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B to $D(D^*)e\nu_e$ transitions at finite temperature in QCD

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In this article, we work out the properties of the B, D, and D^* mesons as well as the $B \to D(D^*)e\nu_e$ decay properties at finite temperature QCD. The behavior of the masses, decay constants and widths of the B, D, and D^* mesons in terms of the temperature is studied. The temperature dependency of the form factors responsible for such decays are also obtained. These temperature-dependent form factors are used to investigate the variation of the branching ratios with respect to the temperature. It is shown that the branching ratios do not change up to $T/T_c = 0.3$, however they start to diminish with increasing the temperature after this region and vanish at the critical or deconfinement temperature.

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I. INTRODUCTION

A flood of papers with different approaches have been dedicated to the investigation of the $B \to D(D^*)e\nu_e$ transition and calculating the corresponding form factors in a vacuum at $T = 0$ temperature (as an example see [1–7] and references therein). These channels are results of the most dominant transition of the b quark, $b \to c$, and could play an essential role in the probing new physics charged Higgs contributions in low energy observables as well as exploring heavy quark dynamics and the origin of the CP violation. The QCD sum rules method [8] has been one of the most applicable tools to hadron physics at zero temperature. This approach was expended to include the hadronic spectroscopy at finite temperature called thermal QCD sum rules [9] assuming that the operator product expansion (OPE) and the quark-hadron duality assumption are valid, but the vacuum condensates are replaced by their thermal expectation values. The aim of this extension was to understand the results of the heavy ion collision experiments. Plenty of works have been devoted to mainly the calculation of vacuum condensates, mass and decay constant of mesons, and some properties of the nucleons at finite temperature and also other applications of the temperature-dependent QCD sum rules [10–22].

The present work encompasses the calculation of the mass and decay constant of the B, D, and D^* mesons as well as the form factors responsible for $B \to D(D^*)e\nu_e$ decay channels and related branching ratios in thermal QCD. Here, we assume that with replacing the quark and gluon condensates and also the continuum threshold by their thermal version, the sum rules for the observables are valid and the contribution of the additional operators in the Wilson expansion at finite temperature [23] is ignorable. These additional operators would be due to the breakdown of the Lorentz invariance at finite temperature by the choice of the thermal rest frame, where matter is at rest at a definite temperature [21,24]. In such a condition, the residual O(3) invariance may bring these additional operators with the same dimension as the vacuum condensates. Moreover, we have another assumption that ignores the interaction of the currents with the existing particles in the medium. Such interactions could require modifying the hadron spectral densities. To get the exact results, these two new features of the thermal QCD should be considered.

The layout of the paper is as follows: in Sec. II the sum rules for the masses, decay constants, widths, and form factors responsible for the $B \to D(D^*)e\nu_e$ decay channels at thermal QCD are derived. Section III is dedicated to the numerical analysis, an investigation of the dependence of the observables on the temperature, and a comparison of our results with the existing experimental data at zero temperature.

II. THEORETICAL FRAMEWORK

In this section, we obtain the QCD sum rules for the mass, decay constants, widths, and form factors of the $B \to D(D^*)e$ channels. Our calculations will closely follow [3,6,7,25] at zero temperature and [14] at SU(3) symmetry-breaking case. First of all, let us calculate the transition form factors responsible for the $B \to D(D^*)e\nu_e$ channel at finite temperature in terms of the mass and decay constants of the participating particles. The starting point is to consider the following temperature-dependent three-point correlation functions:

(i) Correlation function for the $B \to De\nu$ at finite temperature

$$\Pi^V_\mu(T, p, q) = i^2 \int dxdy e^{ipx-ip'y} \times \langle \mathbf{T}\{\bar{d}(x)\gamma_5 c(x), J^V_\mu(0), \bar{b}(y)\gamma_5 d(y)\}\rangle, \quad (1)$$

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(ii) Temperature-dependent correlation function for $B \to D^* e\nu$

$$\Gamma^{V,A}_{\mu\nu}(T, p, q) = i^2 \int dx dy e^{ipx - ip'y} \times \langle \mathbf{T}\{\bar{d}(x)\gamma_\nu c(x), J^{V,A}_\mu(0), \bar{b}(y)\gamma_5 d(y)\}\rangle, \quad (2)$$

where $q = p - p'$ is the transferred momentum, T and $\mathbf{T}$ are temperature and time ordering operator, respectively, and $J^V_\mu = \bar{c}\gamma_\mu b$ and $J^A_\mu = \bar{c}\gamma_\mu\gamma_5 b$ are vector and axial vector part of the transition currents. Lorentz invariances and parity conservation considerations require that the axial part of the first correlation function does not have any contribution. The general idea in QCD sum rules is to calculate the above correlation functions in two different ways: they can be calculated in the phenomenological or physical side, saturating them by tower of hadrons with the same quantum numbers as the interpolating currents of the initial and final states, and in the QCD or theoretical side where the quark, gluon, and their interactions with each other and with the QCD vacuum are considered. In the QCD side, the time ordering product of the currents in the aforementioned correlation functions are expanded in terms of the operators having different mass dimensions via OPE, where the short and long distance effects are separated. The former is calculated using the QCD perturbation theory, whereas the latter are parametrized in terms of the operators owing different mass dimensions. Here we should stress that we will present only the result of the calculations which are related to our aim. For details of the calculations, see [3,6,7].

In the phenomenological part, we need to define the vacuum to the hadronic state matrix elements in terms of the decay constants and mass of the participating hadrons at finite temperature:

$$\begin{aligned} \langle 0|\bar{d}\gamma_\mu\gamma_5 c|D(p)\rangle &= i f_D(T) p_\mu \\ \langle B(p')|\bar{b}\gamma_5\gamma_\mu d|0\rangle &= -i f_B(T) p'_\mu \\ \langle 0|\bar{d}\gamma_\nu c|D^*(p,\varepsilon)\rangle &= m_{D^*}(T) f_{D^*}(T)\varepsilon_\nu, \end{aligned} \quad (3)$$

where $m_{D^*}(T)$, $f_D(T)$, $f_B(T)$, $f_{D^*}(T)$ are temperature-dependent masses and leptonic decay constants, and ε is the polarization vector of D^* meson. The transition matrix elements are also parametrized in terms of the transition form factors $f_\pm$, $F_{\pm,0,V}$ in the following way:

$$\begin{aligned} \langle D(p)|\bar{c}\gamma_\mu b|B(p')\rangle &= f_+ P_\mu + f_- q_\mu \\ \langle D^*(p,\varepsilon)|\bar{c}\gamma_\mu\gamma_5 b|B(p')\rangle &= i[F_0 \varepsilon^*_\mu + F_+(\varepsilon^* p')P_\mu \\ &\quad + F_-(\varepsilon^* p')q_\mu] \\ \langle D^*(p,\varepsilon)|\bar{c}\gamma_\mu b|B(p')\rangle &= i F_V \varepsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu} P^\alpha q^\beta, \end{aligned} \quad (4)$$

where $P = p' + p$. As we deal with the electron in final state, the form factors f_- and F_- are encountered corresponding to m_e; because of the smallness of the electron mass there is no need to calculate those form factors.

The Lorentz structures entered in the calculations are

$$\Pi^V_\mu(T) = [\Pi^{\text{pert}}_+(T) + \Pi^{\text{nonpert}}_+(T)]P_\mu + [\Pi^{\text{pert}}_-(T) + \Pi^{\text{nonpert}}_-(T)]q_\mu, \quad (5)$$

$$\begin{aligned} \Gamma^A_{\mu\nu}(T) &= [\Gamma^{\text{pert}}_0(T) + \Gamma^{\text{nonpert}}_0(T)]g_{\mu\nu} \\ &\quad + [\Gamma^{\text{pert}}_1(T) + \Gamma^{\text{nonpert}}_1(T)]p_\mu p'_\nu \\ &\quad + [\Gamma^{\text{pert}}_2(T) + \Gamma^{\text{nonpert}}_2(T)]p'_\mu p'_\nu \\ &\quad + [\Gamma^{\text{pert}}_3(T) + \Gamma^{\text{nonpert}}_3(T)]p_\mu p_\nu \\ &\quad + [\Gamma^{\text{pert}}_4(T) + \Gamma^{\text{nonpert}}_4(T)]p_\nu p'_\mu, \end{aligned} \quad (6)$$

$$\Gamma^V_{\mu\nu}(T) = [\Gamma^{\text{pert}}_V(T) + \Gamma^{\text{nonpert}}_V(T)] i\varepsilon_{\alpha\beta\mu\nu} p_\alpha p'_\beta, \quad (7)$$

where pert and nonpert stand for perturbative and nonperturbative contributions, respectively. To calculate the form factors f_+, F_0, F_V, and F_+, we will choose the structures P_μ, $g_{\mu\nu}$, $i\varepsilon_{\alpha\beta\mu\nu}p_\alpha p'_\beta$, and $\frac{p_\mu p'_\nu + p'_\mu p'_\nu}{2}$, respectively, from both sides of the correlation functions. The perturbative part of each Π_+ and $\Gamma_{+,0,V}$ on the QCD side can be written in terms of the double dispersion integrals as

$$\begin{aligned} \Pi^{\text{pert}}_+(T, Q^2) &= -\frac{1}{(2\pi)^2}\int_{m_c^2}^{s_0(T)} ds \int_{s'_{1,2}}^{s'_0(T)} ds' \times \frac{\rho_+[s, s', Q^2]}{[s - p^2][s' - p'^2]} + \text{subtraction terms}, \\ \Gamma^{\text{pert}}_{+,0,V}(T, Q^2) &= -\frac{1}{(2\pi)^2}\int_{m_c^2}^{s_0(T)} ds \int_{s'_{1,2}}^{s'_0(T)} ds' \times \frac{\varrho_{+,0,V}[s, s', Q^2]}{[s - p^2][s' - p'^2]} + \text{subtraction terms}, \end{aligned} \quad (8)$$

where $\rho_+[s, s', Q^2]$ and $\varrho_{+,0,V}[s, s', Q^2]$ are called the corresponding spectral densities and $Q^2 = -q^2 > 0$. The temperature-dependent continuum thresholds $s_0(T)$ and $s'_0(T)$ are defined to a very good approximation as [11]

$$\begin{aligned} s_0(T) &= s_0 \frac{\langle\bar{d}d\rangle(T)}{\langle 0|\bar{d}d|0\rangle}\left[1 - \frac{m_c^2}{s_0}\right] + m_c^2, \\ s'_0(T) &= s'_0 \frac{\langle\bar{d}d\rangle(T)}{\langle 0|\bar{d}d|0\rangle}\left[1 - \frac{m_b^2}{s'_0}\right] + m_b^2, \end{aligned} \quad (9)$$

where the $s_0 = s_0(0)$ and $s'_0 = s'_0(0)$ are the continuum thresholds in vacuum at $D(D^*)$ and B channels, respectively. The $\langle\bar{d}d\rangle(T)$ is d quark condensate at finite temperature. We will use the result of the temperature-dependent light quark condensate valid for all temperatures obtained by Barducci *et al.* [26] (for a discussion about the validity of the results for all temperatures see also [27]). The following parametrization for the T-dependent light quark condensate reproduces quite well the results presented in [26]:

$$\langle\bar{d}d\rangle(T) = \langle 0|\bar{d}d|0\rangle[1 - 915.142T^3]^\gamma, \tag{10}$$

where, $\gamma = 0.337\,35$ and $\langle 0|\bar{d}d|0\rangle$ is the quark condensates at zero temperature. This relation has been obtained at the critical or deconfinement temperature, $T_c = 103$ MeV. Using this value for the critical temperature, we obtain

$$\langle\bar{d}d\rangle(T) = \langle 0|\bar{d}d|0\rangle\left[1 - \left(\frac{T}{T_c}\right)^3\right]^\gamma. \tag{11}$$

As long as results are plotted as a function of T/T_c, the precise value of T_c does not matter very much.

The lower limit of the integration over s' is determined as

$$s'_{1,2} = \frac{1}{2}\left[\frac{s}{m_c^2}(m_c^2 + m_b^2 + Q^2) + (m_b^2 - m_c^2 - Q^2)\right] \pm \frac{s - m_c^2}{2m_c^2}[(m_c^2 + m_b^2 + Q^2)^2 - 4m_c^2m_b^2]^{1/2}. \tag{12}$$

After a calculation of the bare loop contributions, the spectral densities are found

$$\begin{aligned}
\rho_+[s,s',Q^2] &= \frac{3}{2\lambda^{3/2}}\{[\Delta's + m_c m_b\Delta][s - s' + Q^2] + [\Delta s' + m_c m_b\Delta'][s' - s + Q^2]\},\\
\varrho_0[s,s',Q^2] &= \frac{3}{2\lambda^{1/2}}[m_c\Delta' + m_b\Delta] + \frac{3m_b}{\lambda^{3/2}}[s'\Delta^2 + s\Delta'^2 - \Delta\Delta' u],\\
\varrho_+[s,s',Q^2] &= \frac{3}{2\lambda^{3/2}}\{m_c[2s'\Delta - u\Delta'] + m_b[2s\Delta' - u\Delta + 4\Delta\Delta' + 2\Delta^2]\}\\
&\quad + \frac{9m_b}{\lambda^{5/2}}\{4ss'\Delta\Delta' - u[s\Delta'^2 + s'\Delta^2 + 2s\Delta\Delta'] + 2s^2[\Delta'^2 + s'\Delta]\},\\
\varrho_V[s,s',Q^2] &= \frac{3}{\lambda^{3/2}}\{m_c[u\Delta' - 2s'\Delta] + m_b[u\Delta - 2s\Delta']\},
\end{aligned} \tag{13}$$

where $\lambda = [s + s' + Q^2]^2 - 4ss'$, $u = s + s' + Q^2$, $\Delta = s - m_c^2$, and $\Delta' = s' - m_b^2$.

The nonperturbative contributions in lowest order in α_s are obtained as

$$\begin{aligned}
\Pi_+^{\text{nonpert}}(T) &= \Pi_+^{\langle\bar{d}d\rangle}(T) + \Pi_+^{\langle\bar{d}Gd\rangle}(T) + \Pi_+^{\langle\bar{d}d\rangle^2}(T)\\
\Gamma_{0,+,V}^{\text{nonpert}}(T) &= \Gamma_{0,+,V}^{\langle\bar{d}d\rangle}(T) + \Gamma_{0,+,V}^{\langle\bar{d}Gd\rangle}(T) + \Gamma_{0,+,V}^{\langle\bar{d}d\rangle^2}(T),
\end{aligned} \tag{14}$$

where

$$\begin{aligned}
\Pi_+^{\langle\bar{d}d\rangle}(T) &= \frac{1}{2}\langle\bar{d}d\rangle(T)\frac{m_c + m_b}{rr'},\\
\Gamma_0^{\langle\bar{d}d\rangle}(T) &= \frac{1}{2}\langle\bar{d}d\rangle(T)\left[\frac{(m_c + m_b)^2 + Q^2}{rr'} + \frac{1}{r} + \frac{1}{r'}\right],\\
\Gamma_+^{\langle\bar{d}d\rangle}(T) &= -\frac{1}{2}\langle\bar{d}d\rangle(T)\frac{1}{rr'},\\
\Gamma_V^{\langle\bar{d}d\rangle}(T) &= \langle\bar{d}d\rangle(T)\frac{1}{rr'},\\
\Pi_+^{\langle\bar{d}Gd\rangle}(T) &= -\frac{m_0^2\langle\bar{d}d\rangle(T)}{12}\left[\frac{2(2m_c + m_b)}{r^2r'} + \frac{2(2m_b + m_c)}{rr'^2} + \frac{3m_c^2(m_c + m_b)}{r^3r'} + \frac{3m_b^2(m_c + m_b)}{rr'^3}\right.\\
&\quad \left. + \frac{m_c^2(2m_c + m_b) + m_b^2(2m_b + m_c) + 2(m_c + m_b)Q^2}{r^2r'^2}\right],\\
\Gamma_0^{\langle\bar{d}Gd\rangle}(T) &= -\frac{m_0^2\langle\bar{d}d\rangle(T)}{12}\left[\frac{3m_c^2}{r^3r'}(m_c^2 + m_b^2 + 2m_cm_b + Q^2) + \frac{3m_b^2}{rr'^3}(m_c^2 + m_b^2 + 2m_cm_b + Q^2)\right.\\
&\quad + \frac{1}{r^2r'^2}[3m_cm_b(m_c^2 + m_b^2 + Q^2) + 2((m_c^2 + m_b^2 + Q^2)^2 - m_c^2m_b^2)] + \frac{1}{r^2r'}[3m_c(m_c + m_b) + 2(m_b^2 + Q^2)]\\
&\quad \left. + \frac{1}{rr'^2}[3m_b(3m_c + m_b) + 4(m_c^2 + Q^2)] - \frac{2}{rr'}\right],
\end{aligned}$$

$$\Gamma_{+}^{\langle \bar{d}Gd\rangle}(T) = \frac{m_0^2\langle \bar{d}d\rangle(T)}{12}\bigg[\frac{3m_c^2}{r^3 r'} + \frac{3m_b^2}{rr'^3} - \frac{2}{rr'^2} + \frac{1}{r^2r'^2}(2m_c^2 + 2m_b^2 - m_c m_b + 2Q^2)\bigg],$$
$$\Gamma_{V}^{\langle \bar{d}Gd\rangle}(T) = -\frac{m_0^2\langle \bar{d}d\rangle(T)}{6}\bigg[\frac{3m_c^2}{r^3 r'} + \frac{3m_b^2}{rr'^3} + \frac{2}{rr'^2} + \frac{1}{r^2r'^2}(2m_c^2 + 2m_b^2 - m_c m_b + 2Q^2)\bigg],$$
$$\Pi_{+}^{\langle \bar{d}d\rangle^2}(T) = -\frac{\pi}{81}\alpha_s\langle \bar{d}d\rangle^2(T)\bigg\{\frac{12m_c^3(m_c + m_b)}{r^4 r'} + \frac{12m_b^3(m_c + m_b)}{rr'^4} + \frac{4m_c}{r^3r'^2}[m_c^2(2m_c + m_b) + m_b^2(2m_b + m_c)$$
$$+ 2Q^2(m_c + m_b)] + \frac{4m_b}{r^2r'^3}[m_c^2(2m_c + m_b) + m_b^2(2m_b + m_c) + 2Q^2(m_c + m_b)] + \frac{8m_c(7m_b - m_c)}{r^3r'}$$
$$+ \frac{8m_b(7m_c - m_b)}{rr'^3} - \frac{12}{r^2r'} - \frac{12}{rr'^2} - \frac{8}{r^2r'^2}[2m_c(2m_b + m_c) + 2m_b(2m_c + m_b) + Q^2]\bigg\},$$
$$\Gamma_{0}^{\langle \bar{d}d\rangle^2}(T) = -\frac{4\pi}{81}\alpha_s\langle \bar{d}d\rangle^2(T)\bigg[\frac{4(2m_c - m_b)}{r^2 r'} + \frac{2(7m_b - 8m_c)}{rr'^2} + \frac{3m_c^3}{r^4r'}(m_c^2 + m_b^2 + 2m_c m_b + Q^2)$$
$$+ \frac{3m_b^3}{rr'^4}(m_c^2 + m_b^2 + 2m_c m_b + Q^2) + \frac{1}{r^3r'^2}[3m_c^2 m_b(m_c^2 + m_b^2 + Q^2) + 2m_c((m_c^2 + m_b^2 + Q^2)^2 - m_c^2 m_b^2)]$$
$$+ \frac{1}{r^2r'^3}[3m_c m_b^2(m_c^2 + m_b^2 + Q^2) + 2m_b((m_c^2 + m_b^2 + Q^2)^2 - m_c^2 m_b^2)]$$
$$+ \frac{m_c}{r^3r'}(17m_c^2 + 16m_b^2 + 19m_c m_b + 16Q^2) + \frac{m_b}{rr'^3}(16m_c^2 + 15m_b^2 - 3m_c m_b + 16Q^2)$$
$$- \frac{1}{r^2r'^2}[2m_c(m_c^2 + Q^2) - 6m_b(m_b^2 + Q^2) + m_c m_b(11m_c + m_b)]\bigg].$$
$$\Gamma_{+}^{\langle \bar{d}d\rangle^2}(T) = \frac{4\pi}{81}\alpha_s\langle \bar{d}d\rangle^2(T)\bigg[\frac{3m_c^3}{r^4r'} + \frac{3m_b^3}{rr'^4} + \frac{m_c}{r^3r'^2}(2m_c^2 + 2m_b^2 - m_c m_b + 2Q^2) + \frac{m_b}{r^2r'^3}(2m_c^2 + 2m_b^2 - m_c m_b + 2Q^2)$$
$$+ \frac{14m_c}{r^3r'} + \frac{10m_b}{rr'^3} + \frac{2(2m_b - m_c)}{r^2r'^2}\bigg],$$
$$\Gamma_{V}^{\langle \bar{d}d\rangle^2}(T) = -\frac{2\pi}{81}\alpha_s\langle \bar{d}d\rangle^2(T)\bigg[\frac{3m_c^3}{r^4r'} + \frac{3m_b^3}{rr'^4} + \frac{m_c}{r^3r'^2}(2m_c^2 + 2m_b^2 - m_c m_b + 2Q^2) + \frac{m_b}{r^2r'^3}(2m_c^2 + 2m_b^2 - m_c m_b + 2Q^2)$$
$$+ \frac{14m_c}{r^3r'} + \frac{14m_b}{rr'^3} + \frac{2(2m_b - m_c)}{r^2r'^2}\bigg]. \tag{15}$$

In calculations $g_s\langle \bar{d}G_{\mu\nu}\sigma_{\mu\nu}d\rangle(T) = m_0^2\langle \bar{d}d\rangle(T)$ has been used.

After selecting the corresponding structures from both sides of the correlation function and equating them and also applying double Borel transformation with respect to the p^2 and p'^2 to subtract the contribution of the higher states and continuum, we obtain the temperature-dependent sum rules for the form factors

$$f_+(Q^2, T) = -\frac{m_c m_b}{f_D(T)f_B(T)m_D(T)^(T)2m_B(T)^2}\exp\bigg[\frac{m_D(T)^2}{M^2} + \frac{m_B(T)^2}{M'^2}\bigg]$$
$$\times\bigg\{-\frac{1}{(2\pi)^2}\int dsds'\rho_+[s, s', Q^2]\exp\bigg(-\frac{s}{M^2} - \frac{s'}{M'^2}\bigg) + \boldsymbol{B}\Pi_+^{\text{nonpert}}\bigg\}$$
$$F_{0,+}(Q^2, T) = -\frac{m_b}{f_{D^*}(T)f_B(T)m_{D^*}(T)m_B(T)^2}\exp\bigg[\frac{m_{D^*}(T)^2}{M^2} + \frac{m_B(T)^2}{M'^2}\bigg]$$
$$\times\bigg\{-\frac{1}{(2\pi)^2}\int dsds'\varrho_{0,+}[s, s', Q^2]\exp\bigg(-\frac{s}{M^2} - \frac{s'}{M'^2}\bigg) + \boldsymbol{B}\Gamma_{0,+}^{\text{nonpert}}\bigg\} \tag{16}$$
$$F_V(Q^2, T) = -\frac{m_b}{2f_{D^*}(T)f_B(T)m_{D^*}(T)m_B(T)^2}\exp\bigg[\frac{m_{D^*}(T)^2}{M^2} + \frac{m_B(T)^2}{M'^2}\bigg]$$
$$\times\bigg\{-\frac{1}{(2\pi)^2}\int dsds'\varrho_V[s, s', Q^2]\exp\bigg(-\frac{s}{M^2} - \frac{s'}{M'^2}\bigg) + \boldsymbol{B}\Gamma_V^{\text{nonpert}}\bigg\},$$

where, $\boldsymbol{B}$ denotes double Borel transformation with respect to the p^2 and p'^2, and M^2 and M'^2 are Borel mass parameters in $D(D^*)$ and B channels, respectively. Here we should also stress that our sum rules stated above have not included the factor $M^2M'^2$ in front of the $\boldsymbol{B}\Pi_+^{\rm nonpert}$ part (see [3]). Such a factor could not be correct since it breaks down the mass dimension consistency between perturbative and nonperturbative parts. From the above sum rules, it is clear to proceed we need to know the temperature-dependent expressions for the mass and decay constants. Lets first calculate the temperature-dependent mass, decay constant, and width of the pseudo scalar B and D mesons. To obtain the mass sum rules, we consider the following two-point thermal correlation function:

$$\Delta(T, q'^2) = i\int d^4x e^{iq'\cdot x}\langle T\{\bar{d}(x)\gamma_5 Q'(x), \bar{Q}'(0)\gamma_5 d(0)\}\rangle, \tag{17}$$

where, $q' = p$, $Q' = c$ for D and $q' = -p'$, $Q' = b$ for B mesons, respectively. The same as in the method presented above to calculate the form factors, we should calculate this two-point correlation function also in both hadronic and quark-gluon languages. In hadronic language or the phenomenological part, the spectral density in zero-width approximation can be written as

$$\rho(s)|_{\rm hadron} = 2f_{\rm PS}(T)^2 m_{\rm PS}(T)^4\delta[s - m_{\rm PS}(T)^2] + \theta[s - s_0''(T)]\rho(s)|_{\rm PQCD}, \tag{18}$$

where PS stands for pseudo scalar D and B mesons, PQCD denotes the perturbative QCD, and $s_0''(T) = s_0(T)$ and $s_0''(T) = s_0'(T)$ are the continuum thresholds related to the D and B channels, respectively. Using Eq. (18) and the quark-hadron duality assumption and equating phenomenological and theoretical sides of the correlation function, the following sum rules are obtained in zero-width approximation:

$$\frac{f_{\rm PS}(T)^2 m_{\rm PS}(T)^4}{m_{\rm PS}(T)^2 - q'^2} = \int_{m_{Q'}^2}^{s''(T)} ds\frac{\xi[s]}{s - q'^2} + \Delta^{\rm nonpert}(T, q'^2). \tag{19}$$

From the same procedure as stated in the three-point correlation function the spectral density and nonperturbative part are found to be [for SU(3)-breaking case see [14,28]]:

$$\xi[s] = \frac{3m_{Q'}^2}{8\pi^2 s}\eta^2[s]\left[1 + \frac{4\alpha_s}{3\pi}f(x)\right], \tag{20}$$

where $x = \frac{m_{Q'}^2}{s}$, $\alpha_s = \alpha_s(m_{Q'}^2)$ and

$$\eta[s] = s - m_{Q'}^2 \tag{21}$$

$$f[x] = \frac{9}{4} + 2Li_2[x] + \ln x\ln[1-x] - \frac{3}{2}\ln\left[\frac{1}{x} - 1\right] - \ln[1-x] + x\ln\left[\frac{1}{x} - 1\right] - \frac{x}{1-x}\ln x, \tag{22}$$

$$\begin{aligned}\Delta^{\rm nonpert}(T, q'^2) = {}& -m_{Q'}\lambda\langle\bar{d}d\rangle(T) + \frac{1}{12\pi}\lambda\langle\alpha_s G^2\rangle(T)\\ &- \frac{m_0^2}{2m_{Q'}}\langle\bar{d}d\rangle(T)\lambda^2(1-\lambda)\\ &- \frac{32\pi}{27m_{Q'}^2}\alpha_s\langle\bar{d}d\rangle^2(T)\lambda^2(2-\lambda-\lambda^2),\end{aligned} \tag{23}$$

where $\lambda = \frac{m_{Q'}^2}{(m_{Q'}^2 - q'^2)}$. The temperature-dependent gluon condensate valid at all temperatures is calculated in lattice QCD [29]. The obtained result can be approximated by two straight lines [30] which we will use in the present work, i.e.,

$$\langle\alpha_s G^2\rangle(T) = \langle 0|\alpha_s G^2|0\rangle\left[\theta(T^* - T) + \frac{1 - \frac{T}{T_c^*}}{1 - \frac{T^*}{T_c^*}}\theta(T - T^*)\right]. \tag{24}$$

where, $\langle 0|\alpha_s G^2|0\rangle$ is the gluon condensate at zero temperature, $T^* \simeq 150$ MeV is the breakpoint temperature where the condensate begins to decrease appreciably, and $T_c^* \simeq 250$ MeV is the temperature at which the gluon condensate vanish at critical temperature, T_c.

Applying the Borel transformation with respect to the q'^2 to both sides of the Eq. (19) we get the following expression for the decay constant of the pseudo scalar meson:

$$\begin{aligned}&f_{\rm PS}^2(T)m_{\rm PS}^4(T)e^{((-m_{\rm PS}^2(T))/(M''^2))}\\ &= \bar{A}(T) + f_{\rm PS}^2 m_{\rm PS}^4 e^{-((m_{\rm PS}^2)/(M''^2))},\end{aligned} \tag{25}$$

where $M'' = M$ and $M'' = M'$ for D and B mesons, respectively, and

$$\bar{A}(T) = \int_{s_0''}^{s_0''(T)} ds\xi(s)e^{-(s/(M''^2))} + \bar{\Delta}^{\rm nonpert}(M''^2, T), \tag{26}$$

where,

$$\begin{aligned}\bar{\Delta}^{\rm nonpert}(M''^2, T) = {}& -m_{Q'}^3\overline{\langle\bar{d}d\rangle}e^{-\beta} + \frac{1}{12\pi}\overline{\langle\alpha_s G^2\rangle}m_{Q'}^2 e^{-\beta}\\ &- \frac{1}{2}m_0^2 m_{Q'}\beta\overline{\langle\bar{d}d|\rangle}e^{-\beta}\left[1 - \frac{1}{2}\beta\right]\\ &- \frac{16}{81}\pi\alpha_s\overline{\langle\bar{d}d\rangle^2}\beta e^{-\beta}(12 - 3\beta - \beta^2),\end{aligned} \tag{27}$$

where $\beta = m_{Q'}^2/M''^2$ and the bar on the operators means subtractions of their vacuum expectation values from thermal expectation values, i.e., for any operator X, $\bar{X}(M''^2, T) = X(M''^2, T) - X(M''^2, T = 0)$. The mass of the pseudo scalar meson is obtained taking derivative with respect to the $-\frac{1}{M''^2}$ for both sides of the Eq. (25) and dividing by itself, i.e.,

$$m_{\mathrm{PS}}^2(T) = \frac{\frac{-d}{d(1/M''^2)}[\bar{A}(T) + f_{\mathrm{PS}}^2 m_{\mathrm{PS}}^4 e^{-((m_{\mathrm{PS}}^2)/(M''^2))}]}{\bar{A}(T) + f_{\mathrm{PS}}^2 m_{\mathrm{PS}}^4 e^{-((m_{\mathrm{PS}}^2)/(M''^2))}}. \quad (28)$$

Our next task is to calculate the temperature-dependent mass and decay constant of the vector D^* meson in zero-width approximation. The following correlation function is responsible for our aim:

$$\Theta_{\mu\nu}(T, p^2) = i \int d^4x e^{ip\cdot x} \langle T\{\bar{d}(x)\gamma_\mu c(x), \bar{c}(0)\gamma_\nu d(0)\}\rangle. \quad (29)$$

From the similar procedure as the pseudo scalar case, the sum rules for the decay constant of the D^* meson is obtained as

$$\begin{aligned} f_{D^*}^2(T) m_{D^*}^2(T) e^{((-m_{D^*}^2(T))/(M^2))} &= \frac{1}{8\pi^2}\int_{m_c^2}^{s_0(T)} ds \frac{(s - m_c^2)^2}{s}\left(2 + \frac{m_c^2}{s}\right)e^{-s/M^2} \\ &\quad - m_c \langle \bar{d}d\rangle(T) e^{-m_c^2/M^2}, \end{aligned} \quad (30)$$

and its mass is obtained as

$$m_{D^*}^2(T) = \frac{\frac{1}{8\pi^2}\int_{m_c^2}^{s_0(T)} ds (s - m_c^2)^2 (2 + \frac{m_c^2}{s}) e^{-s/M^2} - m_c^3 \langle \bar{d}d\rangle(T) e^{-m_c^2/M^2}}{\frac{1}{8\pi^2}\int_{m_c^2}^{s_0(T)} ds \frac{(s - m_c^2)^2}{s}(2 + \frac{m_c^2}{s}) e^{-s/M^2} - m_c \langle \bar{d}d\rangle(T) e^{-m_c^2/M^2}}, \quad (31)$$

where we have ignored the numerically very small contributions of the two-gluon and quark-gluon condensates.

Here, we should stress that the results stated for the masses and the decay constants are valid only in zero-width approximation, and they can not be used in analysis of the temperature-dependent form factors. Therefore, we should consider all hadrons in finite width. For this aim, one should replace the delta function in Eq. (18) by a more complicated function called the Breit-Wigner parametrization [11], i.e.,

$$\delta[s - m_{\mathrm{had}}(T)^2] \to \mathrm{const} \times \frac{1}{[s - m_{\mathrm{had}}(T)^2]^2 + m_{\mathrm{had}}(T)^2 \Gamma_{\mathrm{had}}(T)^2}, \quad (32)$$

showing the hadrons to develop a sizable width $[\Gamma_{\mathrm{had}}(T)]$ at finite temperature (particle absorption in the thermal bath). The const $= \frac{2m_{\mathrm{had}}(T)\Gamma_{\mathrm{had}}(T)}{\pi}$ is obtained if the integration is in the interval $(0 - \infty)$. Using the Eq. (32), the sum rules for the temperature-dependent mass, decay constant, and width is obtained as

$$\frac{2}{\pi} f_{\mathrm{had}}^2(T) m_{\mathrm{had}}(T)^5 \Gamma_{\mathrm{had}}(T) \int_0^\infty ds \frac{1}{[s - m_{\mathrm{had}}(T)^2]^2 + m_{\mathrm{had}}(T)^2 \Gamma_{\mathrm{had}}(T)^2} \times \frac{1}{s - q'^2} = \mathrm{QCD\ side}, \quad (33)$$

where the QCD side is the same as the zero-width approximation case. After applying the Borel transformation with respect to the q'^2, we obtain

$$\frac{2}{\pi} f_{\mathrm{had}}^2(T) m_{\mathrm{had}}(T)^5 \Gamma_{\mathrm{had}}(T) \int_0^\infty ds \frac{1}{[s - m_{\mathrm{had}}(T)^2]^2 + m_{\mathrm{had}}(T)^2 \Gamma_{\mathrm{had}}(T)^2} e^{-s/M''^2} = \boldsymbol{B}_{q'^2}[\mathrm{QCD\ side}], \quad (34)$$

where we have three unknowns, namely, temperature-dependent mass, $m_{\mathrm{had}}(T)$, decay constant, $f_{\mathrm{had}}(T)$, and width $\Gamma_{\mathrm{had}}(T)$. Two find these three unknowns, we need two more relations, which we can get by applying once and twice the derivative with respect to the $-1/M''^2$ to both sides of Eq. (34), i.e.,

$$\frac{2}{\pi} f_{\mathrm{had}}^2(T) m_{\mathrm{had}}(T)^5 \Gamma_{\mathrm{had}}(T) \int_0^\infty ds \frac{s}{[s - m_{\mathrm{had}}(T)^2]^2 + m_{\mathrm{had}}(T)^2 \Gamma_{\mathrm{had}}(T)^2} e^{-s/M''^2} = \frac{-d}{d(1/M''^2)}\{\boldsymbol{B}_{q'^2}[\mathrm{QCD\ side}]\}, \quad (35)$$

$$\frac{2}{\pi} f_{\mathrm{had}}^2(T) m_{\mathrm{had}}(T)^5 \Gamma_{\mathrm{had}}(T) \int_0^\infty ds \frac{s^2}{[s - m_{\mathrm{had}}(T)^2]^2 + m_{\mathrm{had}}(T)^2 \Gamma_{\mathrm{had}}(T)^2} e^{-s/M''^2} = \frac{-d^2}{d(1/M''^2)^2}\{\boldsymbol{B}_{q'^2}[\mathrm{QCD\ side}]\}. \quad (36)$$

TABLE I. Values of some input parameters entering the sum rules.

$\langle\bar{d}d\rangle = -(240 \pm 10)^3$ MeV^3	$m_0^2 = 0.8$ GeV^2
$m_b = 4.7$ GeV	$m_c = 1.23$ GeV
$s_0 = (6.5 \pm 0.5)$ GeV^2	$s'_0 = (35 \pm 2)$ GeV^2
$\alpha_s = 0.1176$	$\langle 0\vert \frac{1}{\pi}\alpha_s G^2\vert 0\rangle = 0.012$ GeV^4
$V_{bc} = (41.2 \pm 1.1) \times 10^{-3}$	$\tau_B = 1.525 \times 10^{-12}$ s

Solving three Eqs. (34)–(36) simultaneously, after lengthy calculations one can obtain the temperature-dependent mass, decay constant, and width for the considered hadrons. We will solve the equations numerically and show the dependency of these parameters on the temperature in the next section.

III. NUMERICAL RESULTS

In this section, we present the dependency of the masses, decay constants, widths, form factors, as well as the branching ratios on temperature obtained from the sum rules. Some input parameters used in the numerical analysis are depicted in Table I. From the explicit expressions for the sum rules it is also clear that they include also four auxiliary parameters, continuum thresholds at zero temperature, s_0 and s'_0, as well as the Borel mass parameters, M^2 and M'^2. The continuum thresholds are not completely arbitrary but they are related to the energy of the first exited states with the same quantum numbers of the interpolating currents. The values of the continuum thresholds s_0 and s'_0 which are in $D(D^*)$ and B channels, respectively, are chosen as shown also in the Table I. The Borel parameters M^2 and M'^2 are mathematical objects, hence the physical observables (masses, decay constants, widths, and form factors) should be practically independent of them. Therefore, we look for working regions for these parameters such that the dependency of sum rules on these parameters are weak. These working regions can be determined requiring that, on one side, the continuum and higher state contributions should be small, and on the other side, the contribution of the operators with the highest dimensions are small, i.e., the sum rules should converge. As a result of the above procedure, the following regions for Borel parameters are obtained: 4 (GeV^2) $\leq M^2 \leq 8$ (GeV^2) and 10 (GeV^2) $\leq M'^2 \leq 20$ (GeV^2).

The dependence of the ratio of the T-dependent masses, decay constants, and widths to their values at zero temperature on $\frac{T}{T_c}$ are depicted in Figs. 1–9. From these figures it is clear that all of masses and decay constants start to diminish with increasing the $\frac{T}{T_c}$. The masses of the pseudoscalar mesons, m_B and m_D as well as the decay constant of the vector meson, f_{D^*} approximately vanish at critical or deconfinement temperature T_c ($\frac{T}{T_c} = 1$). At $T = T_c$, the mass of the vector meson and decay constants of B and D

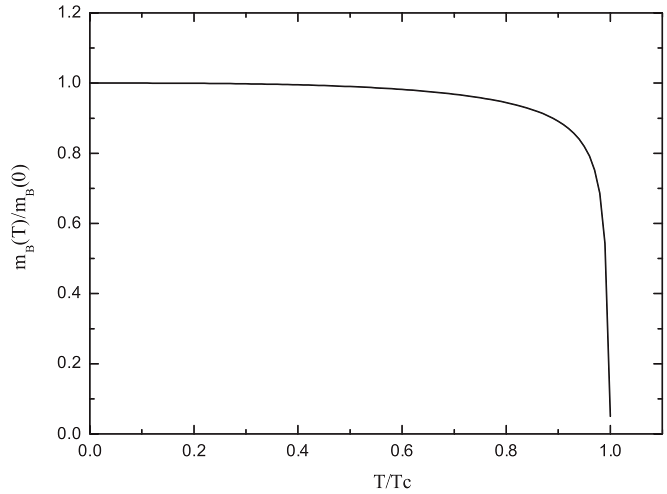

FIG. 1. The dependence of $\frac{m_B(T)}{m_B(0)}$ on $\frac{T}{T_c}$.

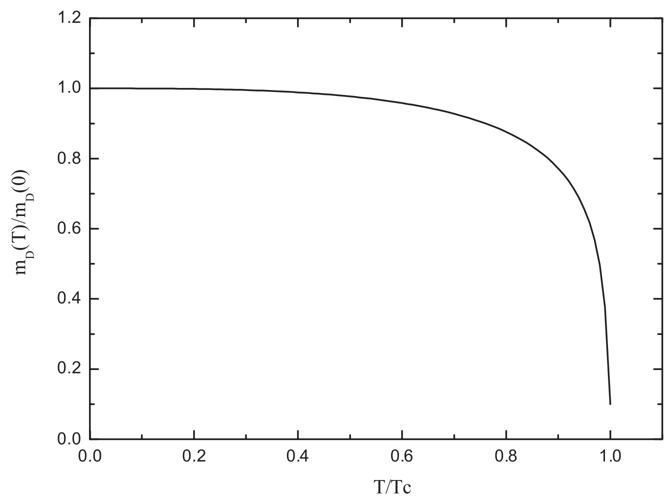

FIG. 2. The dependence of $\frac{m_D(T)}{m_D(0)}$ on $\frac{T}{T_c}$.

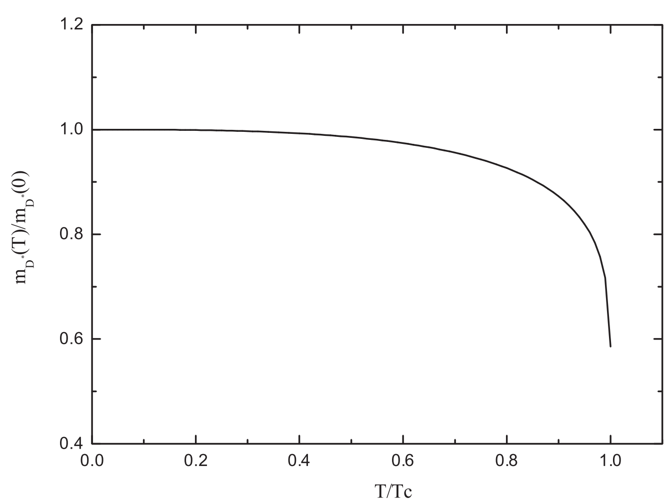

FIG. 3. The dependence of $\frac{m_{D^*}(T)}{m_{D^*}(0)}$ on $\frac{T}{T_c}$.

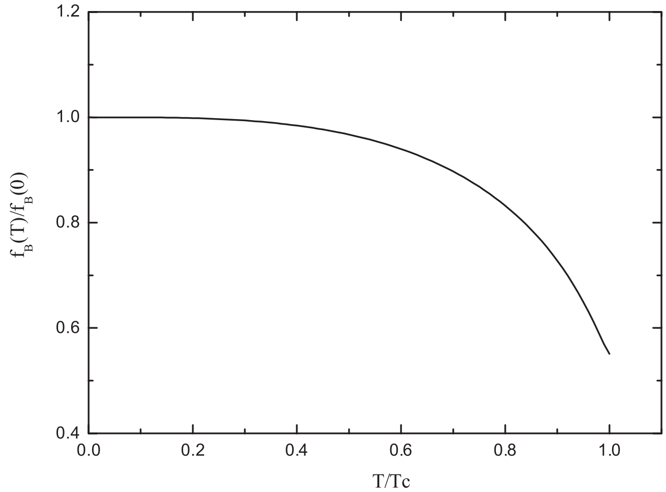

FIG. 4. The dependence of $\frac{f_B(T)}{f_B(0)}$ on $\frac{T}{T_c}$.

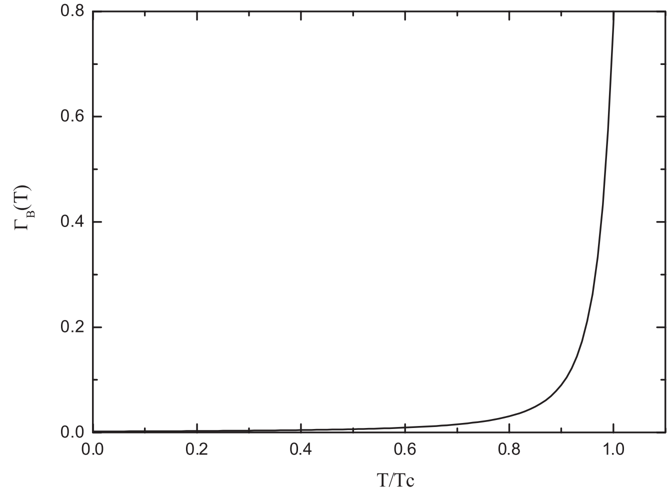

FIG. 7. The dependence of $\Gamma_B(T)$ on $\frac{T}{T_c}$.

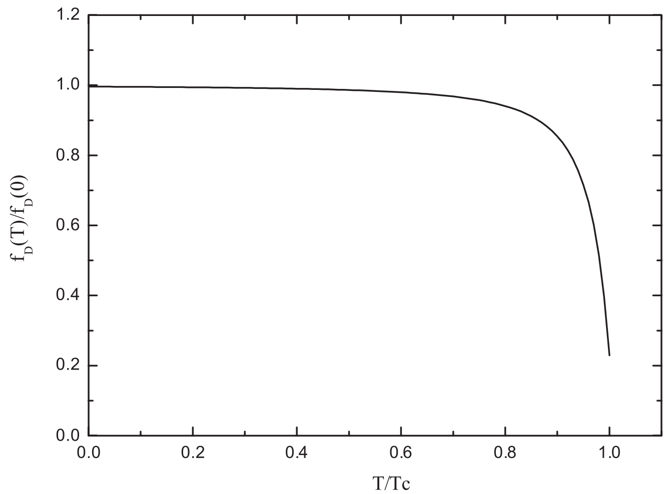

FIG. 5. The dependence of $\frac{f_D(T)}{f_D(0)}$ on $\frac{T}{T_c}$.

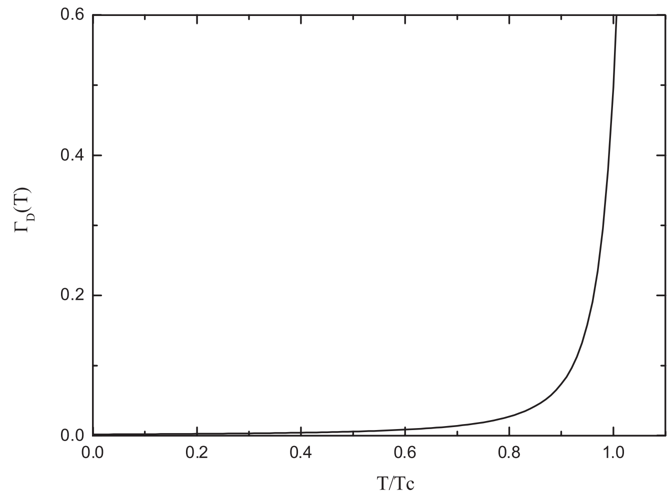

FIG. 8. The dependence of $\Gamma_D(T)$ on $\frac{T}{T_c}$.

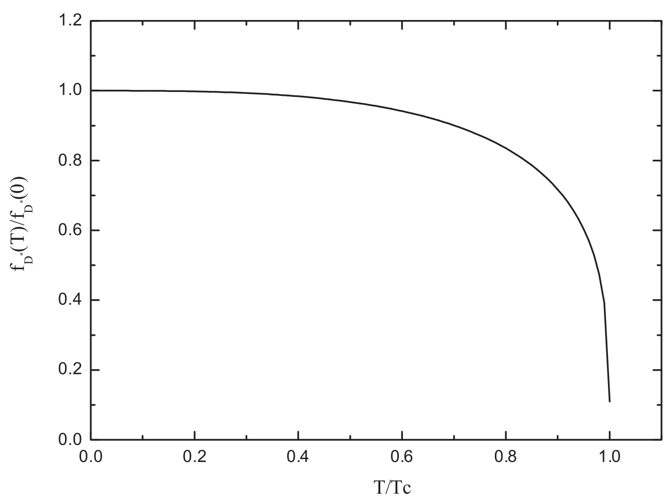

FIG. 6. The dependence of $\frac{f_{D^*}(T)}{f_{D^*}(0)}$ on $\frac{T}{T_c}$.

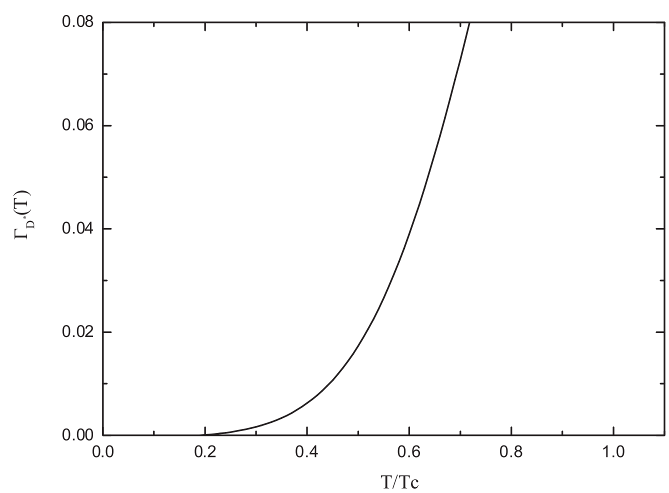

FIG. 9. The dependence of $\Gamma_{D*}(T)$ on $\frac{T}{T_c}$.

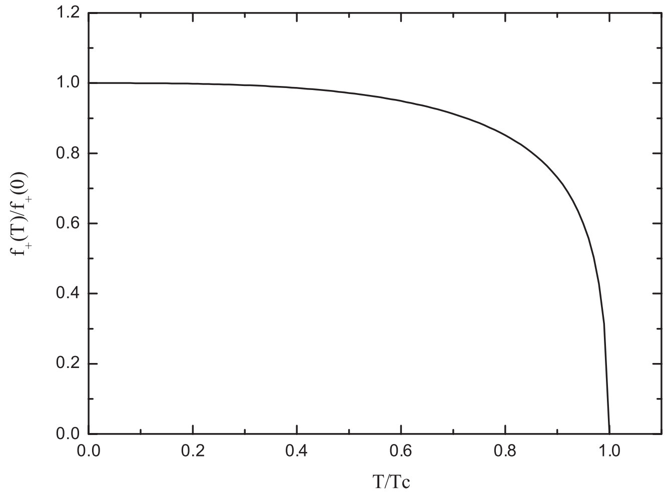

FIG. 10. The dependence of $\frac{f_+(T)}{f_+(0)}$ on $\frac{T}{T_c}$ at $Q^2 = 0$, $M^2 = 6\ \mathrm{GeV}^2$, and $M'^2 = 15\ \mathrm{GeV}^2$.

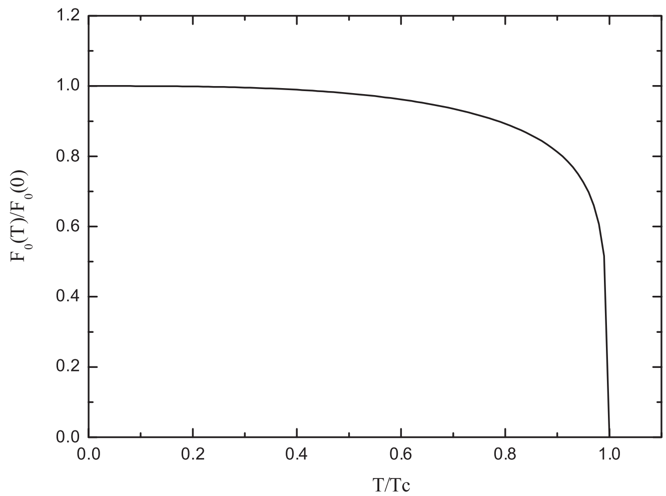

FIG. 11. The dependence of $\frac{F_0(T)}{F_0(0)}$ on $\frac{T}{T_c}$ at $Q^2 = 0$, $M^2 = 6\ \mathrm{GeV}^2$, and $M'^2 = 15\ \mathrm{GeV}^2$.

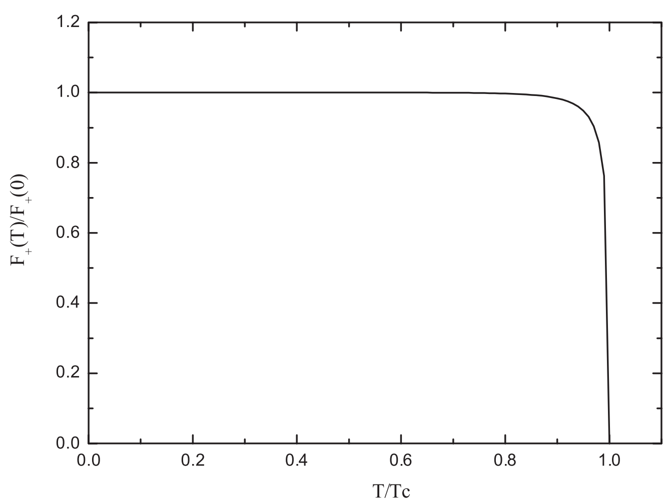

FIG. 12. The dependence of $\frac{F_+(T)}{F_+(0)}$ on $\frac{T}{T_c}$ at $Q^2 = 0$, $M^2 = 6\ \mathrm{GeV}^2$, and $M'^2 = 15\ \mathrm{GeV}^2$.

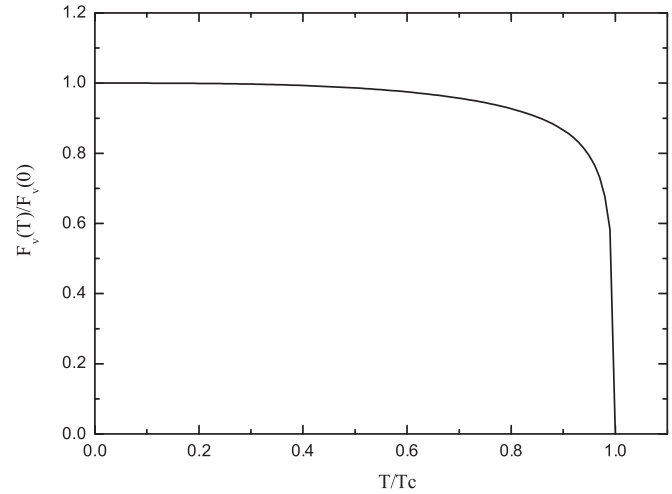

FIG. 13. The dependence of $\frac{F_V(T)}{F_V(0)}$ on $\frac{T}{T_c}$ at $Q^2 = 0$, $M^2 = 6\ \mathrm{GeV}^2$, and $M'^2 = 15\ \mathrm{GeV}^2$.

mesons reach to 58%, 55%, and 24% of their values at zero temperature, respectively. Against the masses and decay constants, the widths grow with increasing temperature. The widths of the pseudoscalar mesons start to grow close to the critical temperature, however, the width of the vector meson starts to grow at $T/T_c \approx 0.5$.

The dependency of the ratio of the form factors to their zero temperature values on $\frac{T}{T_c}$ are presented in Figs. 10–13 at $Q^2 = 0$, as well as $M^2 = 6\ \mathrm{GeV}^2$ and $M'^2 = 15\ \mathrm{GeV}^2$. The form factors f_+, F_0, and F_V show a stability up to $T/T_c \approx (0.4–0.5)$, then they decrease with increasing the temperature and vanish at critical temperature. The form factor F_+, on the other hand, remains unchanged up to very close to the critical temperature; however, suddenly start to decrease and vanish at T_c. Finally, Figs. 14 and 15 show the dependency of the branching ratios of the $B \to De\nu_e$ and $B \to D^*e\nu_e$ channels on the temperature. From these figures, we see that the branching fractions remain unchanged

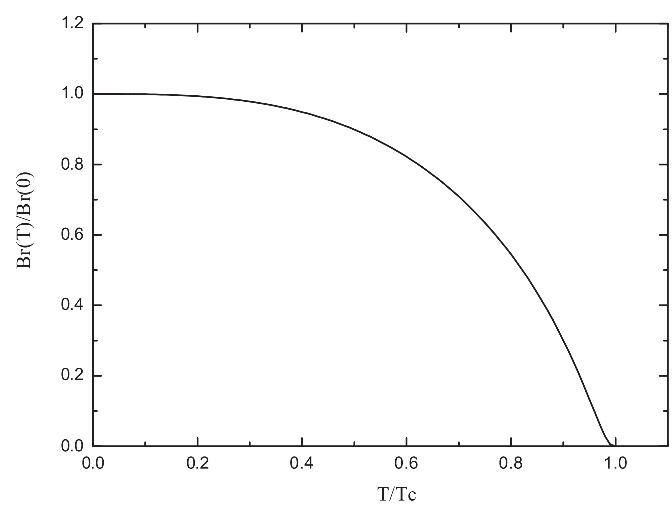

FIG. 14. The dependence of $\frac{\mathrm{Br}(B\to De\nu_e)(T)}{\mathrm{Br}(B\to De\nu_e)(0)}$ on $\frac{T}{T_c}$.

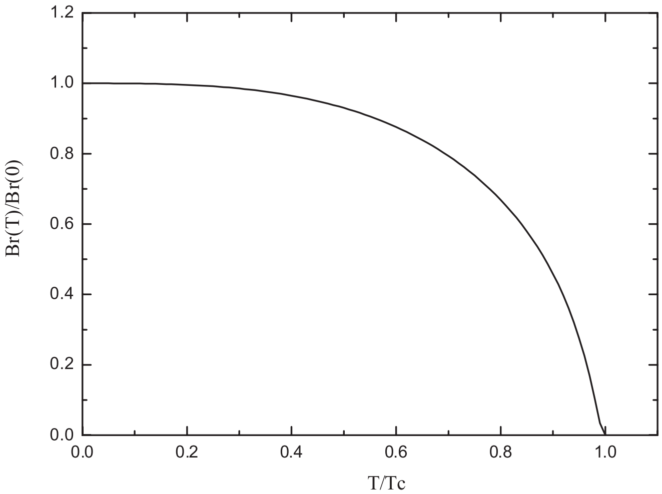

FIG. 15. The dependence of $\frac{\mathrm{Br}(B\to D^* e\nu_e)(T)}{\mathrm{Br}(B\to D^* e\nu_e)(0)}$ on $\frac{T}{T_c}$.

up to $T/T_c \approx 0.3$, however, they start to diminish with increasing temperature after this region and vanish at the critical or deconfinement temperature.

At the end of this section, we collect the values of all observables (masses, decay constants, form factors, as well as the branching fractions at $T = 0$) in Table II. This table shows a good consistency with the experimental values [31] for the masses, decay constants, and the branching ratios. Our results can be checked in real experiments at finite temperature at the LHC in the near future.

TABLE II. Values of observables (masses, decay constants, form factors, and branching ratios) at $T = 0$.

$m_B = (5.28 \pm 0.22)$ GeV
$m_D = (1.87 \pm 0.07)$ GeV
$m_{D^*} = (2.10 \pm 0.09)$ GeV
$f_B = (0.175 \pm 0.031)$ GeV
$f_D = (0.223 \pm 0.045)$ GeV
$f_{D^*} = (0.238 \pm 0.052)$ GeV
$f_+ = 0.375 \pm 0.105$
$F_0 = (3.486 \pm 1.04)$ GeV
$F_+ = (0.056 \pm 0.016)\ \mathrm{GeV}^{-1}$
$F_V = (0.082 \pm 0.025)\ \mathrm{GeV}^{-1}$
$\mathrm{Br}(B \to D e\nu_e) = (2.06 \pm 0.62) \times 10^{-2}$
$\mathrm{Br}(B \to D^* e\nu_e) = (6.00 \pm 1.85) \times 10^{-2}$

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