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$\kappa K^+ \pi^-$ vertex in light cone QCD sum rules

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In this work we study the $\kappa K^+ \pi^-$ vertex in the framework of light cone QCD sum rules. We predict the coupling constant $g_{\kappa K^+ \pi^-}$ to be $g_{\kappa K^+ \pi^-} = (6.0 \pm 1.0)$ GeV and estimate the scalar $f_0 - \sigma$ mixing angle from the experimental ratio $g^2(\kappa \to K\pi)/g^2(\sigma \to \pi\pi)$.

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I. INTRODUCTION

For many years the existence and the nature of light scalar mesons, $\sigma(600)$ and $\kappa(800)$, have been the subject of debates. Considering their constituent structure there have been a lot of discussions and arguments about whether or not they are meson-meson molecular states [1], $q\bar{q}$ states [2,3], or multiquark $q^2\bar{q}^2$ states [4]. These mesons, having such a confused and controversial history, have attracted both theoretical and experimental interests. Having vacuum quantum numbers, they are essential in understanding the mechanism of the symmetry breaking.

The light scalar mesons were studied from the consideration of their two body decays leading to indications about their structure as being a multiquark state [4–7]. Furthermore, the $q\bar{q}$ picture was also utilized to look into some properties of these mesons [2,8]. An investigation on the masses of light scalar mesons was made in Ref. [9], considering the $q\bar{q}$ structure. Taking into account instanton effects leads to consistent predictions for the masses of these mesons. In Ref. [10], the mass and decay constant of the $I = 1/2$ scalar mesons were calculated via QCD sum rules taking their structure as a quark-antiquark pair. The masses of light scalar mesons were also studied in the relativistic quark model [11] as well as the QCD sum rules approach [12,13] considering them as diquark-antidiquark states. Reference [14] presents a review about the masses and the couplings of light scalar mesons considering them as $q\bar{q}$ and multiquark states.

The κ meson has been observed in $K\pi$ scattering, D- and J/ψ-decay processes [15]. There are different values in the literature for the mass and decay width of a κ meson obtained from different data. The mass and width change as $m_\kappa \sim 600$–900 MeV and $\Gamma_\kappa \sim 400$–700 MeV [15,16]. Cherry and Pennington showed that the scalar $\kappa(800)$ above 825 MeV does not exist [17]. On the particle data group the average values are given as $m_\kappa - i\Gamma_\kappa = (672 \pm 40) - i(550 \pm 34)$ MeV [18]. Bugg considered new FOCUS data on $D^+ \to K^-\pi^+\pi^+$ with E791 data on $D^+ \to K^-\pi^+\pi^+$, LASS data on $K\pi$ scattering, and BESS II data on $J/\psi \to \bar{K}^*(890)^\pm \pi^\mp$, and then obtained the value as $m_\kappa - i\frac{\Gamma_\kappa}{2} = (663 \pm 42) - i(329 \pm 27)$ MeV [19]. Moreover, the mass of the κ meson was calculated theoretically in the framework of light cone QCD sum rules as $m_\kappa = (700 \pm 60)$ MeV [20].

Scalar mesons play an important role in different areas of hadron physics. In particular, they provide constructive information in the decay mechanisms of hadrons. The coupling constant $g_{\kappa K^+ \pi^-}$ arises as a κ meson exchange in $\pi K \to \pi K$ scattering and as a virtual scalar κ-meson state in $J/\psi \to \bar{K}^*(890)^\pm \pi^\mp$ decay.

In this work, we study the $\kappa K^+ \pi^-$ vertex in light cone QCD sum rules, calculate the coupling constant $g_{\kappa K^+ \pi^-}$, and discuss the $f_0 - \sigma$ scalar mixing angle θ_s. In our analysis we apply an interpolating current for the κ meson as a $q\bar{q}$ state. This vertex was also considered in [12] using a multiquark structure for the light scalar mesons.

II. FORMALISM AND RESULTS

In this part, the details of the calculation of the coupling constant for the transition $\kappa \to K^+\pi^-$ are presented. We apply the QCD sum rules method from which the hadronic properties can be obtained using the interpolating currents formed out of quark fields [21]. We choose the interpolating currents in terms of quark fields for the κ meson as the scalar current $j_\kappa = \bar{s}d$ and for the K^+ mesons as the axial vector current $j^K_\nu = \bar{s}\gamma_\nu\gamma_5 u$.

To obtain the coupling constant $g_{\kappa K^+ \pi^-}$, we study the $\kappa K^+ \pi^-$ vertex via a two-point correlation function

$$T_\mu(p+q, p) = i\int d^4x e^{ip\cdot x}\langle \pi^-(q)|T\{j^K_\mu(x) j^\dagger_\kappa(0)\}|0\rangle, \quad (1)$$

where p and q are the four-momentum of K^+ and π^- mesons, respectively.

The correlation function in Eq. (1) is calculated in two ways. In the first way, one constructs the physical representation of the correlation function by inserting a complete set of hadronic states. This expresses $T_\mu(p+q, p)$ in terms of hadronic degrees of freedom. In the second way, $T_\mu(p+q, p)$ is calculated in terms of quark-gluon degrees of freedom. Matching these two representations provides the QCD sum rules which allows us to obtain the physical quantity under question. The contributions from the con-

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tinuum and higher states are suppressed via the Borel transformation.

On the physical side, inserting a complete set of hadronic states having the same quantum number with the currents to be studied, we obtain

$$T_\mu(p+q,p) = \frac{\langle 0 \mid j_\mu^K \mid K^+(p)\rangle\langle K^+(p)\pi^-(q) \mid \kappa(p+q)\rangle\langle\kappa| j_\kappa^\dagger \mid 0\rangle}{(p^2-m_{K^+}^2)[(p+q)^2-m_\kappa^2]} + \int_{s_0} ds \int_{s_0'} ds' \frac{\rho_\mu^{\rm cont}(s,s')}{[s-(p+q)^2](s'-p^2)}, \quad (2)$$

where $\rho_\mu^{\rm cont}(s,s')$ is the hadronic spectral density that contains the contributions of higher resonances and the continuum. Here the matrix elements are defined as

$$\begin{aligned} <0 \mid j_\mu^K \mid K^+(p)> &= i f_K p_\mu, \\ <K^+(p)\pi^-(q) \mid \kappa(p+q)> &= g_{\kappa K^+\pi^-}, \\ <\kappa(k+p) \mid j_\kappa \mid 0> &= f_\kappa. \end{aligned} \quad (3)$$

Using Eqs. (2) and (3) we attain the physical side as

$$T_\mu(p+q,p) = \frac{g_{\kappa K^+\pi^-} f_K f_\kappa}{(p^2-m_{K^+}^2)[(p+q)^2-m_\kappa^2]} p_\mu + \ldots. \quad (4)$$

Thus, in the analysis of the coupling constant $g_{\kappa K^+\pi^-}$, the leptonic decay constants f_κ and f_K are needed. We have already calculated f_κ by using the QCD sum rules method in our previous work [20] and we use the experimental value for f_K [18,22].

The theoretical side of the calculation includes the expansion of the correlation function near the light cone $x^2 = 0$, in terms of nonlocal quark-gluon operators and pion distribution amplitudes, in the deep Euclidean region where p^2 and $(p+q)^2$ are large and negative. In our calculation we use the full light quark propagator with both perturbative and nonperturbative contributions [23] as

$$\begin{aligned} iS_q(x,0) &= \langle 0|T\{\bar{q}(x)q(0)\}|0\rangle \\ &= i\frac{\not{x}}{2\pi^2x^4} - \frac{m_q}{4\pi^2x^2} - \frac{\langle\bar{q}q\rangle}{12}\left(1-\frac{im_q}{4}\not{x}\right) \\ &\quad - \frac{x^2}{192}m_0^2\langle\bar{q}q\rangle\left(1-\frac{im_q}{6}\not{x}\right) - ig_s\frac{1}{16\pi^2}\int_0^1 du \\ &\quad \times\left\{\frac{\not{x}}{x^2}\sigma_{\mu\nu}G^{\mu\nu}(ux) - 4iu\frac{x_\mu}{x^2}G^{\mu\nu}(ux)\gamma_\nu \right. \\ &\quad \left. - \frac{im_q}{2}\sigma_{\mu\nu}G^{\mu\nu}(ux)\left[\ln\left(\frac{-x^2\Lambda^2}{4}\right)+2\gamma_E\right]\right\}. \end{aligned} \quad (5)$$

Using the expressions for the interpolating currents and nonlocal quark-gluon operators defined in terms of pion distribution amplitudes, we choose the structure p_μ in Eq. (1) and obtain the following expression for the theoretical side after performing the double Borel transformation with respect to the variables $Q_1^2 = -(p+q)^2$ and $Q_2^2 = -p^2$:

$$\begin{aligned} T &= 2M^2\mu_\pi\int_0^1 d\alpha_q\int_0^1 d\alpha_{\bar{q}}\int_0^1 d\alpha_g \mathcal{T}(\alpha_i)(1-v) \\ &\quad \times\delta'(\alpha_q+\alpha_g(1-v)-u_0) - M^2 f_\pi m_s\varphi_\pi(u_0) \\ &\quad + M^2\mu_\pi u_0\varphi_p(u_0) + M^2\mu_\pi(1-\tilde{\mu}_\pi^2) \\ &\quad \times\left[\frac{1}{3}\varphi_\sigma(u_0)+\frac{1}{6}u_0\varphi'_\sigma(u_0)\right], \end{aligned} \quad (6)$$

where M_1^2 and M_2^2 are the Borel parameters, and $u_0 = \frac{M_2^2}{M_1^2+M_2^2}$ and $M^2 = \frac{M_1^2M_2^2}{M_1^2+M_2^2}$. The pion distribution amplitudes are given in Appendix A.

A similar double Borel transformation on the hadronic side for the same structure p_μ results in

$$T = g_{\kappa K^+\pi^-} f_K f_\kappa e^{-m_\kappa/M_1^2} e^{-m_K/M_2^2} + \int_{s_0} ds \int_{s_0'} ds' \rho^{\rm cont}(s,s') e^{-s/M_1^2} e^{-s'/M_2^2}. \quad (7)$$

Obtaining the coupling constant $g_{\kappa K^+\pi^-}$ via the philosophy of the QCD sum rules requires the match of the two expressions, Eq. (6) and (7). The second term in Eq. (7) represents the continuum contribution. In Refs. [24,25] a prescription has been suggested to subtract their contribution taking into account that the Borel parameters corresponding to channels with different mass scales cannot be constrained to be equal. Following this prescription, which is based on the observation that $\varphi_P(u)$ and $\varphi_\sigma(u)$ are polynomials in $(1-u)$, we write

$$\varphi_\pi(u) = \sum_{k=0}^N b_k(1-u), \qquad \varphi_p(u) = \sum_{k=0}^N b'_k(1-u), \quad (8)$$

to compute their contribution in the duality region. This leads to the following form for the coupling constant $g_{\kappa K\pi}$:

$$g_{\kappa K\pi} = \frac{1}{f_K f_\kappa} e^{m_\kappa^2/M_1^2} e^{m_K^2/M_2^2} \bigg\{ 2M^2 \mu_\pi f_0(s_0/M^2) \times \int_0^1 d\alpha_q \int_0^1 d\alpha_{\bar q} \int_0^1 d\alpha_g \mathcal{T}(\alpha_i)(1-v)\delta'(\alpha_q + \alpha_g(1-v) - u_0) - M^2 f_\pi m_s \sum_{k=0}^{N} b_k \bigg(\frac{M^2}{M_2^2}\bigg) \bigg[1 - e^{-A} \sum_{i=0}^{k} \frac{A^i}{i!} + e^{-A} \frac{M^2 m_\pi^2}{M_1^2 M_2^2} \frac{A^{k+1}}{(k+1)!} \bigg] + M^2 \mu_\pi u_0 \sum_{k=0}^{N} b'_k \bigg(\frac{M^2}{M_2^2}\bigg) \bigg[1 - e^{-A} \sum_{i=0}^{k} \frac{A^i}{i!} + e^{-A} \frac{M^2 m_\pi^2}{M_1^2 M_2^2} \frac{A^{k+1}}{(k+1)!} \bigg] + M^2 f_0(s_0/M^2) \mu_\pi (1 - \tilde\mu_\pi^2) \bigg[\frac{1}{3} \varphi_\sigma(u_0) + \frac{1}{6} u_0 \varphi'_\sigma(u_0) \bigg] \bigg\}, \tag{9}$$

where $f_0(s_0/M^2) = 1 - e^{-s_0/M^2}$ and $A = s_0/M^2$, with s_0 being the smallest continuum threshold.

In the numerical analysis of the sum rules we use the following values for the required parameters: $m_K = 0.494$ GeV with $f_K = (156.6 \pm 1 \pm 3.6)$ MeV [18,22], $m_\kappa = (672 \pm 40)$ MeV [18], $f_\kappa = (0.13 \pm 0.02)$ GeV^2 [20], $f_\pi = 0.132$ GeV [24], and $m_s(1\text{ GeV}) = 0.14$ GeV [26]. The pion distribution amplitudes used in the numerical evaluation of the sum rules are given in Appendix A.

The coupling constant should be independent of the Borel parameters M_1^2 and M_2^2 since they are not physical quantities. First, we determine a stability region where the coupling constant practically remains unchanged with respect to variations of M_1^2 and M_2^2. We find that the coupling constant $g_{\kappa K^+\pi^-}$ is quite stable in the regions $1.0 < M_1^2 < 2.0$ GeV^2, $0.8 < M_2^2 < 1.2$ GeV^2. The continuum threshold parameter s_0 for $\kappa(800)$ is used as $1.6 < s_0 < 2.2$ GeV^2, which is related to the mass square of the next possible excited state in the channel of the interpolating current for κ. We chose a few values of M_2^2 from its interval and we try to understand the dependence of the coupling constant $g_{\kappa K^+\pi^-}$ on the M_1^2 for a fixed value of M_2^2. In Fig. 1, we show the coupling constant $g_{\kappa K^+\pi^-}$ as a function of the Borel parameter M_1^2 for a fixed value $M_2^2 = 1.0$ GeV^2 at different values of the threshold parameter s_0 in the interval. In Fig. 2, we use the limits of M_2^2 to find the variation of the coupling constant with M_2^2. From Fig. 1 and 2, we estimate the coupling constant $g_{\kappa K^+\pi^-}$ as $g_{\kappa K^+\pi^-} = (6.0 \pm 1.0)$ GeV, where the uncertainties in the result arises because of the variations in the continuum threshold, variations in the Borel parameters, and uncertainties in the QCD parameters.

The decay width of the κ meson can be calculated in terms of the coupling constant $g_{\kappa K^+\pi^-}$ as

$$\Gamma(\kappa \to K^+\pi^-) = \frac{g_{\kappa K^+\pi^-}^2}{16\pi m_\kappa^3} \sqrt{\lambda(m_\kappa^2, m_{K^+}^2, m_{\pi^-}^2)}. \tag{10}$$

where $\lambda(m_\kappa^2, m_{K^+}^2, m_{\pi^-}^2) = m_\kappa^4 + m_{K^+}^4 + m_{\pi^-}^4 - 2m_\kappa^2 m_{K^+}^2 - 2m_\kappa^2 m_{\pi^-}^2 - 2m_{K^+}^2 m_{\pi^-}^2$, and the experimental width is given by $\Gamma(\kappa \to K\pi) = \frac{3}{2}\Gamma(\kappa \to K^+\pi^-)$. Using a mean value of the observed results $\Gamma(\kappa \to K\pi) = (550 \pm 34)$ MeV with $m_\kappa = (672 \pm 40)$ MeV [18], the coupling constant $g_{\kappa K^+\pi^-}$ is obtained from Eq. (10) as $g_{\kappa K^+\pi^-} = (6.6 \pm 0.8)$ GeV. The coupling constant $g_{\kappa K^+\pi^-}$ estimated by the light cone QCD sum rules method is consistent with the value determined from the experimental result. The coupling constant $g_{\kappa K^+\pi^-}$ was calculated in the QCD sum rule approach by considering the κ meson as a four-quark state [12]. The value $g_{\kappa K^+\pi^-} = 3.6 \pm 0.3$ GeV obtained in that paper is not consistent with recent experimental data.

On the other hand, the coupling constant $g_{\sigma\pi\pi}$ was calculated in light cone QCD sum rules by considering $f_0 - \sigma$ mixing as $g_{\sigma\pi\pi} = \cos\theta_s g'_{\sigma\pi\pi}$ where θ_s is the

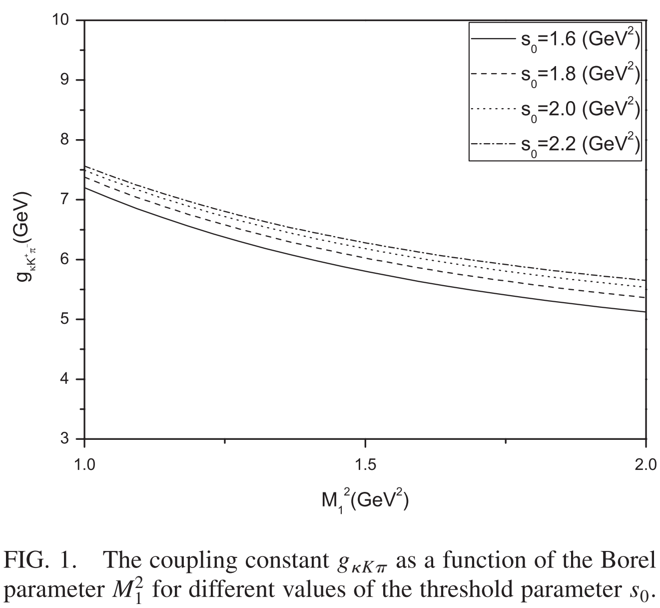

FIG. 1. The coupling constant $g_{\kappa K\pi}$ as a function of the Borel parameter M_1^2 for different values of the threshold parameter s_0.

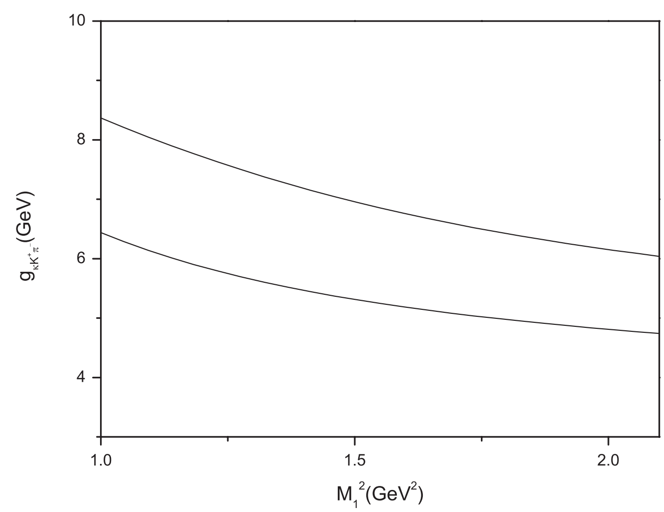

FIG. 2. The coupling constant $g_{\kappa K\pi}$ as a function of the Borel parameter M_1^2 for limit values of the Borel parameter M_2^2 that corresponds to the limits of stability region.

scalar mixing angle [27]. The coupling constant was estimated in that paper as $3.2 \leq g'_{\sigma\pi\pi} \leq 3.9$ GeV. The ratio $g^2(\kappa \to K\pi)/g^2(\sigma \to \pi\pi)$ in terms of scalar mixing angle can be written as

$$\frac{g^2(\kappa \to K\pi)}{g^2(\sigma \to \pi\pi)} = \frac{g^2_{\kappa K\pi}}{\cos^2\theta_s g'^2_{\sigma\pi\pi}}. \tag{11}$$

The value of the ratio is found as $g^2(\kappa \to K\pi)/g^2(\sigma \to \pi\pi) = 4.4$ for the experimental decay widths $\Gamma_\sigma = (504 \pm 84)$ MeV and $\Gamma_\kappa = (550 \pm 34)$ MeV. If the values of $g'_{\sigma\pi\pi}$ and $g_{\kappa K^+\pi^-}$, that both are calculated in light cone QCD sum rules, are used in Eq. (11), the scalar mixing angle θ_s is estimated as $\theta_s = (35 \pm 15)^\circ$. Our estimation is consistent with the analysis of the experimental results that are obtained from the decays $J/\psi \to f_0(980)\phi$ and $J/\psi \to f_0(980)\omega$ as $\theta_s = (34 \pm 6)^\circ$ [28], and from the decays $D_S^+ \to f_0(980)\pi^+$ and $D_S^+ \to \phi\pi^+$ as $35^\circ \leq \theta_s \leq 55^\circ$ [29].

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APPENDIX A

Pion matrix elements which are used in the present work are given as [30]

$$\begin{aligned}
\langle\pi(p)|\bar{q}(x)\gamma_\mu\gamma_5 q(0)|0\rangle &= -if_\pi p_\mu \int_0^1 du e^{i\bar{u}px}\Big(\varphi_\pi(u) + \frac{1}{16}m_\pi^2 x^2 \mathbb{A}(u)\Big) - \frac{i}{2} f_\pi m_\pi^2 \frac{x_\mu}{px}\int_0^1 du e^{i\bar{u}px}\mathbb{B}(u),\\
\langle\pi(p)|\bar{q}(x)i\gamma_5 q(0)|0\rangle &= \mu_\pi \int_0^1 du e^{i\bar{u}px}\varphi_P(u),\\
\langle\pi(p)|\bar{q}(x)\sigma_{\alpha\beta}\gamma_5 q(0)|0\rangle &= \frac{i}{6}\mu_\pi(1-\tilde{\mu}_\pi^2)(p_\alpha x_\beta - p_\beta x_\alpha)\int_0^1 du e^{i\bar{u}px}\varphi_\sigma(u),\\
\langle\pi(p)|\bar{q}(x)\sigma_{\mu\nu}\gamma_5 g_s G_{\alpha\beta}(vx)q(0)|0\rangle &= i\mu_\pi\Big[p_\alpha p_\mu\Big(g_{\nu\beta} - \frac{1}{px}(p_\nu x_\beta + p_\beta x_\nu)\Big) - p_\alpha p_\nu\Big(g_{\mu\beta} - \frac{1}{px}(p_\mu x_\beta + p_\beta x_\mu)\Big)\\
&\quad - p_\beta p_\mu\Big(g_{\nu\alpha} - \frac{1}{px}(p_\nu x_\alpha + p_\alpha x_\nu)\Big) + p_\beta p_\nu\Big(g_{\mu\alpha} - \frac{1}{px}(p_\mu x_\alpha + p_\alpha x_\mu)\Big)\Big]\\
&\quad \times \int \mathcal{D}\alpha e^{i(\alpha_{\bar{q}} + v\alpha_g)px}\mathcal{T}(\alpha_i),
\end{aligned} \tag{A1}$$

where $\mu_\pi = f_\pi \frac{m_\pi^2}{m_u+m_d}$, $\tilde{\mu}_\pi = \frac{m_u+m_d}{m_\pi}$, $\mathcal{D}\alpha = d\alpha_{\bar{q}} d\alpha_q d\alpha_g \delta(1-\alpha_{\bar{q}}-\alpha_q-\alpha_g)$, and the $\varphi_\pi(u)$, $\mathbb{A}(u)$, $\mathbb{B}(u)$, $\varphi_P(u)$, $\varphi_\sigma(u)$, and $\mathcal{T}(\alpha_i)$ are functions of a definite twist.

The pion wave functions are [30]

$$\begin{aligned}
\varphi_\pi(u) &= 6u\bar{u}(1 + a_1^\pi C_1(2u-1) + a_2^\pi C_2^{3/2}(2u-1)), \qquad \mathcal{T}(\alpha_i) = 360\eta_3\alpha_{\bar{q}}\alpha_q\alpha_g^2\Big(1 + w_3\frac{1}{2}(7\alpha_g - 3)\Big),\\
\varphi_P(u) &= 1 + \Big(30\eta_3 - \frac{5}{2}\mu_\pi^2\Big)C_2^{1/2}(2u-1) + \Big(-3\eta_3 w_3 - \frac{27}{20}\mu_\pi^2 - \frac{81}{10}\mu_\pi^2 a_2^\pi\Big)C_4^{1/2}(2u-1),\\
\varphi_\sigma(u) &= 6u\bar{u}\Big[1 + \Big(5\eta_3 - \frac{1}{2}\eta_3 w_3 - \frac{7}{20}\mu_\pi^2 - \frac{3}{5}\mu_\pi^2 a_2^\pi\Big)C_2^{3/2}(2u-1)\Big], \qquad \mathbb{B}(u) = g_\pi(u) - \varphi_\pi(u),\\
g_\pi(u) &= g_0 C_0^{1/2}(2u-1) + g_2 C_2^{1/2}(2u-1) + g_4 C_4^{1/2}(2u-1),\\
\mathbb{A}(u) &= 6u\bar{u}\Big[\frac{16}{15} + \frac{24}{35}a_2^\pi + 20\eta_3 + \frac{20}{9}\eta_4 + \Big(-\frac{1}{15} + \frac{1}{16} - \frac{7}{27}\eta_3 w_3 - \frac{10}{27}\eta_4\Big)C_2^{3/2}(2u-1)\\
&\quad + \Big(-\frac{11}{210}a_2^\pi - \frac{4}{135}\eta_3 w_3\Big)C_4^{3/2}(2u-1)\Big] + \Big(-\frac{18}{5}a_2^\pi + 21\eta_4 w_4\Big)[2u^3(10 - 15u + 6u^2)\ln u\\
&\quad + 2\bar{u}^3(10 - 15\bar{u} + 6\bar{u}^2)\ln\bar{u} + u\bar{u}(2 + 13u\bar{u})],
\end{aligned} \tag{A2}$$

where $C_n^k(x)$ are the Gegenbauer polynomials,

$$g_0 = 1, \qquad g_2 = 1 + \frac{18}{7}a_2^\pi + 60\eta_3 + \frac{20}{3}\eta_4, \qquad g_4 = -\frac{9}{28}a_2^\pi - 6\eta_3 w_3. \tag{A3}$$

The constants in Eqs. (A2) and (A3) are calculated at the renormalization scale $\mu = 1$ GeV^2 and are given as $a_1^\pi = 0$, $a_2^\pi = 0.44$, $\eta_3 = 0.015$, $\eta_4 = 10$, $w_3 = -3$, and $w_4 = 0.2$.

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