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Two-particle correlations in high-energy collisions and the gluon four-point function

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We derive the rapidity evolution equation for the gluon four-point function in the dilute regime and at small x from the JIMWLK functional equation. We show that beyond leading order in N_c the mean field (Gaussian) approximation where the four-point function is factorized into a product of two-point functions is violated. We calculate these factorization breaking terms and show that they contribute at leading order in N_c to correlations of two produced gluons as a function of their relative rapidity and azimuthal angle, for generic (rather than back-to-back) angles. Such two-particle correlations have been studied experimentally at the BNL-RHIC collider and could be scrutinized also for pp (and, in the future, also AA) collisions at the CERN-LHC accelerator.

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I. PRODUCTION OF TWO CORRELATED PARTICLES

The evolution of QCD amplitudes with energy is described by the Balitsky hierarchy [1] or, equivalently, by the JIMWLK [2] functional renormalization group equations. They essentially represent generalizations of the well-known Balitsky-Fadin-Kuraev-Lipatov (BFKL) equation [3] for the evolution of the two-point function to evolution equations for arbitrary n-point functions including the nonlinear effects due to high gluon density. In the unitarity limit of high parton density the Balitsky hierarchy is not closed: the derivative of any n-point function with respect to energy (or rapidity $Y \sim \log E$) involves all m-point functions ($m \geq n$). In the dilute regime, however, the hierarchy can be truncated to obtain closed evolution equations for each n-point function.

Prior work in this field has mostly focused on the evolution of the two-point function and its perturbative unitarization at high energies. The purpose of this paper is to point out that information on the four-point function could be obtained from two-particle correlations in inelastic high-energy collisions in a certain kinematic regime (see below). Computation of multiparticle production in high energy collisions [4] relies on the use of factorization theorems [5] which ensure that the small-x divergences of the observables can be absorbed into the JIMWLK evolution of the n-point functions.

We argue that even in the dilute regime the B-JIMWLK equation for the four-point function cannot be factorized as a product of two BFKL two-point functions. We show that the terms that violate this factorization actually contribute to the correlation function at leading order in N_c.

We consider the correlation of two particles with transverse momenta $p_\perp, q_\perp$ (we shall drop the subscript $\perp$ from now on to avoid cluttering of notation) and rapidities y_p, y_q, respectively:

$$C(\mathbf{p},\mathbf{q}) = \left\langle \frac{dN_2}{d^2p dy_p d^2q dy_q} \right\rangle - \left\langle \frac{dN}{d^2p dy_p} \right\rangle \left\langle \frac{dN}{d^2q dy_q} \right\rangle. \tag{1}$$

When p and q are on the order of a few GeV it is necessary to subtract the background of uncorrelated particle pairs to reveal the structure of the correlation function. The brackets denote an average over events and the momentum distributions shall be normalized according to

$$\int d^2p dy_p \left\langle \frac{dN}{d^2p dy_p} \right\rangle = \langle N \rangle, \tag{2}$$

$$\int d^2p dy_p d^2q dy_q \left\langle \frac{dN_2}{d^2p dy_p d^2q dy_q} \right\rangle = \langle N^2 \rangle, \tag{3}$$

where $\langle N \rangle$ is the total average multiplicity per event. It has been argued in Ref. [6] that in the high-energy limit (but fixed p, q, y_p, y_q) the leading contribution to $C(\mathbf{p},\mathbf{q})$ is due to diagrams such as the one depicted in Fig. 1. For these diagrams the hard amplitudes are disconnected but the correlations arise because for either one (or both) of the colliding hadrons the ladders in the amplitude and/or the conjugate amplitude connect to the same color source. These two-point functions are essentially the unintegrated gluon distributions of the hadrons; they are of order $1/g^2$ when the transverse momentum in the ladder is below the saturation momentum Q_s of the corresponding hadron.

Diagrams such as Fig. 1 should dominate $C(p, q)$ even at high (but not asymptotically high) transverse momentum, $p, q \gtrsim Q_s$, provided one considers *generic* relative angles $\cos\phi \equiv p \cdot q/(|p||q|)$ (in particular, away from the region of "back-to-back" jets, $\phi \simeq \pi$). On the other hand, at leading order in α_s, when $p, q \gg Q_s$ the gluon pair should originate from the same ladder. When the rapidity difference between the two produced gluons and the two beams are smaller than $\sim 1/\alpha_s$, the ladder is DGLAP-ordered

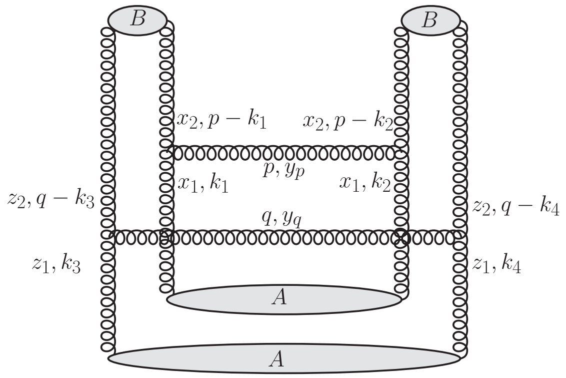

FIG. 1. Correlated production of two particles with generic relative azimuthal angle at leading order. The blobs denote the unintegrated gluon distribution of the projectile A or target B, respectively, and the light-cone momenta are $x_{1,2} = (p/\sqrt{s}) \times \exp(\pm y_p)$, $z_{1,2} = (q/\sqrt{s}) \exp(\pm y_q)$.

which would lead to $C(p, q)$ being dominated by contribution of back-to-back jets [$\delta(p + q)$]. When $|y_p - y_q| \gtrsim 1/\alpha_s$ the delta-function gets smeared out by a BFKL-ordered ladder in between the produced gluons (Mueller-Navelet jets [7]). Instead, here we consider the situation where p, q are somewhat larger than but on the order of Q_s; also, $|y_p - y_q|$ should be significantly smaller than the total rapidity window between the two beams; and, most importantly, the relative azimuthal angle is $\phi \ll \pi$, such that the transverse momenta of the produced gluons do *not* cancel.

We note that two-particle correlations away from the back-to-back regime have recently been measured at the BNL-RHIC accelerator at $\sqrt{s} = 200$ GeV (per colliding nucleon pair) for proton-proton, deuteron-gold, and gold-gold collisions [8–10]. For the former systems only a narrow peak due to fragmentation of the triggered parton have been observed. For collisions of heavy ions, on the other hand, $C(p, q)$ exhibits a "ridge"-like structure: it is narrow in ϕ but extends over several units in $\Delta y = |y_p - y_q|$. The absence of measurable correlations in pp and $d + Au$ collisions may be due to the smallness of the saturation momentum Q_s for a proton or deuteron at RHIC energy. Also, the measurements from RHIC might be expected to be rather sensitive to the initial conditions for the evolution equation at moderately small x_0. At the higher energies of CERN's LHC collider, the saturation momentum of a proton measured from the central rapidity region is expected to be on the order of 1 GeV and such correlations could be sufficiently strong to provide information about the QCD four-point function at small x.

The diagrams like the one from Fig. 1 arise from factorization of the four-point functions in the field of the projectile/target into products of two-point functions [6] (unintegrated gluon distributions). Doing so, however, picks up only the leading-N_c contribution to the four-point function. More generally, $C(\mathbf{p}, \mathbf{q})$ is given by

$$\left\langle \frac{dN_2}{d^2p\,dy_p\,d^2q\,dy_q} \right\rangle = \frac{g^{12}}{64(2\pi)^6} (f_{gaa'} f_{g'bb'} f_{gcc'} f_{g'dd'}) \int \prod_{i=1}^{4} \frac{d^2k_i}{(2\pi)^2 k_i^2} \frac{L_\mu(p, k_1) L^\mu(p, k_2)}{(p - k_1)^2 (p - k_2)^2} \frac{L_\nu(q, k_3) L^\nu(q, k_4)}{(q - k_3)^2 (q - k_4)^2} \times \langle \rho^{*a}{}_A(k_2) \rho^{*b}{}_A(k_4) \rho_A{}^c(k_1) \rho_A{}^d(k_3) \rho^{*a'}{}_B(p - k_2) \rho^{*b'}{}_B(q - k_4) \rho_B{}^{c'}(p - k_1) \rho_B{}^{d'}(q - k_3) \rangle \quad (4)$$

$$= \frac{g^{12}}{64(2\pi)^6} (f_{gaa'} f_{g'bb'} f_{gcc'} f_{g'dd'}) \int \prod_{i=1}^{4} \frac{d^2k_i}{(2\pi)^2 k_i^2} \frac{L_\mu(p, k_1) L^\mu(p, k_2)}{(p - k_1)^2 (p - k_2)^2} \frac{L_\nu(q, k_3) L^\nu(q, k_4)}{(q - k_3)^2 (q - k_4)^2} \times \langle \rho^{*a}{}_A(k_2) \rho^{*b}{}_A(k_4) \rho_A{}^c(k_1) \rho_A{}^d(k_3) \rangle \langle \rho^{*a'}{}_B(p - k_2) \rho^{*b'}{}_B(q - k_4) \rho_B{}^{c'}(p - k_1) \rho_B{}^{d'}(q - k_3) \rangle \quad (5)$$

In the second step we have assumed factorization of the wave functions of projectile and target. L^μ denotes the Lipatov vertex which satisfies

$$L_\mu(p, k_1) L^\mu(p, k_2) = -\frac{4}{p^2} [\delta^{ij} \delta^{nm} + \epsilon^{ij} \epsilon^{nm}] \times k_1^i (p - k_1)^j k_2^n (p - k_2)^m \quad (6)$$

$$L_\mu(p, k) L^\mu(p, k) = -\frac{4k^2}{p^2} (p - k)^2. \quad (7)$$

The expression (5) is depicted in Fig. 2. Here, $\rho(r)$ denotes the color charge density per unit transverse area at a transverse coordinate r and $\rho(k)$ is its Fourier transform. Its two-point function is related to the unintegrated gluon distribution $\Phi(x, k^2)$ via

$$\langle \rho^{*a}(k) \rho^b(k') \rangle(x) = \frac{1}{\alpha_s} \frac{\delta^{ab}}{N_c^2 - 1} (2\pi)^3 \delta(k - k') \Phi(x, k^2). \quad (8)$$

With this normalization one recovers the LO $k_\perp$-factorization formula for the single-inclusive distribution from the diagram 3 with the standard prefactor [11]:

$$\frac{dN}{d^2p\,dy} = 4\alpha_s \frac{N_c}{N_c^2 - 1} \frac{\sigma_0}{p^2} \int d^2k \frac{\Phi_A(x_1, k^2)}{k^2} \times \frac{\Phi_B(x_2, (p - k)^2)}{(p - k)^2}, \quad (9)$$

where σ_0 is the transverse area of the collision (note that in

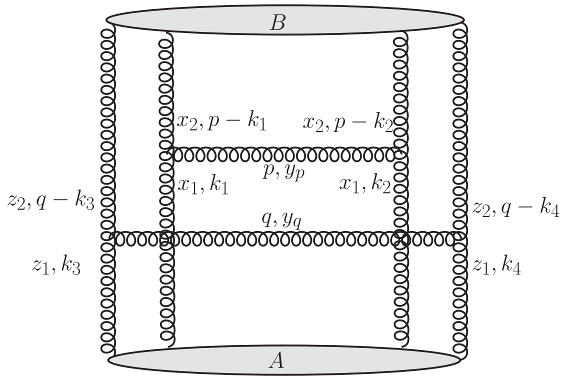

FIG. 2. Correlated production of two particles with generic relative azimuthal angle at leading order. The blobs denote the four-point functions for the projectile A or target B, respectively.

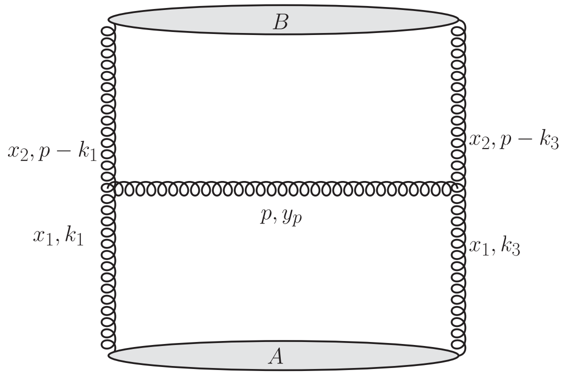

FIG. 3. Single-particle production from $k_\perp$-factorization at leading order. The blobs denote the unintegrated gluon distribution of the projectile A or target B, respectively.

our convention $\Phi(x, k^2)$ is the density of gluons per unit transverse area and it therefore contains a factor of $1/\sigma_0$).

In a mean field (and large N_c) approximation one may factorize the four-point functions from Eq. (5) into products of two-point functions,

$$\langle\rho^a\rho^b\rho^c\rho^d\rangle = \delta^{ab}\delta^{cd}(\rho^2)^2 + \delta^{ac}\delta^{bd}(\rho^2)^2 + \delta^{ad}\delta^{bc}(\rho^2)^2 + \cdots, \tag{10}$$

where $\rho^2 \equiv \langle\rho\rho\rangle$, and the momentum dependence of the two-point function has been suppressed. Then, one of the nine contractions corresponds to the square of the single-inclusive distribution: contract the first ρ with the third and the second with the fourth, for both projectile and target. The color factor for this diagram is[1]

$$f_{gaa'}f_{g'bb'}f_{gcc'}f_{g'dd'}\langle\rho^{*a}{}_A\rho_A{}^c\rangle\langle\rho^{*b}{}_A\rho_A{}^d\rangle\langle\rho^{*a'}{}_B\rho_B{}^{c'}\rangle \times\langle\rho^{*b'}{}_B\rho_B{}^{d'}\rangle \tag{11}$$

$$\sim f_{gaa'}f_{g'bb'}f_{gcc'}f_{g'dd'}\delta^{ac}\delta^{bd}\delta^{a'c'}\delta^{b'd'} = N_c^2(N_c^2-1)^2. \tag{12}$$

The remaining eight diagrams correspond to a color factor of (we take Fig. 1 as an example)

$$f_{gaa'}f_{g'bb'}f_{gcc'}f_{g'dd'}\langle\rho^{*a}{}_A\rho_A{}^c\rangle\langle\rho^{*b}{}_A\rho_A{}^d\rangle\langle\rho^{*a'}{}_B\rho^{*b'}{}_B\rangle \times\langle\rho_B{}^{c'}\rho_B{}^{d'}\rangle \tag{13}$$

$$\sim f_{gaa'}f_{g'bb'}f_{gcc'}f_{g'dd'}\delta^{ac}\delta^{bd}\delta^{a'b'}\delta^{c'd'} = N_c^2(N_c^2-1). \tag{14}$$

Thus, two-particle correlations are suppressed by a factor of $N_c^2 - 1$ as compared to uncorrelated production. For this reason, the leading-N_c *ansatz* (10) may not capture the complete result for $C(p, q)$. Below, we derive the evolution equation for the four-point function from JIMWLK. We determine the corrections beyond the mean-field and large-N_c approximations to the right-hand side of (10) and show that these corrections contribute at the same order in N_c to the correlation function.

In this regard, we should point out that N_c corrections to the two-point function in the dense regime were found to be exceptionally small [12]. However, this needs not be true for the four-point function. In fact, we shall argue below that we do not expect N_c corrections to the four-point function to be anomalously small, even in the dilute regime. A verification or falsification of this expectation via exact numerical solutions would be very valuable.

II. EVOLUTION EQUATION FOR THE FOUR-POINT FUNCTION

In this section we present the equation describing the rapidity evolution of the four-point function $\langle\alpha_r^a\alpha_{\bar{r}}^b\alpha_s^c\alpha_{\bar{s}}^d\rangle$ obtained from the JIMWLK equations, which include terms of subleading order in N_c. In this context it is more natural to work in coordinate space, so r, s, $\bar{r}$, $\bar{s}$ denote transverse coordinates; the four-point function in momentum space can be obtained by Fourier transform. We also find it preferable to work with the fields α rather than the color charge densities ρ; at leading order and in covariant gauge, they are related in coordinate space by

$$A^\mu(x^+, r) \equiv \delta^{\mu-}\alpha(x^+, r) = -g\delta^{\mu-}\delta(x^+)\frac{1}{\nabla_\perp^2}\rho(x^+, r), \tag{15}$$

for a hadron moving at the speed of light in the negative z-direction. Since this field also satisfies $A^+ = 0$, the only nonvanishing field strength is $F^{-i} = -\partial^i\alpha$. In momentum space we have the relation $k^2\alpha(k) = g\rho(k)$.

The JIMWLK evolution equation for the four-point function to lowest order in the fields can be shown to be

[1]Not including factors of N_c which will enter once $\langle\rho\rho\rangle$ is expressed through Φ via Eq. (8).

$$\begin{aligned}\frac{d}{dY}\langle\alpha_r^a\alpha_{\bar r}^b\alpha_s^c\alpha_{\bar s}^d\rangle = {}&\frac{g^2N_c}{(2\pi)^3}\int d^2z\bigg\langle\frac{\alpha_z^a\alpha_{\bar r}^b\alpha_s^c\alpha_{\bar s}^d}{(r-z)^2}+\frac{\alpha_r^a\alpha_z^b\alpha_s^c\alpha_{\bar s}^d}{(\bar r-z)^2}+\frac{\alpha_r^a\alpha_{\bar r}^b\alpha_z^c\alpha_{\bar s}^d}{(s-z)^2}+\frac{\alpha_r^a\alpha_{\bar r}^b\alpha_s^c\alpha_z^d}{(\bar s-z)^2}-4\frac{\alpha_r^a\alpha_{\bar r}^b\alpha_s^c\alpha_{\bar s}^d}{z^2}\bigg\rangle\\
&+\frac{g^2}{\pi}\int\frac{d^2z}{(2\pi)^2}\bigg\langle f^{e\kappa a}f^{f\kappa b}\frac{(r-z)\cdot(\bar r-z)}{(r-z)^2(\bar r-z)^2}[\alpha_r^e\alpha_{\bar r}^f-\alpha_r^e\alpha_z^f-\alpha_z^e\alpha_{\bar r}^f+\alpha_z^e\alpha_z^f]\alpha_s^c\alpha_{\bar s}^d\\
&+f^{e\kappa a}f^{f\kappa c}\frac{(r-z)\cdot(s-z)}{(r-z)^2(s-z)^2}[\alpha_r^e\alpha_s^f-\alpha_r^e\alpha_z^f-\alpha_z^e\alpha_s^f+\alpha_z^e\alpha_z^f]\alpha_{\bar r}^b\alpha_{\bar s}^d\\
&+f^{e\kappa a}f^{f\kappa d}\frac{(r-z)\cdot(\bar s-z)}{(r-z)^2(\bar s-z)^2}[\alpha_r^e\alpha_{\bar s}^f-\alpha_r^e\alpha_z^f-\alpha_z^e\alpha_{\bar s}^f+\alpha_z^e\alpha_z^f]\alpha_{\bar r}^b\alpha_s^c\\
&+f^{e\kappa b}f^{f\kappa c}\frac{(\bar r-z)\cdot(s-z)}{(\bar r-z)^2(s-z)^2}[\alpha_{\bar r}^e\alpha_s^f-\alpha_{\bar r}^e\alpha_z^f-\alpha_z^e\alpha_s^f+\alpha_z^e\alpha_z^f]\alpha_r^a\alpha_{\bar s}^d\\
&+f^{e\kappa b}f^{f\kappa d}\frac{(\bar r-z)\cdot(\bar s-z)}{(\bar r-z)^2(\bar s-z)^2}[\alpha_{\bar r}^e\alpha_{\bar s}^f-\alpha_{\bar r}^e\alpha_z^f-\alpha_z^e\alpha_{\bar s}^f+\alpha_z^e\alpha_z^f]\alpha_r^a\alpha_s^c\\
&+f^{e\kappa c}f^{f\kappa d}\frac{(s-z)\cdot(\bar s-z)}{(s-z)^2(\bar s-z)^2}[\alpha_s^e\alpha_{\bar s}^f-\alpha_s^e\alpha_z^f-\alpha_z^e\alpha_{\bar s}^f+\alpha_z^e\alpha_z^f]\alpha_r^a\alpha_{\bar r}^b\bigg\rangle. \qquad (16)\end{aligned}$$

This expression neglects contributions from higher n-point functions on the right-hand side; in the dilute regime, i.e. when the transverse momenta of the produced particles are higher than the saturation momenta of the colliding hadrons, this approximation should be justified.

In order to derive the color structure of corrections beyond the large-N_c approximation, we factorize the product of four-point functions on the right-hand side of Eq. (16) into products of two-point functions. This Gaussian approximation reduces the evolution equation for the four-point function to a product of two BFKL equations (for the two-point function) plus extra terms which provide corrections to the factorization (10). The result is

$$\frac{d}{dY}\langle\alpha_r^a\alpha_{\bar r}^b\alpha_s^c\alpha_{\bar s}^d\rangle=\frac{d}{dY}[\delta^{ac}\delta^{bd}\alpha^2_{\bar r-\bar s}\alpha^2_{r-s}+\delta^{ab}\delta^{cd}\alpha^2_{s-\bar s}\alpha^2_{r-\bar r}+\delta^{ad}\delta^{bc}\alpha^2_{r-\bar s}\alpha^2_{\bar r-s}]-\frac{\alpha_s}{2\pi^2}\int d^2z[F_0^{abcd}+F_1^{abcd}+F_2^{abcd}] \qquad (17)$$

where

$$\begin{aligned}F_0^{abcd}&\equiv f^{a\kappa b}f^{c\kappa d}\frac{(r-s)^2}{(r-z)^2(s-z)^2}\alpha^2_{r-\bar r}\alpha^2_{s-\bar s}+f^{a\kappa d}f^{b\kappa c}\bigg[\frac{(r-\bar r)^2}{2(r-z)^2(\bar r-z)^2}-\frac{(r-s)^2}{2(r-z)^2(s-z)^2}+(r\leftrightarrow s,\bar s\leftrightarrow\bar r)\bigg]\\
&\quad\times\alpha^2_{r-\bar s}\alpha^2_{\bar r-s}\\
F_1^{abcd}&\equiv f^{a\kappa b}f^{c\kappa d}\bigg[\bigg(\frac{1}{(r-z)^2}-\frac{(s-r)^2}{(r-z)^2(s-z)^2}\bigg)\alpha^2_{r-\bar r}\alpha^2_{z-\bar s}+(r\leftrightarrow\bar s,s\leftrightarrow\bar r)\bigg]\\
&\quad+f^{a\kappa d}f^{b\kappa c}\bigg[\bigg(\frac{1}{(r-z)^2}-\frac{(\bar r-r)^2}{(r-z)^2(\bar r-z)^2}-\frac{1}{(\bar s-z)^2}+\frac{(\bar s-\bar r)^2}{(\bar r-z)^2(\bar s-z)^2}\bigg)\alpha^2_{r-\bar s}\alpha^2_{z-s}+(r\leftrightarrow s,\bar s\leftrightarrow\bar r)\bigg]\\
F_2^{abcd}&\equiv f^{a\kappa b}f^{c\kappa d}\bigg[\bigg[\bigg(\frac{(r-s)^2}{(r-z)^2(s-z)^2}-\frac{1}{(r-z)^2}-\frac{1}{(s-z)^2}\bigg)\alpha^2_{z-\bar r}\alpha^2_{z-\bar s}-(s\leftrightarrow\bar s)\bigg]-(r\leftrightarrow\bar r)\bigg].\end{aligned} \qquad (18)$$

In (18) all terms in F_0, F_1, and F_2 are to be duplicated with the substitutions indicated explicitly in the brackets. Second, all terms in F_0 and F_1 are to be duplicated again substituting $a\leftrightarrow b$ and $r\leftrightarrow\bar r$. Then, all terms in F_0 and F_1 should be duplicated a third time exchanging $c\leftrightarrow d$ and $s\leftrightarrow\bar s$. Furthermore, all terms in F_2 are to be duplicated while letting $b\leftrightarrow c$ and $r\leftrightarrow\bar s$. Finally, the terms obtained in the last substitution (only) in F_2 should be duplicated exchanging $c\leftrightarrow d$ and $r\leftrightarrow\bar r$.

The first term in (17) provides the leading-N_c contribution to the four-point function. The second term gives corrections beyond the large-N_c factorization (10). Since an analytic solution to the evolution equation for the four-point function is not within our reach, a numerical investigation of these terms and their magnitude would be extremely useful. Nevertheless, from

$$\partial_Y\langle\rho^a\rho^b\rho^c\rho^d\rangle\sim\alpha_sN_c\delta^{ab}\delta^{cd}(\rho^2)^2+\alpha_sf^{ac\kappa}f^{bd\kappa}(\rho^2)^2,$$
$$\text{with}\quad\rho^2(Y)\sim e^{\alpha_sN_cY} \qquad (19)$$

one might expect that, generically, the solution to this equation has the following color structure:

$$\langle \rho^a \rho^b \rho^c \rho^d \rangle \sim \delta^{ab}\delta^{cd}(\rho^2)^2 + \delta^{ac}\delta^{bd}(\rho^2)^2 + \delta^{ad}\delta^{bc}(\rho^2)^2 + \tag{20}$$

$$\frac{1}{N_c} f^{ac\kappa} f^{bd\kappa}(\rho^2)^2 + \frac{1}{N_c} f^{ab\kappa} f^{cd\kappa}(\rho^2)^2 + \frac{1}{N_c} f^{ad\kappa} f^{bc\kappa}(\rho^2)^2. \tag{21}$$

(Note that the various two-point functions depend on different coordinates/momenta and so each of the above terms is distinct.) The color factors emerging from the products of the Kronecker tensors have already been discussed above, Eqs. (12) and (14). However, some of the products of a leading-N_c term from the first line (20) with a subleading-N_c term from the second line (21) also contribute at the same order $N_c^2(N_c^2 - 1)$. For example,

$$\frac{1}{N_c} \delta^{a'c'} \delta^{b'd'} f_{gaa'} f_{g'bb'} f_{gcc'} f_{g'dd'} f^{ab\kappa} f^{cd\kappa} = N_c \delta^{ac}\delta^{bd} f^{ab\kappa} f^{cd\kappa} = N_c^2(N_c^2 - 1). \tag{22}$$

This shows that some of the subleading-N_c contributions from the four-point function actually enter $C(p, q)$ at leading order, compare to Eq. (14). Previous results from the literature [6] (also see [13]) are therefore not complete. Nevertheless, the correlations described here should still extend over several units in $|y_p - y_q|$ [13]. Quantitative results for the JIMWLK four-point function and for the corresponding two-particle correlations $C(\mathbf{p}, \mathbf{q})$ as functions of the transverse momenta p, q, relative azimuth ϕ and relative rapidity $|y_p - y_q|$ remain to be found.

In summary, we have argued that two-particle correlations from high-energy collisions may provide some insight into the QCD four-point function. This should be the case, in particular, when the transverse momenta of the produced particles are not very much higher than the saturation momenta of the colliding hadrons and when their relative azimuthal angle is sufficiently less than π. The narrow (in both azimuthal and polar angle) jetlike fragmentation peak should sit on top of a "background" which is broader in the relative rapidity $|y_p - y_q|$.

If expanded in powers of N_c, the leading contribution to the four-point function is given by the product of two BFKL two-point functions. However, we find that genuine B-JIMWLK subleading-N_c corrections also appear in the correlation function $C(\mathbf{p}, \mathbf{q})$, at leading nonvanishing order in N_c. The correlations mentioned here represent an interesting opportunity to study the nontrivial structure of the four-point function of the B-JIMWLK hierarchy, both theoretically and experimentally.

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