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# Calculating phases between $B \to K^*\pi$ amplitudes

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A phase $\Delta\Phi$ between amplitudes for $B^0 \to K^{*0}\pi^0$ and $B^0 \to K^{*+}\pi^-$ plays a crucial role in a method for constraining Cabibbo-Kobayashi-Maskawa parameters. We present a general argument for destructive interference between amplitudes for $B^0 \to K^{*+}\pi^-$ and $B^0 \to K^{*0}\pi^0$ forming together a smaller $I(K^*\pi) = 3/2$ amplitude. Applying flavor SU(3) and allowing for conservative theoretical uncertainties, we obtain lower limits on $|\Delta\Phi|$ and its charge conjugate. Values of these two phases favored by the *BABAR* collaboration are in good agreement with our bounds.

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## I. INTRODUCTION

Charmless hadronic $B$ meson decays from $b \to s$ transitions, including $B \to K\pi$, provide useful information about the weak phase $\gamma$ [1–3]. A method for constraining another angle in the $(\bar{\rho}, \bar{\eta})$ plane, formed by the $\bar{\rho}$ axis and a line going through the apex of the unitarity triangle intersecting the $\bar{\rho}$ axis at $\bar{\rho} = 0.24 \pm 0.03$, is based on Dalitz analyses of $B^0 \to K^+\pi^-\pi^0$ and $B^0 \to K_S\pi^+\pi^-$ [4,5]. The first process enables one to determine a phase $\Delta\Phi$ between quasi–two-body decay amplitudes for $B^0 \to K^{*0}\pi^0$ and $B^0 \to K^{*+}\pi^-$,

$$\Delta\Phi \equiv \mathrm{Arg}[A(K^{*0}\pi^0)A^*(K^{*+}\pi^-)]. \tag{1}$$

While this phase appears as a purely experimental quantity in Refs. [4,5], the purpose of this work is to obtain bounds on $|\Delta\Phi|$ and its charge conjugate. Values of these two phases favored by a recent *BABAR* Dalitz analysis of $B \to K^\pm\pi^\mp\pi^0$ [6,7] are in agreement with our bounds, once sign conventions for $K^*$ decays are taken into account. These results are relevant to extraction of the $I = 3/2$ $B \to K^*\pi$ amplitude $A_{3/2}$, whose phase (along with that of the corresponding charge-conjugate amplitude) determines the above-mentioned angle in the $(\bar{\rho}, \bar{\eta})$ plane. We will show that as a result of destructive interference found between $A(K^{*0}\pi^0)$ and $A(K^{*+}\pi^-)$, the magnitude of $A_{3/2}$ is not well enough known to carry out this program.

This paper will be divided into several short sections. Section II introduces conventions for defining two quasi-two-body resonant amplitudes, $A(K^{*+}\pi^-)$ and $A(K^{*0}\pi^0)$, contributing to $B^0 \to K^+\pi^-\pi^0$. In Sec. III we present a qualitative argument for destructive interference between these two amplitudes forming together an $I(K^*\pi) = 3/2$ amplitude. Crude estimates for $\Delta\Phi$ and its charge conjugate $\Delta\bar{\Phi}$ obtained in Sec. IV assuming flavor SU(3) are improved in Sec. V by including uncertainties from SU(3) breaking and small contributions. Section VI concludes by comparing our bounds on $\Delta\Phi$ and $\Delta\bar{\Phi}$ with recent experimental results obtained by the *BABAR* collaboration.

## II. CONVENTIONS FOR RESONANT AMPLITUDES IN $B^0 \to K^+\pi^-\pi^0$

The conventions for three-body decays, stated explicitly below Eq. (8) of Ref. [6], are illustrated in Fig. 1. Each quasi–two-body subsystem of the three-body decay $B^0 \to K^+\pi^-\pi^0$, as viewed in the rest frame of the vector meson, contains pseudoscalar decay products of the vector meson with momenta $\mathbf{q}$ and $-\mathbf{q}$ and a bachelor pseudoscalar with momentum $\mathbf{p}$. Specifically, the phase conventions adopted in Ref. [6] are such that (a) for $B^0 \to K^{*0}\pi^0$, the $K^{*0}$ decay particle with momentum $\mathbf{q}$ is a $\pi^-$, while the bachelor particle with momentum $\mathbf{p}$ is a $\pi^0$; (b) for $B^0 \to K^{*+}\pi^-$ the $K^{*+}$ decay product with momentum $\mathbf{q}$ is a $K^+$, while the bachelor particle with momentum $\mathbf{p}$ is a $\pi^-$; and (c) for $B^0 \to \rho^-K^+$ the $\rho^-$ decay product with momentum $\mathbf{q}$ is a $\pi^0$, while the bachelor particle with momentum $\mathbf{p}$ is a $K^+$. These enter into a tensor $T = -2\mathbf{p}\cdot\mathbf{q}$ describing the matrix element in the Zemach formalism [8].

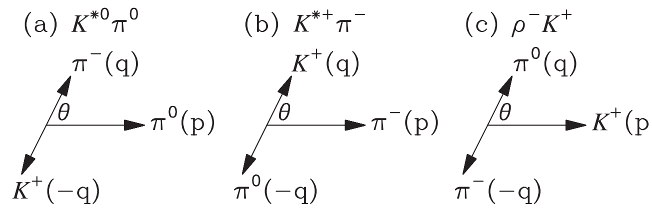


FIG. 1. Convention of Ref. [6] for quasi–two-body subsystems in the three-body decays $B^0 \to K^+\pi^-\pi^0$. (a) $B^0 \to K^{*0}\pi^0$; (b) $B^0 \to K^{*+}\pi^-$; (c) $B^0 \to \rho^-K^+$.

 

One must be careful to use Clebsch-Gordan coefficients appropriate to these phase conventions when constructing $B^0 \to K^{*0}\pi^0$ and $B^0 \to K^{*+}\pi^-$ amplitudes from Dalitz-plot fits. The interchange of the two final-state particles in $K^* \to K\pi$ causes a sign change as a result of the property

$$(j_2 m_2 j_1 m_1 | jm) = (-1)^{j-j_1-j_2}(j_1 m_1 j_2 m_2 | jm) \qquad (2)$$

of the Clebsch-Gordan coefficients [9]. As we choose to use the same order ($j_1 = 1$, $j_2 = 1/2$) in describing both $K^{*0}$ and $K^{*+}$ decays, our relative phases of $A(K^{*+}\pi^-)$ and $A(K^{*0}\pi^0)$ and their charge conjugates will be those of Refs. [6,7] shifted by 180°. In our convention a combination of amplitudes for an $I = 3/2$ final $K^*\pi$ state may be written as

$$3A_{3/2} \equiv A(K^{*+}\pi^-) + \sqrt{2}A(K^{*0}\pi^0), \qquad (3)$$

whose magnitude is determined by measuring the magnitudes of the two amplitudes on the right-hand side and their relative phase. We will argue, first qualitatively and then quantitatively, that in the Cabibbo-Kobayashi-Maskawa (CKM) framework, these two amplitudes add destructively in (3), implying that the amplitude $3A_{3/2}$ is smaller in magnitude than either of these two amplitudes.

## III. AN ARGUMENT FOR DESTRUCTIVE INTERFERENCE

Destructive interference in (3) follows qualitatively in the CKM framework from the cancellation of a $\Delta I = 0$ penguin amplitude dominating the two $B \to K^*\pi$ amplitudes on the right-hand side [10]. The remaining terms on the right-hand side, consisting of electroweak penguin (EWP) and tree amplitudes, are considerably smaller than the penguin amplitude. This is demonstrated by decomposing physical amplitudes into graphical contributions representing distinct flavor topologies [11,12], each of which involves an unknown strong phase,

$$-A(K^{*+}\pi^-) = \lambda_t^{(s)}\left(P_{tc,P} + \tfrac{2}{3}P^C_{\mathrm{EW},P}\right) + \lambda_u^{(s)}(P_{uc,P} + T_P),$$
$$\sqrt{2}A(K^{*0}\pi^0) = \lambda_t^{(s)}\left(P_{tc,P} - P_{\mathrm{EW},V} - \tfrac{1}{3}P^C_{\mathrm{EW},P}\right) + \lambda_u^{(s)}(P_{uc,P} - C_V). \qquad (4)$$

This implies

$$3A_{3/2} = -\lambda_t^{(s)}(P_{\mathrm{EW},V} + P^C_{\mathrm{EW},P}) - \lambda_u^{(s)}(T_P + C_V). \qquad (5)$$

The two CKM factors $\lambda_q^{(q')} \equiv V^*_{qb}V_{qq'}(q = u, t; q' = d, s)$ have a very small ratio $|\lambda_u^{(s)}|/|\lambda_t^{(s)}| \simeq 0.02$ [9]. The dominant term multiplying $\lambda_t^{(s)}$ in the two $B \to K^*\pi$ amplitudes is the penguin contribution $P_{tc,P}$, while the EWP contributions $P_{\mathrm{EW},V}$ and $P^C_{\mathrm{EW},P}$ are smaller, as they are higher order in the electroweak coupling. Thus the dominant penguin contributions cancel in $3A^{K^*\pi}_{3/2}$, which consists of two smaller contributions: EWP terms multiplying $\lambda_t^{(s)}$ and a combination of tree amplitudes $T_P + C_V$ multiplying a very small CKM factor $\lambda_u^{(s)}$.

## IV. AN APPROXIMATE CALCULATION OF $\Delta\Phi$ AND $\Delta\bar{\Phi}$

In order to study quantitatively the interference between the two $B \to K^*\pi$ amplitudes in (3), we make use of two model-independent relations:

(i) Proportionality relations between tree and EWP operators in the $|\Delta S| = |\Delta I| = 1$ effective Hamiltonian, in which one neglects EWP operators $\mathcal{O}_7$ and $\mathcal{O}_8$ with tiny Wilson coefficients, imply the following expression for the EWP $I(K^*\pi) = 3/2$ amplitude in terms of tree amplitudes [13,14],

$$P_{\mathrm{EW},V} + P^C_{\mathrm{EW},P} = -\frac{3\mathcal{K}}{2}(T_V + C_P). \qquad (6)$$

Here, $\mathcal{K}$ is a ratio of Wilson coefficients [15], $\mathcal{K} \equiv (c_9 + c_{10})/(c_1 + c_2) \approx (c_9 - c_{10})/(c_1 - c_2) = -0.0087$.

(ii) In the flavor SU(3) limit, amplitudes for $B \to \rho\pi$ decays are given in terms of the same reduced SU(3) amplitudes (i.e., the same graphical amplitudes) contributing to $B \to K^*\pi$, but involve different CKM factors. Thus, neglecting tiny EWP amplitudes and annihilation contributions $A_P - A_V$ [16,17], one has

$$-\sqrt{2}A(\rho^+\pi^0) = \lambda_u^{(d)}(T_P + C_V) - \lambda_t^{(d)}(P_V - P_P),$$
$$-\sqrt{2}A(\rho^0\pi^+) = \lambda_u^{(d)}(T_V + C_P) + \lambda_t^{(d)}(P_V - P_P). \qquad (7)$$

In the same limit, amplitudes for $\Delta S = 0$ $B^+ \to K^*K$ decays are expressed in terms of penguin amplitudes $P_P$, $P_V$ (again after neglecting small EWP and annihilation contributions),

$$A(\bar{K}^{*0}K^+) = \lambda_t^{(d)}P_P, \qquad A(K^{*+}\bar{K}^0) = \lambda_t^{(d)}P_V. \qquad (8)$$

Here, $P_P \equiv P_{tc,P}$ contributes to $B \to K^*\pi$ amplitudes in (4). Contributions of annihilation amplitudes and terms $\lambda_u^{(d)}P_{uc,P}$, $\lambda_u^{(d)}P_{uc,V}$ which have been omitted in (7) and (8), respectively, will be included later on.

In order to obtain first a rough estimate for $3A_{3/2}$, $\Delta\Phi$, and their charge conjugates we will work at this point in the SU(3) symmetry approximation, which is expected to introduce an uncertainty of about 20%–30% in amplitudes. For now, we will also neglect penguin contributions in (7) which can be estimated to be of the same order,

$$\frac{|\lambda_t^{(d)}P_P|}{|\lambda_u^{(d)}T_P|} \simeq \sqrt{\frac{\mathcal{B}(\bar{K}^{*0}K^+)}{r_\tau\mathcal{B}(\rho^+\pi^-)}} = 0.20 \pm 0.03. \qquad (9)$$

We have used decay branching ratios and a lifetime ratio,

$r_\tau \equiv \tau_{B^+}/\tau_{B^0} = 1.071 \pm 0.009$, from Ref. [18]. Thus we take

$$-\sqrt{2}A(\rho^+\pi^0) \simeq \lambda_u^{(d)}(T_P + C_V),$$
$$-\sqrt{2}A(\rho^0\pi^+) \simeq \lambda_u^{(d)}(T_V + C_P). \quad (10)$$

Flavor SU(3) symmetry breaking in the amplitudes $T_P + C_V$ and $T_V + C_P$ and uncertainties caused by neglecting penguin amplitudes will be included in the analysis at a later point.

We denote $\tilde{\lambda} \equiv \lambda/(1 - \lambda^2/2) = 0.232$, where $\lambda$ is the Wolfenstein parameter [19], and use the central value for CKM parameters [9],

$$\frac{3\mathcal{K}}{2}\frac{\lambda_t^{(s)}}{\lambda_u^{(s)}} = 0.61e^{-i\gamma}. \quad (11)$$

We checked that uncertainties of 10% in the magnitude of this ratio and a few degrees in its strong phase [20] have an insignificant effect on the subsequent analysis. Combining Eqs. (5), (6), (10), and (11), we obtain in this approximation,

$$3A_{3/2} \simeq \tilde{\lambda}\sqrt{2}(A(\rho^+\pi^0) - 0.61e^{-i\gamma}A(\rho^0\pi^+)). \quad (12)$$

We will now use this approximate expression in order to evaluate the magnitude of $3A_{3/2}$ and its $CP$ conjugate.

$CP$-averaged branching ratios and $CP$ asymmetries for relevant $B \to K^*\pi$ and $B \to \rho\pi$ decays are given in Table I [7,18]. The $CP$ asymmetries in $B^+ \to \rho^+\pi^0$ and $B^+ \to \rho^0\pi^+$ are consistent with zero within errors and will be taken to vanish at this point. We quote $B^+ \to \rho\pi$ amplitudes in units of $10^{-3}$, given by square roots of central values for branching ratios divided by the lifetime ratio $\tau_B$. The relative phase between these two amplitudes, which is dominantly a strong phase as shown in (10), will be denoted by

$$\phi \equiv \mathrm{Arg}[A(\rho^0\pi^+)A^*(\rho^+\pi^0)]. \quad (13)$$

Omitting an overall phase of $A(\rho^+\pi^0)$, we obtain numerically

TABLE I. Branching fractions and $CP$ asymmetries for $B \to K^*\pi$, $\rho\pi$. For $B \to K^{*+}\pi^-$, we calculate averages of recent *BABAR* measurements [7] and Belle measurements [21]; for $B^0 \to K^{*0}\pi^0$, we take values from Ref. [7] as Belle has so far obtained only a loose upper limit on this mode [22], while for $B \to \rho\pi$ we quote values in [18].

| Mode | $\mathcal{B}(10^{-6})$ | $A_{CP}$ |
|---|---|---|
| $B^0 \to K^{*+}\pi^-$ | $8.2 \pm 1.0$ | $-0.26 \pm 0.08$ |
| $B^0 \to K^{*0}\pi^0$ | $3.3 \pm 0.6$ | $-0.15 \pm 0.13$ |
| $B^+ \to \rho^+\pi^0$ | $10.9^{+1.4}_{-1.5}$ | $0.02 \pm 0.11$ |
| $B^+ \to \rho^0\pi^+$ | $8.3^{+1.2}_{-1.3}$ | $0.18^{+0.09}_{-0.17}$ |

$$3A_{3/2} = 1.05 - 0.56e^{i(\phi-\gamma)},$$
$$3\bar{A}_{3/2} = 1.05 - 0.56e^{i(\phi+\gamma)}, \quad (14)$$

where $\bar{A}_{3/2}$ is the corresponding amplitude for $\bar{B}^0$ decays.

The phase difference $\phi$ is measurable by constructing geometrically an isospin pentagon for the five $B^{0,+} \to 3\pi$ decay amplitudes [23,24]. The measured $CP$-averaged $B \to \rho\pi$ branching ratios are consistent with an approximately flat pentagon [10] which would correspond to $\phi \simeq 0$. However, these branching ratios permit also a nonflat pentagon. Moreover, large values of $\phi$ cannot be excluded because of sizable experimental errors [25,26]. Theoretically, one expects this phase to be small. QCD factorization predicts its suppression by $\alpha_s(m_b)$ and $1/m_b$. Taking $\phi = 0$ and using a value $\gamma = 65°$ favored by fits to CKM parameters [27,28], one obtains

$$3|A_{3/2}| = 3|\bar{A}_{3/2}| = 0.96, \quad \text{for } \phi = 0. \quad (15)$$

For nonzero values of $\phi$, one of these amplitudes decreases while the other increases. For instance,

$$3|A_{3/2}| = 0.59, \qquad 3|\bar{A}_{3/2}| = 1.57, \quad \text{for } \phi = 90°. \quad (16)$$

The maximal value of $3|A_{3/2}|$ (or $3|\bar{A}_{3/2}|$) is 1.61.

In order to calculate $|\Delta\Phi|$ and $|\Delta\bar{\Phi}|$, the above values of $3|A_{3/2}|$ and $3|\bar{A}_{3/2}|$ may be combined with $|A(K^{*+}\pi^-)|$, $\sqrt{2}|A(K^{*0}\pi^0)|$ and their charge conjugates, also expressed in units of $10^{-3}$. Using central values of corresponding branching ratios in Table I and neglecting $CP$ asymmetries in these processes, one has

$$|A(K^{*-}\pi^+)| = |A(K^{*+}\pi^-)| = 2.86,$$
$$\sqrt{2}|A(\bar{K}^{*0}\pi^0)| = \sqrt{2}|A(K^{*0}\pi^0)| = 2.57. \quad (17)$$

Comparing the smaller amplitudes (15) for $\phi = 0$ with the larger amplitudes (17), we conclude there is a strong destructive interference in (3) and in its charge conjugate, corresponding to phase differences

$$|\Delta\Phi| = |\Delta\bar{\Phi}| = 161°. \quad (18)$$

For $\phi \neq 0$, one of this phases becomes larger than this value while the other phase becomes smaller reaching a minimum value of 146°.

## V. INCLUDING SU(3) BREAKING AND PENGUINS IN $B \to \rho\pi$

The values of $I = 3/2$ amplitudes (15) and (16) and the phases (18) were obtained neglecting several corrections. These include penguin amplitudes which have been neglected in (7) and consequently in (12), and effects of SU(3) breaking in relations between tree amplitudes in $\Delta S = 1$ and $\Delta S = 0$ decays. Including these corrections, Eq. (12) is now replaced by

$$3A_{3/2} = \tilde{\lambda}\sqrt{2}(A(\rho^+\pi^0)R_1 - 0.61e^{-i\gamma}A(\rho^0\pi^+)R_2) + \tilde{\lambda}(1 + 0.61e^{-i\gamma})(A(\bar{K}^{*0}K^+) - A(K^{*+}\bar{K}^0)). \tag{19}$$

$R_{1,2}$ are SU(3) breaking parameters, while the second line describes penguin contributions. We do not include similar SU(3) breaking factors in the latter contributions. We checked that such factors would have a very small effect on constraining $\Delta\Phi$ and $\Delta\bar{\Phi}$ once uncertainties in penguin amplitudes are maximized as discussed below. Although in the above derivation we seem to have neglected annihilation amplitudes and penguin contributions $P_{uc,P}$ and $P_{uc,V}$ involving a CKM factor $\lambda_u^{(s)}$, Eq. (19) is exact in the SU(3) limit $R_1 = R_2 = 1$ and does not neglect any amplitude.

We start by discussing the uncertainty caused by neglecting the contribution of penguin amplitudes. As shown in (9), $P_P$ contributes to $A(B^+ \to \rho^+\pi^0)$ about 20% of its magnitude. One may assume $|P_V| \simeq |P_P|$ [29] on the basis of approximately equal branching ratios measured for $B^+ \to K^0\rho^+$ and $B^+ \to K^{*0}\pi^+$ [18]. To be most conservative, we will maximize the uncertainty caused by the combination $P_V - P_P$ by assuming that the two penguin amplitudes involve a relative minus sign, $P_V \simeq -P_P$ [30]. Thus, neglecting $P_V - P_P$ in the two $B^+ \to \rho\pi$ amplitudes (7) introduces a maximal uncertainty of about 40% in each amplitude. Including the CKM factors in (19), we find that the penguin amplitudes may contribute at most 50% of the contribution of the first line in Eq. (19).

In the presence of these penguin contributions, the phase $\phi$ defined in (13) is not a purely $CP$-invariant strong phase as we have assumed when obtaining the structure (14). Denoting

$$\bar{\phi} \equiv \mathrm{Arg}[A(\rho^0\pi^-)A^*(\rho^-\pi^0)], \tag{20}$$

$\bar{\phi}$ now replaces $\phi$ in the expression for $3\bar{A}_{3/2}$. In general, one has $\bar{\phi} \neq \phi$. The difference between these two phases is suppressed by the ratio of penguin and tree amplitudes in $B^+ \to \rho\pi$. As mentioned, $\phi$ and $\bar{\phi}$ are measurable by constructing the $B \to \rho\pi$ isospin pentagons for $B$ and $\bar{B}$.

SU(3) breaking in $T_P + C_V$ and $T_V + C_P$ may be estimated using naive factorization. We use this estimate as an example illustrating the small effect of SU(3) breaking on the values of $\Delta\Phi$ and $\Delta\bar{\Phi}$. In $B \to K^*\pi$ one has

$$\begin{aligned} T_P + C_V &\propto a_1 f_{K^*} F_0^{B\pi} + a_2 f_\pi A_0^{BK^*}, \\ T_V + C_P &\propto a_2 f_{K^*} F_0^{B\pi} + a_1 f_\pi A_0^{BK^*}, \end{aligned} \tag{21}$$

where [15]

$$\begin{aligned} a_1 &= c_1 + c_2/3, \qquad a_2 = c_2 + c_1/3, \\ c_1 &= 1.079, \qquad c_2 = -0.178, \end{aligned} \tag{22}$$

and $f_\pi = 131$ MeV, $f_{K^*} = 218 \pm 4$ MeV, $F_0^{B\pi} = 0.28 \pm 0.05$, $A_0^{BK^*} = 0.45 \pm 0.07$ [9,31]. The corresponding tree amplitudes for $B \to \rho\pi$ are given by similar expressions replacing $K^* \to \rho$. The relevant decay constant and form factor are $f_\rho = 209 \pm 1$ MeV and $A_0^{B\rho} = 0.37 \pm 0.06$. Using central values for form factors, we obtain

$$\begin{aligned} R_1 &\equiv \frac{(T_P + C_V)_{K^*\pi}}{(T_P + C_V)_{\rho\pi}} = 1.07, \\ R_2 &\equiv \frac{(T_V + C_P)_{K^*\pi}}{(T_V + C_P)_{\rho\pi}} = 1.19. \end{aligned} \tag{23}$$

These SU(3) breaking factors multiply $A(\rho^+\pi^0)$ and $A(\rho^0\pi^+)$ in Eq. (19). As we will see below, these SU(3) breaking corrections do not affect significantly constraints on the phases $\Delta\Phi$ and $\Delta\bar{\Phi}$. Therefore, we will not include in these constraints errors caused by uncertainties in $B$ decay form factors.

We will now study constraints on $|\Delta\Phi|$ and $|\Delta\bar{\Phi}|$ which include experimental errors in branching ratios and $CP$ asymmetries in $B^+ \to \rho^+\pi^0$, $B^+ \to \rho^0\pi^+$, $B^0 \to K^{*+}\pi^-$, $B^0 \to K^{*0}\pi^0$. We take a range for $\gamma$ [27], $\gamma = (68 \pm 4)°$, and theoretical uncertainties from penguin amplitudes in $B^+ \to \rho\pi$ decays as described above. All errors are added in quadrature. The numerical SU(3) breaking factors in (23) will be used. Figure 2 shows resulting plots for bounds on $|\Delta\Phi|$ and $|\Delta\bar{\Phi}|$ as functions of $\phi$ and $\bar{\phi}$, respectively, in the ranges $-180° \le \phi$, $\bar{\phi} \le 180°$. The three solid lines in each plot describe lower, central, and upper values at $1\sigma$ for the two phases. The plots were obtained by taking symmetric errors in $\cos(\Delta\Phi)$ and $\cos\Delta\bar{\Phi}$. This assumes that these two variables are linear functions of the input parameters. The broken lines in Fig. 2 describe central values of $|\Delta\Phi|$ and $|\Delta\bar{\Phi}|$ for the SU(3) symmetric case. The few degree difference between the broken line and the central solid line demonstrates the small effect of SU(3) breaking on the allowed ranges of $|\Delta\Phi|$ and $|\Delta\bar{\Phi}|$.

Using Fig. 2 and assuming normal distributions for $\cos\Phi$ and $\cos\bar{\Phi}$ as functions of the input parameters, we conclude the following lower limits at 95% confidence level:

$$|\Delta\Phi| \ge 131°, \qquad |\Delta\bar{\Phi}| \ge 119°. \tag{24}$$

These lower bounds correspond to the minimal values of $|\Delta\Phi|$, $|\Delta\bar{\Phi}|$ in Fig. 2 which are obtained at $\phi = -180° + \gamma$, $\bar{\phi} = 180° - \gamma$. The bounds should be considered conservative as the magnitudes of the measurable phases $\phi$ and $\bar{\phi}$ are not expected to be larger than 90°.

For completeness, we plot in Fig. 3 the predicted amplitudes $3|A_{3/2}|$ and $3|\bar{A}_{3/2}|$ as functions of $\phi$ and $\bar{\phi}$, respectively. Amplitudes in units of $10^{-3}$ are given by square roots of corresponding branching ratios. We note that the two $I = 3/2$ amplitudes are different from zero except for restricted ranges of the phases $\phi$ and $\bar{\phi}$, $\phi \sim 50°$–$80°$, $\bar{\phi} \sim (-80°) - (-50°)$.

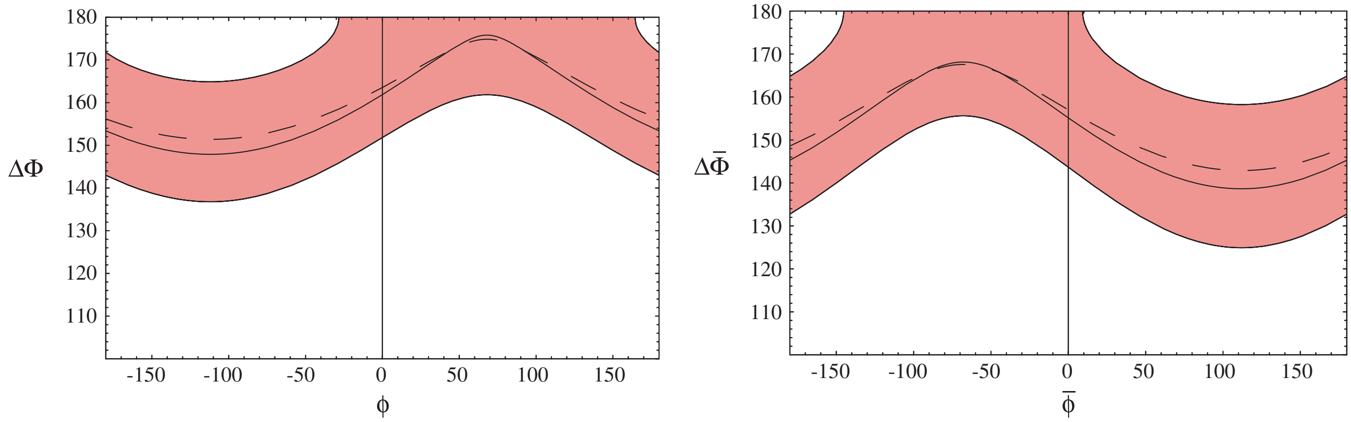


FIG. 2 (color online). Bounds on $\Delta\Phi$ as function of $\phi$ (left), and on $\Delta\bar{\Phi}$ as function of $\bar{\phi}$ (right). Three solid lines describe lower, central, and upper values of $\Delta\Phi$ at $1\sigma$. Broken line corresponds to central value for the SU(3) symmetric case.

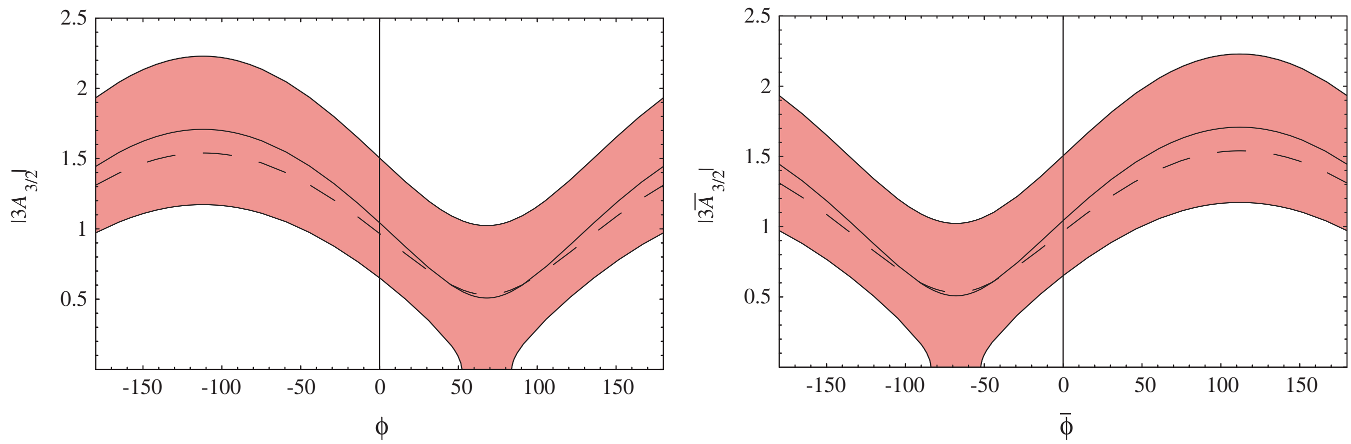


FIG. 3 (color online). Magnitude of isospin amplitude $|3A_{3/2}|$ (left) and $|3\bar{A}_{3/2}|$ (right) in units of $10^{-3}$ as function of $\phi$. Lines as in Fig. 2.

## VI. CONCLUSION: COMPARISON WITH *BABAR* RESULTS

We now compare our lower bounds on $|\Delta\Phi|$ and $|\Delta\bar{\Phi}|$ with values reported by *BABAR* in Ref. [6] and in a recent update [7]. Performing a maximum likelihood fit to 4583 $B \to K^\pm\pi^\mp\pi^0$ events, four solutions were found for $\Delta\Phi' \equiv \Delta\Phi - \pi$ and $\Delta\bar{\Phi}' \equiv \Delta\bar{\Phi} - \pi$ with minimum values of the negative likelihood function (NLL). Results of the two analyses are presented in Table II, quoting for each of the four solutions values for $\Delta\Phi'$, $\Delta\bar{\Phi}'$, and $\Delta$(NLL), the difference in units of NLL with respect to the most likely solution (I). We will compare our bounds to the updated results in Ref. [7].

Solution I with the highest probability favors small values of $\Delta\Phi'$ and $\Delta\bar{\Phi}'$ consistent with zero, or large

TABLE II. Four solutions for $\Delta\Phi' \equiv \Delta\Phi - \pi$ and $\Delta\bar{\Phi}' \equiv \Delta\bar{\Phi} - \pi$ with minimum values of the NLL measured in $B \to K^\pm\pi^\mp\pi^0$. Statistical and systematic errors are added in quadrature. The first three values in each column are taken from [6]. The last three values are the results of a very recent update [7].

| | | Solution I | Solution II | Solution III | Solution IV |
|---|---|---|---|---|---|
| Reference [6] | $\Delta\Phi'$ | $(-21 \pm 35)°$ | $(-134 \pm 30)°$ | $(-22 \pm 30)°$ | $(-139 \pm 30)°$ |
| | $\Delta\bar{\Phi}'$ | $(-5 \pm 34)°$ | $(-5 \pm 33)°$ | $(-163 \pm 33)°$ | $(-163 \pm 33)°$ |
| | $\Delta$(NLL) | 0 | 3.94 | 7.77 | 10.57 |
| Reference [7] | $\Delta\Phi'$ | $(-22 \pm 39)°$ | $(-139 \pm 40)°$ | $(-22 \pm 39)°$ | $(-140 \pm 40)°$ |
| | $\Delta\bar{\Phi}'$ | $(-5 \pm 36)°$ | $(-4 \pm 36)°$ | $(-163 \pm 35)°$ | $(-163 \pm 35)°$ |
| | $\Delta$(NLL) | 0 | 5.43 | 7.04 | 12.33 |

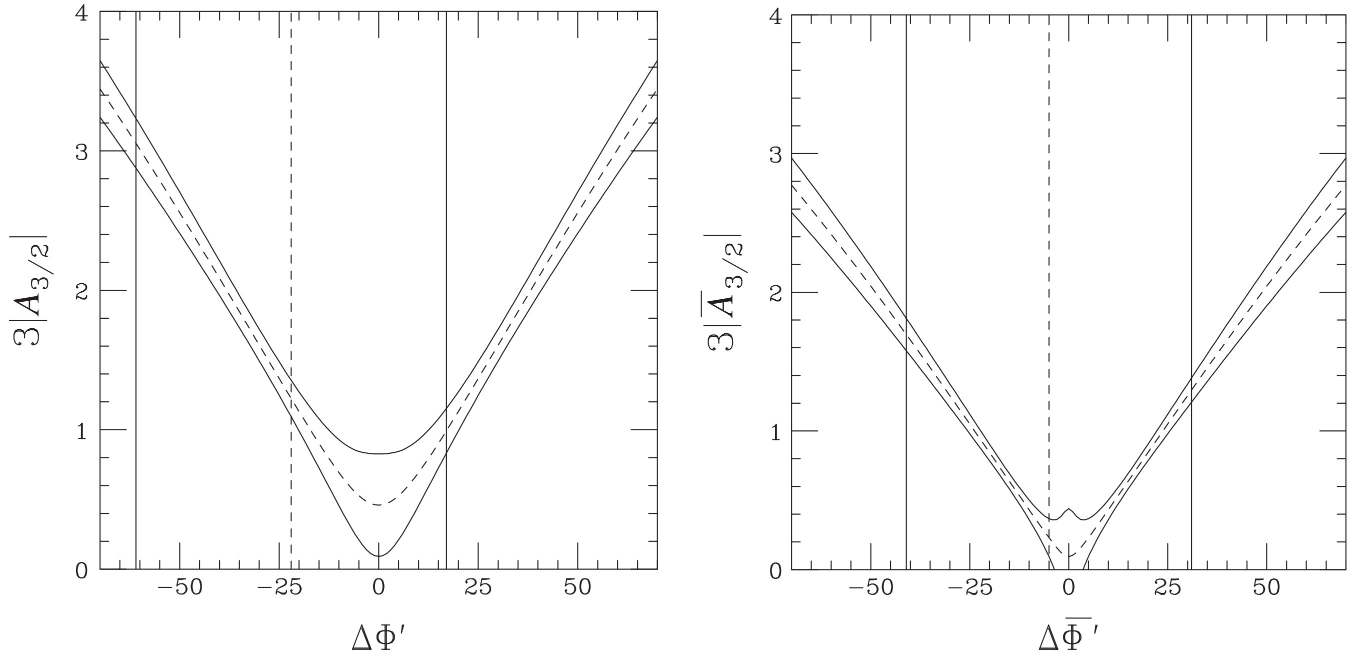


FIG. 4. Magnitudes of $I = 3/2$ amplitudes as functions of relative phases between $K^{*+}\pi^-$ and $K^{*0}\pi^0$ amplitudes, extracted from $B \to K^*\pi$ branching ratios and asymmetries given in Table I. Left: $3|A_{3/2}|$; right: $3|\bar{A}_{3/2}|$. Vertical lines show central value and $1\sigma$ limits of phases $\Delta\Phi'$ or $\Delta\bar{\Phi}'$ quoted in Ref. [7]. Curves are shown for central values of branching ratios and $CP$ asymmetries with bands denoting $1\sigma$ errors added in quadrature.

values of $\Delta\Phi$ and $\Delta\bar{\Phi}$ near 180° in agreement with our lower bounds (24). The next likely Solution II and Solution III (disfavored by $3.3\sigma$ and $3.8\sigma$) involve one large phase and one small phase, while the most unlikely solution (disfavored by $5\sigma$) consists of large values for both $\Delta\Phi'$ and $\Delta\bar{\Phi}'$. The highly favored Solution I, using a different convention than ours for the two phases, is in agreement with our bounds, corresponding to destructive interference between $A(K^{*0}\pi^0)$ and $A(K^{*+}\pi^-)$ in (3) and between their charge conjugates.

Using $B \to K^*\pi$ branching ratios and $CP$ asymmetries quoted in Table I (where *BABAR* and Belle results for $B \to K^{*+}\pi^-$ have been averaged), values of $\Delta\Phi'$ and $\Delta\bar{\Phi}'$ for the favored Solution I [7], and assuming no correlations between these measurements, we calculate for central values of branching ratios and $CP$ asymmetries

$$3|A_{3/2}| = 1.22^{+1.83}_{-1.22}, \qquad 3|\bar{A}_{3/2}| = 0.23^{+1.46}_{-0.23}, \tag{25}$$

where the errors are due to the uncertainties in $\Delta\Phi'$ and $\Delta\bar{\Phi}'$. The dependence of these amplitudes on $\Delta\Phi'$ or $\Delta\bar{\Phi}'$ and on errors in branching ratios and $CP$ asymmetries is illustrated in Fig. 4.

The values of the two isospin 3/2 amplitudes are consistent with zero within large errors. Improvement in errors on the relative phases $\Delta\Phi'$ and $\Delta\bar{\Phi}'$ (depending on their values) may be able to permit determination of $3|A_{3/2}|$ and $3|\bar{A}_{3/2}|$ with sufficient accuracy to constrain their relative phase so as to provide a new constraint on CKM parameters [4,5]. Also, the $CP$ rate asymmetry, $\Delta((K^*\pi)_{I=3/2}) \equiv (3|\bar{A}_{3/2}|)^2 - (3|A_{3/2}|)^2$, has been shown to be equal to a sum combining eight $CP$ rate asymmetries in all possible $B \to K^*\pi$ and $B \to \rho K$ decays [10]. A potential violation of this sum rule would provide evidence for new physics.

Improvements in the measurements of $|A_{3/2}|$, $\Delta\Phi$ and their charge conjugates may be achieved in the near future. The latest results for $B \to K^\pm\pi^\mp\pi^0$ published by the Belle collaboration used a data sample from an integrated luminosity of only 78 fb$^{-1}$ [22]. By now, Belle has accumulated about 10 times more data for this decay mode, approximately twice the amount studied by *BABAR*. Belle should be encouraged to analyze their full set of data in order to improve the measurements of $3|A_{3/2}|$, $\Delta\Phi$, and their charge conjugates.

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