PHYSICAL REVIEW D 81, 095011 (2010)

Constraining scalar leptoquarks from the K and B sectors

Jyoti Prasad Saha,^{1[,*](#page-0-0)} Basudha Misra,^{2[,†](#page-0-1)} and Anirban Kundu^{3[,‡](#page-0-2)}

¹Department of Physics, Serampore College, Serampore 712201, India²Institute of Mathematical Sciences, Channel 600113, India and Centre for High Ex

²Institute of Mathematical Sciences, Chennai 600113, India and Centre for High Energy Physics,

Indian Institute of Science, Bangalore 560012, India ³

 3 Department of Physics, University of Calcutta, 92 A.P.C. Road, Kolkata 700009, India

(Received 10 March 2010; published 18 May 2010)

Upper bounds at the weak scale are obtained for all $\lambda_{ij}\lambda_{im}$ type product couplings of the scalar leptoquark model which may affect $K^0 - \bar{K}^0$, $B_d - \bar{B}_d$, and $B_s - \bar{B}_s$ mixing, as well as leptonic and semilentonic K and B decays. Constraints are obtained for both real and imaginary parts of the couplings semileptonic K and B decays. Constraints are obtained for both real and imaginary parts of the couplings. We also discuss the role of leptoquarks in explaining the anomalously large CP-violating phase in B_s – \bar{B}_s mixing.

DOI: [10.1103/PhysRevD.81.095011](http://dx.doi.org/10.1103/PhysRevD.81.095011) PACS numbers: 14.80.Sv, 13.25.Hw, 14.40.Nd

I. INTRODUCTION

The standard model (SM), in all probability, is just an effective theory valid up to a scale which is much below the Planck scale, and hopefully in the range of a few hundreds of GeV, so that the physics beyond SM can be explored at the LHC. Direct production of new particles will definitely signal new physics (NP); while it is an interesting problem to find out what type of NP is there (commonly known as the ''inverse problem''), it is also well known that indirect data from low-energy experiments will help to pin down the exact structure of NP, including its flavor sector. The low-energy data, in particular, the data coming from the B factories as well as from CDF, DØ, LHCb (and also from the general purpose ATLAS and CMS experiments) are going to play a crucial role in that. There are already some interesting hints; just to name a few [[1\]](#page-10-0): (i) the large mixing phase in $B_s - \bar{B}_s$ mixing; (ii) the fraction of longitudinally
polarized final states in channels like $B \to \phi K^*$ and $B \to$ polarized final states in channels like $B \to \phi K^*$ and $B \to \phi K^*$. (iii) the anomalous direct CP-asymmetries in $B \to \phi K^*$. ρK^* ; (iii) the anomalous direct CP-asymmetries in $B \to \pi K$ decays: (iv) the discrepancy in the extracted value of πK decays; (iv) the discrepancy in the extracted value of $\sin(2\beta)$ from $B_d \to J/\psi K_S$ and $B_d \to \phi K_S$; (v) the larger
branching fraction of $R^+ \to \tau^+ \nu$ compared to the SM branching fraction of $B^+ \rightarrow \tau^+ \nu$ compared to the SM expectation; and (vi) the discrepancy in the extracted values of V_{ub} from inclusive and exclusive modes. While none of them are conclusive proof of any NP, there is a serious tension with the SM when all the data are taken together. If all, or most, of them survive the test of time and attain more significance, this will indicate a new physics whose flavor sector is definitely of the nonminimal flavor violating (NMFV) type.

In this paper, as an example of an NMFV new physics, we focus upon the model of scalar leptoquarks (LQ). In general, as in any NMFV model, we expect possibly large deviations from the SM in the flavor sector observables. LQs that violate both baryon number B and lepton number L must be massive at the level of $\sim 10^{15}$ GeV to avoid proton decay, and are of no interest to us. (There are exceptions; one can construct models where LQs violate both B and L and yet do not mediate proton decay. These LQs can be light. For example, see [[2\]](#page-10-1).) On the other hand, LQs conserving either B or L or both can be light, $\mathcal{O}(100 \text{ GeV})$, and we discuss the phenomenology of only those models that conserve both B and L ; one can find extensive discussions on these models in [[3](#page-10-2)[–5](#page-10-3)]. Vector LQs, as well as some gauge-nonsinglet scalar ones, couple to neutrinos, and their couplings should be very tightly constrained from neutrino mass and mixing data.

Another phenomenological motivation for a LQ model is that this is one of the very few models (R-parity violating supersymmetry is another) where the neutral meson mixing diagram gets a new contribution to the absorptive part. This, for example, may lead to an enhancement in the width difference $\Delta \Gamma$ in the B_s system [\[6\]](#page-10-4), contrary to what hannens in more nopular NP models that can only what happens in more popular NP models that can only decrease $\Delta \Gamma$ [[7\]](#page-10-5). The NP also changes the *CP*-violating
phase in $B \to I/\psi \phi$ and hence can belp reducing the phase in $B_s \rightarrow J/\psi \phi$ and hence can help reducing the tension [8] of SM expectation and the Tevatron data on tension [[8](#page-10-6)] of SM expectation and the Tevatron data on the CP-violating phase and width difference for B_s .

All flavor-changing observables constrain the product of at least two different LQ couplings, one linked with the parent flavor and another with the daughter flavor. The product couplings may be complex and it is generally impossible to absorb the phase just by a simple redefinition of the LQ field. We use the data from $K^0 - \bar{K}^0$, $B_s - \bar{B}_s$,
and $B_t - \bar{B}_s$, mixing to constrain the relevant product and $B_d - \bar{B}_d$ mixing to constrain the relevant product
couplings, generically denoted as $\lambda \lambda$. For the B system couplings, generically denoted as $\lambda \lambda$. For the B system, we use the data on $\Delta M_{d,s}$ and the mixing phase $\sin(2\beta_{d,s})$,
and for the K system, we use ΔM_{u} and s.g. We do not and for the K system, we use ΔM_K and ε_K . We do not discuss other CP violating parameters like ε'/ε since that discuss other CP violating parameters like ε'/ε , since that has large theoretical uncertainties. We also discuss the correlated leptonic and semileptonic decays, i.e., the decays mediated by the same LQ couplings. While decays to most of the semileptonic channels have been observed, the clean leptonic channels only have an upper bound for

[^{*}j](#page-0-3)yotiprasadsaha@gmail.com

[[†]](#page-0-3) basudha@imsc.res.in

[[‡]](#page-0-3) akphy@caluniv.ac.in

almost all the cases, except the already observed leptonic decays $K_L \rightarrow e^+e^-$, $\mu^+\mu^-$. Note that the final state must have leptons and hence the bounds are more robust compared to those coming from models with only hadrons in the final state.

A similar exercise have also been undertaken in [[3,](#page-10-2)[9–](#page-10-7) [12](#page-10-8)]. We update these bounds with new data from the B factories and other collider experiments. In particular, in the subsequent sections, the previous bounds that we quote have been taken from [\[3](#page-10-2),[4](#page-10-9)]. The $D^0 - \bar{D}^0$ system has not been considered due to the large theoretical uncertainties and dominance of long-distance contributions. We refer the reader to [\[13\]](#page-10-10) for a discussion on the bounds coming from $D^0 - \bar{D}^0$ mixing. Leptonic and semileptonic D and D_s decays have been used to put constraints on LQs that couple to the up-type quarks. In particular, LQ contribution might be interesting to explain the D_s leptonic decay anomaly [\[2](#page-10-1),[14](#page-10-11)[,15](#page-10-12)]. The couplings that we constrain are generically of the type $\lambda_{ij} \lambda_{ik}^*$, where the kth quark flavor changes to the jth, but there is no flavor change in the lepton sector. One can, in principle, consider flavor changes in the lepton sector too; for an analysis of that type of processes, see [[16](#page-10-13)]. However, if one has a $\nu\bar{\nu}$ pair in the final state, as in $K_L \rightarrow \pi^0 \nu \bar{\nu}$, there is a chance that lepton flavor is also violated.

The couplings, which are in general complex, may be constrained from a combined study of CP-conserving and CP-violating observables. For neutral mesons, these mean $\sum_{i=1}^{n}$ and ΔM as well as ϵ_K and sin $(2\beta_{d,s})$. However, for most of the vide the better bound. The analysis has been done keeping both the SM and LQ contributions, which keeps the possibility of a destructive intereference, and hence larger possible values of the LQ amplitudes, open.

In Sec. II we briefly state the relevant formulae necessary for the analysis. Sec. III deals with the numerical inputs. In Sec. IV, we take up the analysis, first of the neutral meson mixing, and then the correlated leptonic and semileptonic decays. We conclude and summarize in Sec. V.

II. RELEVANT EXPRESSIONS

A. Neutral meson mixing

For the neutral meson system generically denoted by M^0 and \overline{M}^0 , the mass difference between the two mass eigenstates ΔM is given by

$$
\Delta M = 2 \operatorname{Re} \left[\left(M_{12} - \frac{i}{2} \Gamma_{12} \right) \left(M_{12}^* - \frac{i}{2} \Gamma_{12}^* \right) \right]^{1/2},
$$

\n
$$
\Delta \Gamma = -4 \operatorname{Im} \left[\left(M_{12} - \frac{i}{2} \Gamma_{12} \right) \left(M_{12}^* - \frac{i}{2} \Gamma_{12}^* \right) \right]^{1/2}.
$$
 (1)

For the B system, $|M_{12}| \gg |\Gamma_{12}|$ and $\Delta M = 2|M_{12}|$. For
the K system if the decay is dominantly to the $I = 0$ the K system, if the decay is dominantly to the $I = 0$, $Im\Gamma_{12}$ can be neglected and one can write

$$
|\varepsilon_K| = \frac{1}{2\sqrt{2}} \frac{\text{Im} M_{12}}{\text{Re} M_{12}} = \frac{1}{\sqrt{2}} \frac{\text{Im} M_{12}}{\Delta M_K}.
$$
 (2)

Let the SM amplitude be

$$
M_{12}^{\rm SM} = |M_{12}^{\rm SM}| \exp(-2i\theta_{\rm SM}), \tag{3}
$$

where $\theta_{SM} = \beta_d$ for the $B_d - B_d$ system and approximately zero for $K^0 - \bar{K}^0$ and $B - \bar{B}$ systems mately zero for $K^0 - \bar{K}^0$ and $B_s - \bar{B}_s$ systems.
If we have *n* number of new physics (NP)

If we have *n* number of new physics (NP) amplitudes with weak phases θ_n , one can write

$$
M_{12} = |M_{12}^{SM}| \exp(-2i\theta_{SM}) + \sum_{i=1}^{n} |M_{12}^{i}| \exp(-2i\theta_{i}).
$$
 (4)

This immediately gives the effective mixing phase θ_{eff} as

$$
\theta_{eff} = \frac{1}{2} \arctan \frac{|M_{12}^{SM}| \sin(2\theta_{SM}) + \sum_{i} |M_{12}^{i}| \sin(2\theta_{i})}{|M_{12}^{SM}| \cos(2\theta_{SM}) + \sum_{i} |M_{12}^{i}| \cos(2\theta_{i})},
$$
\n(5)

and the mass difference between mass eigenstates as

$$
\Delta M = 2[|M_{12}^{SM}|^2 + \sum_{i} |M_{12}^{i}|^2
$$

+ 2|M_{12}^{SM}| $\sum_{i} |M_{12}^{i}| \cos 2(\theta_{SM} - \theta_{i})$
+ 2 $\sum_{i} \sum_{j>i} |M_{12}^{j}| |M_{12}^{i}| \cos 2(\theta_{j} - \theta_{i})]^{1/2}$. (6)

For the $K^0 - \bar{K}^0$ system, the dominant part of the shortdistance SM amplitude is

$$
M_{12}^{SM} = \frac{\langle \bar{K}^0 | H_{eff} | K^0 \rangle}{2m_K}
$$

$$
\approx \frac{G_F^2}{6\pi^2} (V_{cd} V_{cs}^*)^2 \eta_K m_K f_K^2 B_K m_W^2 S_0(x_c), \quad (7)
$$

where $x_j = m_j^2/m_W^2$, f_K is the K meson decay constant, and η_K (also called η_{cc} in the literature) and B_K parametrize the short- and the long-distance QCD corrections, respectively. The top-quark loop dependent part, which is tiny due to the CKM suppression, but responsible for CP violation, has been neglected. The function S_0 is given by

$$
S_0(x) = \frac{4x - 11x^2 + x^3}{4(1 - x)^2} - \frac{3x^3 \ln x}{2(1 - x)^3}.
$$
 (8)

For the $B_q^0 - \bar{B}_q^{q_0}$ system $(q = d \text{ for } B_d - \bar{B}_d \text{ and } q = s \text{ for } B_q - \bar{B}_q$ $B_s - \bar{B}_s$), we have an analogous equation, dominated by the top quark loop: the top quark loop:

$$
M_{12}^{SM} = \frac{\langle \bar{B}_{q}^{0} | H_{eff} | B_{q}^{0} \rangle}{2m_{B_{q}}} = \frac{G_{F}^{2}}{6\pi^{2}} (V_{tq} V_{tb}^{*})^{2} \eta_{B_{q}} m_{B_{q}} f_{B_{q}}^{2} B_{B_{q}} m_{W}^{2} S_{0}(x_{t}).
$$
 (9)

B. Leptonic and semileptonic decays

For almost all the cases, the SM leptonic decay widths for neutral mesons are way too small to be taken into account, and we can safely saturate the present bound with the NP amplitude alone, except for the K_L sector. For example, the branching ratio of $B_s \to \mu^+ \mu^-$ is about 3.4×10^{-9} and that of $B_d \rightarrow \mu^+ \mu^-$ is about 1.0×10^{-10}
in the SM, while the experimental limits are at the ballpark of $4-6 \times 10^{-8}$. Another exception is the $B^- \rightarrow l^- \bar{\nu}$ decay, which proceeds through the annihilation channel in the which proceeds through the annihilation channel in the

SM:

$$
Br(B^{-} \to l^{-} \bar{\nu}) = \frac{1}{8\pi} G_F^2 m_B m_l^2 f_B^2 |V_{ub}|^2 \tau_B \left(1 - \frac{m_l^2}{m_B^2}\right)^2,
$$
\n(10)

where τ_B is the lifetime of the B meson.

For the semileptonic decays, we use the following stan-dard convention [[17](#page-10-14)], given for the $B \to K^{(*)}\ell^+\ell^-$ transition: tion:

$$
\langle K(p_2)|\bar{b}\gamma_\mu s|B(p_1)\rangle = P_\mu F_1(q^2) + q_\mu \frac{m_B^2 - m_K^2}{q^2} (F_0(q^2) - F_1(q^2)),
$$

$$
\langle K^*(p_2, \epsilon)|\bar{b}\gamma_\mu (1 \mp \gamma_5)s|B(p_1)\rangle = \mp i q_\mu \frac{2m_{K^*}}{q^2} \epsilon^* \cdot q[A_3(q^2) - A_0(q^2)] \pm i \epsilon_\mu^*(m_B + m_{K^*})A_1(q^2)
$$

$$
\mp \frac{i}{m_B + m_{K^*}} P_\mu(\epsilon^* \cdot q)A_2(q^2) - \epsilon_{\mu\nu\alpha\beta} \epsilon^{*\nu} p_2^{\alpha} q^{\beta} \frac{2V(q^2)}{m_B + m_{K^*}},
$$
(11)

where $P = p_1 + p_2$, and $q = p_1 - p_2$. The pole dominance ensures that $A_3(0) = A_0(0)$, and $A_3(q^2)$ can be expressed in terms of A_1 and A_2 .

C. Leptoquarks

Leptoquarks are color-triplet bosons that can couple to a quark and a lepton at the same time, and can occur in a number of grand unified theories (GUTs) [\[18\]](#page-10-15), composite models [\[19\]](#page-10-16), and superstring-inspired E_6 models [[9\]](#page-10-7). In fact, the R-parity violating squarks of supersymmetry, as far as their couplings with fermions go, are nothing but LQs. Model-independent constraints on their properties are available [\[3](#page-10-2)–[5\]](#page-10-3).

We focus on the scalar LQ model, which conserves both B and L. The relevant part of the Lagrangian [\[3](#page-10-2)] can be written as

$$
\mathcal{L}_{S} = \{ (\lambda_{LS_{0}} \bar{q}_{L}^{c} i \sigma_{2} l_{L} + \lambda_{RS_{0}} \bar{u}_{R}^{c} e_{R}) S_{0}^{\dagger} + \lambda_{R \bar{S}_{0}} \bar{d}_{R}^{c} e_{R} \bar{S}_{0}^{\dagger} \n+ (\lambda_{LS_{1/2}} \bar{u}_{R} l_{L} + \lambda_{RS_{1/2}} \bar{q}_{L} i \sigma_{2} e_{R}) S_{1/2}^{\dagger} \n+ \lambda_{L \bar{S}_{1/2}} \bar{d}_{R} l_{L} \bar{S}_{1/2}^{\dagger} + \lambda_{LS_{1}} \bar{q}_{L}^{c} i \sigma_{2} \sigma^{a} l_{L} \cdot S_{1}^{a\dagger} \} + \text{H.c.}
$$
\n(12)

where (S_0, \tilde{S}_0) , $(S_{1/2}, \tilde{S}_{1/2})$, and S_1^a $(a = 1, 2, 3)$ represent
the *SU(2)* singlet doublet and triplet I Os respectively the $SU(2)$ singlet, doublet, and triplet LQs, respectively. λ^{ij} is the coupling strength of a leptoquark to an ith generation lepton and a jth generation quark, which is in general complex. σ 's are the Pauli spin matrices. Note that all the four terms that couple a neutrino with a LQ can have potential constraints on neutrino mass and mixing. For example, $\tilde{S}_{1/2}$ can generate the observed neutrino mixing pattern through a type-II seesaw mechanism [\[20,](#page-10-17)[21\]](#page-10-18). However, the constraints also depend on the vacuum expectation value of a higher-representation scalar field. That is why we show the non-neutrino constraints for these couplings too, keeping in mind that the neutrino constraints may be stronger.

In this work, we focus only on those processes that involve down-type quarks. Thus, there is no way to constrain λ_{RS_0} and $\lambda_{LS_1/2}$ from these processes. In fact, these two sets of coupling can be constrained from processes like $D^0 - \bar{D}^0$ mixing and $\ell_i \rightarrow \ell_i + \gamma$. The latter can be constrained from neutrino mixing too, but as we have just mentioned, the limits would depend on other model parameters. We constrain only five types of scalar LQ couplings here: λ_{LS_0} , $\lambda_{R\tilde{S}_0}$, $\lambda_{R S_{1/2}}$, $\lambda_{L\tilde{S}_{1/2}}$, and λ_{LS_1} .

While we have not explicitly shown the generation indices in Eq. ([12](#page-2-0)), it is assumed that the LQs can couple with fermions from two different generations. There is another approach which we should mention here. In this approach [\[4,](#page-10-9)[5](#page-10-3)], one takes the LQ coupling for a single generation, say the third, so that only the third generation fermions are affected. However, these couplings are taken to be in the weak basis, and when one rotates to the mass basis of the quarks, the cross-generational couplings are generated with a mechanism similar to that of the Cabibbo-Kobayashi-Maskawa (CKM) mixing. This controls the relative magnitudes, as well as the phases, of the LQ couplings, and given a texture in the weak basis, all the couplings in the mass basis are present, with predicted magnitudes and phases (the LQ coupling phase also comes from the CKM phase).

While one gets comparable bounds to that of [\[3](#page-10-2)] in this scheme as well, one should note that: (i) The mixing matrix for the down-type quarks is not known. What one knows is the misalignment between the u_L and the d_L bases. Thus, one is forced to consider only the charged-current processes where the misalignment (and not the individual rotation matrices) matters. (ii) The CKM scheme does

not say anything about the rotation matrices for the rightchiral quark sector. Whatever one uses there is at best an assumption. A similar analysis has been done for R-parity violating supersymmetric models too [[22\]](#page-10-19).

Thus, we will assume that whatever couplings are nonzero, are so in the physical basis of the quark fields, and the phase is arbitrary and not a function of the CKM parameters.

D. Direct production limits

The direct production limits depend on the LQ model, as well as the SM fermions these LQs can couple to. The best limits are as follows [\[23\]](#page-10-20): $m_{LQ} > 256$, 316, 229 GeV for 1st, 2nd, and 3rd generation LQs, respectively, when they are pair produced, and $m_{LQ} > 298$, 73 GeV for 1st and 2nd generation LQs when there is single production. These are the absolute lower bounds at 95% CL, but the constraints are tighter if the LQ coupling to the SM fermions, denoted here by λ , is large. For example, if λ is of the order of the electromagnetic coupling, the first generation LQs were found to have a limit of 275–325 GeV by both the HERA experiments [[24](#page-10-21)]. For even larger couplings ($\lambda = 0.2{\text -}0.5$) and above) the LEP experiments exclude a much wider mass range [\[25\]](#page-10-22), but such strong couplings are already almost ruled out if LQ signals are to be observed at the LHC.

The present-day limits for pair-produced LQs are due to the Tevatron experiments. For a summary, we refer the reader to [\[26\]](#page-10-23). The first generation LQs are searched in $2e + 2j$ or $1e + 2j + \text{MET}$ channel; the second generation ones are in $2\mu + 2j$ or $1\mu + 2j + \text{MET}$ channel; and the third generation ones are in $2\tau + 2b$ or $2b + \text{MET}$ channel. We refer to the original papers [[27](#page-10-24)] for the details of the analysis.

The production of LQ states, either single, associated with a lepton (from $qg \rightarrow LQ + \ell$), or in pair, from $q\bar{q}$, $gg \to LQ + \overline{LQ}$, has been studied in detail; for example, the reader may look at [\[21](#page-10-18)[,28–](#page-10-25)[30](#page-10-26)]. At $\sqrt{s} = 14$ TeV, the cross-section of pair production of scalar leptoquarks is cross-section of pair production of scalar leptoquarks is about 1 pb [\[30\]](#page-10-26). While this goes down significantly for the initial LHC run of $\sqrt{s} = 7$ TeV, one expects to see LQ
signals upto $m_{\text{ice}} = 500$ GeV even with 5 fb⁻¹ of lumisignals upto $m_{\text{LO}} = 500 \text{ GeV}$ even with 5 fb⁻¹ of luminosity. The cross-section for single production depends on the value of λ . For $\lambda = \sqrt{4\pi\alpha}$, the electric charge, the cross-section for LO plus charged lepton production is cross-section for LQ plus charged lepton production is about 100 fb for $m_{\text{LO}} = 500$ GeV. The cross-section is proportional to λ^2 , so we expect events even with \sqrt{s} = 7 TeV and $\lambda \approx 0.05$ Obviously for smaller values of λ 7 TeV and $\lambda \approx 0.05$. Obviously, for smaller values of λ , pair production will be more favored, and we expect the preliminary run of LHC to establish a limit of the order of 500 GeV.

In this analysis, we will use a somewhat conservative reference mass value of 300 GeV for every LQ state, independent of the quantum numbers and generation. The bounds on the product couplings scale as m_{LQ}^2 , so the bounds that we show should be multiplied by $(300/m_{\rm LQ})^2$.

E. Constraints from meson mixing

Consider the neutral meson $M^0 \equiv q_i \bar{q}_k$. The oscillation can have a new LQ-mediated amplitude, with i-type leptons and some scalar LQs in the box, as shown in Fig. [1](#page-3-0). The amplitude is proportional to $(\lambda_{ik}^* \lambda_{ij})^2$. We consider, as
in the standard practice, a bierarchical coupling scheme, so in the standard practice, a hierarchical coupling scheme, so that we may consider only two LQ couplings to be nonzero at the most. Also, we consider any one type of LQ to be present at the same time. This keeps the discussion simple and the numerical results easily tractable; however, this may not be the case where we have some high-energy texture of the couplings and there can be a number of nonzero couplings at the weak scale.

For the LQ box, one must consider the same type of lepton flowing inside the box if we wish to restrict the number of LQ couplings to 2. The effective Hamiltonian contains the operator \tilde{O}_1 , defined as

$$
\tilde{O}_1 = [\bar{b}\gamma^\mu P_R d]_1 [\bar{b}\gamma_\mu P_R d]_1,\tag{13}
$$

(where the subscript 1 indicates the $SU(3)_c$ singlet nature of the current), and is given by

$$
\mathcal{H}_{\text{LQ}} = \frac{(\lambda_{ik}^* \lambda_{iq})^2}{128 \pi^2} \left[\frac{c_1}{m_{\text{LQ}}^2} \left\{ I \left(\frac{m_l^2}{m_{\text{LQ}}^2} \right) \right\} + \frac{c_2}{m_{\text{LQ}}^2} \right] \tilde{O}_1, \quad (14)
$$

where $c_1 = 1$, $c_2 = 0$ for S_0 , \tilde{S}_0 and $S_{1/2}$, $c_1 = c_2 = 1$ for $S_{1/2}$, and $c_1 = 4$, $c_2 = 1$ for S_1 . Therefore, if we are allowed to neglect the SM, the limits on the product allowed to neglect the SM, the limits on the product couplings for $(\lambda_{LS_0}, \lambda_{RS_0}, \lambda_{RS_1/2}), \lambda_{LS_1/2},$ and λ_{LS_1} should be at the ratio of $1: \frac{1}{\sqrt{2}}: \frac{1}{\sqrt{5}}$. The operator \tilde{O}_1 is multiplicatively renormalized and the LQ couplings are those obtained at the weak scale. The function $I(x)$, defined as

$$
I(x) = \frac{1 - x^2 + 2x \log x}{(1 - x)^3},
$$
 (15)

is always very close to $I(0) = 1$; note that we have taken all LQs to be degenerate at 300 GeV.

F. Constraints from leptonic and semileptonic decays

The LQ couplings which may contribute to $K^0 - \bar{K}^0$, $B_d - \bar{B}_d$ and $B_s - \bar{B}_s$ mixing should also affect various
LO-mediated semilentonic $(b \rightarrow d(s)l^+l^- \rightarrow \rightarrow dl^+l^-)$ LQ-mediated semileptonic $(b \rightarrow d(s)l^+l^-$, $s \rightarrow dl^+l^-$)

FIG. 1. Leptoquark contributions to $M^0 - \bar{M}^0$ mixing.

TABLE I. Effective four-fermion operators for scalar leptoquarks. G generically stands for $\lambda^2/m_{\text{LO}}^2$.

Interaction	4-fermion vertex	Fierz-transformed vertex
$(\lambda_{LS_0}\bar{q}_L^c i\sigma_2 l_L + \lambda_{RS_0}\bar{u}_R^c e_R)S_0^{\dagger}$ $\lambda_{R\tilde{S}_o} \overline{d}_R^c e_R \tilde{S}_0^{\dagger}$ $(\lambda_{LS_{1/2}}\bar{u}_{R}l_{L} + \lambda_{RS_{1/2}}\bar{q}_{L}i\sigma_{2}e_{R})S_{1/2}^{\dagger}$ $\lambda_{L\tilde{S}_{1/2}}\bar{d}_{R}l_{L}\tilde{S}_{1/2}^{\dagger}$ $\lambda_{LS_1}\bar{q}_L^c i\sigma_2\vec{\sigma}l_L\cdot\vec{S}_1^\dagger$	$G(\bar{d}_L^c \nu_L)(\bar{\nu}_L d_L^c)$ $G(\bar{d}_R^c e_R)(\bar{e}_R d_R^c)$ $G(d_L e_R)(\bar{e}_R d_L)$ $G(\bar{d}_R \nu_L)(\bar{\nu}_L d_R)$ $G(d_Re_L)(\bar{e}_Ld_R)$ $G(\bar{\nu}_L d_L^c)$	$\frac{1}{2}G(\bar{d}_{L}^{c}\gamma^{\mu}d_{L}^{c})(\bar{\nu}_{L}\gamma_{\mu}\nu_{L})$ $\frac{1}{2}G(\bar{d}_R^c\gamma^{\mu}d_R^c)(\bar{e}_R\gamma_{\mu}e_R)$ $\frac{1}{2}G(\bar{d}_L\gamma_\mu d_L)(\bar{e}_R\gamma_\mu e_R)$ $\frac{1}{2}G(\bar{d}_R\gamma^{\mu}d_R)(\bar{\nu}_L\gamma_{\mu}\nu_L)$ $\frac{1}{2}G(\bar{d}_R\gamma^{\mu}d_R)(\bar{e}_L\gamma_{\mu}e_L)$ $\frac{1}{2}G(\bar{d}_L^c\gamma^{\mu}d_L^c)(\bar{\nu}_L\gamma_{\mu}\nu_L)$
	$2G(\bar{d}_L^c e_L)(\bar{e}_L d_L^c)$	$G(\bar{d}_L^c \gamma^{\mu} d_L^c)(\bar{e}_L \gamma_{\mu} e_L)$

and purely leptonic $(B_{d(s)}^0 \to l^+l^-$, $K^0 \to l^+l^-$) decays.
The estimated PPs of laptonic flavor conserving $\Lambda P(S)$ The estimated BRs of leptonic flavor conserving $\Delta B(S) = 1$ processes within SM are very small compared to their 1 processes within SM are very small compared to their experimental numbers or upper bounds, except for $K_L \rightarrow$ e^+e^- , $\mu^+\mu^-$. Therefore, it is quite reasonable to ignore the SM effects for these channels while constraining the LQ couplings. For these mixing correlated decays, the final state leptons must be of the same flavor. The leptonic decay modes are theoretically clean and free from any hadronic uncertainties. The semileptonic modes have the usual form-factor uncertainties, and the SM contribution cannot be neglected here.

To construct four-fermion operators from λ type couplings which mediate leptonic and semileptonic B and K

TABLE II. Branching ratios for some leptonic decays of K and B mesons [\[23\]](#page-10-20). The limits are at 90% confidence level. The SM expectation is negligible.

Mode	Branching ratio Mode	Branching ratio
	$K_s \to e^+e^- \quad \langle 9 \times 10^{-9} \quad K_s \to \mu^+\mu^-$	$<$ 3.2 \times 10 ⁻⁷
		$K_L \rightarrow e^+e^ 9_{-4}^{+6} \times 10^{-12}$ $K_L \rightarrow \mu^+\mu^-(6.84 \pm 0.11) \times 10^{-9}$
	$B_d \rightarrow \mu^+ \mu^-$ <1.0 × 10 ⁻⁸ $B_d \rightarrow \mu^+ \mu^-$	$< 1.0 \times 10^{-8}$
	$B_d \rightarrow \tau^+ \tau^-$ <4.1 × 10 ⁻³ $B_s \rightarrow e^+ e^-$	$< 5.4 \times 10^{-5}$
	$B_s \rightarrow \mu^+ \mu^-$ <3.3 × 10 ⁻⁸	

decays, one needs to integrate out the LQ field. The effective 4-fermi Hamiltonians and vertices which are related to the mixing is given in Table [I.](#page-4-0) The vertices show that the limits coming from leptonic or semileptonic decays will be highly correlated. For charged leptons in the final state, one can constrain RS_0 , $RS_{1/2}$, $LS_{1/2}$, or LS_1 type LQs. The bounds for the first three will be the same, which is just twice that of LS_1 . Similarly, if we have neutrinos in the final state, LS_0 , $LS_{1/2}$, or LS_1 type LQ couplings are
bounded all limits being the same bounded, all limits being the same.

The product LQ coupling may in general be complex. If we neglect the SM, there is no scope of CP violation and the data constrains only the magnitude of the product, so we can, if we wish, take the product to be real. In fact, if we assume CP invariance, K_S decay channels to $\ell_i^+ \ell_i^-$ constrain only the real part of the product couplings, and K_L constrains the imaginary part. For the processes where the SM contribution cannot be neglected, we have saturated the difference between the highest experimental prediction and the lowest SM expectation with an incoherently summed LQ amplitude. For a quick reference, the data on the leptonic and semileptonic channels is shown in Tables [II](#page-4-1) and [III](#page-4-2). Note that these bounds are almost free from QCD uncertainties except for the decay constants of the mesons, and hence are quite robust. There are other

TABLE III. Branching ratios for some semileptonic K and B decays $[23,31-34]$ $[23,31-34]$ $[23,31-34]$ $[23,31-34]$ $[23,31-34]$. The limits are at 90% confidence level. Also shown are the central values for the SM. For the SM expectations shown with an error margin, we have taken the lowest possible values, so that the LQ bounds are most conservative. The systematic and statistical errors have been added in quadrature.

Mode	Branching ratio	SM expectation	Mode	Branching ratio	SM expectation
$K_S \rightarrow \pi^0 e^+ e^-$	$3.0^{+1.5}_{-1.2} \times 10^{-9}$	2.1×10^{-10}	$K_S \rightarrow \pi^0 \mu^+ \mu^-$	$2.9^{+1.5}_{-1.2}\times10^{-9}$	4.8×10^{-10}
$K_L \rightarrow \pi^0 e^+ e^-$	$< 2.8 \times 10^{-10}$	2.4×10^{-11}	$K_L \rightarrow \pi^0 \mu^+ \mu^-$	$<$ 3.8 \times 10 ⁻¹⁰	4.4×10^{-12}
$K_L \rightarrow \pi^0 \nu \bar{\nu}$	$\leq 6.7 \times 10^{-8}$	$(2.8 \pm 0.6) \times 10^{-11}$	$K^+ \rightarrow \pi^+ \nu \bar{\nu}$	$(17.3^{+11.5}_{-10.5}) \times 10^{-11}$	$(8.5 \pm 0.7) \times 10^{-11}$
$K^+ \to \pi^+ e^+ e^-$	$(2.88 \pm 0.13) \times 10^{-7}$	$(2.74 \pm 0.23) \times 10^{-7}$	$K^+ \to \pi^+ \mu^+ \mu^-$	$(8.1 \pm 1.4) \times 10^{-8}$	$(6.8 \pm 0.6) \times 10^{-8}$
$B_d \rightarrow \pi^0 e^+ e^-$	\leq 1.4 \times 10 ⁻⁷	3.3×10^{-8}	$B_d \rightarrow \pi^0 \mu^+ \mu^-$	$\leq 1.8 \times 10^{-7}$	3.3×10^{-8}
$B_d \rightarrow K^0 e^+ e^-$	$(1.3^{+1.6}_{-1.1}) \times 10^{-7}$	2.6×10^{-7}	$B_d \rightarrow K^0 \mu^+ \mu^-$	$(5.7^{+2.2}_{-1.8}) \times 10^{-7}$	$(3.3 \pm 0.7) \times 10^{-7}$
$B_d \rightarrow K^* \mu^+ \mu^-$	$(1.06 \pm 0.17) \times 10^{-6}$	$(1.0 \pm 0.4) \times 10^{-6}$	$B_d \rightarrow K^* e^+ e^-$	1.39×10^{-6}	$(1.3 \pm 0.4) \times 10^{-6}$
$B_d \rightarrow \pi^0 \nu \bar{\nu}$	$< 2.2 \times 10^{-4}$	$(8.5 \pm 3.5) \times 10^{-8}$	$B_d \to K^0 \nu \bar{\nu}$	$< 1.6 \times 10^{-4}$	$(1.35 \pm 0.35) \times 10^{-5}$
$B_d \to K^{*0} \nu \bar{\nu}$	$< 1.2 \times 10^{-4}$	3.8×10^{-6}	$B^+ \to \pi^+ e^+ e^-$	$< 8.0 \times 10^{-6}$	$(2.03 \pm 0.23) \times 10^{-8}$
$B^{+} \to \pi^{+} \mu^{+} \mu^{-}$	$<\!\!6.9\times10^{-6}$	$(2.03 \pm 0.23) \times 10^{-8}$	$B^+\to\pi^+\nu\bar{\nu}$	$<$ 100 \times 10 ⁻⁶	$(9.7 \pm 2.1) \times 10^{-6}$
$B^+ \to K^+ e^+ e^-$	$< 1.25 \times 10^{-5}$	6.0×10^{-7}	$B^+ \to K^+ \mu^+ \mu^-$	$< 8.3 \times 10^{-6}$	6.0×10^{-7}
$B^+\to K^{+\ast}\nu\bar{\nu}$	$\leq 8.0 \times 10^{-5}$	$(12.0 \pm 4.4) \times 10^{-6}$	$B^+ \to K^+ \nu \bar{\nu}$	$<$ 14 \times 10 ⁻⁶	$(4.5 \pm 0.7) \times 10^{-6}$
$B_s \rightarrow \phi \mu^+ \mu^-$	$(1.44 \pm 0.57) \times 10^{-6}$	1.6×10^{-6}	$B_s \rightarrow \phi \nu \bar{\nu}$	$< 5.4 \times 10^{-3}$	$(13.9 \pm 5.0) \times 10^{-6}$

semileptonic channels which we do not show here, e.g., $B \to K^* \nu \bar{\nu}$, because they yield less severe bounds.
The constraints coming from the decay $M^0 (= a^2)$

The constraints coming from the decay $M^0 (\equiv q_i \bar{q}_k) \rightarrow$ $\ell_i \ell_i$ can be expressed as

$$
|\lambda_{ij}\lambda_{ik}^*| < 2\sqrt{F_M}m_{\text{LQ}}^2\tag{16}
$$

for $R\bar{S}_0$, $RS_{1/2}$, and $L\bar{S}_{1/2}$ types, and without the factor of 2
on the right hand side for the LS, type LOs Here on the right-hand side for the LS_1 type LQs. Here

$$
F_M = \frac{1}{G_M} \text{Br}(M^0 \to \ell_i \bar{\ell}_i),
$$

\n
$$
G_M = \frac{1}{32\pi} f_{M^0} \tau_{M^0} M_{M^0}^3 m_{\ell} \sqrt{1 - 4 \frac{m_{\ell}^2}{M_{M^0}^2}},
$$
\n(17)

 τ and f_{M^0} being the lifetime and the decay constant of M^0 respectively. Note that K_L has a lifetime 2 orders of magnitude larger than that of K_S and hence the bounds coming from K_L decays are going to be tighter by that amount.

III. NUMERICAL INPUTS

The numerical inputs have been taken from various sources and listed in Table [IV.](#page-5-0) We use the BSW form factors [\[17\]](#page-10-14) with a simple pole dominance, and the relevant form factors at zero momentum transfer $q^2 = 0$ are taken as follows [\[35\]](#page-10-29):

$$
F_0^{B\to K}(0) = F_1^{B\to K}(0) = 0.38,
$$

\n
$$
F_0^{B\to \pi}(0) = F_1^{B\to \pi}(0) = 0.33,
$$

\n
$$
B \to K^*: V(0) = 0.37,
$$

\n
$$
A_1(0) = A_2(0) = 0.33,
$$

\n
$$
A_0(0) = 0.32,
$$
\n(18)

while we take $F_0^{K \to \pi}(0) = 0.992$. This is not incompatible with the lattice QCD result of 0.9560(84) [[36](#page-10-30)]. The theoretical uncertainty comes mostly from the form factors, but is never more than 10% for the LQ coupling bounds. The bounds are not a sensitive function of the exact values of the form factors, and remain more or less the same even when one uses the light-cone form factors.

The mass differences ΔM are all pretty well-measured;
consistency, we use the UTfit values [37]. We use for consistency, we use the UTfit values [\[37\]](#page-10-31). We use $\sin(2\beta_d)$ as measured in the charmonium channel [[1\]](#page-10-0). The SM prediction is taken from the measurement of the UT sides only since that is least likely to be affected by new physics. (However, this need not be true always. For example, if there is a new physics contributing in the $B_d - \bar{B}_d$
mixing amplitude, the extracted value of V, may not be mixing amplitude, the extracted value of V_{td} may not be equal to its SM value.) For β_s , which is defined as $\arg(-V_{ts}V_{tb}^*/V_{cs}V_{cb}^*)$, the errors are asymmetric:

$$
\beta_s = (0.47^{+0.13}_{-0.21}) \cup (1.09^{+0.21}_{-0.13}), \tag{19}
$$

which we show in a symmetrized manner. The decay constants $f_{B_{ds}}$ are taken from [\[38\]](#page-10-32) as a lattice average of various groups. The same holds for $f_B\sqrt{B_B}$ and ξ , defined as $\xi = f_{B_s} \sqrt{B_{B_s}} / f_{B_d} \sqrt{B_{B_d}}$, whose value we take to be
1.258 + 0.020 + 0.043 $1.258 \pm 0.020 \pm 0.043.$

IV. ANALYSIS

A. Neutral meson mixing

While our bounds are shown in Table [V](#page-6-0) following the procedure outlined in Sec. II E, let us try to understand the origin of these bounds.

Take Fig. [2\(a\)](#page-6-1) as an example, which shows the bounds on the real and imaginary parts of $\lambda_{i1} \lambda_{i2}^*$. This is shown for
the triplet LO S.: all LOs produce a similar diagram, with the triplet LQ S_1 ; all LQs produce a similar diagram, with the limits properly scaled. To get an idea of the scaling, one may again look at Table [V,](#page-6-0) and scale accordingly.

For the K system, we use ΔM_K and $|\varepsilon_K|$ as the con-
aints. The SM part is assumed to be dominated by the straints. The SM part is assumed to be dominated by the short-distance contributions only. Note the spokelike structure; this is because $|\varepsilon_K|$ gives a very tight constraint on $\text{Im}(M_{12})$ and only those points are chosen for which $(\lambda \lambda^*)^2$
is almost real. However, as we will see later, all the bounds $\frac{1}{2}$ and $\frac{1}{2}$ and $\frac{1}{2}$ and $\frac{1}{2}$ and $\frac{1}{2}$ $\frac{1$ except those for the LS_0 type LQs will be superseded by those coming from leptonic and semileptonic K decays; however, $i = 3$ bounds will stand.

A similar analysis is shown for the $B_d - \bar{B}_d$ system in
g 2(b) and Table V. Note that the bounds on the real and Fig. [2\(b\)](#page-6-1) and Table [V.](#page-6-0) Note that the bounds on the real and the imaginary parts of any product coupling are almost the same. This is, of course, no numerical accident. To understand this, let us analyze the origin of these bounds. There are two main constraints for the B_d system: ΔM_d and $\sin(2\beta_d)$. There will be a region centered around the origin $\sin(2\beta_d)$. There will be a region, centered around the origin of Re $(\lambda \lambda)$ – Im $(\lambda \lambda)$ plane (since ΔM_d can be explained

TABLE IV. Input parameters. For the form factors, see text.

Observable	Value	Observable	Value
ΔM_K	5.301×10^{-3} ps ⁻¹	$ \varepsilon_K $	$(2.228 \pm 0.011) \times 10^{-3}$
ΔM_{B_d}	(0.507 ± 0.005) ps ⁻¹	B_K	0.75 ± 0.07
ΔM_{B_s}	$(17.77 \pm 0.12) \text{ ps}^{-1}$	η_{B_K}	1.38 ± 0.53
	0.55 ± 0.01	f_K	160 MeV
$\frac{\eta_{B_{B_d}(B_{B_s})}}{\sin(2\beta_d)_{\rm exp}}$	0.668 ± 0.028	$J\,B$	(228 ± 17) MeV
$\sin(2\beta_d)_{\rm SM}$	0.731 ± 0.038	f_{B_s}/f_{B_d}	$(1.199 \pm 0.008 \pm 0.023)$
$(\beta_s)_{exp}$	$(0.43 \pm 0.17) \cup (1.13 \pm 0.17)$	$f_{B_s} \sqrt{B_{B_s}}$	$(257 \pm 6 \pm 21)$ MeV

TABLE V. Bounds from the neutral meson mixing. The third column shows the bounds when the couplings are assumed to be real. The last three columns are for complex couplings.

Process & indices	LQ Type	Real only	Real part of complex	Img part of complex	$ \lambda \lambda^* $
$K^0 - \bar{K}^0$	$LS_0, R\tilde{S}_0, RS_{1/2}$	0.008	0.008	0.008	0.008
$(i1)(i2)^*$	$L\tilde{S}_{1/2}$	0.0055	0.0055	0.0055	0.0055
	LS_1	0.0036	0.0036	0.0036	0.0036
$B_d - \bar{B}_d$	LS_0 , $R\ddot{S}_0$, $RS_{1/2}$	0.009	0.022	0.022	0.027
$(i1)(i3)^*$	$L\tilde{S}_{1/2}$	0.0063	0.016	0.016	0.019
	LS_1	0.004	0.010	0.010	0.012
$B_s - \bar{B}_s$	LS_0 , $R\ddot{S}_0$, $RS_{1/2}$	0.05	0.13	0.13	0.18
$(i2)(i3)^*$	$L\tilde{S}_{1/2}$	0.034	0.09	0.09	0.13
	LS_1	0.02	0.06	0.06	0.08

by the SM alone), where $|\lambda \lambda|$ is small and the phase can be arbitrary. At the 1σ level, this region appears to be small, because the measured value of $sin(2\beta_d)$ from the charmonium channels is just barely compatible with that obtained from a measurement of the sides of the unitarity triangles. The region expands if we take the error bars to be larger. This is the SM-dominated region, where LQ creeps in to whatever place is left available. Any analysis, taking both SM and LQ but assuming incoherent sum of amplitudes, should generate this region only.

However, there is always scope for fully constructive or destructive interference between SM and any NP. Consider a situation where the LQ contribution is large, so large that even after a destructive interference with the SM amplitude, enough is left to saturate ΔM_d . This LQ-dominated region (this is true for all NP models in general) gives us region (this is true for all NP models in general) gives us the bounds, and in the limit where the SM can be neglected, the bounds on $\text{Re}(\lambda \lambda)$ are almost the same as on Im $(\lambda \lambda)$. The alignment of the fourfold symmetric structure is differ-ent from Fig. [2\(b\)](#page-6-1) because of the sizable value of $sin(2\beta_d)$.

The limits for the B_s system are shown in Fig. [3.](#page-7-0) Note that the origin is excluded at the 1σ level; this is due to the large observed values of β_s : $\beta_s = (25 \pm 10)^{\circ}$ in the first quadrant and a mirror image ge observed values of p_s : $p_s = (23 \pm 10)$ O(63 \pm
° in the first quadrant and a mirror image in the second 10Þ quadrant.

The magnitude of the product is bounded to be less than 0.08 at the 1σ level for the triplet LO, and scaled according to Table [V.](#page-6-0) For $i = 2$, the relevant coupling mediates the leptonic decay $B_s \to \mu^+ \mu^-$ and semileptonic $B_d \to$ $K^{(*)}\mu^+\mu^-$ decays. We will see in the next part, just like the K system, that the constraints coming from such decays are much stronger. The same observation is true for $i = 1$. Again, only for λ_{LS_0} type couplings, there is no leptonic or semileptonic contributions (the down-type quark current couples with the neutrino current only), and the bounds coming from the mixing stand. Thus, for $i = 1, 2$ and any other LQ except S_0 , it is extremely improbable that the LQ contribution explains the large mixing phase.

What happens for $i = 3$? This will mediate the decays $B_s \to \tau^+ \tau^-$ and $B \to X_s \tau^+ \tau^-$. While there is no data on these channels yet, we may have a consistency check with the lifetime of B_s . This tells us that couplings as large as 0.05 are allowed, but the decay $B_s \rightarrow \tau^+ \tau^-$ should be close to the discovery limit. This will be an interesting channel to

FIG. 2 (color online). (a) Allowed parameter space for $\lambda_{i1}\lambda_{i2}^*$ for λ_{LS_1} type couplings. (b) The same for $\lambda_{i1}\lambda_{i3}^*$.

FIG. 3 (color online). (a) Allowed parameter space for $\lambda_{i2}\lambda_{i3}^*$ (b) The reach for the angle β_s . For more details, see text.

explore at the LHC. There is an exception: if we consider λ_{LS_0} type couplings, neutrinos flow inside the box, and then we have final-state neutrinos, and not τ leptons.

Note that the box diagram with leptoquarks and leptons has a nonzero absorptive part, which is responsible for the corresponding correlated decays. This affects the width differences $\Delta \Gamma_{d,s}$. As has been shown in [[6](#page-10-4)], NP that contributes to $\Delta \Gamma$ may enhance the mixing phase in the contributes to $\Delta \Gamma$ may enhance the mixing phase in the $B = \bar{B}$ box contrary to the Grossman theorem [7] which $B_s - \bar{B}_s$ box, contrary to the Grossman theorem [[7\]](#page-10-5), which tells that the mixing phase in the R, system must decrease tells that the mixing phase in the B_s system must decrease due to NP if there is no absorptive amplitude in the box diagram. The effect on $\Delta \Gamma_d / \Gamma_d$ is negligible; with the bounds that we get here it is never more than 1% or bounds that we get here, it is never more than 1%, or even less (note that [[6](#page-10-4)] uses a LQ mass of 100 GeV and we need to scale their results). For B_s , $\Delta \Gamma_s / \Gamma_s$ may go up
to 30% without significantly enhancing the leptonic to 30% without significantly enhancing the leptonic branching ratios like $B_s \rightarrow \tau^+\tau^-$, and one can also get a significant nonzero phase in the $B_s - \bar{B}_s$ mixing that is indicated by the present experiments [8] indicated by the present experiments [[8](#page-10-6)].

B. Leptonic and semileptonic decays

We have assumed only two LQ couplings to be present simultaneously, with identical lepton indices. Thus we will be interested only in lepton flavor conserving processes. A similar analysis was done in [\[39](#page-10-33)] for vector LQs. Our bounds are shown in Tables [VI](#page-7-1) and [VII](#page-8-0).

Apart from the leptonic K_L decays, the SM amplitudes can be neglected as a first approximation. Thus, one may saturate the experimental bounds with the LQ amplitude alone. This generates most of the numbers in Table [VI.](#page-7-1) For K_L decays, we consider the SM part too, and add the amplitudes incoherently. Note that K_L decays only constrain the imaginary part of the LQ coupling. This can be understood as follows. Consider the decay $K_L \rightarrow \mu^+ \mu^-$. While in the limit of CP invariance, one can write K_L = $(K^0 - \bar{K}^0)/\sqrt{2}$, it is $\lambda_{21}\lambda_{22}^*$ that mediates K^0 decay and λ^* λ_{22} that mediates \bar{K}^0 decay. Taking the combination $\lambda_{21}^* \lambda_{22}$ that mediates \overline{K}^0 decay. Taking the combination, the imaginary part of the coupling is responsible for K_L decays, and the real part is responsible for K_S decays. As mentioned earlier, $B_s \rightarrow \tau^+ \tau^-$ does not have a limit yet, but the SM expectation is about $\mathcal{O}(10^{-6})$, and if $|\lambda_{32}\lambda_{33}| \sim$ 10^{-2} , one expects the BR to be of the order of 4×10^{-5} .

For the channel $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, the outgoing neutrino can have any flavor, and so the bound is valid for $i = 1, 2, 3$. However, these bounds are valid when one can have a neutrino in the final state, i.e., for LQs of the L category, which couple with lepton doublets.

Semileptonic decays give the best bounds, but they are the least robust one, considering the uncertainty in the form factors. While we take the BSW form factors [\[17](#page-10-14)], the lattice QCD or light-cone sum rules based form factors may change the final results by at most 10%. To be con-

TABLE VI. Bounds from the correlated leptonic $K_{L(S)}$ and $B_{d(s)}$ decays. The LQs are either of $R\bar{S}_0$, $RS_{1/2}$, or $L\bar{S}_{1/2}$ type. For LS_1 type LQ, the bounds are half of that shown here.

$K_{L(S)}$ Decay	Coupling	$ \lambda \lambda^* $	$B_{d(s)}$ Decay	Coupling	$ \lambda \lambda^* $
$K_S \rightarrow e^+e^-$ $K_S \rightarrow \mu^+ \mu^-$ $K_L \rightarrow e^+e^-$ $K_L \rightarrow \mu^+ \mu^-$	$(12)(11)^*$ $(22)(21)^*$ $(12)(11)^*$ $(22)(21)^*$	1.8×10^{-1} 5.5×10^{-3} 2.4×10^{-4} 6.4×10^{-6}	$B_d \rightarrow \mu^+ \mu^-$ $B_d \rightarrow \tau^+ \tau^-$ $B_s \rightarrow \mu^+ \mu^-$	$(21)(23)^*$ $(31)(33)^{*}$ $(22)(23)^*$	2.8×10^{-3} 1.2×10^{-1} 4.3×10^{-3}

TABLE VII. Bounds from the correlated semileptonic B and K decays. The LQs are either of $R\ddot{S}_0$, $R\dot{S}_{1/2}$, or $L\ddot{S}_{1/2}$ type. For $L\dot{S}_1$ type LQ, the bounds are half of that shown here. For the final-state neutrino channels, the LQ can be LS_0 , $LS_{1/2}$, or LS_1 type, all giving the some bound the same bound.

Decay channel	Coupling	$ \lambda \lambda^* $	Decay channel	Coupling	$ \lambda \lambda^* $
$K_S \rightarrow \pi^0 e^+ e^-$	$(11)(12)^*$	2.8×10^{-3}	$K_L \rightarrow \pi^0 e^+ e^-$	$(11)(12)^*$	2.8×10^{-5}
$K_S \rightarrow \pi^0 \mu^+ \mu^-$	$(21)(22)^*$	4.6×10^{-3}	$K_L \rightarrow \pi^0 \mu^+ \mu^-$	$(21)(22)^*$	5.9×10^{-5}
$K^+ \to \pi^+ e^+ e^-$	$(11)(12)^*$	1.2×10^{-3}	$K^+ \to \pi^+ \mu^+ \mu^-$	$(21)(22)^*$	9.5×10^{-4}
$K_L \rightarrow \pi^0 \nu \bar{\nu}$	$(i1)(i2)^*$	4.6×10^{-4}			
$B_d \rightarrow \pi^0 e^+ e^-$	$(11)(13)^*$	1.1×10^{-3}	$B^{+} \to \pi^{+} e^{+} e^{-}$	$(11)(13)^*$	5.0×10^{-4}
$B_d \rightarrow \pi^0 \mu^+ \mu^-$	$(21)(23)^*$	1.2×10^{-3}	$B^+ \to \pi^+ \mu^+ \mu^-$	$(21)(23)^*$	4.6×10^{-4}
$B_d \rightarrow K^0 e^+ e^-$	$(12)(13)^*$	3.6×10^{-4}	$B^+ \to K^+ e^+ e^-$	$(12)(13)^*$	7.0×10^{-3}
$B_d \rightarrow K^* e^+ e^-$	$(12)(13)^*$	9.7×10^{-4}	$B_d \rightarrow K^* \mu^+ \mu^-$	$(22)(23)^*$	1.1×10^{-3}
$B_d \rightarrow K^0 \mu^+ \mu^-$	$(22)(23)^*$	1.5×10^{-3}	$B^+\to K^+\mu^+\mu^-$	$(22)(23)^*$	5.6×10^{-3}
$B_d \rightarrow \pi^0 \nu \bar{\nu}$	$(i1)(i3)^*$	4.4×10^{-2}	$B^+\to\pi^+\nu\bar{\nu}$	$(i1)(i3)^*$	2.0×10^{-2}
$B_d \to K^0 \nu \bar{\nu}$	$(i2)(i3)^*$	2.6×10^{-2}	$B^+ \to K^+ \nu \bar{\nu}$	$(i2)(i3)^*$	6.5×10^{-3}
$B_d \to K^{0*} \nu \bar{\nu}$	$(i2)(i3)^*$	1.5×10^{-2}	$B^+ \to K^{+*} \nu \bar{\nu}$	$(i2)(i3)^*$	1.2×10^{-2}
$B_s \rightarrow \phi \mu^+ \mu^-$	$(22)(23)^{*}$	7.9×10^{-4}	$B_s \rightarrow \phi \nu \bar{\nu}$	$(i2)(i3)^*$	9.1×10^{-2}

servative, we saturate the difference between the SM prediction and the maximum of the data by LQ contributions.

Let us just say a few words about $B^- \rightarrow \tau^- \bar{\nu}$. In the SM, the branching ratio can be worked out from Eq. ([10](#page-2-1)) and is $(9.3^{+3.4}_{-2.3}) \times 10^{-5}$, where the major sources of uncertainty are $|V_{ub}|$ and f_B . The observed number, $(14.3 \pm 3.7) \times 10^{-5}$ [1] is a bit above the SM prediction. The tension can 10^{-5} 10^{-5} [1] is a bit above the SM prediction. The tension can be alleviated with LS_0 or LS_1 type leptoquarks; the necessary combination is $\lambda_{31} \lambda_{33}^*$, and the bounds that we have
obtained on this particular combination in Table IX can obtained on this particular combination in Table [IX](#page-9-0) can easily jack up the branching ratio to the observed level. A similar exercise has been done for the leptonic D_s decays in [[14](#page-10-11)].

We have summarized our bounds in Tables [VIII](#page-8-1), [IX,](#page-9-0) and [X](#page-9-1). These tables contain no new information, but just shows the best bound for a given LQ type and a given set of indices. They further show that (i) Except for R-type LQs with indices (31)(33), semileptonic, and in a few cases leptonic, decays give the best constraints. In most of the cases they are one or more orders of magnitude stronger than those coming from the mixing, so with those LQs, one should not expect much discernible effects from CP asym-

TABLE VIII. Bounds coming from $K^0 - \bar{K}^0$ mixing and correlated decays. The better bounds have been emphasized. Note that K_S decays constrain $\text{Re}(\lambda_i \lambda_{i2}^*)$ while K^+ decays constrain only the magnitudes; however, in view of a tight constraint on the imaginary
part the bound from K^+ decay can be taken to be on the real part of the prod part, the bound from K^+ decay can be taken to be on the real part of the product coupling. Here and in the next two tables, all numbers in the "Previous bound" column are taken from either [\[3\]](#page-10-2) or [\[4\]](#page-10-9), with scaling the LQ mass to 300 GeV.

LQ type	Indices	Previous bound	This analysis				
			From mixing		From decay		
			Real part	Imag. part	Channel	Bound	
LS_0	$(i1)(i2)^*$	1.8×10^{-4}	8×10^{-3}	8×10^{-3}	$K_L \rightarrow \pi^0 \nu \bar{\nu}$	4.6×10^{-4}	
$R\tilde{S}_0, RS_{1/2}$	$(11)(12)^{*}$	2.7×10^{-3}	8×10^{-3}	8×10^{-3}	$K^+ \rightarrow \pi^+ e^+ e^-$	1.2×10^{-3}	
					$(K_L \to \pi^0 e^+ e^-)$	(2.8×10^{-5})	
	$(21)(22)^*$	5.4×10^{-5}	8×10^{-3}	8×10^{-3}	$K^+ \rightarrow \pi^+ \mu^+ \mu^-$	9.5×10^{-4}	
					$(K_L \rightarrow \mu^+ \mu^-)$	(6.4×10^{-6})	
	$(31)(32)^{*}$	0.018	8×10^{-3}	8×10^{-3}	\mathbf{z} , \mathbf{z} , \mathbf{z} , \mathbf{z}	$\mathbf{1}$, and $\mathbf{1}$	
$L\tilde{S}_{1/2}$	$(11)(12)^*$	1.8×10^{-4}	5.6×10^{-3}	5.6×10^{-3}	$K^+ \to \pi^+ e^+ e^-$	1.2×10^{-3}	
					$(K_L \to \pi^0 e^+ e^-)$	(2.8×10^{-5})	
	$(21)(22)^*$	1.8×10^{-4}	5.6×10^{-3}	5.6×10^{-3}	$K^+ \to \pi^+ \mu^+ \mu^-$	9.5×10^{-4}	
					$(K_L \rightarrow \mu^+ \mu^-)$	(6.4×10^{-6})	
	$(31)(32)^{*}$	5.4×10^{-5}	5.6×10^{-3}	5.6×10^{-3}	$K_L \rightarrow \pi^0 \nu \bar{\nu}$	4.6×10^{-4}	
LS_1	$(11)(12)^{*}$	1.8×10^{-4}	3.6×10^{-3}	3.6×10^{-3}	$K^+ \to \pi^+ e^+ e^-$	6.0×10^{-4}	
					$(K_L \to \pi^0 e^+ e^-)$	(1.4×10^{-5})	
	$(21)(22)^*$	2.7×10^{-5}	3.6×10^{-3}	3.6×10^{-3}	$K^+ \to \pi^+ \mu^+ \mu^-$	4.8×10^{-4}	
					$(K_L \rightarrow \mu^+ \mu^-)$	(3.2×10^{-6})	
	$(31)(32)^{*}$	1.8×10^{-4}	3.6×10^{-3}	3.6×10^{-3}	$K_L \rightarrow \pi^0 \nu \bar{\nu}$	4.6×10^{-4}	

TABLE IX. Bounds coming from $B_d - \bar{B}_d$ mixing and correlated decays. The better bounds have been emphasized.

LQ type	indices Previous bound	This analysis				
				From mixing		From decay
			Real part	Imag. part	Channel	Bound
LS_0	$(i1)(i3)^*$	0.036	0.022	0.022	$B^+ \rightarrow \pi^+ \nu \bar{\nu}$	2.0×10^{-2}
$R\tilde{S}_0, RS_{1/2}$	$(11)(13)^{*}$	0.054	0.022	0.022	$B^{+} \to \pi^{+} e^{+} e^{-}$	5.0×10^{-4}
	$(21)(23)^*$	7.2×10^{-3}	0.022	0.022	$B^+ \to \pi^+ \mu^+ \mu^-$	4.6×10^{-4}
	$(31)(33)^{*}$	0.054	0.022	0.022	$B_d \rightarrow \tau^+ \tau^-$	1.2×10^{-1}
$L\tilde{S}_{1/2}$	$(11)(13)^*$	0.054	0.016	0.016	$B^+\rightarrow \pi^+e^+e^-$	5.0×10^{-4}
	$(21)(23)^*$	7.2×10^{-3}	0.016	0.016	$B^+\to\pi^+\mu^+\mu^-$	4.6×10^{-4}
	$(31)(33)^*$	0.054	0.016	0.016	$B^+\to\pi^+\nu\bar{\nu}$	2.0×10^{-2}
LS_1	$(11)(13)^*$	0.036	0.010	0.010	$B^+\rightarrow \pi^+e^+e^-$	2.5×10^{-4}
	$(21)(23)^*$	3.6×10^{-3}	0.010	0.010	$B^+ \to \pi^+ \mu^+ \mu^-$	2.3×10^{-4}
	$(31)(33)^{*}$	0.027	0.010	0.010	$B_d \rightarrow \tau^+ \tau^-$	6.2×10^{-2}

TABLE X. Bounds coming from $B_s - \bar{B}_s$ mixing and correlated decays. The better bounds have been emphasized.

metries. (ii) While the bounds coming from decays are only on the magnitude of the product couplings, information on the complex weak phases of these couplings must come from mixing data, unless one makes a careful study of semileptonic CP asymmetries.

V. SUMMARY AND CONCLUSIONS

In this paper we have computed the bounds on several scalar leptoquark coupling combinations coming from $M^0 - \bar{M}^0$ mixing as well as leptonic and semileptonic decays. Though such an analysis is not new, we have implemented several features in this analysis which were not been taken into account in earlier studies. Apart from the improved data on the B system, we have also used the data on CP violating phases, and obtained nontrivial constraints on the real and imaginary parts of the couplings. We note that for the gauge-singlet LQ S_0 with λ_{LS_0} type couplings, it is possible to alleviate the mild tension between the measured and predicted values of $sin(2\beta_d)$, as well as to explain the large mixing phase in the B_s system. For this type of LQs, there are no modifications in leptonic or semileptonic channels, unless we consider final-state neutrinos.

For all other type of LQs, leptonic and semileptonic decays provide the better constraints (the exceptions are final-state τ channels). We do not expect any effects on nonleptonic final states like those coming from, say, Rparity violating supersymmetry with λ' type couplings. While the bounds coming from the leptonic channels are quite robust (apart from the mild uncertainty in the meson decay constants), those coming from semileptonic decays have an inherent uncertainty of the order of $10\%-15\%$, whose origin is the imprecise nature of the form factors.

ACKNOWLEDGMENTS

We thank Fabio Bossi and Jernej Kamenik for helpful comments. The work of A. K. was supported by BRNS, Government of India; CSIR, Government of India; and the DRS program of the University Grants Commission.

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