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Nature of the $a_0(980)$ meson in the light of photon-photon collisions

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New high-statistics Belle data on the reaction $\gamma\gamma \to \pi^0\eta$ are analyzed to clarify the two-photon production mechanisms and the nature of the $a_0(980)$ meson. The obtained solution for the amplitude $\gamma\gamma \to \pi^0\eta$ is consistent with the chiral theory expectation for the $\pi\eta$ scattering length, with the strong coupling of the $a_0(980)$ to the $\pi\eta$, $K\bar{K}$, and $\pi\eta'$ channels, and with a key role of the rescattering mechanisms $a_0(980) \to (K\bar{K} + \pi^0\eta + \pi^0\eta') \to \gamma\gamma$ in the $a_0(980) \to \gamma\gamma$ decay. Such a picture argues in favor of the $q^2\bar{q}^2$ nature of the $a_0(980)$ meson and is in agreement with the properties of its partners, the $\sigma_0(600)$ and $f_0(980)$ mesons, in particular, with those that manifest themselves in $\gamma\gamma \to \pi\pi$. An important role of the vector meson exchanges in the formation of the nonresonant background in $\gamma\gamma \to \pi^0\eta$ is also revealed. The preliminary information on the reaction $\pi^0\eta \to \pi^0\eta$ is obtained.

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I. INTRODUCTION

Recently, the Belle Collaboration obtained new high-statistics data on the reaction $\gamma\gamma \to \pi^0\eta$ at the KEKB e^+e^- collider [1]. The statistics collected in the Belle experiment is 3 orders of magnitude higher than in the earlier experiments performed by the Crystal Ball (336 events) [2] and JADE (291 events) [3] Collaborations. The experiments revealed a specific feature of the $\gamma\gamma \to \pi^0\eta$ cross section. It turned out sizable in the region between the $a_0(980)$ and $a_2(1320)$ resonances (see Fig. 1), which certainly indicates the presence of additional contributions. These contributions must be coherent with the resonance ones because the $\gamma\gamma \to \pi^0\eta$ amplitude in the $\pi^0\eta$ invariant mass region $\sqrt{s} < 1.4$ GeV is dominated by two lowest partial waves [1]: S and D_2 waves ($D_2 \equiv D_{\lambda=2}$, where λ is the absolute value of the difference between the helicities of the initial photons). The Belle Collaboration carried out the fit to the $\gamma\gamma \to \pi^0\eta$ data, taking into account interference between resonance and background contributions [1]. In so doing the simplified Breit-Wigner functions were used to describe the resonance contributions with spin $J = 0$ and 2. For example, in propagators of the $a_0(980)$ and the putative heavy $a_0(Y)$ resonance only the coupling to the $\pi\eta$ channel was taken into account, and the $a_0(980) \to \gamma\gamma$ and $a_0(Y) \to \gamma\gamma$ transition amplitudes were approximated by constants [1]. The background contributions were approximated by second order polynomials in $\sqrt{s}$. It turned out that the description of the S wave requires a smooth background with the amplitude comparable in magnitude with the $a_0(980)$ resonance amplitude in its maximum and having the large imaginary part [1]. As a result the background leads, practically, to quadrupling the cross section in the $a_0(980)$ peak region and to filling the dip between the $a_0(980)$ and $a_2(1320)$ resonances. The origin of such a considerable background in the S wave is unknown. The imaginary part of the background amplitude is determined by the contributions of the real intermediate states $\pi\eta$, $K\bar{K}$, $\pi\eta'$, and certainly requires the distinct dynamical decoding.

In this work we shall show that the experimentally observed pattern is the result of the combination of many dynamical factors. To analyze the data, we have significantly developed the model proposed previously in Ref. [4]. The basis for this model is an idea of what the $a_0(980)$ resonance can be as a suitable candidate in four-quark states [5,6]. There exists a set of important evidence in favor of the four-quark nature of the $a_0(980)$; see, for example, Refs. [6–11]. As a result of the performed analysis, we have elucidated that the Belle data are in close agreement with the case of strong coupling the $a_0(980)$ to the $\pi\eta$, $K\bar{K}$, and $\pi\eta'$ channels, and with a key role of the

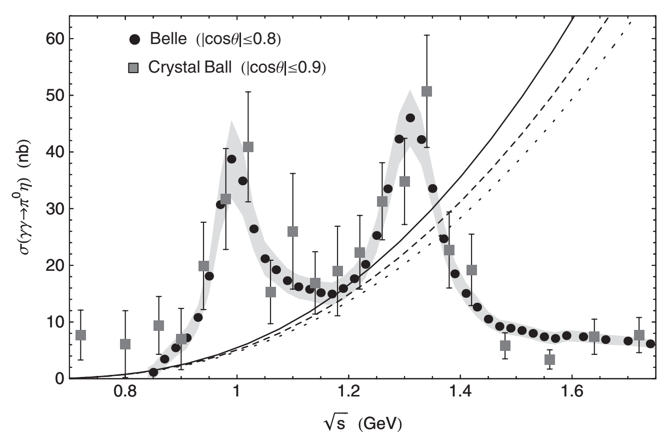

FIG. 1. The Belle and Crystal Ball data for the $\gamma\gamma \to \pi^0\eta$ cross section. θ denotes the output angle of the π^0 (or η) in the $\gamma\gamma$ center-of-mass system. The average statistical error of the Belle data is approximately ± 0.4 nb, the shaded band shows the size of their systematic error. The solid, dashed, and dotted lines correspond to the total, helicity 0, and S wave $\gamma\gamma \to \pi^0\eta$ cross sections caused by the elementary ρ and ω exchanges for $|\cos\theta| \leq 0.8$.

rescattering mechanisms $a_0(980) \to (K\bar{K} + \pi^0\eta + \pi^0\eta') \to \gamma\gamma$, i.e., four-quark transitions, in the decay $a_0(980) \to \gamma\gamma$ [12–17]. Furthermore, the nontrivial evidence has been found for the important role of the nonresonant production $\gamma\gamma \to \pi^0\eta$ via vector meson exchanges. The model information on the reaction $\pi^0\eta \to \pi^0\eta$, which we have extracted from the Belle data, is in reasonable agreement with the expectations based on chiral dynamics [19–21].

II. DYNAMICAL MODEL FOR $\gamma\gamma \to \pi^0\eta$

To analyze the data, we use a model for the helicity, M_λ, and corresponding partial, $M_{\lambda J}$, amplitudes of the reaction $\gamma\gamma \to \pi^0\eta$, where the electromagnetic Born contributions from ρ, ω, K^*, and K exchanges modified by form factors and strong elastic and inelastic final-state interactions in $\pi^0\eta$, $\pi^0\eta'$, K^+K^-, and $K^0\bar{K}^0$ channels, as well as the contributions due to the direct interaction of the resonances with photons, are taken into account:

$$\begin{aligned} M_0(\gamma\gamma \to \pi^0\eta; s, \theta) = {} & M_0^{\mathrm{Born}V}(\gamma\gamma \to \pi^0\eta; s, \theta) \\ & + \tilde{I}^V_{\pi^0\eta}(s) T_{\pi^0\eta\to\pi^0\eta}(s) \\ & + \tilde{I}^V_{\pi^0\eta'}(s) T_{\pi^0\eta'\to\pi^0\eta}(s) \\ & + (\tilde{I}^{K^{*+}}_{K^+K^-}(s) - \tilde{I}^{K^{*0}}_{K^0\bar{K}^0}(s) \\ & + \tilde{I}^{K^+}_{K^+K^-}(s)) T_{K^+K^-\to\pi^0\eta}(s) \\ & + M^{\mathrm{direct}}_{\mathrm{res}}(s), \end{aligned} \tag{1}$$

$$\begin{aligned} M_2(\gamma\gamma \to \pi^0\eta; s, \theta) = {} & M_2^{\mathrm{Born}V}(\gamma\gamma \to \pi^0\eta; s, \theta) \\ & + 80\pi d^2_{20}(\theta) M_{\gamma\gamma\to a_2(1320)\to\pi^0\eta}(s), \end{aligned} \tag{2}$$

$d^2_{20}(\theta) = (\sqrt{6}/4)\sin^2\theta$; the diagrams corresponding to these amplitudes are shown in Figs. 2 and 3.

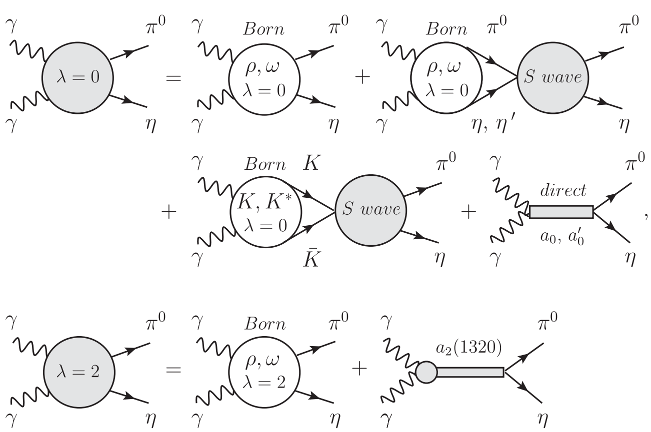

FIG. 2. Diagrammatical representation for the helicity amplitudes $\gamma\gamma \to \pi^0\eta$.

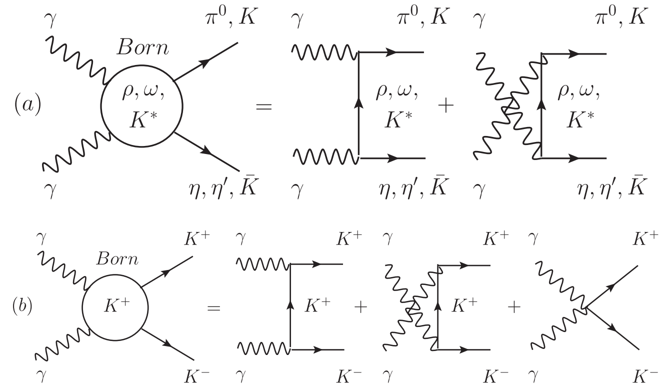

FIG. 3. The Born ρ, ω, K^*, K exchange diagrams for $\gamma\gamma \to \pi^0\eta$, $\gamma\gamma \to \pi^0\eta'$, and $\gamma\gamma \to K\bar{K}$.

The first terms in the right-hand parts of Eqs. (1) and (2) represent the real Born helicity amplitudes caused by the contributions of the ρ and ω exchange mechanisms. These contributions are equal in magnitude and in sign. With this note we write [4,22]

$$\begin{aligned} M_0^{\mathrm{Born}V}(\gamma\gamma \to \pi^0\eta; s, \theta) = {} & 2 g_{\omega\pi\gamma} g_{\omega\eta\gamma} \frac{s}{4}\left[\frac{tG_\omega(s,t)}{t - m^2_\omega}\right. \\ & \left. + \frac{uG_\omega(s,u)}{u - m^2_\omega}\right], \end{aligned} \tag{3}$$

$$\begin{aligned} M_2^{\mathrm{Born}V}(\gamma\gamma \to \pi^0\eta; s, \theta) = {} & 2 g_{\omega\pi\gamma} g_{\omega\eta\gamma} \frac{m^2_\pi m^2_\eta - tu}{4} \\ & \times \left[\frac{G_\omega(s,t)}{t - m^2_\omega} + \frac{G_\omega(s,u)}{u - m^2_\omega}\right], \end{aligned} \tag{4}$$

where t and u are the Mandelstam variables for the reaction $\gamma\gamma \to \pi^0\eta$, $g_{\omega\eta\gamma} = \frac{1}{3} g_{\omega\pi\gamma}\sin(\theta_i - \theta_P)$, $g^2_{\omega\pi\gamma} = 12\pi\Gamma_{\omega\to\pi\gamma}[(m^2_\omega - m^2_\pi)/(2m_\omega)]^{-3} \approx 0.519\ \mathrm{GeV}^{-2}$ [23], the "ideal" mixing angle $\theta_i = 35.3°$, the pseudoscalar mixing angle θ_P is a free parameter, $G_\omega(s,t)$ and $G_\omega(s,u)$ are the form factors in the t and u channels [for the elementary ρ and ω exchanges $G_\omega(s,t) = G_\omega(s,u) = 1$]. In the corresponding Born amplitudes for $\gamma\gamma \to \pi^0\eta'$, $g_{\omega\eta'\gamma} = \frac{1}{3} g_{\omega\pi\gamma}\cos(\theta_i - \theta_P)$. The amplitudes $M_\lambda^{\mathrm{Born}K^*}(\gamma\gamma \to K\bar{K}; s, \theta)$ for the K^* exchanges result from Eqs. (3) and (4) with the use of substitutions $m_\omega \to m_{K^*}$, $G_\omega \to G_{K^*}$, $m_\pi \to m_K$, $m_\eta \to m_K$, and $2g_{\omega\pi\gamma}g_{\omega\eta\gamma} \to g^2_{K^*K\gamma}$, where $g^2_{K^{*+}K^+\gamma} \approx 0.064\ \mathrm{GeV}^{-2}$ and $g^2_{K^{*0}K^0\gamma} \approx 0.151\ \mathrm{GeV}^{-2}$ [23].

Note that the Born sources of the reaction $\gamma\gamma \to \pi^0\eta$ corresponding to the t and u channel exchanges with the quantum numbers of the vector ρ and ω mesons [as well as with those of the axial-vector $b_1(1235)$ and $h_1(1170)$ mesons] are poorly understood in the nonasymptotic energy region of interest. One can say only that the elementary ρ and ω exchanges, whose contributions to the $\gamma\gamma \to \pi^0\eta$ cross section (primarily to the S wave) increase very rapidly with the energy, are not observed in the experiments; see Fig. 1. This fact was explained in Ref. [22]. The appropriate Reggeization of the elementary exchanges with higher spins strongly reduces dangerous contributions. Such a reduction has to take place already in the region of 1–1.5 GeV. Therefore, in applications to the real case, it is natural to use the form factors of the Regge type: $G_\omega(s,t) = \exp[(t - m_\omega^2)b_\omega(s)]$, $G_\omega(s,u) = \exp[(u - m_\omega^2)b_\omega(s)]$, where we put $b_\omega(s) = b_\omega^0 + (\alpha'_\omega/4) \times \ln[1 + (s/s_0)^4]$, $b_\omega^0 = 0$, $\alpha'_\omega = 0.8\ \mathrm{GeV}^{-2}$, and $s_0 = 1\ \mathrm{GeV}^2$ (form factors for the K^* exchange result from the above by substitution of K^* for ω).

As for the $b_1(1235)$ and $h_1(1170)$ exchanges, their amplitudes have the form analogous to expressions (3) and (4) except for the common sign in the amplitude with helicity 0. The available (rather poor) information on the coupling constant gives $g^2_{h_1\pi\gamma} \approx 9g^2_{b_1\pi\gamma} \approx 0.34\ \mathrm{GeV}^{-2}$ [23]. The coupling constant, higher mass, and more damping form factor lead to the fivefold (at least) suppression of the axial-vector exchanges in comparison with the vector ones, which is why we neglect their contributions.

The terms in Eq. (1), proportional to the S wave amplitudes of hadronic reactions, $T_{\pi^0\eta\to\pi^0\eta}(s)$, $T_{\pi^0\eta'\to\pi^0\eta}(s)$, and $T_{K^+K^-\to\pi^0\eta}(s)$, are due to the rescattering mechanisms. In amplitudes T we take into account the contribution from mixed $a_0(980)$ and (heavy) $a_0(Y)$ resonances (hereinafter referred to as a_0 and a_0', respectively), and the background contributions. The amplitudes T are given by

$$T_{\pi^0\eta\to\pi^0\eta}(s) = T_0^1(s) = \frac{\eta_0^1(s)e^{2i\delta_0^1(s)} - 1}{2i\rho_{\pi\eta}(s)} = T^{\mathrm{bg}}_{\pi\eta}(s) + e^{2i\delta^{\mathrm{bg}}_{\pi\eta}(s)}T^{\mathrm{res}}_{\pi^0\eta\to\pi^0\eta}(s), \quad (5)$$

$$T_{\pi^0\eta'\to\pi^0\eta}(s) = T^{\mathrm{res}}_{\pi^0\eta'\to\pi^0\eta}(s)e^{i[\delta^{\mathrm{bg}}_{\pi\eta'}(s)+\delta^{\mathrm{bg}}_{\pi\eta}(s)]}, \quad (6)$$

$$T_{K^+K^-\to\pi^0\eta}(s) = T^{\mathrm{res}}_{K^+K^-\to\pi^0\eta}(s)e^{i[\delta^{\mathrm{bg}}_{K\bar{K}}(s)+\delta^{\mathrm{bg}}_{\pi\eta}(s)]}, \quad (7)$$

where $T^{\mathrm{bg}}_{\pi\eta}(s) = (e^{2i\delta^{\mathrm{bg}}_{\pi\eta}(s)} - 1)/(2i\rho_{\pi\eta}(s))$, $T^{\mathrm{res}}_{\pi^0\eta\to\pi^0\eta}(s) = (\eta_0^1(s)e^{2i\delta^{\mathrm{res}}_{\pi\eta}(s)} - 1)/(2i\rho_{\pi\eta}(s))$, $\delta_0^1(s) = \delta^{\mathrm{bg}}_{\pi\eta}(s) + \delta^{\mathrm{res}}_{\pi\eta}(s)$, $\rho_{ab}(s) = \sqrt{(1 - m_{ab}^{(-)2}/s)(1 - m_{ab}^{(+)2}/s)}$, $m_{ab}^{(\pm)} = m_b \pm m_a$ ($ab = \pi\eta, K^+K^-, K^0\bar{K}^0, \pi\eta'$), $\delta^{\mathrm{bg}}_{\pi\eta}(s)$, $\delta^{\mathrm{bg}}_{\pi\eta'}(s)$, and $\delta^{\mathrm{bg}}_{K\bar{K}}(s)$ are the phase shifts of the elastic background contributions in the channels $\pi\eta$, $\pi\eta'$, and $K\bar{K}$ with isospin $I = 1$, respectively.

The amplitudes of the a_0–a_0' resonance complex in Eqs. (5)–(7) have the form [6,14,24,25]

$$T^{\mathrm{res}}_{ab\to\pi^0\eta}(s) = \frac{g_{a_0ab}\Delta_{a_0'}(s) + g_{a_0'ab}\Delta_{a_0}(s)}{16\pi[D_{a_0}(s)D_{a_0'}(s) - \Pi^2_{a_0a_0'}(s)]}, \quad (8)$$

where $\Delta_{a_0'}(s) = D_{a_0'}(s)g_{a_0\pi^0\eta} + \Pi_{a_0a_0'}(s)g_{a_0'\pi^0\eta}$ and $\Delta_{a_0}(s) = D_{a_0}(s)g_{a_0'\pi^0\eta} + \Pi_{a_0a_0'}(s)g_{a_0\pi^0\eta}$, g_{a_0ab} and $g_{a_0'ab}$ are the coupling constants, the propagator of the a_0 (and similarly a_0') resonance is

$$\frac{1}{D_{a_0}(s)} = \frac{1}{m^2_{a_0} - s + \sum_{ab}[\mathrm{Re}\Pi^{ab}_{a_0}(m^2_{a_0}) - \Pi^{ab}_{a_0}(s)]}, \quad (9)$$

where $\mathrm{Re}\Pi^{ab}_{a_0}(s)$ is determined by a singly subtracted dispersion integral of $\mathrm{Im}\Pi^{ab}_{a_0}(s) = \sqrt{s}\Gamma_{a_0\to ab}(s) = g^2_{a_0ab}\rho_{ab}(s)/(16\pi)$; $\Pi_{a_0a_0'}(s) = C + \sum_{ab}(g_{a_0'ab}/g_{a_0ab}) \times \Pi^{ab}_{a_0}(s)$, where C is the mixing parameter. The explicit form of the polarization operators $\Pi^{ab}_{a_0}(s)$ has been written out in Refs. [6,10,25–27].

The amplitude

$$M^{\mathrm{direct}}_{\mathrm{res}}(s) = s\frac{g^{(0)}_{a_0\gamma\gamma}\Delta_{a_0'}(s) + g^{(0)}_{a_0'\gamma\gamma}\Delta_{a_0}(s)}{D_{a_0}(s)D_{a_0'}(s) - \Pi^2_{a_0a_0'}(s)}e^{i\delta^{\mathrm{bg}}_{\pi\eta}(s)} \quad (10)$$

in Eq. (1) describes the $\gamma\gamma \to \pi^0\eta$ transition caused by the direct coupling constants of the a_0 and a_0' resonances to photons $g^{(0)}_{a_0\gamma\gamma}$ and $g^{(0)}_{a_0'\gamma\gamma}$; the factor s appears due to the gauge invariance.

Equation (1) implies that the amplitudes $T_{\pi^0\eta\to\pi^0\eta}(s)$, $T_{\pi^0\eta'\to\pi^0\eta}(s)$, and $T_{K\bar{K}\to\pi^0\eta}(s)$ in $\gamma\gamma \to \pi^0\eta \to \pi^0\eta$, $\gamma\gamma \to \pi^0\eta' \to \pi^0\eta$, and $\gamma\gamma \to K\bar{K} \to \pi^0\eta$ rescattering loops (see Fig. 2) lie on the mass shell [28]. In so doing, the functions $\tilde{I}^V_{\pi^0\eta}(s)$, $\tilde{I}^V_{\pi^0\eta'}(s)$, $\tilde{I}^{K^*}_{K\bar{K}}(s)$, and $\tilde{I}^{K^+}_{K^+K^-}(s)$ are the amplitudes of the triangle loop diagrams describing the transitions $\gamma\gamma \to ab \to$ (scalar state with a mass $= \sqrt{s}$), in which the meson pairs $\pi^0\eta$, $\pi^0\eta'$, and $K\bar{K}$ are produced by the electromagnetic Born sources (see Fig. 3). For example, the function

$$\tilde{I}^{K^+}_{K^+K^-}(s) = \frac{s}{\pi}\int_{4m^2_{K^+}}^{\infty}\frac{\rho_{K^+K^-}(s')M^{\mathrm{Born}K^+}_{00}(\gamma\gamma \to K^+K^-; s')}{s'(s' - s - i\varepsilon)} \times ds', \quad (11)$$

where $M^{\mathrm{Born}K^+}_{00}(\gamma\gamma \to K^+K^-; s')$ is the S wave of the Born charged one-kaon exchange amplitude (see Refs. [29–33] for details). The functions $\tilde{I}^V_{\pi^0\eta}(s)$, $\tilde{I}^V_{\pi^0\eta'}(s)$, and $\tilde{I}^{K^*}_{K\bar{K}}(s)$ are calculated in a similar way. The amplitude $M_0(\gamma\gamma \to \pi^0\eta; s, \theta)$ constructed in such a way satisfies the Watson theorem in the elastic region [13,14].

For the $a_2(1320)$ production amplitude [see Eq. (2)], we use the following simple parametrization:

$$M_{\gamma\gamma\to a_2(1320)\to\pi^0\eta}(s) = \frac{G_{a_2}(s)\sqrt{s\Gamma^{\mathrm{tot}}_{a_2}(s)B(a_2\to\pi\eta)/\rho_{\pi\eta}(s)}}{m^2_{a_2} - s - i\sqrt{s}\Gamma^{\mathrm{tot}}_{a_2}(s)}. \quad (12)$$

Here

$$G_{a_2}(s) = \sqrt{\Gamma^{(0)}_{a_2\to\gamma\gamma}(s)} + i\frac{M^{\mathrm{Born}V}_{22}(\gamma\gamma\to\pi^0\eta; s)}{16\pi} \times \sqrt{\rho_{\pi\eta}(s)\Gamma^{\mathrm{tot}}_{a_2}(s)B(a_2\to\pi\eta)}, \quad (13)$$

$$\Gamma^{\mathrm{tot}}_{a_2}(s) = \Gamma^{\mathrm{tot}}_{a_2}\frac{m^2_{a_2}}{s}\frac{q^5_{\pi\eta}(s)}{q^5_{\pi\eta}(m^2_{a_2})}\frac{D_2(q_{\pi\eta}(m^2_{a_2})r_{a_2})}{D_2(q_{\pi\eta}(s)r_{a_2})}, \quad (14)$$

where $q_{\pi\eta}(s) = \sqrt{s}\rho_{\pi\eta}(s)/2$, $D_2(x) = 9 + 3x^2 + x^4$, and r_{a_2} is the interaction radius. By definition $\Gamma_{a_2\to\gamma\gamma}(s) = |G_{a_2}(s)|^2$ and $\Gamma^{(0)}_{a_2\to\gamma\gamma}(s) = (\frac{\sqrt{s}}{m_{a_2}})^3\Gamma^{(0)}_{a_2\to\gamma\gamma}$. The second term in $G_{a_2}(s)$ corresponds to the $a_2(1270)\to\pi^0\eta\to\gamma\gamma$ transition with the real π^0 and η mesons in the intermediate state. The estimates show that this term is less than 3% in the amplitude and can be omitted. Recall that the observed two-photon decays widths of the $f_2(1270)$ and $a_2(1320)$ mesons [23] satisfy rather well the relation $\Gamma_{f_2\to\gamma\gamma}/\Gamma_{a_2\to\gamma\gamma} = 25/9$, which holds in the naive $q\bar{q}$ model for the direct annihilation transitions $q\bar{q}\to\gamma\gamma$.

The normalization of the amplitudes M_λ is specified by their relation with the cross section $\sigma = \sigma_0 + \sigma_2$, where $\sigma_\lambda = \frac{\rho_{\pi\eta}(s)}{64\pi s}\int|M_\lambda|^2 d\cos\theta$.

III. RESULTS OF THE DATA DESCRIPTION

Figures 4(a)–4(c) illustrate the results of one of the fitting variants to the Belle data on the cross section $\gamma\gamma\to\pi^0\eta$ (hereafter variant 1); the corresponding parameter values are collected in Table I. As is seen from Fig. 4(c), the resulting fitted curve agrees quite satisfactorily with the data within their systematic errors. Such an agreement allows definite conclusions concerning the main dynamical constituents of the $\gamma\gamma\to\pi^0\eta$ reaction mechanism. The contributions of these constituents are shown in detail in Figs. 4(a) and 4(b).

We begin with the contribution of the inelastic rescattering $\gamma\gamma\to K^+K^-\to\pi^0\eta$, where the intermediate K^+K^- pair is produced via the charged one-kaon exchange mechanism [see Fig. 3(b)]. This mechanism predicts the natural scale for the $a_0(980)$ production cross section in $\gamma\gamma\to\pi^0\eta$ [4,22,27,34]. The maximum of the cross section $\gamma\gamma\to K^+K^-\to a_0(980)\to\pi^0\eta$ is basically controlled by the product of the parameter $R_{a_0} = g^2_{a_0K^+K^-}/g^2_{a_0\pi\eta}$ and the value $|\tilde{I}^{K^+}_{K^+K^-}(4m^2_{K^+})|^2$. Its estimate gives $\sigma(\gamma\gamma\to K^+K^-\to a_0(980)\to\pi^0\eta; |\cos\theta|\le 0.8)\approx 0.8\times 1.4\alpha^2 R_{a_0}/m^2_{a_0}\approx 24\,\mathrm{nb}\times R_{a_0}$ (here we neglect the heavy a'_0 resonance contribution). The specific feature of this mechanism is the strong energy dependence of the $a_0(980)\to K^+K^-\to\gamma\gamma$ decay width, which is conditioned by the function $\tilde{I}^{K^+}_{K^+K^-}(s)$ [4,13,14,27,29]. $\tilde{I}^{K^+}_{K^+K^-}(s)$ decreases sharply just below the K^+K^- threshold that leads to the noticeable additional narrowing of the $a_0(980)$ peak in the $\gamma\gamma\to\pi^0\eta$ channel; namely, its effective width turns out to be about 40 MeV at $\Gamma_{a_0\to\pi\eta}(m^2_{a_0})\approx$

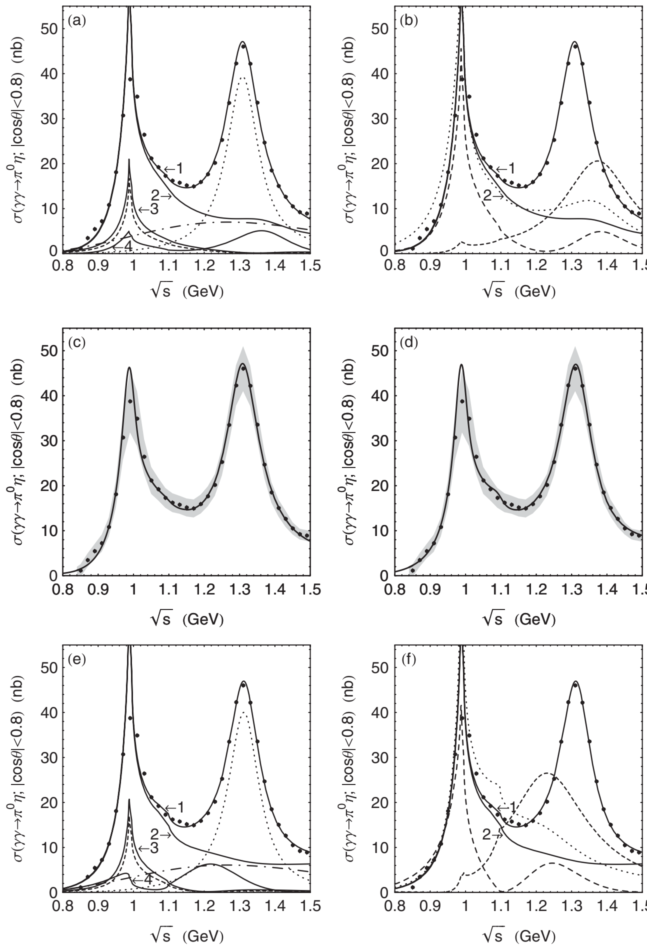

FIG. 4. Fits to the Belle data. The curves in (a), (b), and (c) correspond to variant 1. (a) The total cross section $\gamma\gamma\to\pi^0\eta$ (solid line 1), its helicity 0 (solid line 2), and helicity 2 (dotted line) parts, the contribution from the $\gamma\gamma\to K^+K^-\to\pi^0\eta$ rescattering with intermediate K^+K^- produced via the Born K exchange (solid line 3), the contribution from the $\gamma\gamma\to K\bar{K}\to\pi^0\eta$ rescattering with intermediate $K\bar{K}$ produced via the Born K and K^* exchanges (dashed line), the contribution from the Born ρ and ω exchange amplitude with $\lambda = 0$ (dot-dashed line), and the combined contribution from the Born one and the S wave $\gamma\gamma\to(\pi^0\eta+\pi^0\eta')\to\pi^0\eta$ rescattering (solid line 4); see Figs. 2 and 3. (b) The solid lines 1 and 2 are the same as in (a), the short-dashed line corresponds to the contribution from the $\gamma\gamma\to\pi^0\eta$ direct transition amplitude for the $a_0 - a'_0$ resonance complex [see Eq. (10)], the dotted line shows the total contribution from the $a_0 - a'_0$ resonance one, and the long-dashed line corresponds to the helicity 0 cross section without the contribution from the direct transition amplitude. The resulting solid line in (c) corresponds to the solid line 1 in (a) [or (b)], folded, in the region $0.84\ \mathrm{GeV} < \sqrt{s} < 1.15\ \mathrm{GeV}$, with a Gaussian with $\sigma = 10$ MeV mass resolution; the shaded band shows the size of the systematic error of the data. (d), (e), and (f) The same as (c), (a), and (b), respectively, for variant 2.

200 MeV (see Refs. [4,6] for details). The $\gamma\gamma\to K^+K^-\to\pi^0\eta$ rescattering contribution to the cross section $\gamma\gamma\to\pi^0\eta$ is shown by solid line 3 in Fig. 4(a). The

TABLE I. Fitted parameters. The systematic errors of the data are taken into account.

Variant	1	2
m_{a_0} (GeV)	$0.9845^{+0.001}_{-0.004}$	$0.9855^{+0.0015}_{-0.002}$
$g_{a_0\pi\eta}$ (GeV)	$4.23^{+0.25}_{-0.08}$	$3.99^{+0.17}_{-0.08}$
$g_{a_0K^+K^-}$ (GeV)	$3.79^{+0.14}_{-0.71}$	3.58 ± 0.28
$g_{a_0\pi\eta'}$ (GeV)	$-2.13^{+0.4}_{-0.28}$	$-2.34^{+0.32}_{-0.25}$
$g^{(0)}_{a_0\gamma\gamma}$ (GeV^{-1})	$(1.77^{+0.29}_{-0.11}) \times 10^{-3}$	$(1.77^{+0.13}_{-0.11}) \times 10^{-3}$
$m_{a_0'}$ (GeV)	$1.4^{+0.02}_{-0.01}$	1.276 ± 0.01
$g_{a_0'\pi\eta}$ (GeV)	3.3 ± 0.16	$3.55^{+0.14}_{-0.1}$
$g_{a_0'K^+K^-}$ (GeV)	0.28 ± 0.3	0.48 ± 0.32
$g_{a_0'\pi\eta'}$ (GeV)	$2.91^{+0.35}_{-0.49}$	$2.91^{+0.29}_{-0.32}$
$g^{(0)}_{a_0'\gamma\gamma}$ (GeV^{-1})	$(-11.5^{+0.3}_{-0.4}) \times 10^{-3}$	$(-11.5^{+0.3}_{-0.2}) \times 10^{-3}$
C (GeV^2)	$0.06^{+0.025}_{-0.03}$	0.041 ± 0.01
c_0	-0.6 ± 0.03	-0.63 ± 0.03
c_1 (GeV^{-2})	$-6.48^{+1.5}_{-0.65}$	$-1.46^{+0.15}_{-0.32}$
c_2 (GeV^{-4})	$0.12^{+0.3}_{-0.12}$	$0.095^{+0.30}_{-0.095}$
$f_{K\bar{K}}$ (GeV^{-1})	$-0.37^{+0.13}_{-0.3}$	$-0.37^{+0.13}_{-0.25}$
$f_{\pi\eta'}$ (GeV^{-1})	$0.28^{+0.45}_{-0.3}$	$0.3^{+0.4}_{-0.6}$
θ_P (degree)	-24^{+3}_{-5}	-18^{+3}_{-6}
m_{a_2} (GeV)	$1.322^{+0.005}_{-0.003}$	$1.323^{+0.004}_{-0.003}$
$\Gamma^{\rm tot}_{a_2}$ (GeV)	$0.116^{+0.011}_{-0.007}$	$0.111^{+0.007}_{-0.009}$
$\Gamma^{(0)}_{a_2\to\gamma\gamma}$ (keV)	1.053 ± 0.048	1.047 ± 0.005
r_{a_2} (GeV^{-1})	$1.9^{+0.9}_{-1.9}$	$3^{+2}_{-0.7}$
χ^2/ndf	$5.1/13 = 0.39$	$4.8/13 = 0.37$

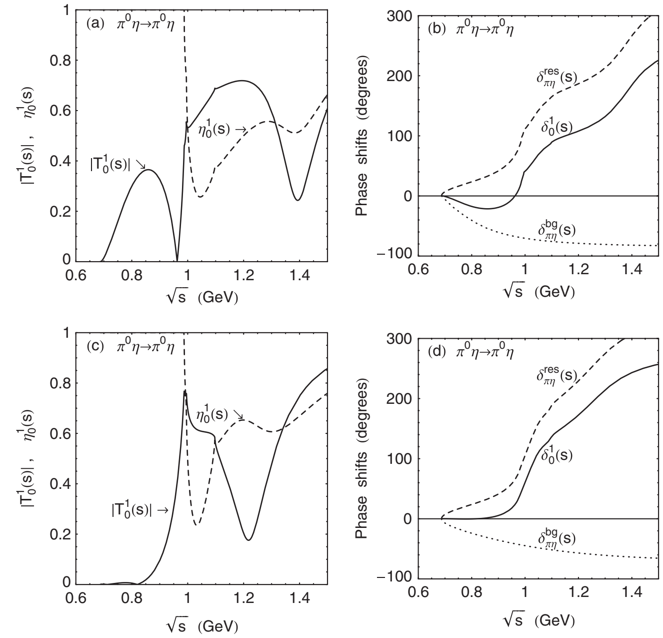

FIG. 5. Modulus of $T_0^1(s)$, inelasticity $\eta_0^1(s)$ (a), and phase shifts (b) of the S wave amplitude $\pi^0\eta \to \pi^0\eta$ for variant 1 (here $a_0^1 = 0.0098$); (c) and (d) the same for variant 2 (here $a_0^1 = 0.0066$).

K^* exchange narrows slightly the $a_0(980)$ peak [see the dashed line under solid line 3 in Fig. 4(a)].

It is clear that the $\gamma\gamma \to K\bar{K} \to \pi^0\eta$ rescattering mechanism alone cannot describe the data in the $a_0(980)$ resonance region. The observed cross section can be obtained by adding the Born ρ and ω exchange contribution, modified by the S wave rescattering $\gamma\gamma \to (\pi^0\eta + \pi^0\eta') \to \pi^0\eta$, and the amplitude of Eq. (10), caused by the direct transitions of the a_0 and a_0' resonances into photons. Each of the contributions of these two mechanisms are not too large in the $a_0(980)$ region (see solid line 4 in Fig. 4(a) for the first of them and the short-dashed line in Fig. 4(b) for the second). But the main thing is that their coherent sum with the contribution of the inelastic rescattering $\gamma\gamma \to K\bar{K} \to \pi^0\eta$ (see the diagrams for the amplitude with $\lambda = 0$ in Fig. 2) leads to the considerable enhancement of the $a_0(980)$ resonance manifestation [see solid line 2 in Fig. 4(a)]. Recall that all the S wave contributions to the amplitude $\gamma\gamma \to \pi^0\eta$ have the equal phase for $\sqrt{s}$ below the K^+K^- threshold, in accord with the Watson theorem.

The important role played by the background elastic amplitude of $\pi^0\eta$ scattering, $T^{\rm bg}_{\pi\eta}(s)$, should be noted; see Eq. (5). First, the choice of the negative background phase shift $\delta^{\rm bg}_{\pi\eta}(s)$ in $T^{\rm bg}_{\pi\eta}(s)$ [see Fig. 5(b)] allows the agreement of the $\pi\eta$ scattering length in the considered model with the estimates based on current algebra [19] and chiral perturbation theory [20], according to which a_0^1 (in units of m_π^{-1}) ≈ 0.005–0.01. There takes place a compensation in a_0^1 between the resonance contribution (about 0.3) and the background one [35,36]. Second, the considerable negative value of $\delta^{\rm bg}_{\pi\eta}(s)$ near 1 GeV provides the resonancelike behavior of the cross section shown by solid line 4 in Fig. 4(a). The characteristics for the S wave amplitude $\pi^0\eta \to \pi^0\eta$ corresponding to variant 1 are represented in Figs. 5(a) and 5(b).

For the background phase shifts we use the simplest parametrizations, which are suitable in the physical region of the reaction $\gamma\gamma \to \pi^0\eta$:

$$e^{i\delta^{\rm bg}_{ab}(s)} = [(1 + iF_{ab}(s))/(1 - iF_{ab}(s))]^{1/2}, \tag{15}$$

where

$$F_{\pi\eta}(s) = \frac{\sqrt{1 - m^{(+)2}_{\pi\eta}/s}\,(c_0 + c_1(s - m^{(+)2}_{\pi\eta}))}{1 + c_2(s - m^{(+)2}_{\pi\eta})^2}, \tag{16}$$

$$F_{K\bar{K}}(s) = f_{K\bar{K}}\sqrt{s}(\rho_{K^+K^-}(s) + \rho_{K^0\bar{K}^0}(s))/2, \tag{17}$$

$$F_{\pi\eta'}(s) = f_{\pi\eta'}\sqrt{s - m^{(+)2}_{\pi\eta'}}. \tag{18}$$

We now turn to the curves shown in Fig. 4(b) and discuss the contribution from the heavy (probably existent [23]) a_0' resonance with a mass $m_{a_0'} \approx 1.4$ GeV (see variant 1 in Table I). There is a distinct enhancement in the region of 1.4 GeV in the cross section corresponding to the amplitude $M_{\rm res}^{\rm direct}(s)$ (see the short-dashed line). This enhancement turns into the shoulder in the cross section that corresponds to the total resonance contribution (see the dotted line), i.e., the combined contribution from the amplitude $M_{\rm res}^{\rm direct}(s)$ and the rescattering amplitudes proportional to $T^{\rm res}_{ab\to\pi^0\eta}(s)$; see Eq. (8). The proper resonance signal is practically absent in the region of 1.4 GeV in the total cross section σ_0 (see solid line 2) involving additionally the $\gamma\gamma\to\pi^0\eta$ Born contribution and the rescattering $\gamma\gamma\to\pi^0\eta\to\pi^0\eta$ caused by the background elastic amplitude $\pi^0\eta\to\pi^0\eta$. Thus, there is strong destructive interference between the different contributions, which hides the a_0' resonance in the $\gamma\gamma\to\pi^0\eta$ cross section [37]. Nevertheless, in many respects owing to the a_0', we succeed in modeling a large smooth background required by the Belle data [1] both under the $a_2(1320)$ and between $a_0(980)$ and $a_2(1320)$ resonances.

Note that the a_0' mass is not well determined for the existing compensations and the correlations between the fitted parameters. Even if one puts $m_{a_0'}\approx 1.28$ GeV (instead of 1.4 GeV), then one can obtain a reasonable description of the data. The fit variant (denoted as variant 2) demonstrates this fact by means of Figs. 4(d)–4(f), 5(c), and 5(d) and Table I. Moreover, this variant gives some idea of model uncertainties of the other parameter values and possible variations of the pattern of the $\pi\eta$ scattering amplitude [38].

It is interesting to consider the $\gamma\gamma\to\pi^0\eta$ cross section attributed only to the resonance contributions and, following [4,13,14,27], to determine the width of the $a_0(980)\to\gamma\gamma$ decay averaged over the resonance mass distribution in the $\pi\eta$ channel:

$$\langle\Gamma_{a_0\to\gamma\gamma}\rangle_{\pi\eta}=\int_{0.9\ \rm GeV}^{1.1\ \rm GeV}\frac{s}{4\pi^2}\sigma_{\rm res}(\gamma\gamma\to\pi^0\eta;s)d\sqrt{s} \quad (19)$$

[the integral is taken over the region occupied by the $a_0(980)$ resonance]. This quantity is an adequate characteristic of the coupling of the $a_0(980)$ resonance with a $\gamma\gamma$ pair. Taking into account in $\sigma_{\rm res}$ the contributions from all the rescatterings and direct transitions into $\gamma\gamma$, we obtain $\langle\Gamma_{a_0\to(K\bar K+\pi\eta+\pi\eta'+{\rm direct})\to\gamma\gamma}\rangle_{\pi\eta}\approx 0.4$ keV. For the rescatterings only $\langle\Gamma_{a_0\to(K\bar K+\pi\eta+\pi\eta')\to\gamma\gamma}\rangle_{\pi\eta}\approx 0.23$ keV and for the direct transitions only $\langle\Gamma^{\rm direct}_{a_0\to\gamma\gamma}\rangle_{\pi\eta}\approx 0.028$ keV. These estimates correspond to variant 1. Variant 2 gives the close values.

IV. CONCLUSION

We have analyzed in detail the Belle data on the cross section for the reaction $\gamma\gamma\to\pi^0\eta$ with the use of a sufficiently full dynamical model and have shown that the experimentally observed pattern is the result of the combination of many dynamical factors. In particular, the performed analysis allows the definite conclusion concerning the dominance of the rescattering mechanisms $a_0(980)\to(K\bar K+\pi^0\eta+\pi^0\eta')\to\gamma\gamma$, i.e., four-quark transitions, in the decay $a_0(980)\to\gamma\gamma$. In turn, this gives a new argument in favor of the $q^2\bar q^2$ nature of the $a_0(980)$ resonance. As to the ideal $q\bar q$ model prediction for the two-photon decay widths of the $f_0(980)$ and $a_0(980)$ mesons, $\Gamma_{f_0\to\gamma\gamma}/\Gamma_{a_0\to\gamma\gamma}=25/9$, it is excluded by experiment. One more result of our analysis consists in the preliminary information obtained for the first time on the S wave amplitude of the reaction $\pi^0\eta\to\pi^0\eta$.

The investigations of the mechanisms of the reactions $\gamma\gamma\to\pi^+\pi^-$, $\gamma\gamma\to\pi^0\pi^0$, $\gamma\gamma\to\pi^0\eta$, $\gamma\gamma\to K^+K^-$, and $\gamma\gamma\to K^0\bar K^0$ are undoubtedly an important constituent of physics of light scalar mesons. The Belle Collaboration has investigated the reactions $\gamma\gamma\to\pi^+\pi^-$, $\gamma\gamma\to\pi^0\pi^0$, and $\gamma\gamma\to\pi^0\eta$ with the highest statistics. However, similar information is still lacking for the processes $\gamma\gamma\to K^+K^-$ and $\gamma\gamma\to K^0\bar K^0$. The S wave contributions near thresholds of these two channels are not clearly understood [15,22,33].

To conclude, we point to the promising possibility of investigating the nature of the light scalar mesons $\sigma(600)$, $f_0(980)$, and $a_0(980)$ in the $\gamma\gamma^*$ collisions, where γ^* is a virtual photon with virtuality Q^2. If they are four-quark states, their contributions to the $\gamma\gamma^*\to\pi^+\pi^-$, $\gamma\gamma^*\to\pi^0\pi^0$, and $\gamma\gamma^*\to\pi^0\eta$ cross sections should decrease with increasing Q^2 more rapidly than the contributions from the classical tensor mesons $f_2(1270)$ and $a_2(1320)$. A similar behavior of the contribution from the $q^2\bar q^2$ exotic resonance state with $I^G(J^{PC})=2^+(2^{++})$ [7] to the $\gamma\gamma^*\to\rho^0\rho^0$ and $\gamma\gamma^*\to\rho^+\rho^-$ cross sections was recently observed by the L3 Collaboration [39].

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[1] S. Uehara *et al.* (Belle Collaboration), Phys. Rev. D **80**, 032001 (2009).

[2] D. Antreasyan *et al.* (Crystal Ball), Phys. Rev. D **33**, 1847 (1986).

[3] T. Oest *et al.* (JADE Collaboration), Z. Phys. C **47**, 343 (1990).

[4] N. N. Achasov and G. N. Shestakov, Z. Phys. C **41**, 309 (1988).

[5] R. L. Jaffe, Phys. Rev. D **15**, 267 (1977); **15**, 281 (1977).

[6] N. N. Achasov, S. A. Devyanin, and G. N. Shestakov,

Phys. Lett. **96B**, 168 (1980); Usp. Fiz. Nauk **142**, 361 (1984) [Sov. Phys. Usp. **27**, 161 (1984)].

[7] N. N. Achasov, S. A. Devyanin, and G. N. Shestakov, Phys. Lett. **108B**, 134 (1982); Z. Phys. C **16**, 55 (1982); **27**, 99 (1985); N. N. Achasov and G. N. Shestakov, Usp. Fiz. Nauk **161**, 6, 53 (1991) [Sov. Phys. Usp. **34**, 471 (1991)].

[8] N. N. Achasov and V. N. Ivanchenko, Nucl. Phys. **B315**, 465 (1989).

[9] N. N. Achasov, Usp. Fiz. Nauk **168**, 1257 (1998) [Phys. Usp. **41**, 1149 (1998)]; Nucl. Phys. **A728**, 425 (2003).

[10] N. N. Achasov and A. V. Kiselev, Phys. Rev. D **68**, 014006 (2003).

[11] N. N. Achasov, *15th International Seminar on High Energy Physics: Quarks-2008, Sergiev Posad, Russia*, http://quarks.inr.ac.ru/2008/.

[12] Such an interpretation of the $\gamma\gamma \to \pi^0\eta$ data testifies in favor of the $q^2\bar{q}^2$ nature of the $a_0(980)$ state and agrees with the previous interpretation [13–15] of the high-statistics Belle data on the reactions $\gamma\gamma \to \pi^+\pi^-$ [16] and $\gamma\gamma \to \pi^0\pi^0$ [17] and with the properties of the $\sigma(600)$ and $f_0(980)$ resonances that are the partners of the $a_0(980)$. In particular, in Refs. [13,14] it was shown that the $\sigma(600) \to \gamma\gamma$ and $f_0(980) \to \gamma\gamma$ decays are due basically to the rescattering mechanisms, i.e., to four-quark transitions such as $\sigma(600) \to \pi^+\pi^- \to \gamma\gamma$ and $f_0(980) \to K^+K^- \to \gamma\gamma$. As for the $\sigma(600)$ and $f_0(980)$ direct coupling constants to $\gamma\gamma$, they are small. A similar conclusion about the direct coupling of the $\sigma(600)$ to $\gamma\gamma$ has also been obtained in Ref. [18]. Thus, the mechanisms of the $\gamma\gamma$ decays of light scalars $\sigma(600)$, $f_0(980)$, and $a_0(980)$ differ in principle from the mechanism of the $\gamma\gamma$ decays of classical P wave tensor $q\bar{q}$ mesons $f_2(1270)$, $a_2(1320)$, and $f_2'(1525)$, which is the direct annihilation transition $q\bar{q} \to \gamma\gamma$ [11].

[13] N. N. Achasov and G. N. Shestakov, Phys. Rev. D **72**, 013006 (2005); Phys. Rev. Lett. **99**, 072001 (2007).

[14] N. N. Achasov and G. N. Shestakov, Phys. Rev. D **77**, 074020 (2008); Pis'ma Zh. Eksp. Teor. Fiz. **88**, 345 (2008) [JETP Lett. **88**, 295 (2008)].

[15] N. N. Achasov and G. N. Shestakov, arXiv:0905.2017.

[16] T. Mori *et al.* (Belle Collaboration), Phys. Rev. D **75**, 051101(R) (2007); J. Phys. Soc. Jpn. **76**, 074102 (2007).

[17] S. Uehara *et al.* (Belle Collaboration), Phys. Rev. D **78**, 052004 (2008).

[18] G. Mennessier, S. Narison, and W. Ochs, Phys. Lett. B **665**, 205 (2008).

[19] H. Osborn, Nucl. Phys. **B15**, 501 (1970); J. L. Petersen, Phys. Rep. **2**, 155 (1971).

[20] V. Bernard, N. Kaiser, and Ulf-G. Meissner, Phys. Rev. D **44**, 3698 (1991).

[21] D. Black, A. H. Fariborz, and J. Schechter, Phys. Rev. D **61**, 074030 (2000).

[22] N. N. Achasov and G. N. Shestakov, Yad. Fiz. **55**, 2999 (1992) [Sov. J. Nucl. Phys. **55**, 1677 (1992)].

[23] C. Amsler *et al.* (Particle Data Group), Phys. Lett. B **667**, 1 (2008).

[24] N. N. Achasov, S. A. Devyanin, and G. N. Shestakov, Phys. Lett. B **88**, 367 (1979); N. N. Achasov and G. N. Shestakov, Phys. Rev. D **58**, 054011 (1998).

[25] N. N. Achasov and V. V. Gubin, Phys. Rev. D **56**, 4084 (1997); **63**, 094007 (2001); N. N. Achasov and A. V. Kiselev, Phys. Rev. D **73**, 054029 (2006); Yad. Fiz. **70**, 2005 (2007) [Phys. At. Nucl. **70**, 1956 (2007)].

[26] N. N. Achasov, S. A. Devyanin, and G. N. Shestakov, Yad. Fiz. **32**, 1098 (1980) [Sov. J. Nucl. Phys. **32**, 566 (1980)].

[27] N. N. Achasov and G. N. Shestakov, Pis'ma Zh. Eksp. Teor. Fiz. **90**, 355 (2009) [JETP Lett. **90**, 313 (2009)].

[28] Properly calculating the accuracy of this approximation is rather difficult. From the chiral Lagrangian standpoint, the off-shell effects are related to the higher order corrections. The phenomenological form factors introduced into the Born exchange amplitudes effectively take into account the deviations caused by such corrections.

[29] For the elementary one-kaon exchange $\tilde{I}^{K^+}_{K^+K^-}(s) = 8\alpha(\frac{m^2_{K^+}}{s}[\pi + i\ln\frac{1+\rho_{K^+K^-}(s)}{1-\rho_{K^+K^-}(s)}]^2 - 1)$ for $s \geq 4m^2_{K^+}$ and $\tilde{I}^{K^+}_{K^+K^-}(s) = 8\alpha(\frac{m^2_{K^+}}{s}[\pi - 2\arctan|\rho_{K^+K^-}(s)|]^2 - 1)$ for $0 \leq s \leq 4m^2_{K^+}$ [4]; $\alpha = 1/137$. In this work, to calculate the function $\tilde{I}^{K^+}_{K^+K^-}(s)$ we use the amplitude $M^{\mathrm{Born}K^+}_{00}(s) = \frac{16\pi\alpha m^2_{K^+}}{s\rho_{K^+K^-}(s)}[\ln\frac{1+\rho_{K^+K^-}(s)}{1-\rho_{K^+K^-}(s)} - \ln\frac{1+\rho_{K^+K^-}(s;x_2)}{1-\rho_{K^+K^-}(s;x_2)}]$, where the second term in square brackets appears for the form factor $G_{K^+}(t,u) = \frac{1}{s}[\frac{m^2_{K^+}-t}{1-(u-m^2_{K^+})/x_2^2} + \frac{m^2_{K^+}-u}{1-(t-m^2_{K^+})/x_2^2}]$, which suppresses the elementary one-kaon exchange amplitudes with increasing $\sqrt{s}$; $\rho_{K^+K^-}(s;x_2) = \rho_{K^+K^-}(s)/(1 + 2x_2^2/s)$ and x_2 is an input parameter. The form factor of such a type was proposed in the work [30]. Note that the form factor is introduced through replacing the elementary one-kaon exchange amplitudes $\hat{M}^{\mathrm{Born}K^+}_0(s,\theta) = (32\pi\alpha m^2_{K^+}/s)/[1 - \rho^2_{K^+K^-}(s)\cos^2\theta]$, $\hat{M}^{\mathrm{Born}K^+}_2(s,\theta) = 8\pi\alpha\rho^2_{K^+K^-}(s)\sin^2\theta/[1 - \rho^2_{K^+K^-}(s) \times \cos^2\theta]$ by $M^{\mathrm{Born}K^+}_\lambda(s,\theta) = G_{K^+}(t,u)\hat{M}^{\mathrm{Born}K^+}_\lambda(s,\theta)$, which keeps gauge invariance beyond the tree approximation [30]. The necessity of the inclusion of the form factors in the Born amplitudes of the elementary one-pion and one-kaon exchanges in the reactions $\gamma\gamma \to \pi^+\pi^-$ and $\gamma\gamma \to K^+K^-$ was recently discussed in Refs. [14,27]; see also Refs. [22,30–33]. In particular, the estimate $x_2 \approx 1.75$ GeV was obtained in Ref. [14] from the simultaneous analysis of the Belle data on the $\gamma\gamma \to \pi^+\pi^-$ and $\gamma\gamma \to \pi^0\pi^0$ cross sections; this estimate is used in this work.

[30] M. Poppe, Int. J. Mod. Phys. A **1**, 545 (1986).

[31] R. P. Johnson, Ph.D. thesis, Stanford University [SLAC-Report-294, 1986].

[32] D. Morgan and M. R. Pennington, Phys. Lett. B **192**, 207 (1987); J. Boyer *et al.* (Mark II Collaboration), Phys. Rev. D **42**, 1350 (1990); H. J. Behrend *et al.* (CELLO Collaboration), Z. Phys. C **56**, 381 (1992).

[33] N. N. Achasov and G. N. Shestakov, Mod. Phys. Lett. A **9**, 1351 (1994).

[34] The analogous rescattering mechanism $\gamma\gamma \to K^+K^- \to f_0(980) \to \pi\pi$ dominates in the $f_0(980)$ resonance production in the reactions $\gamma\gamma \to \pi\pi$ [13,14].

[35] The negative background phase is not news. It was shown in Ref. [36] that in the linear σ model there is a negative background phase in the S wave $\pi\pi$ scattering amplitude with $I = 0$, which hides the σ meson with the result that the $\pi\pi$ scattering phase shift δ_0^0 does not pass over 90° at the putative resonance mass. It has been made clear that

shielding wide lightest scalar mesons in chiral dynamics is very natural [11,13,14,25].
[36] N. N. Achasov and G. N. Shestakov, Phys. Rev. D **49**, 5779 (1994).
[37] A similar compensation between the background and a_0' resonance contributions has been also obtained in the phenomenological fit performed by the authors of the Belle experiment [1].
[38] It should be emphasized that the predictions for the elastic $\pi\eta$ scattering amplitude $T_0^1(s)$ are very preliminary for the large theoretical uncertainties associated with the vector exchanges. The strong model dependence of $T_0^1(s)$ is adequately illustrated in Fig. 5.
[39] P. Achard *et al.* (L3 Collaboration), Phys. Lett. B **568**, 11 (2003); **597**, 26 (2004); **604**, 48 (2004); **615**, 19 (2005).