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Low-energy $U(1) \times USp(2M)$ gauge theory from simple high-energy gauge group

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We give an explicit example of the embedding of a near-BPS low-energy $(U(1) \times USp(2M))/\mathbb{Z}_2$ gauge theory into a high-energy theory with a simple gauge group and adjoint matter content. This system possesses degenerate monopoles arising from the high-energy symmetry breaking as well as non-Abelian vortices due to the symmetry breaking at low energies. These solitons of different codimensions are related by the exact homotopy sequences.

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I. INTRODUCTION

Topological solitons are important in many areas of physics, ranging from high-energy (elementary particle) physics, condensed matter physics and string theory to cosmology. In this paper, we shall focus on a system possessing non-Abelian vortices and monopoles: a supersymmetric gauge theory with a $G = USp(2M)$ gauge group, which is broken to $H = U(1) \times USp(2M-2)$ by the vacuum expectation value (VEV) of an adjoint scalar field. This breaking gives rise to regular non-Abelian 't Hooft-Polyakov monopoles. According to Goddard-Nuyts-Olive-Weinberg [1–4], the non-Abelian monopoles transform according to the dual group of H, which in this case is $\tilde{H} = U(1) \times SO(2M-1)$. Several difficulties in the naïve idea of non-Abelian monopoles have been known for some time, i.e. the global H group suffering from a topological obstruction and non-normalizable zero-modes do not allow the standard quantization and construction of the H multiplets of monopoles [5–9]. These problems arise in the Coulomb phase of the theory.

As was done in a series of investigations [10,11] for $SU(N)$ gauge theories, we take one step further here and break the remaining gauge symmetry completely at a much lower mass scale. This can be realized by the introduction of an $\mathcal{N} = 2$ breaking term in the superpotential, giving rise to an effective Fayet-Iliopoulos term. In systems with such a hierarchical gauge symmetry breaking, the homotopy group-maps relate the regular monopoles to the non-Abelian vortices arising at low energies, allowing for a better understanding of the concept of the non-Abelian monopole itself. Also, this kind of system provides a (dual) model of a non-Abelian color-confining superconductor, further motivating its study.

Besides the cases of $SU(N)$ gauge theories extensively studied in the last several years, this type of analysis has so far been made only in the case of $SO(N)$ gauge theories [12], i.e., with a hierarchical breaking, $SO(N) \to U(1) \times SO(N-2) \to \mathbb{1}$. In the $SO(N)$ systems the adjoint matter in the high-energy system yields at low energies exactly the right matter content—a system with light fundamental matter, all charged with respect to a common $U(1)$ factor.

As gauge systems with hierarchical symmetry breaking $G \to H \to \mathbb{1}$ and a color-flavor locked symmetry H_{C+F} have been constructed to date only for the $SU(N)$ and $SO(N)$ gauge groups [10–12], one might wonder to which extent our idea of defining non-Abelian monopoles through better-understood non-Abelian *vortices* is general. The central aim—and the result—of the present paper is to construct explicitly an analogous system with the unitary symplectic gauge group, strengthening further our belief that this kind of approach is of quite a general validity.

Among the many remarkable developments which followed the discovery of genuine non-Abelian vortices (vortices with continuous non-Abelian moduli) in Refs. [10,13] (for reviews, see [14,15]) is the moduli matrix formalism [16,17] (see review [18]), first constructed for domain walls. This formalism made it possible to uncover the full moduli space of these non-Abelian vortices, first in the $U(N) \sim (U(1) \times SU(N))/\mathbb{Z}_N$ theories and subsequently in models with generic gauge groups [19]. Finally, in Ref. [20] an in-depth study of the non-Abelian vortices, including the cases of the $(U(1) \times USp(2M))/\mathbb{Z}_2$ gauge group, has been carried out.

The system considered in this paper reduces at low energies, as we shall show, to the $(U(1) \times USp(2M))/\mathbb{Z}_2$ models investigated in Ref. [20]; the properties of the vortex moduli space found there then give detailed exact information about the massive non-Abelian monopoles.

II. $USp(2M)$ THEORY WITH MATTER IN THE FUNDAMENTAL REPRESENTATION

Let us first briefly review the superpotential for N_f fundamental hypermultiplets in the $USp(2M)$ gauge theory with $\mathcal{N} = 2$ extended supersymmetry in 3 + 1 dimensions

$$\sqrt{2} \sum_{i=1}^{N_f} \tilde{q}_a^i \Phi^{ab} q_b^i, \qquad (1)$$

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where i denotes the flavor index and a, $b = 1, \ldots, 2M$ denote the color indices. Because of the pseudoreal nature of USp matter fields, we can by a change of basis

$$q^i = \frac{1}{\sqrt{2}}(Q^i + iQ^{N_f+i}), \qquad \tilde{q}^i = \frac{1}{\sqrt{2}}(Q^i - iQ^{N_f+i}), \tag{2}$$

write the superpotential as

$$\frac{1}{\sqrt{2}} \sum_{i=1}^{2N_f} Q_a^i \Phi^{ab} Q_b^i, \tag{3}$$

where we have used the fact that $\Phi^{ab} = \Phi^{ba}$ is symmetric and we use a notation where the color indices are raised and lowered with the invariant rank-two tensor of $USp(2M)$

$$J^{\mathrm{T}} = -J, \qquad J^\dagger J = \mathbf{1}_{2M}, \tag{4}$$

which we choose to be the skew-diagonal matrix as usual. The (global) flavor symmetry which the theory at hand possesses is $O(2N_f)$. The mass term is

$$\sum_{i,j=1}^{2N_f} \frac{m_{ij}}{2} Q_a^i J^{ab} Q_b^j, \tag{5}$$

where $m_{ij} = \hat{m}_i J_{ij}$ is antisymmetric. The flavor symmetry is now $O(2N_f) \cap USp(2N_f) \sim U(N_f)$.

III. $USp(2M)$ THEORY WITH MATTER IN THE ADJOINT REPRESENTATION

To construct a system with a hierarchical gauge symmetry breaking as explained in the Introduction we use the matter fields (squarks) in the adjoint representation rather than in the fundamental representation. As in the previous case we start with the matter fields in the basis

$$\sqrt{2} \sum_{i=1}^{N_f} \mathrm{Tr}\{\tilde{q}^i [\Phi, q^i]\}, \tag{6}$$

while by the change of basis (2) we obtain

$$\begin{aligned} \mathcal{W}_{\mathrm{Adj,Yukawa}} &= \frac{i}{\sqrt{2}} \sum_{i,j=1}^{2N_f} J_{ij} \mathrm{Tr}\{Q^i [\Phi, Q^j]\} \\ &= i\sqrt{2} \sum_{i,j=1}^{2N_f} J_{ij} \mathrm{Tr}\{Q^i \Phi Q^j\}, \end{aligned} \tag{7}$$

with $J^{\mathrm{T}} = -J$, $J^\dagger J = \mathbf{1}_{2N_f}$ being the rank-two invariant tensor of $USp(2N_f)$ [21], whereas the mass term is now

$$\sum_{i,j=1}^{2N_f} \frac{m_{ij}}{2} \mathrm{Tr}\{Q^i Q^j\}, \tag{8}$$

and needs to be symmetric in order not to vanish. We shall choose $m_{ij} = \hat{m}_i \tilde{J}_{ij}$, where $\tilde{J}$ is the symmetric invariant tensor of $SO(2N_f)$

$$\tilde{J}^{\mathrm{T}} = \tilde{J}, \qquad \tilde{J}^\dagger \tilde{J} = \mathbf{1}_{2N_f}, \tag{9}$$

where we again use the skew-diagonal basis. The global flavor symmetry of our system is thus $USp(2N_f) \cap O(2N_f) \sim U(N_f)$.

IV. $\mathcal{N} = 1$ DEFORMATION

Finally, we will add a soft supersymmetry breaking term as $\mu \, \mathrm{Tr}\Phi^2$ to the adjoint theory and hence we have the superpotential

$$\begin{aligned} \mathcal{W}_{\mathrm{Adj}} = i\sqrt{2} \sum_{i,j=1}^{2N_f} J_{ij} \mathrm{Tr}\{Q^i \Phi Q^j\} + \sum_{i,j=1}^{2N_f} \frac{m_{ij}}{2} \mathrm{Tr}\{Q^i Q^j\} \\ + \frac{\mu}{2} \mathrm{Tr}\{\Phi^2\}, \end{aligned} \tag{10}$$

which gives rise to the following vacuum equations

$$J_{ij}[Q^j, \Phi] + \frac{i}{\sqrt{2}} m_{ij} Q^j = 0, \qquad i = 1, \ldots, 2N_f, \tag{11}$$

$$J_{ij}[Q^i, Q^j] + i\sqrt{2}\mu\Phi = 0, \tag{12}$$

(repeated indices are summed over) together with the D-term conditions.

First a word on what we expect. From group theory we know that the adjoint representation of $USp(2M)$ splits as [22]

$$\begin{aligned} &USp(2M) \supset SU(2) \times USp(2M-2), \\ &\mathrm{Adj} = (\mathrm{Adj}, \mathbb{1}) + (\mathbb{1}, \mathrm{Adj}) + (\square, \square), \end{aligned} \tag{13}$$

$(M > 1)$. Actually, we are interested only in the $U(1)$ subgroup of $SU(2)$ so the relevant decomposition reads

$$\begin{aligned} &USp(2M) \supset U(1) \times USp(2M-2), \\ &\mathrm{Adj} = 3(0, \mathbb{1}) + (0, \mathrm{Adj}) + (1, \square) + (-1, \square). \end{aligned} \tag{14}$$

We require the system to be such that only the fields in the fundamental representation in the low-energy $USp(2M-2)$ remain light, other fields with no $U(1)$ charges all becoming massive, with a mass of the order $\mathcal{O}(m)$. Furthermore, only one set of fundamentals will remain light, either the one with positive $U(1)$ charge or the one with negative charge in Eq. (14).

We choose the VEV of Φ as

$$\langle\Phi\rangle = \epsilon \, \mathrm{diag}(m, \underbrace{0, \ldots, 0}_{M-1}, -m, \underbrace{0, \ldots, 0}_{M-1}) \equiv \epsilon\Phi_0, \tag{15}$$

and the mass parameters as

$$m_{ij} = -i\sqrt{2} m \tilde{J}_{ij}, \qquad \mu = -i\sqrt{2}\nu, \tag{16}$$

where again $\tilde{J} = \tilde{J}^{\mathrm{T}}$ is the invariant tensor of $SO(2N_f)$. In order to have a separation of scales in the hierarchical gauge symmetry breaking, we take $m \gg \nu$. $\epsilon = \pm$ is the sign that will select which fundamental fields will become light, with positive or negative $U(1)$ charge, respectively. Accordingly, we make an ansatz $Q^{N_f+i} = (Q^i)^\dagger$, which solves the D-flatness conditions. This ansatz together with the masses taken as in Eq. (16) reduces the vacuum equations to

$$[\Phi_0, Q^i] + \epsilon m Q^i = 0, \qquad i = 1, \ldots, N_f, \tag{17}$$

$$\sum_{i=1}^{N_f}[Q^i, Q^{i\dagger}] + \epsilon\nu\Phi_0 = 0. \tag{18}$$

The light fields are then seen to correspond to the nontrivial eigenvectors of $[\Phi_0, \cdot]$ with eigenvalue $-\epsilon m$ and they in turn condense by Eq. (18). Without loss of generality, we can choose the light fields to be the ones with positive $U(1)$ charge and set $\epsilon := +$. Such eigenvectors are found to be $h^i(x)$ where

$$Q^i = Q^i_a t^a = h^i_\alpha K^\alpha + h^i_{M-1+\alpha}L^\alpha, \tag{19}$$

where $i = 1, \ldots, N_f$ is the flavor index and $a = 1, \ldots, M(2M+1)$ is the adjoint color index and finally $\alpha = 1, \ldots, M-1$ is half of the fundamental color index for $USp(2M-2)$. The matrices $K, L \in \mathfrak{usp}(2M)^{\mathbb{C}}$ are

$$(K^\alpha)_i{}^j = \frac{1}{2}(\delta_{1+\alpha,i}\delta^{1,j} - \delta_{M+1,i}\delta^{M+1+\alpha,j}), \tag{20}$$

$$(L^\alpha)_i{}^j = \frac{1}{2}(\delta_{M+1+\alpha,i}\delta^{1,j} + \delta_{M+1,i}\delta^{1+\alpha,j}). \tag{21}$$

If we instead wanted the fundamental fields with negative $U(1)$ charge to be the light fields, we should set $\epsilon := -$ and the eigenvectors would be

$$Q^i = Q^i_a t^a = h^i_\alpha (K^\alpha)^{\mathrm{T}} + h^i_{M-1+\alpha}(L^\alpha)^{\mathrm{T}}. \tag{22}$$

See the Appendix for details.

Calculating now explicitly the commutator, Eq. (18) gives rise to the D-flatness conditions of the $U(1)\times USp(2M-2)$ low-energy theory with fundamental matter content. Let us make the following definition:

$$h^i = \begin{pmatrix} h^i_\alpha \\ h^i_{M-1+\alpha}\end{pmatrix} \equiv \begin{pmatrix} k^i_\alpha \\ \ell^i_\alpha \end{pmatrix}, \tag{23}$$

with k, ℓ being $(M-1)$-vectors of color and i is the flavor index. Then, independently of the choice of the sign ϵ, $(4\times)$ Eq. (18) reads

$$\begin{pmatrix} -h^{i\dagger}h^i + 4\nu m & 0 & 0 & 0 \\ 0 & \mathbf{A} & 0 & \mathbf{B}^\dagger \\ 0 & 0 & h^{i\dagger}h^i - 4\nu m & 0 \\ 0 & \mathbf{B} & 0 & -\mathbf{A}^{\mathrm{T}} \end{pmatrix} = 0, \tag{24}$$

from which the Abelian D-term constraint (in the low-energy $\mathcal{N}=1$ theory) is easily read off. Now for the non-Abelian part, we find the form of the matrices $\mathbf{A}$, $\mathbf{B}$:

$$\mathbf{A} \equiv k^i k^{i\dagger} - (\ell^i\ell^{i\dagger})^{\mathrm{T}}, \qquad \mathbf{B} \equiv \ell^i k^{i\dagger} + (\ell^i k^{i\dagger})^{\mathrm{T}}, \tag{25}$$

where $\mathbf{B}^{\mathrm{T}} = \mathbf{B}$ is manifest. Using that

$$h^i h^{i\dagger} = \begin{pmatrix} k^i k^{i\dagger} & k^i\ell^{i\dagger} \\ \ell^i k^{i\dagger} & \ell^i\ell^{i\dagger}\end{pmatrix}, \tag{26}$$

together with the explicit form of the generators

$$t^n = \begin{pmatrix} \alpha & \beta_S \\ \beta_S^\dagger & -\alpha^{\mathrm{T}}\end{pmatrix}, \qquad \alpha^\dagger = \alpha, \qquad \beta_S^{\mathrm{T}} = \beta_S, \tag{27}$$

we obtain

$$0 = \mathrm{Tr}\{h^i h^{i\dagger} t^n\} = \mathrm{Tr}\{\mathbf{A}\alpha\} + \frac{1}{2}\mathrm{Tr}\{\mathbf{B}\beta_S\} + \frac{1}{2}\mathrm{Tr}\{\mathbf{B}^\dagger\beta_S^\dagger\}, \tag{28}$$

for *all* α, β_S, which forces $\mathbf{A} = \mathbf{B} = 0$, where we have used the fact that $\mathbf{B}$ is symmetric. Now as a check, we can count the number of constraints of $\mathbf{A} = \mathbf{B} = 0$ yielding $M'(2M'+1)$ with $M' \equiv M-1$, which indeed coincides with the number of constraints in Eq. (28). Hence, using a color-flavor matrix notation $(hh^\dagger)_\alpha{}^{\alpha'} = h_\alpha{}^i (h^\dagger)_i{}^{\alpha'} = h^i h^{i\dagger}$ we can write the Eqs. (17) and (18) as

$$\mathrm{Tr}\{hh^\dagger\} = 4\nu m, \tag{29}$$

$$\mathrm{Tr}\{hh^\dagger t^n\} = 0, \tag{30}$$

which are the D-term conditions appropriate for constructing non-Abelian Bogomol'nyi-Prasad-Sommerfield (BPS) vortices and $t^n \in \mathfrak{usp}(2M-2)$ and $n = 1, \ldots, (M-1)\times(2(M-1)+1)$ and specifically for the fundamental representation, as we intended. These vortices have already been studied in the low-energy theory in Ref. [20]. A comment is in store to emphasize the importance of identifying the "light mass" degrees of freedom in the symmetry breaking.

In order to have a vacuum that breaks completely the local gauge symmetry, allowing at the same time for an intact global color-flavor symmetry, we shall choose the number of flavor multiplets to be $N_f = 2M-2$. Thus h is a square matrix with the following VEV:

$$\langle h\rangle = \frac{\sqrt{\xi}}{\sqrt[4]{M-1}}\mathbf{1}_{2M-2}. \tag{31}$$

For completeness, let us write down the low-energy effective action for the light fundamental fields:

$$\mathcal{L} = -\frac{1}{4g^2}F^n_{\mu\nu}F^{n\mu\nu} - \frac{1}{4e^2}F^0_{\mu\nu}F^{0\mu\nu} + \mathrm{Tr}(\mathcal{D}_\mu h)(\mathcal{D}^\mu h)^\dagger - \frac{e^2}{2}|\mathrm{Tr}(hh^\dagger t^0) - \xi|^2 - \frac{g^2}{2}|\mathrm{Tr}(hh^\dagger t^n)|^2, \tag{32}$$

where we have rescaled the fields $h \to \sqrt{2}gh$ and $\xi \equiv \nu m \to e\xi/(\sqrt{2M-2})$ and defined the $U(1)$ generator

$$t^0 \equiv \frac{\mathbf{1}_{2M-2}}{2\sqrt{M-1}}, \tag{33}$$

and finally the index $n = 1, \ldots, (M-1)(2(M-1)+1)$. Because of different renormalization effects of the subgroups after the gauge symmetry breaking, we use e to denote the coupling for $U(1)$ and g for $USp(2M-2)$. Note that we have neglected higher-order terms in ν/m, which will give rise to non-BPS terms in the low-energy action for vortices; hence as already mentioned it is a near-BPS system.

As a final remark, let us note that in the strictly BPS limit our low-energy system would have a large vortex-moduli space including the so-called semilocal vortices [20]. The latter do not have a definite transverse size, and would not confine the monopole at their ends. However, our system (with $m \gg \nu$) is almost, but not exactly, BPS. When small non-BPS corrections arising from the high-energy gauge symmetry breaking are taken into account, we expect the vortex moduli, which are not related to the exact global symmetry of the system, to disappear. This has been explicitly shown [23] in the case of the vortex moduli in the $SU(N+1)$ theory with $N_f > N$, spontaneously broken at two scales, $SU(N+1) \to U(N) \to \mathbb{1}$.

V. CONCLUSION

Our system is characterized by the hierarchical gauge symmetry breaking

$$G \xrightarrow{m} H \xrightarrow{2\sqrt{\nu m}} \mathbb{1}. \tag{34}$$

As all the fields in the underlying theory are in the adjoint representation, we actually have $G = USp(2M)/\mathbb{Z}_2$. The (light) matter content of the low-energy theory shows also that $H = (U(1) \times USp(2M-2))/\mathbb{Z}_2$. Since $\pi_1(G) = \mathbb{Z}_2$, the exact homotopy sequence tells us that

$$\pi_2(G/H) \sim \pi_1(H)/\mathbb{Z}_2: \tag{35}$$

the regular monopoles arising at the high-mass scale breaking are confined by the doubly-wound vortices of the low-energy theory. The results of Ref. [20], which hold in a vacuum with the color-flavor locked phase, indicate that the minimal winding vortices of the low-energy $U(1) \times USp(2M-2)$ system, which are stable in the full theory as $\pi_1(G) = \mathbb{Z}_2$, appear classified according to the *spinor representation* of a dual (color-flavor) $SO(2M-1)$ symmetry group. The regular monopoles of our system, associated with the doubly-wound vortices, are then predicted to transform according to various representations including the *vector representation* of the $SO(2M-1)$ group, reminiscent of the Goddard-Nuyts-Olive duality. These group-theoretic features of our vortex-monopole complex are under a careful scrutiny at present, and will be presented elsewhere.

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APPENDIX

It is also possible, though more elaborate, to use the real algebra $\mathfrak{usp}(2M)$ instead of the complexified algebra $\mathfrak{usp}(2M)^{\mathbb{C}}$ as we have utilized in the calculation. However, it requires us to change the basis. Using the definitions we already have made, we can write

$$Q_a^i t^a = H_\alpha^i \kappa^\alpha + \tilde{H}_\alpha^i \tilde{\kappa}^\alpha + H_{M-1+\alpha}^i \lambda^\alpha + \tilde{H}_{M-1+\alpha}^i \tilde{\lambda}^\alpha,$$

with

$$\kappa^\alpha \equiv K^\alpha + (K^\alpha)^{\mathrm{T}}, \qquad \tilde{\kappa}^\alpha \equiv iK^\alpha - i(K^\alpha)^{\mathrm{T}}, \tag{A1}$$

$$\lambda^\alpha \equiv L^\alpha + (L^\alpha)^{\mathrm{T}}, \qquad \tilde{\lambda}^\alpha \equiv iL^\alpha - i(L^\alpha)^{\mathrm{T}}, \tag{A2}$$

where $\kappa, \tilde{\kappa}, \lambda, \tilde{\lambda} \in \mathfrak{usp}(2M)$. Now to obtain the eigenvectors in this basis, we find the following linear combination

$$h_\epsilon^i = \frac{1}{\sqrt{2}}(H^i + \epsilon i \tilde{H}^i). \tag{A3}$$

We recognize h_ϵ^i as $\sqrt{2}$ times the eigenvectors found in the text and ϵ again selects the $U(1)$ charge. Thus it is an advantage to work directly with the complexified algebra.

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