Model for QCD ground state with magnetic disorder

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We explore an ansatz for the QCD vacuum in the Coulomb gauge that describes gauge field fluctuations in the presence of a weakly interacting gas of Abelian monopoles. Such magnetic disorder leads to longrange correlations which are manifested through the area law for the Wilson loop. In particular we focus on the role of the residual monopole-monopole interactions in providing the mechanism for suppression of the gluon propagator at low momenta which also leads to low-momentum enhancement in the ghost propagator.

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I. INTRODUCTION

A condensate of magnetic monopoles screens the chromoelectric field between fundamental color charges and leads to formation of flux tubes between static quarks. Consequently condensation of magnetic degrees of freedom in the QCD vacuum has been proposed as the mechanism underlying color confinement [\[1–](#page-4-0)[4](#page-4-1)] and recently strong evidence for magnetic dominance has emerged from lattice gauge simulations [\[5–](#page-4-2)[8](#page-4-3)]. In QCD monopoles emerge as solutions of classical equations of motion [[9\]](#page-4-4), albeit having infinite energy, the underlying ultraviolet singularity is expected to be regularized by quantum fluctuations [[10](#page-4-5)[,11](#page-4-6)] and in any case should not play a role in the low-energy domain. Monopoles create vortices [\[5,](#page-4-2)[12](#page-4-7)[,13\]](#page-4-8) and all together lead to percolation of monopole-antimonopole chains [[14](#page-4-9)]. Even though there is ample evidence for the presence of magnetic domains it is still an open issue which of the many possible monopole-vortex geometries dominates the QCD vacuum [\[8\]](#page-4-3). It should be noted, however, that vortexlike configurations of long monopole chains are needed to achieve the confining scenario between charges associated with the center of the gauge group as expected for QCD [\[15\]](#page-4-10).

Recently lattice simulations in both Landau [[16](#page-4-11)–[24](#page-5-0)] and Coulomb gauge [\[25–](#page-5-1)[27](#page-5-2)] have significantly advanced our knowledge on the infrared (IR) properties of QCD Green's functions. And together with studies in the continuum [\[28–](#page-5-3) [38](#page-5-4)] have reinvigorated discussion on the role of Green's functions in probing the long-range properties of the QCD vacuum [\[39](#page-5-5)[,40\]](#page-5-6). For example, in the Gribov-Zwanziger (GZ) scenario [\[41–](#page-5-7)[44](#page-5-8)], confinement is related to the presence of large field configurations near the boundary of the Gribov region, the Gribov horizon. The Gribov region is defined as the domain in the gauge field space that satisfies a particular gauge condition. Within the Gribov region the Faddeev-Popov (FP) operator is positive and it vanishes on the horizon. Thus in the GZ scenario one expects that the vacuum expectation value (VEV) of the inverse of the FP operator, referred to as the ghost propagator, is IR enhanced. On the other hand, the gluon propagator, which represents propagation of color charges is expected to be suppressed. Hereafter, by a suppressed propagator we mean a low-momentum behavior which is softer than that of perturbation theory, i.e. in Landau gauge a propagator $D(k) \propto k^{2\alpha}$ with $\alpha > -1$. In particular, the so-called massive solution corresponds to $\alpha = 0$ with $D(0) = \text{const}$, and the scaling solution has $\alpha > 0$ and $D(0) = 0$. Similarly our definition of an IR enhanced ghost propagator $d(k)/k^2$ includes both, a propagator that has an IR finite or IR divergent dressing function $d(k)$. It is still debatable which type of solutions, massive or scaling, is emerging from lattice simulations [\[45\]](#page-5-9). In covariant gauges scaling solution is motivated by the Kugo-Ojima confinement condition [\[46\]](#page-5-10), which relies on a global Becchi-Rouet-Stora-Tyutin (BRST) charge. It is different from the GZ confinement picture discussed above, with the latter based on a nonlocal condition. Nevertheless, in the Coulomb gauge the GZ picture implies a scaling solution. As discussed recently in [[47](#page-5-11)], however, scaling solution in the Coulomb gauge does not necessitate a scaling solution in a covariant gauge due to ambiguities with a nonperturbative definition of the charge.

Regardless the exact IR limit of the gluon and ghost propagators, lattice simulations quoted above, find unambiguously the IR suppression of the gluon propagator in both Landau and Coulomb gauges. This is to be contrasted with some of the phenomenological models where it is assumed that the quark-antiquark potential originates from the nonperturbative gluon exchange. Thus a confining quark-antiquark potential would necessitate an IR enhanced gluon propagator [[48,](#page-5-12)[49](#page-5-13)].

In this paper we explore the possibility that the lowmomentum suppression of the gluon propagator may not necessarily be related to confinement but to color screening. In the latter case IR suppression of the gluon propagator implies that colored, physical gluons do not propagate and, as follows from the GZ conjecture, it would not be the role of the gluon propagator but of the IR enhanced ghost propagator to carry the distinct signatures

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of confinement. It is a known phenomenon that a dynamically generated gluon mass and IR suppression of the gluon propagator can emerge as a result of screening e.g. induced by condensation of magnetic domains (vortices, monopoles) [[50](#page-5-14),[51](#page-5-15)]. It can be well illustrated in models [[8\]](#page-4-3), i.e. the Fradkin-Shenker model [\[52\]](#page-5-16) where the confined and Higgs phase are smoothly connected and there is also a smooth transition in the gauge propagators [[53\]](#page-5-17). Similarly in the Landau gauge, solutions of QCD Dyson-Schwinger equations that display screening behavior have recently been studied in [[33](#page-5-18)].

In the following section we examine the effect of condensation of magnetic monopoles in the Coulomb gauge. In particular we study an ansatz for the vacuum wave functional that contains a weakly interacting gas of monopoles. The recent studies of various models for the vacuum wave function [[34](#page-5-19)[–38](#page-5-4)[,54–](#page-5-20)[57\]](#page-5-21) were shown to be quite successful phenomenologically. In particular analytical calculations can be done with an ansatz that describes the vacuum in terms of Gaussian fluctuations of the transverse, vector potential around the zero-field configurations. Since such a simple vacuum does not explicitly contain magnetic degrees of freedom it leads to a perimeter law falloff of the VEV of a Wilson loop. Nevertheless onset of confinement could be seen through the IR enhancement of the ghost propagator and the non-Abelian Coulomb potential between static charges. This is because a Gaussian ansatz can support large fluctuations of the fields, which may be nearing the boundary of the Gribov region. We begin with the construction of our vacuum ansatz followed by a discussion of the gluon and ghost propagators as well as the Wilson loop. A summary and outlook are given in Sec. III.

II. MODEL FOR THE MONOPOLE DOMINATED VACUUM

In the following we consider $SU(2)$ Coulomb gauge QCD, in the Shrödinger picture represented by the transverse field variables, $\nabla \cdot A^a = 0$, where $a = 1 ... N_c^2$
 $1 = 3$ is the color index. The Gaussian approximation verse field variables, $v \cdot A = 0$, where $u = 1...v_C$
1 = 3 is the color index. The Gaussian approximation to
the vacuum discussed earlier is given by a wave function in the vacuum discussed earlier is given by a wave function in the form

$$
\Psi_T^w[A_T] \propto e^{-(1/2)} \int dx dy A_T(x) \omega_T(x-y) A_T(y). \tag{1}
$$

We will use the $T(L)$ subscript to represent components in the color algebra transverse (longitudinal) to a chosen direction, $w = w^a$, $(w^2 = 1)$, i.e. $A_T^{ia} \equiv A^{ia} - (w^b A^{ib})w^a$,
 $A_T^i \equiv A^{ia}w^a$. The meaning of w and the role of fields along $A_L^i \equiv A^{ia} w^a$. The meaning of w and the role of fields along w will be discussed below. The 2-point function (hereafter w will be discussed below. The 2-point function (hereafter referred to as the gluon propagator) of color-transverse gluons is then given by $(V$ is the three-dimensional volume)

$$
\mathcal{V}^{-1}\langle A_T(k)A_T(-k)\rangle = \delta_T(k)\delta_T(w)D_T(k),\qquad(2)
$$

where $\delta_T(n) = \delta_T^{ij}(n) = \delta_{ij} - n^i n^j/n^2$ is the transverse

projector in three dimensions and $D_T(k) = 1/(2\omega_T(k))$ where $\omega_T(k)$ is the Fourier transform of $\omega_T(x)$ that appears in Eq. [\(1\)](#page-1-0). Our ansatz wave function absorbs the Coulomb gauge functional measure [\[58\]](#page-5-22) i.e.

$$
\int DA \mathcal{J}[A] \langle \Psi_{\text{QCD}} | A \rangle^2 \to \int DA \langle \Psi_{\text{ansatz}} | A \rangle^2 = 1. \quad (3)
$$

With the Gaussian wave function to (approximately) restrict field configurations to the inside of the Gribov volume where the FP operator $\mathcal J$ is positive, $\omega_T(k)$ should be taken such that it diverges in the IR limit, i.e. $\omega_T(k \to 0) \propto$ $k^{1+\alpha_{G,T}}$ with $\alpha_{G,T} < -1$. For large momenta, the choice $\omega_T(k) \rightarrow k$ makes the vacuum wave function match that of the free theory.

The separation of transverse and longitudinal (with respect to w) components is motivated by the assumed dominance of Abelian monopoles. The classical field of an Abelian monopole centered at c is given by [\[59\]](#page-5-23) $(x_n \equiv$ $n \cdot x, x_{\perp}^i \equiv x^i - n^i x_n$

$$
a^{ia}(x, \alpha) = qa^i(x - c, n)w^a,
$$
 (4)

where

$$
a^{i}(x - c, n) = \frac{g}{4\pi} \frac{[(x - c) \times n]^{i}}{|x - c| [(x - c] - (x_{n} - c_{n})]},
$$
 (5)

 $\alpha = (q, c, n)$ denotes collectively the monopole coordinates, $q \pm 1$ is the monopole charge in units of the magnetic charge g, c represents its location, and n is a unit vector that defines the orientation of the (straight) Dirac string. In the following we will neglect fluctuations in the relative orientation of the monopole color orientations, i.e. in Eq. (1) (1) we set w to be the common orientation for all monopoles. This restriction can be easily removed, however, analytical calculations, even in the weak coupling limit that we discuss below, would not be possible. The Abelian component of the gauge field A_L is then assumed to fluctuate over a background of monopole-antimonopole gas. For N monopoles we thus write $(\alpha = (\alpha_1, \dots \alpha_N)),$

$$
\Psi_{L,N}^{w,\alpha}[A_L] \propto e^{-(1/2)\int dx dy A_L(x,\alpha)\omega_L(x-y)A_L(y,\alpha)},\tag{6}
$$

where

$$
A_L(x, \alpha) \equiv A_L(x) - \sum_{i=1}^{N} a(x, \alpha_i). \tag{7}
$$

Finally, after summing over the monopole coordinates, averaging over their (common) color orientation, and summing over the N configurations we obtain the ansatz for the vacuum wave functional given by

$$
|\Psi[A]|^2 = \frac{1}{Z[0]} \int \frac{dw}{4\pi} D\alpha \sum_{N=0}^{\infty} \Psi_m(\alpha) |\Psi_{L,N}^{w,\alpha}[A_L] \Psi_T^{w}[A_T]|^2.
$$
\n(8)

The integration measure over monopole coordinates is

given by $D\alpha = \prod_{i=1}^{N} [d c_i (d n_i/4\pi) (1/2 \sum_{q_i = \pm 1})]$. The distribution of monopoles is specified by the wave function $\Psi_m(\{\alpha\})$. We first discuss the noninteracting approximation,

$$
\Psi_m(\alpha) = \frac{\rho^N}{N!} = \text{const},\tag{9}
$$

with ρ being the density of monopole pairs. The gluon propagator in this case is simply given by

$$
\mathcal{V}^{-1}\langle A^{ia}(k)A^{jb}(-k)\rangle = \delta_T^{ij}(k)\delta^{ab}D(k),
$$

$$
D(k) = \frac{2}{3}D_T(k) + \frac{1}{3}D_L(k),
$$
(10)

with

$$
D_L(k) = \frac{1}{2\omega_L(k)} + \rho \int \frac{dn}{4\pi} [a^i(k, n)a^i(-k, n)]
$$

=
$$
\frac{1}{2\omega_L(k)} + \frac{g^2 \rho}{2k^4} \int_{-1}^1 dx \frac{1 - x^2}{x^2 + \epsilon^2}.
$$
 (11)

The monopole contribution is singular in the limit $\epsilon \rightarrow 0$ due to the collinear singularity associated with the momentum component along the Dirac string [[59](#page-5-23),[60](#page-5-24)]. Even if this divergence was to be regularized, for example, by combining monopole-antimonopole pairs into closed chains, the contribution remains strongly enhanced in the IR due to the $1/k⁴$ behavior which originates from the long-range, Coulomb, monopole field. Thus a simple model with noninteracting monopoles (same result is obtained in the case of vortices) cannot be adequate since it gives as strongly enhanced gluon propagator that is inconsistent with all lattice results.

The IR suppression of gluon propagator must therefore originate from screening by the interacting monopoles. To this extent we introduce an effective interaction which is repulsive (attractive) between monopole (antimonopole), respectively,

$$
V_{ij} = V(\alpha_i, \alpha_j) = q_i q_j \delta^2 (n_i - n_j) V(c_i - c_j, n_i), \quad (12)
$$

and replace Ψ_m by the corresponding partition function

$$
\Psi_m(\alpha) = \frac{\rho^N}{N!} e^{-(1/4)\sum_{i,j=1}^N V_{ij}}.
$$
\n(13)

We choose the potential V in Eq. ([12](#page-2-0)) in the form

$$
V(k, n) = \int dx V(x, n)e^{ik \cdot x} = \frac{4\pi M^{1+2\gamma}}{k_n^2 k^{2+2\gamma}}.
$$
 (14)

As will be seen below, the $1/k_n^2$ $(k_L = n \cdot k)$ term will be responsible for screening in the direction along the string responsible for screening in the direction along the string while the critical exponent γ will control the IR behavior of the propagator, and M will be related to the inverse Debye length.

In calculation of matrix elements, $\langle 0 \rangle = D_A \langle 0 \rangle [A] \Psi[A]]^2$ with the wave function given by $\int DAO[A] |\Psi[A]|^2$ with the wave function given by
Eq. (8) the summation over the number of monopoles is Eq. [\(8](#page-1-1)), the summation over the number of monopoles is done with the help of an auxiliary sine-Gordon field [[3\]](#page-4-12), $\phi(x, n)$. In particular for a generating functional

$$
Z[J] \equiv \int DA e^{\int dx \left[J_L^i(x)A_L^i(x) + J_T^{ia}(x)A_T^{ia}(x)\right]} |\Psi[A]|^2, \quad (15)
$$

one finds

$$
Z[J] = Z_T[J_T] \int D\phi e^{-S_L[\phi, J_L]}, \qquad (16)
$$

with
$$
Z_T[J_T] = \exp(\frac{1}{4} \int dx dy J_T(x) \omega_T^{-1}(x - y) J_T(y))
$$
 and
\n
$$
S_L[\phi, J_L] = \frac{1}{4\pi M^{1+2\gamma}} \int dx dn [\phi(x, n) \partial_L^2 (\partial^2)^{1+\gamma} \phi(x, n) + \lambda_D^{-(4+2\gamma)} [1 - \cos(\phi(x, n) - v_L(x, n))]],
$$
\n(17)

where

$$
\nu_L(x, n) = i \int dy J_L^i(y) a^i(y - x, n). \tag{18}
$$

We defined the Debye screening length λ_D = $(M^{1+2\gamma}\rho)^{-1/(4+2\gamma)}$. In terms of the generating functional
the gluon propagator is given by $\partial^2 \ln Z[L = 0]/\partial x \cup \partial y$. the gluon propagator is given by $\partial^2 \ln Z[J=0]/\partial_{J(k)} \partial_{J(-k)}$. In the mean-field approximation, which is valid in the limit $\lambda_D > \rho^{-1/3}$, where $\rho^{-1/3}$ is the average separation between the monopoles, the functional integral over ϕ can be computed analytically using the saddle-point approximation. At this level it is clear that interaction between monopole pairs screens their charges and thus regularizes the IR divergent part of the gluon propagator, leading to a propagator which is of the form given by Eq. [\(11\)](#page-2-1) but with the last term on the right-hand side replaced by

$$
\frac{g^2 \rho}{2} \int_{-1}^1 dx (1 - x^2) \frac{k^{2\gamma}}{k_n^2 k^{2 + 2\gamma} + \lambda^{-4 - 2\gamma}}
$$

= $k_D^{2\gamma} (g^2 \rho \lambda_D^4) \left[\frac{(1 + k_D^4) \arctan(k_D^2) - k_D^2}{k_D^6} \right],$ (19)

where $k_D \equiv k\lambda_D$. In [[27](#page-5-2)] lattice simulation of the Coulomb propagator in $D = 4$ was performed and shown to be well approximated by the Gribov formula, $D(k) =$ $k/(2\sqrt{k^4 + M^4})$ with $M = 0.890$ GeV. In [[55](#page-5-25)[–57\]](#page-5-21) it was shown that the $D = 3$ Yang-Mills (YM) action in I and shown that the $D = 3$ Yang-Mills (YM) action in Landau gauge may be a good approximation to $D = 4$ Coulomb gauge wave function, and recently the corresponding Coulomb propagator was evaluated and shown to compare favorably with the exact $D = 4$ propagator [[54](#page-5-20)]. For comparison we then use results obtained with the $D = 3$ YM action since it allows one to separate the nonmagnetic and magnetic contributions. In Fig. [1](#page-3-0) circles represent the Coulomb propagator from [\[54\]](#page-5-20) with vortices removed. We use this propagator to fix ω_L and ω_T under a simplifying assumption $\omega_L(k) = \omega_T(k)$ which we parametrize as $(k/m)^a/2((k/m)^2 + b^2)^{1/2+a/2}$ so that at low momentum
 $\omega_{\lambda} = k^a$ and in the UV $\omega_{\lambda} = \gamma (k)$. The fit of ω to the $\omega_{LT} \sim k^a$ and in the UV, $\omega_{LT} \sim 1/k$. The fit of ω to the

FIG. 1. Comparison of our gluon propagator with that obtained from lattice computations [\[54\]](#page-5-20).

propagator from [\[54\]](#page-5-20) with vortices removed yields $a =$ 1.21, $b = 0.29$, $m = 3.88$ GeV. The result of the fit is shown by the dashed line. We then fix the three parameters that describe our propagator [cf. Eqs. ([11](#page-2-1)) and ([19\)](#page-2-2)], $L \equiv$ $g^2 \rho \lambda_D^4$, λ_D , and γ by fitting the full gluon propagator from [\[54\]](#page-5-20) (squares in Fig. [1](#page-3-0)) and obtain $\lambda_D = 1.51 \text{ GeV}^{-1}$, $L =$ 6.46 GeV⁻¹, and $\gamma = 0.32$. The result is shown by the solid line. And the condition of applicability of the meanfield approximation requires week monopole-monopole interaction (i.e. strong chromoelectric coupling, $e =$ $4\pi/g$) with

$$
g < \sqrt{\frac{L}{\lambda_D}} \lesssim 2. \tag{20}
$$

As discussed in Sec. I, and shown above, while the IR suppression of the gluon might be the result of screening of magnetic charges, IR enhancement of the ghost propagator comes from YM field distribution near the Gribov horizon. We postpone a detailed numerical study of the ghost propagator and just notice that because monopoles introduce orientation in color, the diagonal (D_L) and offdiagonal (D_T) gluon propagators are different, and the mean-field relation between the ghost and gluon propagators becomes more complicated. In particular, defining the longitudinal and transverse ghost form factors $d_L(k)$ and $d_T(k)$ by

$$
\frac{d_L(k)}{k^2} \equiv \mathcal{V}^{-1} \int DAw^a w^b M^{-1}[A] |\Psi^w[A]|^2
$$
\n
$$
\frac{d_T(k)}{k^2} \equiv \mathcal{V}^{-1} \int DA \frac{\delta_T^{ab}(w)}{2} M^{-1}[A] |\Psi^w[A]|^2,
$$
\n(21)

where $M^{-1}[A] = M^{-1}[A](k, a, -k, b) = -e(\nabla \cdot D[A])^{-1}$
is the inverse of the Faddeev-Poppoy operator. In the meanis the inverse of the Faddeev-Poppov operator. In the meanfield approximation, one obtains

$$
d_L^{-1}(k) = e^{-1} - I[D_T, d_T]
$$

\n
$$
d_T^{-1}(k) = e^{-1} - \frac{1}{2}I[D_L, d_T] - \frac{1}{2}I[D_T, d_L],
$$
\n(22)

where [\[61\]](#page-5-26)

$$
I[D,d] = \frac{N_C}{2} \int d\mathbf{p} D(p) \frac{pk - p \cdot k}{pk(p-k)^2} d(p-k).
$$
 (23)

In the case $D_L = D_T = D$ these give $d_L = d_T = d$ with d given by

$$
d_L^{-1}(k) = e^{-1} - I[D, d],
$$
 (24)

that has been studied in the past [\[34–](#page-5-19)[38](#page-5-4)]. In general, after averaging over monopole color directions,

$$
d(k) = \frac{2}{3}d_T(k) + \frac{1}{3}d_L(k).
$$
 (25)

The IR analyst of Eq. [\(24\)](#page-3-1) leads to a relation between gluon and ghost critical exponents. That is assuming the IR behavior, of the form $D(k) \propto k^{2\gamma}$ and $d(k) \propto k^{2\delta}$ one finds $\delta = -1/4 - \gamma/2$. Thus an IR enhanced ghost propagator $\delta = -1/4 - \gamma/2$. Thus an IR enhanced ghost propagator $(\delta < 0)$ pecessitates a screened and IR suppressed $(\gamma > 0)$ $(\delta \le 0)$ necessitates a screened and IR suppressed $(\gamma > 0)$
gluon propagator. In the case of Eq. (22), with $D_{L/m}(k) \propto$ gluon propagator. In the case of Eq. ([22](#page-3-2)), with $D_{L(T)}(k) \propto$ $k^{2\gamma_{L(T)}}$ and $d_{L(T)}(k) \propto k^{2\delta_{L(T)}}$, respectively, one finds δ
-1/4 – γ_L /2 and $\delta_L = -1/4 - (\gamma_L - \gamma_L/2)$. In a $T =$ ddi $-1/4 - \gamma_L/2$ and $\delta_L = -1/4 - (\gamma_T - \gamma_L/2)$. In addition one should consider the Coulomb form factor and tion one should consider the Coulomb form factor and the gap equation. It was found in [\[38\]](#page-5-4) that the coupled set of Dyson equations for these functions admitted only IR finite solutions. With the addition of monopoles, however, a preliminary analysis of Eq. [\(21\)](#page-3-3) indicates that IR critical solutions are possible.

Finally we comment on the role of monopoles in suppressing the large Wilson loop,

$$
W_J[C] = \frac{1}{2J+1} \operatorname{Tr} \langle \Psi | P \exp \left(i e \oint_C dx^i A^{i,a} T^a \right) | \Psi \rangle, \tag{26}
$$

where T^a are the $SU(2)$ color generators in the J-th representation. The integration over A is computed by shifting A_L according to Eq. ([7\)](#page-1-2). In the limit of large loops, the contribution from the nonmonopole component (and A_T) is determined by the gluon propagator

$$
\ln W_J[C] \sim -\int_{C \to \infty} dx^i dy^j \delta_T^{ij}(\mathbf{k}) \int \frac{dk}{(2\pi)^3} D(k) e^{ik \cdot x}
$$

$$
\sim O(R^{-1}D(k \sim R^{-1})), \tag{27}
$$

where R is the perimeter of the loop. An IR suppressed gluon propagator with $D(k) \sim k^{2\gamma}$ and $\gamma \ge 0$ leads to screening of the WIlson loop, i.e. the loop is dominated by shot range correlations and has at most perimeter dependence. Thus if the long-range correlations are to dominate they must come from the monopole gas and it is possible to ignore the contributions from the fluctuating field. This leads to

$$
W_J[C] = \frac{1}{2J+1} \sum_{m=-J}^{J} \int D\phi e^{-S[\phi, m\eta[C]]}, \quad (28)
$$

where S is given by Eq. ([17](#page-2-3)) and $\eta[C](c, n) =$ $\oint_C dy^i a^i(y - c, n)$. Here c is the location of a single mono-
pole. In particular for a large loop in xy plane with perimepole. In particular for a large loop in xy plane with perimeter, $R \gg \lambda_D$, $\eta \rightarrow 2\pi \text{sgn}(c \cdot n)$. It immediately follows
that N-ality zero (Linteger) loops are screened and all that N-ality zero (J-integer) loops are screened and all nonzero N-ality loops behave equivalently to the loop in the fundamental representation $J = 1/2$. The Casimir scaling presumably comes from the neglected effects of the fluctuating field. In the weakly interacting limit $(\lambda_D >$ $\rho^{-1/3}$), the path integral in Eq. ([28\)](#page-4-13) can also be evaluated in the saddle-point approximation; this time however the saddle point does not correspond to $\phi = 0$ but centers around η [[3](#page-4-12),[62](#page-5-27)] and is given by the solution of

$$
\frac{(-\partial_L)^2(-\partial)^{2+\gamma}}{\lambda_D^{-(4+2\gamma)}}\phi(x,n) + \sin(\phi(x,n) + m\eta[C](x,n)) = 0.
$$
\n(29)

For large loops $R \gg \lambda_D$ in the xy plane, the equation becomes effectively one-dimensional, $\phi(x, n) \rightarrow \phi(z_n)$, $z_n = z \cdot n$ with $\phi(z)$ interpolating smoothly the discontinuity in *n* across the Wilson loop between $-\pi$ and π over nuity in η across the Wilson loop, between $-\pi$ and π over a distance of the order of λ_D . Substituting such a saddlepoint solution into the action in Eq. [\(28\)](#page-4-13) one finds

$$
S\left[\phi, \frac{1}{2}\eta\right] \sim R^2 \lambda \frac{\lambda^{-4+2\gamma}}{M^{1+2\gamma}}
$$
 (30)

and thus

$$
\ln W_{1/2}[C] \sim \lambda_D \rho A[C],\tag{31}
$$

where A is the area of the loop.

III. SUMMARY AND OUTLOOK

Since condensation of magnetic degrees of freedom is known to be present in the QCD it becomes essential to include magnetic degrees of freedom when constructing models of the QCD vacuum. In our construction we have assumed that these can be represented by a weakly interacting gas of aligned (in color) Abelian monopoles. Such a state is known to reproduce the confining properties for large, fundamental Wilson loops, however it does suffer from yielding $U(1)^N$ rather than Z_N charge dependence.
The latter could originate from vortex configurations of The latter could originate from vortex configurations of long monopole chains and a construction of an ansatz for the corresponding vacuum sate would be highly desirable. Here we have shown how magnetic monopoles influence the gluon propagator and have argued that the IR suppression is the result of screening of magnetic charge and not confinement. Suppression of the low-momentum gluon propagator leads to suppression of long-range gluon fluctuations and restricts the field variables to the inside of the Gribov region. A self-consistent calculation of gluon and ghost propagators and the Coulomb form factor is currently underway.

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