# $\mathcal{N} = (0, 2)$ deformation of the $\mathcal{N} = (2, 2)$ Wess-Zumino model in two dimensions

M. Shifman[1] and A. Yung[1,2]

[1]*William I. Fine Theoretical Physics Institute, University of Minnesota, Minneapolis, Minnesota 55455, USA*

[2]*Petersburg Nuclear Physics Institute, Gatchina, St. Petersburg 188300, Russia*

(Received 13 January 2010; published 24 May 2010)

We construct a simple $\mathcal{N} = (0, 2)$ deformation of the two-dimensional Wess-Zumino model. In addition to superpotential, it includes a "twisted" superpotential. Supersymmetry may or may not be spontaneously broken at the classical level. In the latter case an extra right-handed fermion field $\zeta_R$ involved in the $\mathcal{N} = (0, 2)$ deformation plays the role of Goldstino.

DOI: 10.1103/PhysRevD.81.105022 PACS numbers: 12.60.Jv, 11.27.+d, 11.30.Pb

Recently it was found [1,2] that non-Abelian string solitons in certain $\mathcal{N} = 1$ bulk gauge theories are described on the world sheet by $\mathcal{N} = (0, 2)$ deformations of the $CP(N-1)$ models. This finding raised interest to $\mathcal{N} = (0, 2)$ deformations of two-dimensional $\mathcal{N} = (2, 2)$ models in general. Here we will consider $\mathcal{N} = (0, 2)$ deformations of the Wess-Zumino model [3]. General elements of $\mathcal{N} = (0, 2)$ deformations were worked out by Witten [4,5]. A broad class of the (0, 2) Landau-Ginzburg models were analyzed, from various perspectives, in [6–8]. The prime interest of these studies was the flow of the (0, 2) Landau-Ginzburg models to nontrivial (0, 2) superconformal field theories [6,7], and $\mathcal{N} = (0, 2)$ analogs of the topological rings in the $\mathcal{N} = (2, 2)$ theories [8].

Here we will consider the $\mathcal{N} = (0, 2)$ deformation of the Wess-Zumino model with the emphasis on an aspect which will be thoroughly studied in a subsequent publication [9], namely, spontaneous breaking/nonbreaking of supersymmetry. Related issues of interest are (i) a nonrenormalization theorem; (ii) Bogomol'nyi-Prasad-Sommerfield (BPS) saturation of possible kinks. We use a formalism which is simple enough and is adequate to the problem. It parallelizes the formalism exploited in [2] to construct the heterotic $CP(N-1)$ model; see also [10]. For simplicity we consider only the simplest version of the Wess-Zumino model, in which interactions come only from the potential term. Generalizations are straightforward; see also [6,7].

We find that, even though the $\mathcal{N} = (0, 2)$ supersymmetry is implemented at the Lagrangian level, generically supersymmetry is spontaneously broken at the tree level. One can fine-tune a free parameter of the model in such a way that it stays unbroken at the tree level, and then (presumably) to any finite order in perturbation theory.

Two space-time coordinates are

$$x^\mu = \{t, z\}, \qquad \mu = 0, 1. \tag{1}$$

The $\mathcal{N} = (2, 2)$ superspace is spanned by[1]

$$\{x^\mu, \theta_\alpha, \bar{\theta}_\beta\}, \qquad \alpha, \beta = 1, 2. \tag{2}$$

In addition to the standard chiral superfields $\Phi^a$ of the conventional Wess-Zumino model, we will introduce $\mathcal{N} = (0, 2)$ superfields

$$\begin{aligned} \mathcal{B} &= \{\zeta_R(x^\mu + i\bar{\theta}\gamma^\mu\theta) + \sqrt{2}\theta_R\mathcal{F}\}\theta_L^\dagger, \\ \mathcal{B}^\dagger &= \theta_L\{\zeta_R^\dagger(x^\mu - i\bar{\theta}\gamma^\mu\theta) + \sqrt{2}\theta_R^\dagger\mathcal{F}^\dagger\}. \end{aligned} \tag{3}$$

Since $\theta_L$ and $\theta_L^\dagger$ enter in Eq. (3) explicitly, $\mathcal{B}$ and $\mathcal{B}^\dagger$ are *not* superfields with regards to the supertransformations with parameters $\epsilon_L$, $\epsilon_L^\dagger$. These supertransformations are absent in the heterotic model. Only those survive which are associated with $\epsilon_R$, $\epsilon_R^\dagger$. Note that $\mathcal{B}$ and $\mathcal{B}^\dagger$ are superfields with regards to the shifts with $\epsilon_R$, $\epsilon_R^\dagger$. As usual, we will introduce a shorthand for the chiral coordinate

$$\tilde{x}^\mu = x^\mu + i\bar{\theta}\gamma^\mu\theta. \tag{4}$$

Then the transformation laws with the parameters $\epsilon_R$, $\epsilon_R^\dagger$ are as follows (we set $\epsilon_L = \epsilon_L^\dagger = 0$):

$$\begin{aligned} \delta\theta_R &= \epsilon_R, \qquad \delta\theta_R^\dagger = \epsilon_R^\dagger, \\ \delta\tilde{x}^0 &= 2i\epsilon_R^\dagger\theta_R, \qquad \delta\tilde{x}^1 = 2i\epsilon_R^\dagger\theta_R. \end{aligned} \tag{5}$$

With respect to such supertransformations, $\mathcal{B}$ and $\mathcal{B}^\dagger$ are superfields. Indeed,

$$\delta\zeta_R = \sqrt{2}\mathcal{F}\epsilon_R, \qquad \delta\mathcal{F} = \sqrt{2}i(\partial_L\zeta_R)\epsilon_R^\dagger, \tag{6}$$

plus Hermitian conjugate transformations.

[1]The gamma matrices are chosen as $\gamma^0 = \gamma^t = \sigma_2$, $\gamma^1 = \gamma^z = i\sigma_1$, $\gamma_5 \equiv \gamma^0\gamma^1 = \sigma_3$. Moreover, $\bar{\theta} = \theta^\dagger\gamma^0$. With these definitions, the $\alpha = 1$ spinor component is right-handed while $\alpha = 2$ is left-handed.

 

Thus, the boson sector of the deformed model coincides with that of the conventional Wess-Zumino model, while the fermion sector is expanded. In addition to the fermion fields $\psi^a_{R,L}$ of the Wess-Zumino model, it includes a right-handed fermion field $\zeta_R$.

The $\mathcal{N}=(0,2)$ action can be written as

$$S=\int d^2x\{d^4\theta\Phi^{a\dagger}\Phi^a+[d^2\theta\mathcal{W}(\Phi^a)+\text{H.c.}]+\Delta\mathcal{L}_h\},$$
$$\Delta\mathcal{L}_h=\left\{\sqrt{2}\kappa\int d\theta_L^\dagger d\theta_R\mathcal{B}+\text{H.c.}\right\}-2\int d^4\theta\mathcal{B}^\dagger\mathcal{B}$$
$$+2\left\{\int d\theta_L^\dagger d\theta_R\theta_L d\theta_L\mathcal{B}\mathcal{S}(\Phi^a)+\text{H.c.}\right\}, \tag{7}$$

where the second term presents the heterotic deformation, $\mathcal{W}$ is the superpotential, while $\mathcal{S}(\Phi^a)$ is a function of the chiral superfield $\Phi$ which couples the heterotic sector to the conventional Wess-Zumino model. (Some generalizations will be considered later.) Let us call it $h$ superpotential. Moreover, $\kappa$ is a constant of dimension of mass. Note that both superpotential and $h$ superpotential have dimensions of mass too. The terms containing $\mathcal{B}$ and given by integrals over a reduced superspace will be referred to as $h$ terms. Adding a constant to the $h$ superpotential is equivalent to shifting $\kappa$ since they enter only in the combination $\mathcal{S}(\phi^a)+\kappa$; see Eq. (9). One can use this freedom to fix the value of $\mathcal{S}$ at some given point, without loss of generality.

In components

$$\mathcal{L}=\partial_\mu\phi^{a\dagger}\partial^\mu\phi^a+\bar{\psi}^a\gamma^\mu i\partial_\mu\psi^a+F^{a\dagger}F^a$$
$$+\{F^a\partial_a\mathcal{W}-(\partial_a\partial_b\mathcal{W})(\psi_L^a\psi_R^b)+\text{H.c.}\}+\Delta\mathcal{L}_h, \tag{8}$$

where

$$\Delta\mathcal{L}_h=\zeta_R^\dagger i\partial_L\zeta_R+\mathcal{F}^\dagger\mathcal{F}+\{\mathcal{F}[\mathcal{S}(\phi^a)+\kappa]$$
$$-\zeta_R\psi_L^a\partial_a\mathcal{S}+\text{H.c.}\}, \tag{9}$$

and

$$\partial_L=\partial_t+\partial_z,\qquad \partial_R=\partial_t-\partial_z. \tag{10}$$

The auxiliary superfield $\mathcal{F}$, as usual, can be eliminated via equations of motion,

$$\mathcal{F}^\dagger=-[\mathcal{S}(\phi^a)+\kappa]. \tag{11}$$

Then the bosonic part of $\mathcal{L}_h$ takes the form

$$\mathcal{L}_{h,\text{bos}}=-|\mathcal{S}(\phi^a)+\kappa|^2. \tag{12}$$

Adding the Wess-Zumino part we obtain the scalar potential,

$$V=|\partial_a\mathcal{W}|^2+|\mathcal{S}(\phi^a)+\kappa|^2. \tag{13}$$

Supersymmetric vacua exist (at the classical level) provided that the set of equations

$$\partial_a\mathcal{W}=0\qquad(\text{all }a),\qquad \mathcal{S}(\phi^a)+\kappa=0 \tag{14}$$

are satisfied at one or more critical points $\phi_*$. Considering $\kappa$ as a free parameter one can always fine-tune it in such a way that at least one vacuum [a solution $\partial_a\mathcal{W}(\phi_*)=0$] will be classically supersymmtric.

It is instructive to derive conserved supercurrents. If in the undistorted model with $\mathcal{N}=(2,2)$ supersymmetry we had four conserved supercurrents, $J_L^\mu$, $J_R^\mu$, and their complex conjugated, now we expect only two of those to survive. The conserved components are

$$J_L^\mu=\sqrt{2}\{i\nu^\mu\psi_R^\dagger F+i\nu^\mu\zeta_R^\dagger\mathcal{F}+\bar{\nu}^\mu\psi_L\partial_L\phi^\dagger\},$$
$$(J_L^\mu)^\dagger=\sqrt{2}\{-i\nu^\mu\psi_R F^\dagger-i\nu^\mu\zeta_R\mathcal{F}^\dagger+\bar{\nu}^\mu\psi_L^\dagger\partial_L\phi\}, \tag{15}$$

where summation over $a$ is implicit, and we defined two conjugated 2-vectors,

$$\nu^\mu=\{1,1\},\qquad \bar{\nu}^\mu=\{1,-1\}. \tag{16}$$

The corresponding superalgebra is as follows:

$$\{Q_L^\dagger Q_L\}=2(H-P^z). \tag{17}$$

From Eq. (17) it is obvious that massless right movers can (and do) form short (single-state) "multiplets."

In the $\mathcal{N}=(2,2)$ Wess-Zumino model ($\mathcal{B}=0$) there is a relation for the dilatation operator

$$(\gamma_\mu J^\mu)_{L,R}=i2\sqrt{2}F(\psi^\dagger)_{L,R}. \tag{18}$$

In the $\mathcal{N}=(0,2)$-deformed model the analog of this relation is

$$\bar{\nu}_\mu J_L^\mu=i2\sqrt{2}(F\psi_R^\dagger+\mathcal{F}\zeta_R^\dagger). \tag{19}$$

*Some generalizations.*—In addition to (9), one can couple the $\mathcal{B}$ field to other fields through a number of extra terms, for instance,

$$\int d^4\theta\mathcal{B}^\dagger\mathcal{B}f(\Phi\Phi^\dagger)\quad\text{or}\quad\int d^4\theta\mathcal{B}\tilde{f}(\Phi\Phi^\dagger)+\text{H.c.} \tag{20}$$

The first term gives, in particular, a coupling of the $\zeta$ kinetic term with the $\phi$, $\phi^\dagger$ fields. As was mentioned, such interactions will not be considered for the time being. The second term was considered in [2]. It generates the $\zeta_R\partial_L\phi^\dagger\psi_R$ interaction and an additional bifermion term $\psi_L^\dagger\psi_R$ in (11), as well as $\zeta_R^\dagger\psi_L$ in $F$, resulting in four-fermion interactions in the Lagrangian.

*Nonrenormalization of h terms.*—As well-known, $F$ terms in the effective Lagrangian in the $\mathcal{N}=(2,2)$ theory are protected from renormalizations by nonrenormalization theorems [11,12]. Thus, the superpotential term, being an integral over a reduced superspace, is unaffected by loops. Since the $h$ terms are also given by integrals over a reduced superspace, similar theorems can be established in the $\mathcal{N}=(0,2)$ models for these terms. An appropriate choice of the background field in this case is

$$\begin{aligned} \Phi_b^\dagger &= 0, \qquad \Phi_b = C_1 + C_2^\alpha \theta_\alpha + C_3 \theta^2, \\ \mathcal{B}_b^\dagger &= 0, \qquad \mathcal{B}_b = C_4 \theta_L^\dagger + C_5 \theta_R \theta_L^\dagger, \end{aligned} \tag{21}$$

where the subscript "b" marks the background fields, and $C_{1,2,3,4,5}$ are $c$-numerical *constants*. This choice assumes that $\Phi$ and $\Phi^\dagger$ are treated as independent variables, not connected by complex conjugation, and so are $\mathcal{B}$ and $\mathcal{B}^\dagger$ (i.e. we keep in mind a kind of analytic continuation). The $x$ independent fields (21) are invariant under the action of $Q_L^\dagger$. Next, to do the calculation of the effective action we decompose the superfields

$$\begin{aligned} \Phi &= \Phi_b + \Phi_{qu}, \qquad \Phi^\dagger = \Phi_b + \Phi_{qu}^\dagger, \\ \mathcal{B} &= \mathcal{B}_b + \Phi_{qu}, \qquad \mathcal{B}^\dagger = \mathcal{B}_b + \mathcal{B}_{qu}^\dagger, \end{aligned} \tag{22}$$

where the subscript "qu" denotes the quantum part of the superfield, expand the action in $\Phi_{qu}$, $\mathcal{B}_{qu}$ dropping the linear terms, and treat the remainder as the action for the quantum fields. We, then, integrate the quantum fields over, order by order, keeping the background field fixed. The crucial point is that in the given background field (a) the terms containing $\mathcal{W}$ and $\mathcal{B}$, *without* $\mathcal{W}^\dagger$ and $\mathcal{B}^\dagger$, do *not* vanish, and (b) there exists an exact supersymmetry under $Q_L^\dagger$-generated supertransformations.

After substituting in loops Green's functions in the given background and integrating over all vertices except the first one, we arrive at an expression of the type

$$\int d\theta_R^\dagger \times (\text{a } \theta_R^\dagger \text{ independent function}) = 0. \tag{23}$$

The $\theta_R^\dagger$ independence follows from the fact that our superspace is homogeneous in the $\theta_R^\dagger$ direction even in the presence of the background field (21). This completes the proof of nonrenormalization of $F$ and $h$ terms in the $\mathcal{N} = (0, 2)$ theory.

A subtle point here is that this proof tacitly assumes the absence of infrared singularities. Thus, it is certainly valid for the Wilsonean effective action [13]. If infrared contributions are included (i.e. the generator of 1-particle irreducible amplitudes is studied) the question should be investigated on a case by case basis.

*Kinks.*—$\mathcal{N} = (2, 2)$ models with two or more supersymmetric vacua (two or more zeros of $\partial_a \mathcal{W}$) support interpolating kink solutions which, typically, are 1/2 BPS saturated, i.e. preserve two out of four supersymmetries of the $\mathcal{N} = (2, 2)$ model under consideration. Adding an $\mathcal{N} = (0, 2)$ deformation can destroy vacuum degeneracy and, thus, eliminate kinks altogether. Even if we choose a deformatioin of a special form which does not break $\mathcal{N} = (0, 2)$ supersymmetry in two or more vacua, BPS-saturated kinks do not exist in such theories. This is readily seen through the Bogomoln'yi completion [14]. With the heterotic deformation switched on the Bogomol'nyi completion of the bosonic part of the energy functional is impossible, generally speaking.

The work of M. S. was supported in part by DOE Grant No. DE-FG02-94ER408. The work of A. Y. was supported by FTPI, University of Minnesota, by RFBR Grant No. 09-02-00457a, and by Russian State Grant for Scientific Schools No. RSGSS-11242003.2.

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