nondemocratic flavor ratio can be found in [13]. We evaluate the magnitude of seasonal variation and discuss the condition under which such an amount of variation can be in practice established. We propose using the seasonal variation measurement as a tool to probe the physics of DM annihilation.

In Sec. II, taking into account the matter effects inside the Sun, we derive the minimum value of the width of the spectrum leading the oscillatory terms to average to zero. We also show that uncertainty in the production point will not lead to vanishing of the oscillatory terms. In Sec. III, we investigate various sources that lead to widening of the spectrum of monochromatic neutrinos. We then show that this widening is too small to make dropping of the oscillatory terms justifiable. In Sec. IV, we demonstrate the observable effects of the oscillatory terms in seasonal variation of the flux. In Sec. V, we anticipate the conclusions that can be reached based on different observational outcomes. The results are summarized in Sec. VI.

II. EFFECTS OF THE OSCILLATING TERMS

In the case where the DM pair annihilates to a neutrino pair and the physics governing dark matter annihilation is flavor blind, neutrinos of all three flavors can be produced in equal numbers. In this case, the neutrino oscillation cannot alter the flavor composition: if $F_e^0=F_\mu^0=F_\tau^0$, $\sum_\alpha P_{\alpha\beta}F_\alpha^0=F_\beta^0$. However, DM interaction can, in general, be flavor sensitive, so neutrinos of different flavors can be produced with different amounts. For example, it has been recently suggested that there might be a correlation between flavor structure of DM couplings and the neutrino mass matrix [11]. In fact, it is possible that a coherent mixture of different flavors is produced at the source, which means that the density matrix can be nondiagonal in the flavor basis. The density matrix, being a Hermitian matrix, can always be diagonalized. Let us denote the eigenstates of the density matrix at the source with $|\nu_\alpha\rangle$, which are in general linear compositions of $|\nu_e\rangle$, $|\nu_\mu\rangle$, and $|\nu_\tau\rangle$.

Let us consider such a state with momentum p produced inside the center of the Sun. This state after propagating a distance L evolves into

$$|\nu_\alpha;p;L\rangle = a_{\alpha1}(L)|1;p\rangle + a_{\alpha2}(L)|2;p\rangle + a_{\alpha3}(L)|3;p\rangle \tag{2}$$

where $|i;p\rangle$ is the neutrino mass eigenstate (in the vacuum). Let us define $(\Delta p^{ij})_{\rm lim}$ as the minimum value of the energy interval for which

$$\arg\left[\frac{[a_{\beta i}(0)(a_{\beta j}(0))^*(a_{\alpha i}(L/c))^* a_{\alpha j}(L/c)]|_{p+(\Delta p^{ij})_{\rm lim}}}{[a_{\beta i}(0)(a_{\beta j}(0))^*(a_{\alpha i}(L/c))^* a_{\alpha j}(L/c)]|_{p}}\right] = 2\pi, \tag{3}$$

where $\alpha\neq\beta$ and L is the distance between the Sun and the Earth. Suppose that the spectrum of the neutrino flux is almost monochromatic with width $\Delta p \ll p$. If the width of the spectrum is smaller than $(\Delta p^{ij})_{\rm lim}$, the average of the oscillatory terms will in general be nonzero. In this case, if the energy resolution of detector δE is larger than Δp, the corresponding oscillatory terms will lead to an error in deriving the initial flux. Let us evaluate the numerical value of $(\Delta p^{ij})_{\rm lim}$. In vacuum, $(\Delta p^{ij})_{\rm lim}$ can be estimated as

$$\frac{(\Delta p^{ij})_{\rm lim}}{p} = \frac{2\pi p}{L\Delta m_{ij}^2}, \tag{4}$$

so taking $L=1.5\times10^{13}$ cm we find $(\Delta p^{12})_{\rm lim}/p = 0.01(p/100\ \mathrm{GeV})$ and $(\Delta p^{13})_{\rm lim}/p = 0.0005(p/100\ \mathrm{GeV})$. Notice that this result is independent of α and β. Of course, the neutrinos produced in the center of the Sun have to pass through the matter inside the Sun before reaching the detectors. Figures 1(a) and 1(b) show $(\Delta p^{12})_{\rm lim}/p$ and $(\Delta p^{13})_{\rm lim}/p$ versus the neutrino momentum, taking into account the matter effects for various values of θ_{13}. To draw these figures, we have numerically obtained the evolution of the neutrino states of different flavors. Considering that the density of matter is sizeable only inside the Sun with radius $R_\odot\sim7\times10^{10}$ cm $\sim L_{\rm osc}$, we expect the numerical values of $(\Delta p)_{\rm lim}/p$ in the presence of matter to be almost similar to that in vacuum. In fact, the results shown in Fig. 1 fulfill this expectation: The order of magnitude of $(\Delta p^{ij})_{\rm lim}/p$ is the same as that in vacuum, and, moreover, independence of $(\Delta p^{ij})_{\rm lim}/p$ from flavor (i.e., α and β) is maintained.

Similarly to Eq. (2), we can write

$$|\bar\nu_\alpha;p;L\rangle = \bar a_{\alpha1}(L)|\bar1;p\rangle + \bar a_{\alpha2}(L)|\bar2;p\rangle + \bar a_{\alpha3}(L)|\bar3;p\rangle, \tag{5}$$

where $|\bar i;p\rangle$ is the antineutrino mass eigenstate (in the vacuum). Because of matter effects inside the Sun, even in the absence of the CP-violating phase δ, $a_{\alpha i}(L)\neq\bar a_{\alpha i}(L)$. However, since averaging of the oscillatory terms is mainly due to the propagation of neutrinos and antineutrinos in the large empty space between the Sun surface and the Earth, the value of $(\Delta p^{ij})_{\rm lim}/p$ for neutrinos and antineutrinos will be similar. Direct numerical calculations confirm this claim.

Another effect that may lead to averaging of the oscillatory terms is the difference in the production point; i.e., the difference in baseline. The dark matter particles are distributed in a radius of

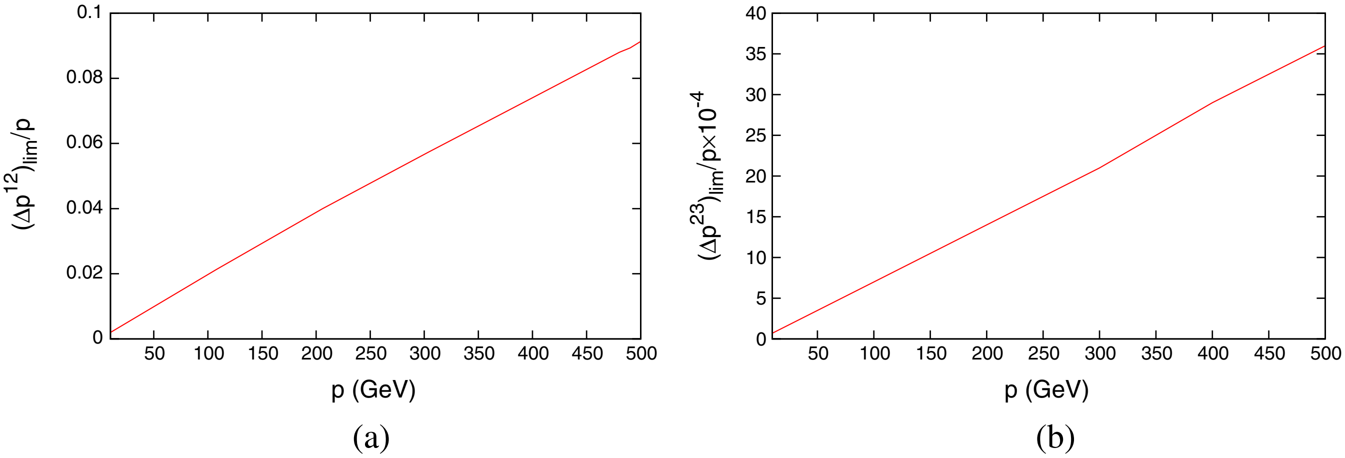

FIG. 1 (color online). The dependence of $(\Delta p^{12})_{\rm lim}/p$ and $(\Delta p^{13})_{\rm lim}/p$ defined in Eq. (3) on the momentum of neutrinos. The behavior shown in these graphs does not change by varying θ_{13} in the allowed range $\theta_{13} < 10°$ or changing the flavor composition.

$$r_{\rm DM} \approx \left(\frac{9T}{8\pi G_N \rho m_{\rm DM}}\right)^{1/2}, \tag{6}$$

where T and ρ are, respectively, the temperature and density inside the Sun center. G_N is the Newton gravitational constant. Inserting the numerical values, we obtain $r_{\rm DM} = 2\times10^8$ cm $(100\ {\rm GeV}/m_{\rm DM})^{1/2} = 0.003R_\odot\,(100\ {\rm GeV}/m_{\rm DM})^{1/2}$. Inside the volume around the Sun center with $r < r_{\rm DM}$, the density is high such that $V_e = \sqrt{2}G_F N_e \gg \Delta m^2_{12}/p$, so ν_e and $\bar{\nu}_e$ in practice correspond to energy eigenstates. As a result, $\overset{(-)}{\nu}_e$ within this volume does not go through oscillation. On the other hand, the oscillation length corresponding to the 2–3 splitting, $4\pi p/\Delta m^2_{\rm atm} \sim 10^{10}$ cm is much larger than $r_{\rm DM}$, so averaging out the corresponding oscillatory terms is not justified. Let us define $\bar{P}_{\alpha\beta}$ as the average of the oscillation probability over the production point with keeping the oscillatory terms. Moreover, let us define $\langle P_{\alpha\beta}\rangle$ as the same quantity with dropping the oscillatory terms. If the ratio $\bar{P}_{\alpha\beta}/\langle P_{\alpha\beta}\rangle$ equals to one, it means when we integrate over the production point, the oscillatory terms average to zero. We have numerically calculated $\bar{P}_{\alpha\beta}/\langle P_{\alpha\beta}\rangle$ and displayed it in Fig. 2. As seen in Fig. 2, $\bar{P}_{e\mu}/\langle P_{e\mu}\rangle$ and $\bar{P}_{\mu\mu}/\langle P_{\mu\mu}\rangle$ can substantially deviate from one, which means dropping the oscillatory terms for the monochromatic spectrum is not justified. Thus, the numerical calculation demonstrated in Fig. 2 confirms our simple analysis made above. Another interesting point is the significant sensitivity to θ_{13}. This is understandable because for $\theta_{13} > 0$, the 1–3 resonance in the Sun can play a role. We repeated the same numerical analysis for the case that the initial neutrino state is a coherent combination of the neutrino flavor states and reached the same conclusion.

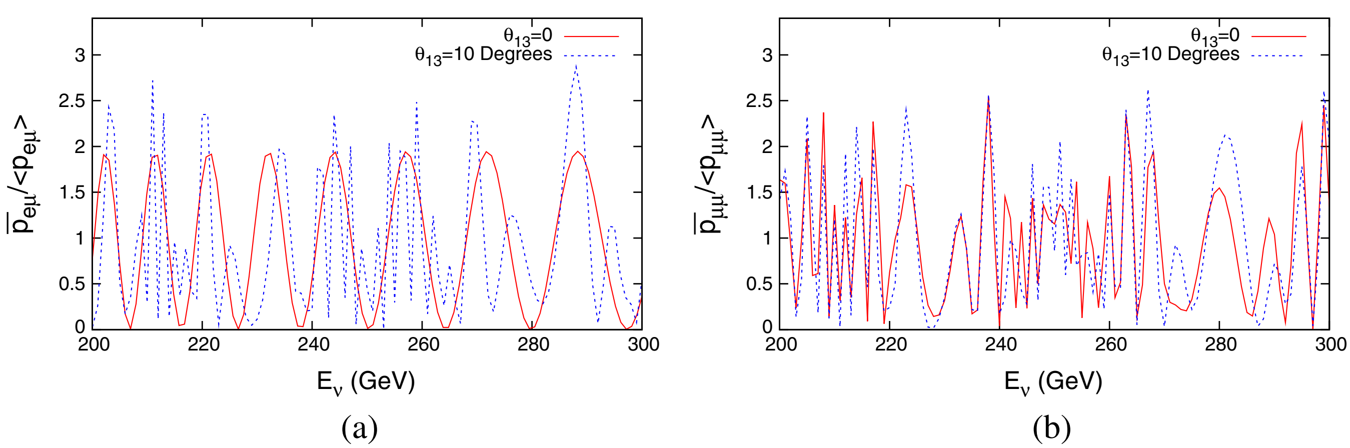

FIG. 2 (color online). The dependence of the ratios $\bar{P}_{e\mu}/\langle P_{e\mu}\rangle$ and $\bar{P}_{\mu\mu}/\langle P_{\mu\mu}\rangle$ on the energy of the produced neutrinos in the annihilation of the DM particles inside the Sun. $\bar{P}_{\alpha\beta}$ is the oscillation probability $\nu_\alpha \to \nu_\beta$ obtained by integrating over the production point of neutrinos inside the Sun taking into account the oscillatory terms. To perform this analysis, we have taken the neutrino mass scheme to be normal hierarchical. The $\langle P_{\alpha\beta}\rangle$ is the same quantity without taking into account the oscillatory terms. Deviation of the ratio $\bar{P}_{\alpha\beta}/\langle P_{\alpha\beta}\rangle$ from one is a measure of the significance of the oscillatory terms.

III. WIDTH OF THE SPECTRUM OF NEUTRINOS

The spectrum of neutrinos from direct annihilation of dark matter particles at rest will be monochromatic. In reality, however, several thermal and quantum mechanical effects lead to widening of the spectrum. Let us consider them one by one.

Thermal widening: The dark matter particles can fall in gravitational potential of the Sun, and despite the very low cross section can eventually be thermalized. The temperature in the Sun center is about 1 keV so the average velocity is given by $\bar{v} = (3T/m_{\rm DM})^{1/2} \simeq 60$ km/sec$(100\ {\rm GeV}/m_{\rm DM})^{1/2}$. The widening due to this velocity is

$$\frac{\Delta E}{E} \sim \frac{\bar{v}}{c} \sim 10^{-4}\left(\frac{T}{1.3\ {\rm keV}}\right)^{1/2}\left(\frac{100\ {\rm GeV}}{m_{\rm DM}}\right)^{1/2}. \qquad (7)$$

Decoherence due to gravitational acceleration: The DM particles become accelerated in the gravitational field of the Sun, so neutrinos from the annihilation of a DM pair will have a coherence length smaller than they would if the acceleration did not exist. We follow the same logic as in [14] to calculate the wave packet size. Notice that this is a quantum mechanical effect originating from the uncertainty principle. Let us hypothetically divide the path of the DM particles to successive segments of size $v\Delta\tau$. At each segment, the wave packet of the emitted neutrinos will have a width of $\Delta p \sim 1/\Delta\tau$. During $\Delta\tau$, the velocity of DM particles is changed by $4\pi G\rho r\Delta\tau/3$ so the average momentum of the neutrino wave packets emitted during the successive segments will differ by $\delta p \sim 4\pi G\rho r\Delta\tau m_{\rm DM}/3$. Now, if $\delta p \ll 1/\Delta\tau$, the two successive wave packets will form a single wave packet with length $2\Delta\tau$. However, for $\delta p > 1/\Delta\tau$, the two successive wave packets are incoherent. That is, the coherence length can be found by equating $\Delta\tau = 1/\delta p$, which leads to

$$\frac{\Delta p}{p} = \left(\frac{4\pi G\rho r_{\rm DM}}{3m_{\rm DM}}\right)^{1/2} \sim 5\times 10^{-17}\left(\frac{100\ {\rm GeV}}{m_{\rm DM}}\right)^{1/2}. \qquad (8)$$

Thus, widening of the spectrum due to acceleration is quite negligible. The width of the spectrum of the neutrino flux is dominated by thermal fluctuations rather than quantum mechanical widening.

Natural width of neutrino wave packet: The annihilation time sets a natural limit on the wave packet size of the produced neutrinos. That is

$$\frac{\Delta p}{p} > \frac{n_{\rm DM}\langle\sigma_{\rm ann}v\rangle}{m_{\rm DM}},$$

where $n_{\rm DM}$ is the density of dark matter in the Sun center. $\langle\sigma_{\rm ann}v\rangle$ is determined by the DM abundance in the universe: $\langle\sigma_{\rm ann}v\rangle \sim 10^{-36}\ {\rm cm}^2$. Evaluating $n_{\rm DM}$ is more model dependent. $n_{\rm DM}$ cannot be larger than $C\tau_\odot/(4\pi r_{\rm DM}^3/3)$, where C is the capture rate of DM particles by the Sun and $\tau_\odot$ is the Sun lifetime. $r_{\rm DM}$ determines the size of volume around the Sun center where the DM density is relatively high [see, Eq. (6)]. Annihilation of course reduces the DM number density but let us take $n_{\rm DM} \sim C\tau_\odot/(4\pi r_{\rm DM}^3/3)$ to obtain a conservative estimate for the natural width. The capture rate is given by [15]

$$C \sim \frac{\rho_{\rm DM}}{m_{\rm DM}v_{\rm DM}}\left(\frac{M_\odot}{m_p}\right)\sigma_{\text{DM-nucleon}}\langle v_{\rm esc}^2\rangle, \qquad (9)$$

where $\rho_{\rm DM} = 0.39\ {\rm GeV\,cm}^{-3}$ [16] and $v_{\rm DM} \sim 270\ {\rm km\,sec}^{-1}$ [17] are, respectively, the local density and velocity of DM particles in our galaxy; $M_\odot$ and m_p are, respectively, the Sun and proton masses. Of course, $\sigma_{\text{DM-nucleon}}$ is unknown but if the interaction is spin dependent, it can be as high as $\mathcal{O}$(pb) [18]. The maximal possible capture rate is therefore $O[10^{24}\ {\rm sec}^{-1}]$. Inserting the numerical values, we find that

$$\frac{n_{\rm DM}\langle v\sigma_{\rm ann}\rangle}{m_{\rm DM}} \sim 10^{-38}$$

so even with overestimating $n_{\rm DM}\langle v\sigma_{\rm ann}\rangle$, the natural lower bound is too weak, and $\Delta p/p$ will be dominated by thermal widening; i.e., Eq. (7).

Widening due to scattering: At energies higher than 100 GeV, some of the produced neutrinos can undergo scattering before leaving the Sun. Neutrinos undergoing charged current interactions convert into charged leptons, which are absorbed in the matter and do not contribute to the neutrino flux. An exception is, of course, $\nu_\tau \to \tau$ because the subsequent decay of the tau lepton regenerates high energy ν_τ. On the other hand, neutrinos undergoing neutral current interactions are converted to another neutrino with somewhat lower energy. Thus, the neutrino spectrum emerging from the Sun surface is composed of a sharp line at $E_\nu = m_{\rm DM}$ superimposed over a continuous spectrum with $E_\nu < m_{\rm DM}$. The ratio of neutrinos with $E_\nu \simeq m_{\rm DM}$ to those with $E_\nu < m_{\rm DM}$ depends on the neutral current cross section, which itself depends on the energy of neutrinos before scattering. This energy is in turn determined by $m_{\rm DM}$. The mean free path of the neutral current interaction of the neutrinos with energy 100 GeV in the Sun center is

$$\ell_{\rm NC} = \frac{1}{n_0\sigma_{\rm NC}} = 1.5\times 10^6\ {\rm km}\left(\frac{5\times 10^{25}\ {\rm cm}^{-3}}{n_0}\right)\left(\frac{1.3\times 10^{-37}\ {\rm cm}^2}{\sigma_{\rm NC}}\right),$$

where we have used the data from [19]. Considering the fact that the matter density in the Sun falls with radius as $e^{-r/(0.1R_\odot)}$, the ratio of neutrinos undergoing neutral current interactions should be of the order of $0.1R_\odot/\ell_{\rm NC} \simeq 5\%$. The ratio increases with energy and at $E_\nu = 500$ GeV reaches 35%. We restrict our analysis to the case $m_{\rm DM} <$ 500 GeV, which is also theoretically and phenomenologically motivated. In this range, the sharp line in the spectrum is quite pronounced so we will consider only this sharp line and will neglect the softened part of the spectrum. If $m_{\rm DM}$ turns to be greater than about 500 GeV, this analysis should be reconsidered taking into account the softening effects due to the neutral current scattering.

IV. OBSERVABLE EFFECTS OF THE OSCILLATORY TERMS: SEASONAL VARIATION

For a monochromatic neutrino flux, the rate of μ-track events in IceCube as a function of time can be estimated as

$$\frac{dN_\mu(t)}{dt} = \int \frac{F^0_{\nu_\alpha} w P_{\alpha\mu}(t)(\sigma^{CC}_{\nu_\mu p}\rho_p + \sigma^{CC}_{\nu_\mu n}\rho_n) R_\mu A_{\rm eff}(\theta[t])}{[L(t)]^2} dV + \int \frac{F^0_{\bar\nu_\alpha} \bar w P_{\bar\alpha\bar\mu}(t)(\sigma^{CC}_{\bar\nu_\mu p}\rho_p + \sigma^{CC}_{\bar\nu_\mu n}\rho_n) R_\mu A_{\rm eff}(\theta[t])}{[L(t)]^2} dV, \tag{10}$$

where integration is over the volume inside the Sun where neutrinos are produced. $F^0_{\nu_\alpha}$ ($F^0_{\bar\nu_\alpha}$) is the flux of ν_α ($\bar\nu_\alpha$) produced in unit volume. ρ_p and ρ_n are, respectively, the number densities of protons and neutrons in the ice. $\sigma^{CC}_{\nu_\mu p}$, $\sigma^{CC}_{\nu_\mu n}$, $\sigma^{CC}_{\bar\nu_\mu p}$, and $\sigma^{CC}_{\bar\nu_\mu n}$ are the cross sections of the charged current interactions of ν_μ and $\bar\nu_\mu$ with proton and neutron, respectively. w and $\bar w$ are suppressions factors, respectively, due to the absorption of neutrino and antineutrino fluxes in the Sun. For the energies that we are interested in, $E_\nu > 100$ GeV, w and $\bar w$ do not depend on the flavor but in general, $w \neq \bar w$. R_μ is the muon range in the detector which is the same for muon and antimuon [20]. $A_{\rm eff}(\theta[t])$ is the effective area of the detector, which depends on the angle between the neutrino momentum and the axis of the array of PMTs in detectors, θ. Because of the tilt of the rotation axis of the Earth, this angle changes as the Earth moves in its orbit around the Sun. $L(t)$ is the distance between the Sun and the Earth which varies about 3% during a year. Finally, $P_{\alpha\mu}$ and $P_{\bar\alpha\bar\mu}$ are, respectively, the oscillation probability of $\nu_\alpha \to \nu_\mu$ and $\bar\nu_\alpha \to \bar\nu_\mu$.[1] These probabilities can be numerically derived from the evolution of neutrino states:

$$i\frac{d|\nu_\gamma\rangle}{dt} = \left[\frac{m_\nu^\dagger \cdot m_\nu}{2p} + {\rm diag}(V_e, 0, 0)\right]_{\gamma\sigma} |\nu_\sigma\rangle \tag{11}$$

and

$$i\frac{d|\bar\nu_\gamma\rangle}{dt} = \left[\frac{m_\nu^T \cdot m_\nu^*}{2p} - {\rm diag}(V_e, 0, 0)\right]_{\gamma\sigma} |\bar\nu_\sigma\rangle, \tag{12}$$

where $V_e = \sqrt{2} G_F N_e$.

The following remarks are in order:

(i) In Eq. (10), we have neglected the subdominant contribution from $\nu_\tau \to \tau \to \mu$, which is suppressed by ${\rm Br}(\tau \to \mu\nu\bar\nu) = 17\%$.

(ii) In the case that the spectrum is continuous, the total flux has to be replaced with differential flux, and an integration over energy has to be taken.

(iii) Because of the tilt of the Earth rotation axis, during Autumn and Winter in the southern hemisphere, neutrinos entering the IceCube pass through the mantle of the Earth before reaching the detectors. However, we can neglect the oscillation of the neutrino inside the Earth mantle. The reason is that $\Delta m^2_{ij}/p \ll V_e$ so the effective mixing inside the Earth vanishes and no oscillation takes place in the constant density of the Earth. (Notice that although V_e in the Sun center where neutrinos originate is much larger than V_e inside the Earth mantle, V_e in the Sun surface is smaller than that in the Earth mantle. That is while the density in the Earth mantle is almost constant. Because of the difference in profile, the matter effects in the Sun and Earth are different.)

(iv) Production can be either lepton number conserving (i.e., DM + DM $\to \nu + \bar\nu$) or lepton number violating (i.e., DM + DM $\to \bar\nu + \bar\nu$ or DM + DM $\to \nu + \nu$). In the former case, the flux of neutrino and antineutrino will be obviously the same. In the latter case, as long as the CP is conserved in annihilation, processes DM + DM $\to \bar\nu + \bar\nu$ and DM + DM $\to \nu + \nu$ take place with the same rate. Even if CP is violated, any asymmetry in the ν and $\bar\nu$ fluxes will be a subdominant effect resulting from the interference of the tree level and loop level contributions. Thus, we can safely take the initial flux of neutrinos and antineutrinos to be the same.

Let us define

$$\tilde N(t_0, \Delta t) \equiv \frac{\int_{t_0}^{t_0+\Delta t}(dN_\mu/dt)dt}{\int_{t_0}^{t_0+\Delta t} A_{\rm eff}(\theta[t])/[L(t)]^2 dt}. \tag{13}$$

Notice that if the oscillatory terms average to zero, $\tilde N(t_0, \Delta t)$ will be constant and independent of t_0 and Δt. Let us therefore define

$$\Delta(t_0, \Delta t) \equiv \frac{\tilde N(t_0, \Delta t) - \tilde N(t_0 + \Delta t, 1\,{\rm year} - \Delta t)}{\tilde N(t_0, \Delta t) + \tilde N(t_0 + \Delta t, 1\,{\rm year} - \Delta t)}. \tag{14}$$

In the absence of the oscillatory terms, $\Delta(t_0, \Delta t)$ vanishes. Deviation of Δ from zero is a measure of the strength of the oscillatory terms. For a continuous spectrum, Δ vanishes

[1] As discussed in the beginning of Sec. II, ν_α ($\bar\nu_\alpha$) can be a coherent combination of different neutrino flavor eigenstates that diagonalize the density matrix. We perform our numerical analysis for the case that ν_α corresponds to a given flavor. However, as discussed, our results apply to the general case, too.

because the oscillatory terms average to zero, so Δ determines if there is a sharp feature in the spectrum.

Tables I and II show values of Δ for different values of $m_{\rm DM}(= E_\nu)$, θ_{13}, $(t_0, \Delta t)$ and for both normal hierarchical (NH) and inverted hierarchical (IH) neutrino mass schemes. To fill the Table I, we have taken the initial flux at source to be composed of merely ν_e and $\bar{\nu}_e$ and in the case of Table II, we have taken the initial flux to be composed of only ν_μ and $\bar{\nu}_\mu$. For one set of the data, we have chosen ($t_0 = 20$ of March, $\Delta t = 186$ days). The twentieth of March corresponds to the spring equinox[2] and $t_0 + \Delta t$ corresponds to the autumn equinox. For another set, we have chosen $t_0 = 3$ of April and $\Delta t = 186$ days, which correspond to the points in the orbit indicated in Fig. 3.

We have taken two values of θ_{13} ($\theta_{13} = 0$ and $\theta_{13} = 10°$) and set the CP-violating phase δ equal to zero. We have calculated $\Delta(t_0, \Delta t)$ for the case of exactly monochromatic spectrum as well as for a narrow Gaussian with width given by the thermal fluctuations of the dark matter [see, Eq. (7)]. As expected from the previous section, the difference in Δ for two cases [monochromatic versus Gaussian with the width given in Eq. (7)] is quite negligible. In both cases, Δ significantly deviates from zero, which means seasonal variation due to oscillatory terms is significant and potentially measurable.

Several features are obvious in the figures of Tables I and II. As can be seen from Table I, in the case $\theta_{13} = 0$, the values of $\Delta(t_0, \Delta t)$ (for both $t_0 = 20$ March and $t_0 = 3$ April) are equal for NH and IH. This equality is a consequence of the fact that for $\theta_{13} = 0$, the contribution of ν_3 to ν_e is zero, so both the NH and IH cases are equivalent. By comparing the figures in Table I with the figures of Table II it can be seen that the value of $\Delta(t_0, \Delta t)$ is typically larger for the annihilation of DM particles to electron neutrinos ($\mathrm{DM} + \mathrm{DM} \to \nu_e \bar{\nu}_e$). Thus, if the measurements of neutrino telescopes (such as IceCube) show a large value for $\Delta(t_0, \Delta t)$, the annihilation of DM particles to ν_e (and $\bar{\nu}_e$) will be favored.

In the real experiments such as IceCube, measurement of $\Delta(t_0, \Delta t)$ will be more tricky and statistical errors and background events have to be taken into account carefully. The statistical error of the ratio $\Delta(t_0, \Delta t)$ in Eq. (14) is given by the following formula

$$\delta\Delta(t_0, \Delta t) = \frac{2(\tilde{N}(t_0, \Delta t)\tilde{N}(t_0 + \Delta t, 1\,\text{year} - \Delta t))^{1/2}}{(\tilde{N}(t_0, \Delta t) + \tilde{N}(t_0 + \Delta t, 1\,\text{year} - \Delta t))^{3/2}}, \tag{15}$$

where we have inserted $\delta\tilde{N} = \sqrt{\tilde{N}}$. To evaluate $\delta\Delta$, we should estimate what is the maximum number of events per year allowed within the present bounds. The bound from present observation on the total number of muon tracks

[2]The spring equinox is called "Norooz" in Farsi and is the beginning of the year in the Iranian calendar.

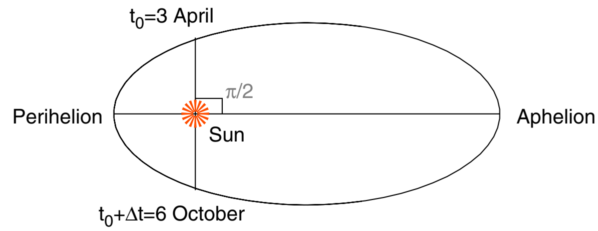

FIG. 3 (color online). Positions of the Earth on $t_0 = 3$ April and $t_0 + \Delta t = 6$ October, where $\Delta t = 186$ days.

depends on the shape of spectrum of the neutrinos. This is understandable because the detection threshold of AMANDA and its augmented version IceCube are not exactly the same, especially once DeepCore is added. In [21], an analysis has been made for the spectrum corresponding to annihilation into W^+W^- as well as into $\tau^+\tau^-$ for various values of the DM mass. The result is that if the bound is saturated, depending on the DM mass or the shape of the spectrum, a km^3 scale detector can observe between 500 to a few thousand events per year. The spectrum for annihilation into neutrino pair is harder so the bound will be stronger. Detailed analysis is beyond the scope of the present paper. Taking the total number of 400/year/km^2, however, sounds reasonable. With 400 muon tracks, the statistical error $\delta\Delta$ from Eq. (15) is less than 0.05. From Eq. (9), we find that this corresponds to nucleon DM cross section of 10^{-4} pb, which is well below the bound on spin-dependent cross sections. Typical values for Δ in Tables I and II can therefore be established after a few years of data taking (i.e., $\delta\Delta \ll \Delta$). In practice, the measurement will suffer from reducible and irreducible backgrounds. A well-known irreducible background comes from the "solar atmospheric neutrinos." That is neutrinos produced by interaction of cosmic rays with the atmosphere of the Sun which amounts to about 10 events per year [22]. The signal can be larger than this background by 1 order of magnitude. Moreover, the solar atmospheric flux, its energy spectrum, as well as its flavor composition can be calculated so its effects can be subtracted. Two other sources of backgrounds exist for the μ-track events from the Sun: the atmospheric neutrinos and the atmospheric

TABLE I. The values of $\Delta(t_0, \Delta t)$ defined in Eq. (14) for both normal and inverted hierarchies of the neutrino's mass spectrum. The initial flux is taken to be composed of ν_e and $\bar{\nu}_e$ with two different values of energies $E_\nu = 100$ and 300 GeV. On $t_0 = 3$ April, the earth reaches the point on its orbit depicted in Fig. 3 and $t_0 = 20$ March corresponds to the spring equinox.

	Δ (20 March, 186 days)				Δ (3 April, 186 days)			
E_ν (GeV)	$\theta_{13} = 0°$		$\theta_{13} = 10°$		$\theta_{13} = 0°$		$\theta_{13} = 10°$	
	NH	IH	NH	IH	NH	IH	NH	IH
100	18%	18%	9%	11%	12%	12%	6%	7%
300	57%	57%	37%	42%	60%	60%	39%	43%

TABLE II. The same as Table I except that the initial flux is taken to be composed of ν_μ and $\bar{\nu}_\mu$.

	Δ (20 March, 186 days)				Δ (3 April, 186 days)			
E_ν (GeV)	$\theta_{13} = 0°$		$\theta_{13} = 10°$		$\theta_{13} = 0°$		$\theta_{13} = 10°$	
	NH	IH	NH	IH	NH	IH	NH	IH
100	9%	6%	4%	1%	7%	4%	3%	0.3%
300	12%	7%	6%	19%	13%	7%	6%	20%

muons. From the spring to autumn equinoxes, the Sun is below the horizon at the IceCube site, and the neutrinos coming from the Sun produce upward-going μ-tracks. The atmospheric muons will be absorbed in this period of time by the surrounding material of the detector. This results in a huge suppression of the atmospheric muon background such that it can be completely neglected. During the autumn to spring equinoxes, the Sun is above the horizon for IceCube, and the number of atmospheric muons increases drastically. However, by using the outer parts of the IceCube as a veto, the atmospheric muon background will be quite low at the DeepCore [23] (the surrounding area can veto atmospheric muon events up to one part in 10^6). Let us now discuss the muon-track events induced by atmospheric neutrinos. As shown in Appendix C of Ref. [24], thanks to the high angular resolution of the IceCube for μ-tracks, by focusing on a cone with half angle 1° around the direction of Sun, the number of background through-going μ-tracks induced by atmospheric neutrinos can be reduced to $\sim 6\ \mathrm{yr}^{-1}$ for the whole energy spectrum above the IceCube energy threshold of the μ-track detection (number of background contained μ-track events in the DeepCore is $\sim 2.5\ \mathrm{yr}^{-1}$). Thus, the background events from the atmospheric neutrinos are not also a limiting factor in the measurement of $\Delta(t_0, \Delta t)$.

In the previous section, we observed that $P_{e\mu}$ is quite sensitive to θ_{13}. The reason is that for $\theta_{13} = 0$, no 1–3 resonance in $\nu_e \to \nu_\mu$ takes place but for $\theta_{13} > 0$, such a resonance plays a significant role. From the tables, we observe that Δ is also quite sensitive to θ_{13}. Considering this high sensitivity, it is tempting to entertain the possibility of extracting θ_{13} from Δ but absence of knowledge on the initial flavor composition renders the method useless.

V. DISCUSSION OF RESULTS

Throughout this paper we have focused on a monochromatic spectrum (i.e., a very narrow line) of neutrinos from the direct annihilation of dark matter particles to neutrino pairs inside the Sun. In reality, as we discussed earlier, the neutral current interactions inside the Sun will smear a part of the sharp line into a continuous spectrum with lower energy. Moreover, along with DM + DM $\to \overset{(-)}{\nu} + \overset{(-)}{\nu}$, DM particles can annihilate into $\tau\bar{\tau}$ and other particles whose subsequent decay will lead to a continuous spectrum with lower energy. Thus, the spectrum will be composed of a sharp line superimposed on the upper edge of a continuous background. Our discussion holds valid about the sharp line part of the spectrum.

In neutrino telescopes such as IceCube, the direction of μ-track can be reconstructed by amazing precision of 1° [25] which means neutrinos from the Sun can be singled out. In practice, the measurement of the spectrum is going to be challenging, especially that a large fraction of muons are produced by interaction of ν_μ outside the detector and lose energy before entering the detector. However, if the statistics is high enough, the spectrum can be reconstructed by measuring the energy of contained muons. Let us consider different situations one by one.

In case that the spectrum is not reconstructed, the measurement of Δ gives invaluable information on the shape of the spectrum. From Tables I and II, we see that Δ can be quite sizeable for both normal and inverted hierarchical mass schemes. Thus, if the number of events is a few hundreds, deviation of Δ from zero can be established. Since we have divided the number of events by $\int L^{-2} A_{\mathrm{eff}} dt$ in the definition of $\tilde{N}(t_0, \Delta t)$, a deviation of Δ from zero indicates that the oscillatory terms in oscillation probability do not average to zero [see, Eqs. (13) and (14)]. This in turn shows that there must be sharp features in the spectrum, originating from direct annihilation of DM pairs to neutrino pairs. To reach such a conclusion, the possibility of other seasonal modulation (like detector performance or seasonal variation of the background [26]) has to be subtracted.

In the case that the spectrum is reconstructed, Δ again provides invaluable information. If the spectrum contains a sharp line, we, in general, expect Δ to be nonzero. If a sharp line is observed in the spectrum but Δ turns out to be zero, the most natural explanation is that the initial flavor ratio is $F^0_{\nu_e} = F^0_{\nu_\mu} = F^0_{\nu_\tau}$ which in turn means the physics of DM annihilation is flavor blind. This may be a unique way to learn about flavor composition as the cascade events most probably will lie below the detection threshold for $m_{\mathrm{DM}} \lesssim 500$ GeV. In general, to analyze the properties of dark matter, seasonal variation Δ provides a powerful tool.

VI. CONCLUDING REMARKS

In the case that DM pairs annihilate into neutrino pairs, there will be a sharp line in the spectrum. We have shown that in the presence of such a line, the oscillatory terms in the oscillation probability do not average to zero and can therefore lead to a seasonal variation of the number of events in neutrino detectors. We have shown that the main cause for widening of the line is the thermal velocity distribution of DM particles inside the Sun but the widening will be too small to lead to vanishing of the oscillatory effects. We have demonstrated that the uncertainty in the production point cannot lead to vanishing of the oscillatory effects, either.

We have defined an observable quantity, Δ, whose deviation from zero is a measure of the significance of the oscillatory terms [see, Eq. (14)]. We have shown that Δ can reach as high as 60% so its deviation from zero can be established by a few hundred muon-track events. We have calculated the background and statistical error and have found that in the case that the flux is close to the present bounds, measuring Δ is doable. Δ contains invaluable information on the properties of DM particles. If $\Delta \neq 0$, even without performing the challenging energy spectrum reconstruction, we may conclude that there is a sharp line in the spectrum so the DM pairs have an annihilation mode into neutrino pairs. If the spectrum is reconstructed and a sharp line is identified, but Δ turns out to be zero, a natural explanation is that at the source all three flavors are produced by equal amounts. This in turn means that DM annihilation is flavor blind. Considering that for low values of E_ν (i.e., for $m_{\rm DM} \lesssim 500$ GeV), the cascade events will be below the detection threshold of neutrino telescopes, Δ might indirectly provide a unique probe of flavor composition.

As discussed, the width of the line is given by thermal fluctuations in the Sun center ($r < r_{\rm DM} < 0.01R_\odot$). Even conventional solar neutrinos from thermonuclear processes are produced mainly in the outer layers, so our information on this region depends merely on solar models. Our results are, however, robust against solar models as $\Delta p/p$ scales as $T^{1/2}$ [see, Eq. (7)]. In order for the oscillatory terms given by Δm^2_{12} to be erased, T has to be 10 000 times larger than the value predicted by solar models, which seems quite unlikely.

ACKNOWLEDGMENTS

We are grateful to A. Yu Smirnov, R. Allahverdi, P. Serpicon and P. Gondolo for useful comments. A. E. appreciates "Bonyad-e-Melli-e-Nokhbegan" for partial financial support.

[1] C. Amsler *et al.* (Particle Data Group), Phys. Lett. B **667**, 1 (2008).

[2] D. Hooper, arXiv:0901.4090; F. D. Steffen, Eur. Phys. J. C **59**, 557 (2009); M. E. Peskin, J. Phys. Soc. Jpn. **76**, 111017 (2007); H. Murayama, arXiv:0704.2276; D. Hooper and S. Profumo, Phys. Rep. **453**, 29 (2007).

[3] G. Jungman, M. Kamionkowski, and K. Griest, Phys. Rep. **267**, 195 (1996); G. Bertone, D. Hooper, and J. Silk, Phys. Rep. **405**, 279 (2005).

[4] W. H. Press and D. N. Spergel, Astrophys. J. **296**, 679 (1985).

[5] G. B. Gelmini, P. Gondolo, and E. Roulet, Nucl. Phys. **B351**, 623 (1991); M. Kamionkowski, Phys. Rev. D **44**, 3021 (1991); S. Ritz and D. Seckel, Nucl. Phys. **B304**, 877 (1988); V. D. Barger, F. Halzen, D. Hooper, and C. Kao, Phys. Rev. D **65**, 075022 (2002); V. Barger, W. Y. Keung, G. Shaughnessy, and A. Tregre, Phys. Rev. D **76**, 095008 (2007); A. de Gouvea, Phys. Rev. D **63**, 093003 (2001).

[6] V. N. Gribov and B. Pontecorvo, Phys. Lett. **28B**, 493 (1969).

[7] R. Lehnert and T. J. Weiler, Phys. Rev. D **77**, 125004 (2008); A. E. Erkoca, M. H. Reno, and I. Sarcevic, Phys. Rev. D **80**, 043514 (2009); M. Cirelli, N. Fornengo, T. Montaruli, I. Sokalski, A. Strumia, and F. Vissani, Nucl. Phys. **B727**, 99 (2005); **B790**, 338(E) (2008).

[8] V. D. Barger, W. Y. Keung, and G. Shaughnessy, Phys. Lett. B **664**, 190 (2008).

[9] M. Blennow, H. Melbeus, and T. Ohlsson, J. Cosmol. Astropart. Phys. 01 (2010) 018;

[10] M. Blennow, J. Edsjo, and T. Ohlsson, J. Cosmol. Astropart. Phys. 01 (2008) 021.

[11] C. Boehm, Y. Farzan, T. Hambye, S. Palomares-Ruiz, and S. Pascoli, Phys. Rev. D **77**, 043516 (2008).

[12] Y. Farzan, Phys. Rev. D **80**, 073009 (2009); Y. Farzan, S. Pascoli, and M. A. Schmidt, arXiv:1005.5323; Y. Farzan (work in progress).

[13] R. Allahverdi, S. Bornhauser, B. Dutta, and K. Richardson-McDaniel, Phys. Rev. D **80**, 055026 (2009). If the masses of the right-handed neutrinos and sneutrinos are degenerate (i.e., the supersymmetric limit), the neutrino flux will emerge to be monochromatic.

[14] Y. Farzan and A. Y. Smirnov, Nucl. Phys. **B805**, 356 (2008).

[15] A. Gould, Astrophys. J. **328**, 919 (1988); **321**, 571 (1987).

[16] R. Catena and P. Ullio, arXiv:0907.0018.

[17] M. Kamionkowski and A. Kinkhabwala, Phys. Rev. D **57**, 3256 (1998).

[18] S. Archambault *et al.*, Phys. Lett. B **682**, 185 (2009).

[19] J. N. Bahcall, M. H. Pinsonneault, and S. Basu, Astrophys. J. **555**, 990 (2001); R. Gandhi, C. Quigg, M. H. Reno, and I. Sarcevic, Astropart. Phys. **5**, 81 (1996).

[20] D. E. Groom, N. V. Mokhov, and S. I. Striganov, At. Data Nucl. Data Tables **78**, 183 (2001).

[21] J. Braun, D. Hubert, (f. t. I. Collaboration), arXiv:0906.1615.

[22] G. L. Fogli, E. Lisi, A. Mirizzi, D. Montanino, and P. D. Serpico, Phys. Rev. D **74**, 093004 (2006); Nucl. Phys. B, Proc. Suppl. **168**, 283 (2007); J. Phys. Conf. Ser. **120**, 052039 (2008).

[23] C. Wiebusch (f. t. I. Collaboration), arXiv:0907.2263.

[24] V. Barger, J. Kumar, D. Marfatia, and E. M. Sessolo, Phys. Rev. D **81**, 115010 (2010).

[25] J. Ahrens *et al.* (IceCube Collaboration), Astropart. Phys. **20**, 507 (2004).

[26] P. Adamson *et al.* (MINOS Collaboration), Phys. Rev. D **81**, 012001 (2010).