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Are the $X(4160)$ and $X(3915)$ charmonium states?

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Inspired by the newly observed states $X(4160)$ and $X(3915)$, we analyze the mass spectra of these states in different quark models and calculate their strong decay widths by the ${}^3P_0$ model. According to the mass spectra of charmonium states predicted by the potential model, the states $\chi_0(3^3P_0)$, $\chi_1(3^3P_1)$, $\eta_{c2}(2^1D_2)$, $\eta_c(4^1S_0)$ all can be candidates for the $X(4160)$. However, only the decay width of the state $\eta_{c2}(2^1D_2)$ in our calculation is in good agreement with the data reported by Belle, and the decay of $\eta_{c2}(2^1D_2) \to D\bar{D}$, which is not seen in experiment, is also forbidden. Therefore, it is reasonable to interpret the charmonium state $\eta_{c2}(2^1D_2)$ as the state $X(4160)$. For the state $X(3915)$, although the mass of $\chi_0(2^3P_0)$ is compatible with the experimental value, the calculated strong decay width is much larger than experimental data. Hence, the assignment of $X(3915)$ to the charmonium state $\chi_0(2^3P_0)$ is disfavored in our calculation.

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I. INTRODUCTION

Many new charmonium-like states, the so-called *XYZ* mesons, have been reported by the Belle and *BABAR* collaborations in recent years. Some of these states can be understood as conventional mesons that are comprised of only pure $c\bar{c}$ quark pairs. However, most of the *XYZ* states do not match well the mass spectrum of $c\bar{c}$ predicted by the QCD-motivated potential models. By considering the effects of virtual mesons loops [1–4] and color screening [5], the masses of some excited charmonium states are smaller than those calculated by the conventional quark model. Therefore, some *XYZ* states [2], including the most robust state $X(3872)$ [2,4,6], may still be compatible with the mass spectrum of charmonium.

Last year, the Belle Collaboration reported a new charmonium-like state, the $X(4160)$ [7], in the processes $e^+e^- \to J/\psi D^{(*)}\bar{D}^{(*)}$ with a significance of 5.1σ. It has mass $M = 4156^{+25}_{-20} \pm 15$ MeV and width $\Gamma = 139^{+111}_{-61} \pm 21$ MeV. Based on the processes $e^+e^- \to J/\psi D\bar{D}$, $e^+e^- \to J/\psi D^*\bar{D}$, and $e^+e^- \to J/\psi D^*\bar{D}^*$, the upper limits of the branch ratios of $X(4160)$ are given as

$$\mathcal{B}_{D\bar{D}}(X(4160))/\mathcal{B}_{D^*\bar{D}^*}(X(4160)) < 0.09, \tag{1}$$

$$\mathcal{B}_{D^*\bar{D}}(X(4160))/\mathcal{B}_{D^*\bar{D}^*}(X(4160)) < 0.22. \tag{2}$$

The $X(4160)$ has possible charge parity $C = +$, since the photons γ and J/ψ have $J^{PC} = 1^{--}$, and $e^+e^- \to \gamma \to J/\psi X(4160)$ is the main process. Hence the $X(4160)$ can have $J^{PC} = 0^{-+}, 0^{++}, 1^{-+}, 2^{-+}, 1^{++}, 2^{++}, \ldots$. In Ref. [8], Chao discussed the possible interpretation of the $X(4160)$ in view of the production rate in $e^+e^- \to J/\psi X(4160)$. He believes that the charmonium states 4^1S_0, 3^3P_0 may be assigned to the state $X(4160)$ by analogy with the cross section of $e^+e^- \to J/\psi\eta_c(1S) \times (\eta_c(2S)\chi_{c0}(1P))$, while the 2^1D_2 [9] cannot be ruled out. According to the mass spectrum [5] predicted by the potential model with color screening, Li and Chao also give some arguments about the $\chi_0(3^3P_0)$ as an assignment for the $X(4160)$.

Using the vector-vector interaction within the framework of the hidden gauge formalism, Molina and Oset [10] suggested that the $X(4160)$ is a molecular state of $D_s^*\bar{D}_s^*$ with $J^{PC} = 2^{++}$.

Very recently, Refs. [11–15] reported the newest charmonium-like state, the $X(3915)$, which is observed by Belle in $\gamma\gamma \to \omega J/\psi$ with a statistical significance of 7.7σ. It has mass and width

$$M = 3915 \pm 3 \pm 2 \text{ MeV}, \qquad \Gamma = 17 \pm 10 \pm 3 \text{ MeV}. \tag{3}$$

The Belle Collaboration determined the $X(3915)$ production rates $\Gamma_{\gamma\gamma}(X(3915))\ \mathcal{B}(X(3915) \to \omega J/\psi) = 61 \pm 17 \pm 8$ eV and $\Gamma_{\gamma\gamma}(X(3915))\ \mathcal{B}(X(3915) \to \omega J/\psi) = 18 \pm 5 \pm 2$ eV for $J^P = 0^+$ or 2^+, respectively. Because the partial width of this state to $\gamma\gamma$ or $\omega J/\psi$ is too large, it is very unlikely to be the charmonium state analyzed by Yuan [13].

The $X(3915)$ also has charge parity $C = +$, because it is observed in the process of $\gamma\gamma \to \omega J/\psi$. In Ref. [16], Liu *et al.* argued that the $\chi_0(2^3P_0)$ can be assigned to the $X(3915)$ if taking $R = 1.8$–1.85 GeV^{-1} in the simple harmonic oscillator (SHO) wave functions approximation.

Up to now, the interpretation of the $X(4160)$ and $X(3915)$ is still unclear. The states $\chi_0(3^3P_0)$, $\chi_1(3^3P_1)$, $\eta_{c2}(2^1D_2)$ listed in Table I can all be interpreted as the $X(4160)$ just on the mass level. Which charmonium state is an assignment for the $X(4160)$? One can answer this question in different ways. We study the $X(4160)$ and $X(3915)$ via strong decay by the ${}^3P_0$ model [18–21] in this work. In

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TABLE I. Theoretical mass spectrum of the charmonium candidates for the $X(4160)$ and $X(3915)$. The masses are in units of MeV. The results are taken from Ref. [5] with the color screening potential model, and from Ref. [17], including the nonrelativistic potential and Godfrey-Isgur relativized potential models.

State J^{PC}	$\chi_0(2^3P_0)$ 0^{++}	$\eta_c(4^1S_0)$ 0^{-+}	$\chi_0(3^3P_0)$ 0^{++}	$\chi_1(3^3P_1)$ 1^{++}	$\eta_{c2}(2^1D_2)$ 2^{-+}
Reference [5], color screening	3842	4250	4131	4178	4099
Reference [17], nonrelativistic	3852	4384	4202	4271	4158
Reference [17], Godfrey-Isgur	3916	4425	4292	4317	4208

the following discussion, we take the $\chi_0(3^3P_0)$, $\chi_1(3^3P_1)$, $\eta_{c2}(2^1D_2)$, $\eta_c(4^1S_0)$, and $\chi_0(2^3P_0)$ as candidates for the $X(4160)$ and $X(3915)$, respectively.

The paper is organized as follows. In the next section we give a brief review of the $^3P_0$ model. Section III is devoted to discussing the possible strong decay channels and giving the corresponding amplitudes of the candidates for the $X(4160)$ and $X(3915)$. In Sec. IV, we present and analyze the results obtained by the $^3P_0$ model. Finally, the summary of the present work is given in the last section.

II. A REVIEW OF THE $^3P_0$ MODEL OF MESON DECAY

The $^3P_0$ decay model, also known as the quark-pair creation model, was originally introduced by Micu [18] and further developed by Le Yaouanc, Ackleh, Roberts *et al.* [19–21]. It is applicable to the Okubo-Zweig-Iizuka (OZI) rule allowed strong decays of a hadron into two other hadrons, which are expected to be the dominant decay modes of a hadron. Because the $^3P_0$ model gives a good description of many observed partial widths of hadrons, it has been widely used to evaluate the strong decays of mesons and baryons composed of u, d, s, c, b quarks [16,22–33].

The $^3P_0$ model of strong decays assumes that quark-antiquark pairs are created with vacuum quantum number $J^{PC} = 0^{++}$ [18]. The diagrams of all possible decay processes $A \to B + C$ of mesons are shown in Fig. 1. In many cases only one of them contributes to the strong decay of a meson. The transition operator of this model takes the form

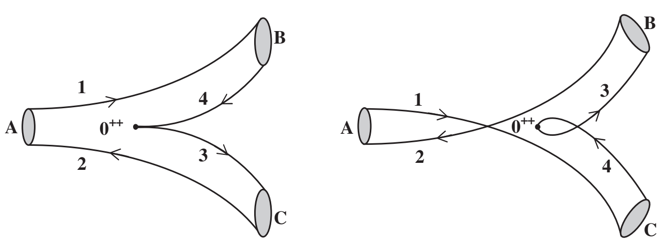

FIG. 1. The two possible diagrams contributing to $A \to B + C$ in the $^3P_0$ model.

$$T = -3\gamma \sum_m \langle 1m1-m|00\rangle \int d\mathbf{p}_3 d\mathbf{p}_4 \delta^3(\mathbf{p}_3+\mathbf{p}_4) \times \mathcal{Y}_1^m\left(\frac{\mathbf{p}_3-\mathbf{p}_4}{2}\right)\chi_{1-m}^{34}\phi_0^{34}\omega_0^{34} b_3^\dagger(\mathbf{p}_3) d_4^\dagger(\mathbf{p}_4), \quad (4)$$

where γ, which is a dimensionless parameter, represents the probability of the quark-antiquark pair created from the vacuum and can be extracted by fitting experimental data. $\phi_0^{34} = (u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3}$, $\omega_0^{34} = (R\bar{R} + G\bar{G} + B\bar{B})/\sqrt{3}$ are flavor and color singlet states, respectively. $\chi_{1,-m}^{34}$ is a spin-triplet state. $\mathcal{Y}_l^m(\mathbf{p}) \equiv |p|^l Y_l^m(\theta_p, \phi_p)$ is the lth solid harmonic polynomial that reflects the momentum-space distribution of the created quark-antiquark pair. $b_3^\dagger(\mathbf{p}_3)$, $d_4^\dagger(\mathbf{p}_4)$ are the creation operators of the quark and antiquark, respectively.

In general, the mock state is adopted to describe the meson with the spatial wave function $\psi_{n_A L_A M_{L_A}}(\mathbf{p}_1, \mathbf{p}_2)$ in the momentum representation [34].

$$|A(n_A^{2S_A+1}L_{AJ_AM_{J_A}})(\mathbf{P}_A)\rangle \equiv \sqrt{2E_A} \sum_{M_{L_A},M_{S_A}} \langle L_A M_{L_A} S_A M_{S_A}|J_A M_{J_A}\rangle \int d\mathbf{p}_A \psi_{n_A L_A M_{L_A}} \times (\mathbf{p}_1,\mathbf{p}_2)\chi_{S_A M_{S_A}}^{12}\phi_A^{12}\omega_A^{12}|q_1(\mathbf{p}_1)\bar{q}_2(\mathbf{p}_2)\rangle, \quad (5)$$

with the normalization conditions

$$\langle A(n_A^{2S_A+1}L_{AJ_AM_{J_A}})(\mathbf{P}_A)|A(n_A^{2S_A+1}L_{AJ_AM_{J_A}})(\mathbf{P}'_A)\rangle = 2E_A\delta^3(\mathbf{P}_A - \mathbf{P}'_A), \quad (6)$$

where n_A represents the radial quantum number of the meson A composed of q_1, $\bar{q}_2$ with momentum $\mathbf{p_1}$ and $\mathbf{p_2}$. E_A is the total energy, $\mathbf{P}_A$ is the momentum of the meson A, and $\mathbf{p}_A = (m_2\mathbf{p}_1 - m_1\mathbf{p}_2)/(m_1+m_2)$ is the relative momentum between the quark and antiquark. $\mathbf{S}_A = \mathbf{s}_{q_1} + \mathbf{s}_{q_2}$, $\mathbf{J}_A = \mathbf{L}_A + \mathbf{S}_A$ stand for the total spin and total angular momentum, respectively. $\mathbf{L}_A$ is the relative orbital angular momentum between q_1 and $\bar{q}_2$. $\langle L_A M_{L_A} S_A M_{S_A}|J_A M_{J_A}\rangle$ denotes a Clebsch-Gordan coefficient, and $\chi_{S_A M_{S_A}}^{12}$, ϕ_A^{12}, and ω_A^{12} are the spin, flavor, and color wave functions, respectively.

The S matrix of the process $A \to B + C$ is defined by

$$\langle BC|S|A\rangle = I - 2\pi i\delta(E_A - E_B - E_C)\langle BC|T|A\rangle, \tag{7}$$

with

$$\langle BC|T|A\rangle = \delta^3(\mathbf{P}_A - \mathbf{P}_B - \mathbf{P}_C)\mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}}, \tag{8}$$

where $\mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}}$ is the helicity amplitude of $A \to B + C$. In the center-of-mass frame of the meson A, $\mathbf{P}_A = 0$, and $\mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}}$ can be written as

$$\begin{aligned}\mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}}(\mathbf{P}) = \gamma\sqrt{8E_AE_BE_C} &\sum_{\substack{M_{L_A},M_{S_A},\\ M_{L_B},M_{S_B},\\ M_{L_C},M_{S_C},m}} \langle L_AM_{L_A}S_AM_{S_A}|J_AM_{J_A}\rangle\langle L_BM_{L_B}S_BM_{S_B}|J_BM_{J_B}\rangle\langle L_CM_{L_C}S_CM_{S_C}|J_CM_{J_C}\rangle \\ &\times \langle 1m1-m|00\rangle\langle\chi^{14}_{S_BM_{S_B}}\chi^{32}_{S_CM_{S_C}}|\chi^{12}_{S_AM_{S_A}}\chi^{34}_{1-m}\rangle[\langle\phi^{14}_B\phi^{32}_C|\phi^{12}_A\phi^{34}_0\rangle I^{M_{L_A},m}_{M_{L_B},M_{L_C}}(\mathbf{P}, m_1, m_2, m_3) \\ &+ (-1)^{1+S_A+S_B+S_C}\langle\phi^{32}_B\phi^{14}_C|\phi^{12}_A\phi^{34}_0\rangle I^{M_{L_A},m}_{M_{L_B},M_{L_C}}(-\mathbf{P}, m_2, m_1, m_3)],\end{aligned} \tag{9}$$

with the momentum-space integral

$$I^{M_{L_A},m}_{M_{L_B},M_{L_C}}(\mathbf{P}, m_1, m_2, m_3) = \int d\mathbf{p}\,\psi^*_{n_BL_BM_{L_B}}\left(\frac{m_3}{m_1+m_3}\mathbf{P}+\mathbf{p}\right)\psi^*_{n_CL_CM_{L_C}}\left(\frac{m_3}{m_2+m_3}\mathbf{P}+\mathbf{p}\right)\psi_{n_AL_AM_{L_A}}(\mathbf{P}+\mathbf{p})\mathcal{Y}^m_1(\mathbf{p}), \tag{10}$$

where $\mathbf{P} = \mathbf{P}_B = -\mathbf{P}_C$, $\mathbf{p} = \mathbf{p}_3$, and m_3 is the mass of the created quark q_3; $\langle\chi^{14}_{S_BM_{S_B}}\chi^{32}_{S_CM_{S_C}}|\chi^{12}_{S_AM_{S_A}}\chi^{34}_{1-m}\rangle$ and $\langle\phi^{14}_B\phi^{32}_C|\phi^{12}_A\phi^{34}_0\rangle$ are the overlaps of the spin and flavor wave function, respectively.

The spin overlap can be given in terms of Wigner's $9j$ symbol,

$$\begin{aligned}\langle\chi^{14}_{S_BM_{S_B}}\chi^{32}_{S_CM_{S_C}}|\chi^{12}_{S_AM_{S_A}}\chi^{34}_{1-m}\rangle &= \sum_{S,M_S}\langle S_BM_{S_B}S_CM_{S_C}|SM_S\rangle \\ &\times\langle S_AM_{S_A}1-m|SM_S\rangle(-1)^{S_C+1} \\ &\times\sqrt{3(2S_A+1)(2S_B+1)(2S_C+1)} \\ &\times\begin{Bmatrix}\frac{1}{2} & \frac{1}{2} & S_A\\ \frac{1}{2} & \frac{1}{2} & 1\\ S_B & S_C & S\end{Bmatrix}.\end{aligned} \tag{11}$$

Generally, one takes the SHO approximation for the momentum-space wave functions of mesons in Eq. (10). The SHO wave function reads

$$\begin{aligned}\Psi_{nLM_L}(\mathbf{p}) = (-1)^n(-i)^LR^{L+(3/2)}\sqrt{\frac{2n!}{\Gamma(n+L+\frac{3}{2})}} \\ \times\exp\left(-\frac{R^2p^2}{2}\right)L^{L+(1/2)}_n(R^2p^2)\mathcal{Y}_{LM_L}(\mathbf{p}),\end{aligned} \tag{12}$$

with $\mathcal{Y}_{LM_L}(\mathbf{p}) = |\mathbf{p}|^LY_{LM_L}(\Omega_p)$. Here R denotes the SHO wave function scale parameter; $\mathbf{p}$ represents the relative momentum between the quark and the antiquark within a meson; $L^{L+(1/2)}_n(R^2p^2)$ is an associated Laguerre polynomial.

The decay width for the process $A \to B + C$ in terms of the helicity amplitude is

$$\Gamma = \pi^2\frac{|\mathbf{P}|}{M_A^2}\frac{1}{2J_A+1}\sum_{M_{J_A},M_{J_B},M_{J_C}}|\mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}}|^2.$$

For comparison with experiments, $\mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}}(\mathbf{P})$ can be converted into the partial amplitude via the Jacob-Wick formula [35]

$$\begin{aligned}\mathcal{M}^{JL}(A\to BC) = \frac{\sqrt{2L+1}}{2J_A+1}&\sum_{M_{J_B},M_{J_C}}\langle L0JM_{J_A}|J_AM_{J_A}\rangle \\ &\times\langle J_BM_{J_B}J_CM_{J_C}|JM_{J_A}\rangle\mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}}(\mathbf{P}),\end{aligned} \tag{13}$$

where $\mathbf{J} = \mathbf{J}_B + \mathbf{J}_C$, $\mathbf{J}_A = \mathbf{J}_B + \mathbf{J}_C + \mathbf{L}$, and $M_{J_A} = M_{J_B} + M_{J_C}$. Then the decay width in terms of the partial wave amplitude is taken as

$$\Gamma = \pi^2\frac{|\mathbf{P}|}{M_A^2}\sum_{JL}|\mathcal{M}^{JL}|^2, \tag{14}$$

where $|\mathbf{P}|$, as mentioned above, is the momentum of the outgoing meson in the rest frame of the meson A. It is obtained as

$$|\mathbf{P}| = \frac{\sqrt{[M_A^2-(M_B+M_C)^2][M_A^2-(M_B-M_C)^2]}}{2M_A},$$

where M_A, M_B, and M_C are the masses of mesons A, B, and C, respectively.

III. POSSIBLE STRONG DECAY CHANNELS AND AMPLITUDES OF THE CANDIDATES FOR THE X(4160) AND X(3915)

As analyzed in Sec. I, we consider the $\eta_c(4^1S_0)$, $\chi_0(3^3P_0)$, $\chi_1(3^3P_1)$, $\eta_{c2}(2^1D_2)$ as the possible candidates

TABLE II. The OZI rule and phase space allowed open-charm strong decay modes of the possible charmonium states for the $X(4160)$ and $X(3915)$.

State	J^{PC}	Decay mode	Decay channel
$\eta_c(4^1S_0)$	0^{-+}	$0^- + 1^-$	$D\bar{D}^*$, $D_s^+D_s^{*-}$
		$1^- + 1^-$	$D^*\bar{D}^*$
$\chi_0(3^3P_0)$	0^{++}	$0^- + 0^-$	$D\bar{D}$, $D_s^+D_s^-$
		$1^- + 1^-$	$D^*\bar{D}^*$
$\chi_1(3^3P_1)$	1^{++}	$0^- + 1^-$	$D\bar{D}^*$, $D_s^+D_s^{*-}$
		$1^- + 1^-$	$D^*\bar{D}^*$
$\eta_{c2}(2^1D_2)$	2^{-+}	$0^- + 1^-$	$D\bar{D}^*$, $D_s^+D_s^{*-}$
		$1^- + 1^-$	$D^*\bar{D}^*$
$\chi_0(2^3P_0)$	0^{++}	$0^- + 0^-$	$D\bar{D}$

TABLE III. The partial wave amplitudes for the strong decays of the relevant charmonium states and the overlap of the flavor wave function $\langle\phi_B^{14}\phi_C^{32}|\phi_A^{12}\phi_0^{34}\rangle = 1/\sqrt{3}$ and $\mathcal{E} = \gamma\sqrt{E_AE_BE_C}$.

State	Decay channel	Decay amplitude
$\eta_c(4^1S_0)$	$0^- + 1^-$	$\mathcal{M}^{11} = \frac{\sqrt{2}}{3}\mathcal{E}\mathcal{I}^{00}$
	$1^- + 1^-$	$\mathcal{M}^{11} = \frac{2}{3}\mathcal{E}\mathcal{I}^{00}$
$\chi_0(3^3P_0)$	$0^- + 0^-$	$\mathcal{M}^{00} = \frac{\sqrt{2}}{3\sqrt{3}}\mathcal{E}(\mathcal{I}^{00} - 2\mathcal{I}^{\pm})$
	$1^- + 1^-$	$\mathcal{M}^{00} = \frac{\sqrt{2}}{9}\mathcal{E}(\mathcal{I}^{00} - 2\mathcal{I}^{\pm})$
		$\mathcal{M}^{22} = \frac{4}{9}\mathcal{E}(\mathcal{I}^{00} + \mathcal{I}^{\pm})$
$\chi_1(3^3P_1)$	$0^- + 1^-$	$\mathcal{M}^{10} = \frac{2}{9}\mathcal{E}(\mathcal{I}^{00} - 2\mathcal{I}^{\pm})$
		$\mathcal{M}^{12} = \frac{\sqrt{2}}{9}\mathcal{E}(\mathcal{I}^{00} + \mathcal{I}^{\pm})$
	$1^- + 1^-$	$\mathcal{M}^{22} = \frac{2}{3\sqrt{3}}\mathcal{E}(\mathcal{I}^{00} + \mathcal{I}^{\pm})$
$\eta_{c2}(2^1D_2)$	$0^- + 1^-$	$\mathcal{M}^{11} = \frac{2}{15}\mathcal{E}(\sqrt{3}\mathcal{I}^{\pm} - \mathcal{I}^{00})$
	$1^- + 1^-$	$\mathcal{M}^{11} = \frac{2\sqrt{2}}{15}\mathcal{E}(\sqrt{3}\mathcal{I}^{\pm} - \mathcal{I}^{00})$
$\chi_0(2^3P_0)$	$0^- + 0^-$	$\mathcal{M}^{00} = \frac{\sqrt{2}}{3\sqrt{3}}\mathcal{E}(\mathcal{I}^{00} - 2\mathcal{I}^{\pm})$

for the $X(4160)$, and assume that the upper limit of the mass is 4156 MeV as observed by Belle. For the $X(3915)$, the charmonium state $\chi_0(2^3P_0)$ with mass 3916 MeV is chosen as the candidate. According to the $^3P_0$ model discussed in the above section, the OZI rule allows open-charm strong decay, and corresponding amplitudes of possible charmonium states are listed in Tables II and III. We replace $\mathcal{I}_{0,0}^{+1-1}$, $\mathcal{I}_{0,0}^{-1+1}$ with $\mathcal{I}^{\pm}$ and $\mathcal{I}_{0,0}^{0,0}$ with $\mathcal{I}^{0,0}$ in Table III. The details of the spatial integral about $\mathcal{I}^{\pm}(\mathbf{P})$ and $\mathcal{I}^{0,0}(\mathbf{P})$ are given in the Appendix.

IV. NUMERICAL RESULTS AND DISCUSSION

There are several parameters that one should input to calculate the strong decay of mesons in the $^3P_0$ model. In the present work, the masses of constituent quarks are taken as $m_u = m_d = 0.22$ GeV, $m_s = 0.419$ GeV, $m_c = 1.6$ GeV [36]. The strength of quark-pair creation $\gamma = 6.95$ has been adopted by many authors [23,28], and is obtained by fitting strong decay widths of light and charmed mesons, charmonium, and baryons observed by experiments. The value of γ is higher than that used in Ref. [37] by a factor of $\sqrt{96\pi}$ due to different field theory conventions. The strength of $s\bar{s}$ creation satisfies $\gamma_s = \gamma/\sqrt{3}$ [38]. References [16,23,24] also use this value to study the strong decay of charmonium, heavy-light mesons, and heavy baryons. The R values of D, D^*, D_s, D_s^* in the SHO are shown in Table IV, and are obtained by meson mass calculations in the nonrelativistic quark model with Coulomb item, linear confinement, and smeared hyperfine interactions.

First of all, we study the strong decay of the $\chi_0(3^3P_0)$, which is discussed by Chao and Li in Refs. [5,8] from the production process of $e^+e^- \to J/\psi + X(4160)$ and the mass spectrum obtained by the potential model with color screening. By solving the Schrödinger equation with the Numerov algorithm [40], we also obtain the mass 4149 MeV from the same potential and parameters as in Ref. [5]. Usually, the width of strong decays is sensitive [16,22,23,25,28,33] to the R value in the SHO. Here the reasonable value of R is obtained by fitting the wave function obtained by solving the Schrödinger equation [5].

By Fourier transformation, Eq. (12) turns into

$$\Psi_{nLM_L}(\mathbf{r}) = R_{nL}(r)Y_{LM_L}(\Omega_r), \tag{15}$$

with the radial wave function

$$R_{nL}(r) = R^{-(L+(3/2))}\sqrt{\frac{2n!}{\Gamma(n+L+\frac{3}{2})}}\exp\left(-\frac{R^{-2}r^2}{2}\right)r^L \times L_n^{L+(1/2)}(R^{-2}r^2). \tag{16}$$

The wave function $u(r) = rR_{nL}(r)$ of charmonium state $3P$ is shown in Fig. 2. By fitting the wave function obtained by the Numerov algorithm (the wave function is denoted as "NAWF" in the following) with Eq. (16), we get $R = 2.5 \sim 2.98$ GeV^{-1}.

The $\chi_0(3^3P_0)$ has decay channels of $0^{++} \to 0^- + 0^-$ with the S wave and $0^{++} \to 1^- + 1^-$ with S, D waves, while $0^{++} \to 0^- + 1^-$ is forbidden. Therefore, it can decay into $D\bar{D}$, D_sD_s, $D^*\bar{D}^*$, which are allowed by the phase space. In Fig. 3, we show the dependence of the partial widths of the strong decay of the $\chi_0(3^3P_0)$ on the R_A. Taking $R_A = 2.5$–2.98 GeV^{-1} discussed above, the total width ranges from 105 to 143 MeV, which falls in the range of experimental data. However, the dominate contribution

TABLE IV. The parameters relevant to the two-body strong decays of the charmonium state in the $^3P_0$ model.

State	Mass (MeV) [39]	R (GeV^{-1}) [36]
D	1869.62(±), 1864.84(0)	1.52
D^*	2021.27(±), 2006.97(0)	1.85
D_s	1968.49(±)	1.41
D_s^*	2112.3(±)	1.69

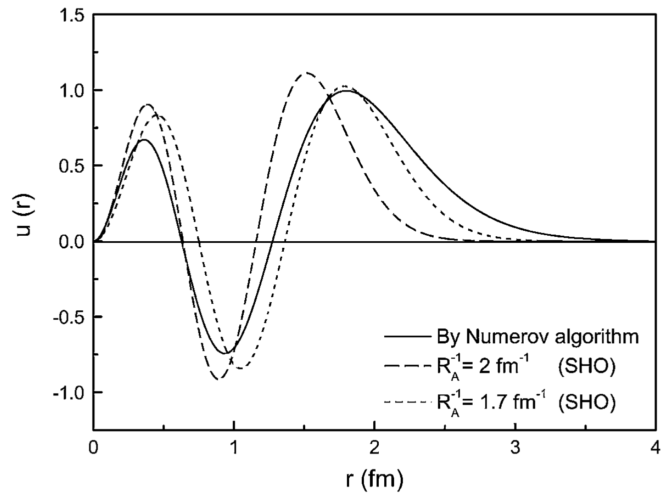

FIG. 2. The wave function of charmonium state $3P$.

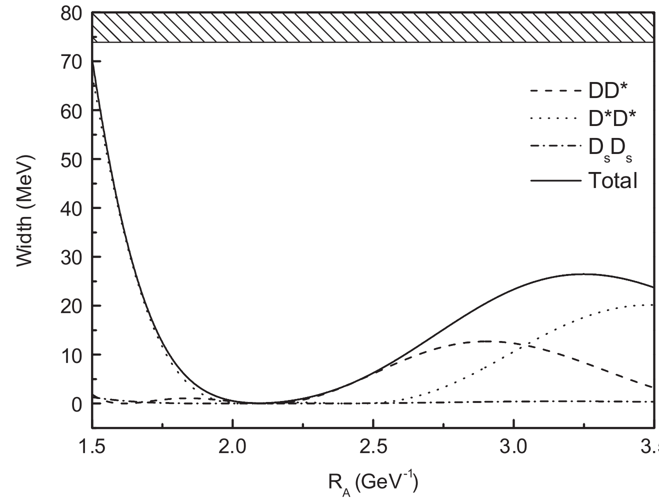

FIG. 4. The possible strong decay of the $\eta_c(4^1S_0)$.

comes from $\chi_0(3^3P_0) \to DD$, which is inconsistent with the experimental result. So the assignment of the charmonium state $\chi_0(3^3P_0)$ to the $X(4160)$ is disfavored.

The $\eta_c(4^1S_0)$ is mostly like the $X(4160)$ for it has high production cross sections in the process of $e^+e^- \to J/\psi + X(4160)$ discussed by Chao [8]. However, it is difficult to understand why the predicted masses 4250 [5], 4384, and 4425 MeV [17] are much higher than 4156 MeV. By considering the effect of the meson loops [41], the mass may be lower than that of Refs. [5,17]. Here, we assume the mass of the $\eta_c(4^1S_0)$ is 4156 MeV. The main decay channels of the $\eta_c(4^1S_0)$ are $0^{-+} \to 0^- + 1^-$ and $0^{-+} \to 1^- + 1^-$ with a P wave between outgoing mesons. Obviously, the $0^{-+} \to 0^- + 0^-$ is forbidden. The decay widths of the main decay channels are shown in Fig. 4. The total width can only reach up to 25 MeV with R_A around 2.9 GeV, which is obtained by fitting to NAWF of the $\eta_c(4^1S_0)$. It is about 3 times smaller than the lower

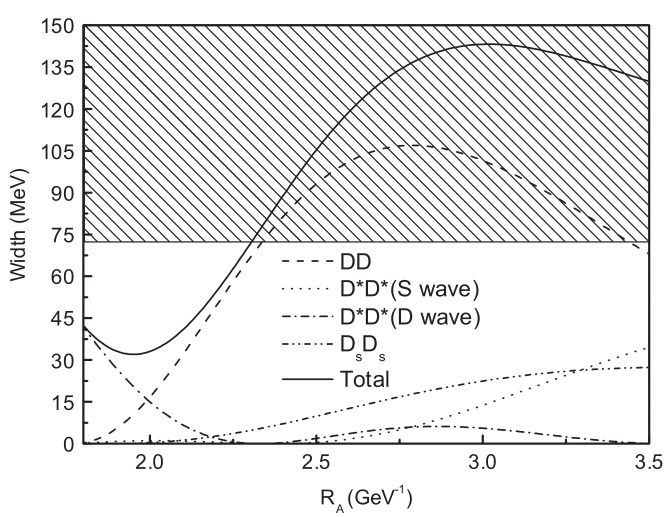

FIG. 3. The possible strong decay of the $\chi_0(3^3P_0)$.

limit of the experimental result of the $X(4160)$. Since the results of some hadron states predicted by the $^3P_0$ model may be a factor of 2–3 off the experimental width due to inherent uncertainties of this model [18–21,28], the assignment of the $X(4160)$ to the $\eta_c(4^1S_0)$ cannot be excluded. Besides, the ratio of the main decay channels $D\bar{D}^*$, $D^*\bar{D}^*$ is

$$\frac{\mathcal{B}(\eta_c(4^1S_0) \to D\bar{D}^*)}{\mathcal{B}(\eta_c(4^1S_0) \to D^*\bar{D}^*)} = 1.25. \tag{17}$$

It is much larger than the 0.22 reported by Belle. If one takes the $\eta_c(4^1S_0)$ as an assignment of $X(4160)$, the precision measurement of the ratio between the width of the $D\bar{D}^*$ and $D^*\bar{D}^*$ is necessary in further experiments.

Because the $\chi_1(3^3P_1)$ has quantum number $J^{PC} = 1^{++}$ and mass 4178 MeV, it is also a possible candidate for the $X(4160)$. The channels $1^{++} \to 0^- + 1^-$ and $1^{++} \to 1^- + 1^-$ with S and D waves are the main decay channels of the $\chi_1(3^3P_1)$. Figure 5 shows our results in the $^3P_0$ model. Taking $R_A = 2.5$–$2.98\ \mathrm{GeV}^{-1}$, the total width is consistent with the range of the $X(4160)$. However, the dominant decay is $\chi_1(3^3P_1) \to D\bar{D}^*$ while the decay width has only a few MeV for the $\chi_1(3^3P_1) \to D^*\bar{D}^*$ channel, which is inconsistent with the experimental data. Therefore, regarding the $X(4160)$ as the $\chi_1(3^3P_1)$ state is impossible.

Another possible candidate for the $X(4160)$ is the charmonium state $\eta_{c2}(2^1D_2)$. First, it has quantum number $J^{PC} = 2^{-+}$ and masses 4099 MeV [5] and 4158 MeV [17], which are compatible with the results of Belle. Second, the $\psi(4160)$ [39] is known to be a good candidate for the $\psi(2^3D_1)$ with $J^{PC} = 1^{--}$, which is discussed in detail by Chao [8]. So the $X(4160)$ may be the D-wave spin-singlet charmonium state $^1D_2(2D)$. Third, $\eta_{c2}(2^1D_2)$ decaying into $D\bar{D}$ is forbidden, and this decay is not seen by Belle either.

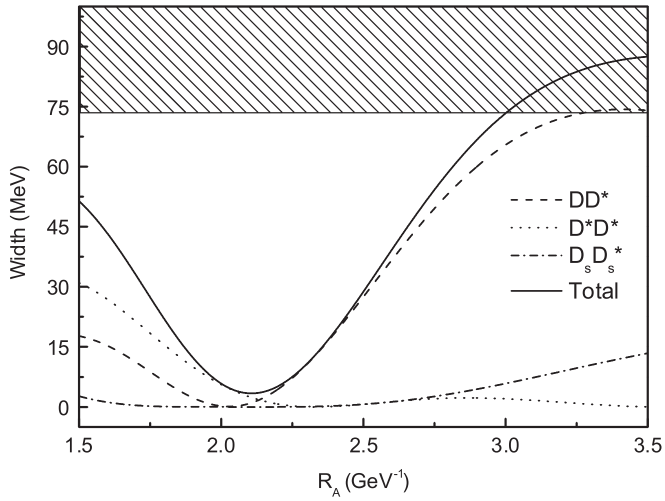

FIG. 5. The possible strong decay of the $\chi_1(3^3P_1)$.

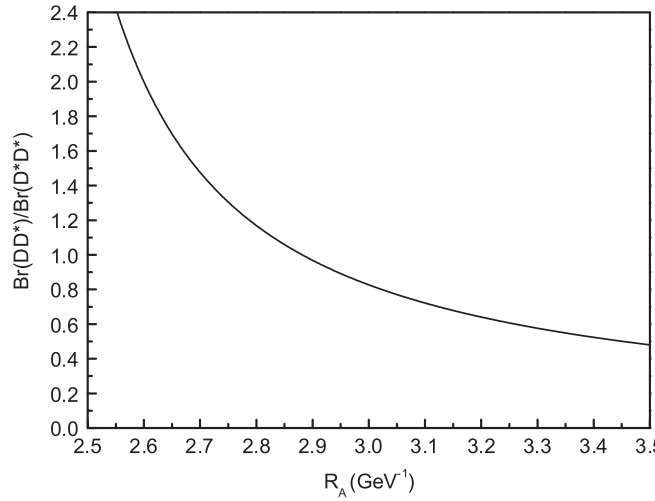

FIG. 7. The ratio of $\frac{\mathcal{B}(\eta_{c2}(2^1D_2)\to D\bar{D}^*)}{\mathcal{B}(\eta_{c2}(2^1D_2))\to D^*\bar{D}^*)}$ with the R_A value of the SHO.

For the strong decay of the $\eta_{c2}(2^1D_2)$, there are $2^{-+}\to 0^- + 1^-$ and $2^{-+}\to 1^- + 1^-$ decay channels with a P wave between outgoing mesons. In this case, final states $D\bar{D}^*$, $D_s\bar{D}_s^*$, and $D^*\bar{D}^*$ are phase space allowed. In Fig. 6, we present the numerical results of the main decay channels for the $\eta_{c2}(2^1D_2)$. By fitting the NAWF of the $\eta_{c2}(2^1D_2)$, we get $R_A = 2.7$–$3.0\ \text{GeV}^{-1}$. The total decay width of the $\eta_{c2}(2^1D_2)$ falls in the range of the $X(4160)$ released by Belle. Taking the reasonable R_A value of the SHO, the ratio of the main decay channel $D\bar{D}^*$, $D^*\bar{D}^*$ is

$$\frac{\mathcal{B}(\eta_{c2}(2^1D_2)\to D\bar{D}^*)}{\mathcal{B}(\eta_{c2}(2^1D_2))\to D^*\bar{D}^*)} = 1.4\text{–}0.76 \qquad (18)$$

and is shown in Fig. 7. However, the result is somewhat larger than the $\mathcal{B}_{D^*\bar{D}}(X(4160))/\mathcal{B}_{D^*\bar{D}^*}(X(4160)) < 0.22$ observed by Belle. We believe that it is very important to measure this ratio since it is independent of the uncertain strength γ of the quark-pair creation from the vacuum.

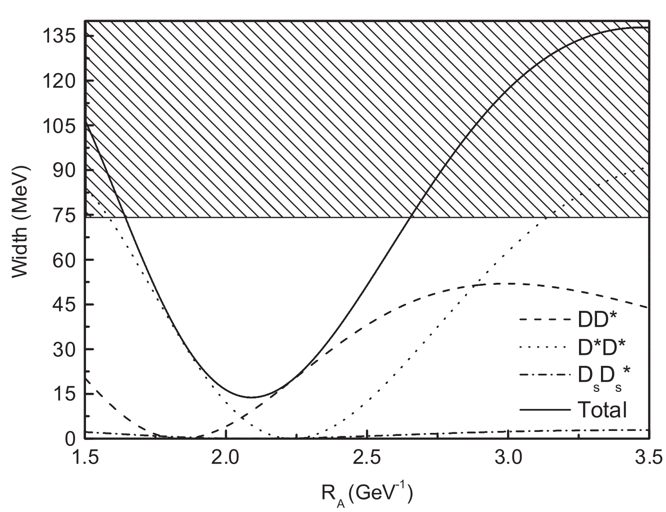

FIG. 6. The possible strong decay of the $\eta_{c2}(2^1D_2)$.

To sum up, the $\eta_{c2}(2^1D_2)$ is a better candidate for the $X(4160)$ in our calculation.

The $X(3915)$, which was observed by Belle in $\gamma\gamma\to\omega J/\psi$ with a statical significance of 7.7σ [11], is the most recent addition to the collection of the XYZ states. This state is similar to the $Y(3940)$ [42,43] previously discovered by Belle and *BABAR* in the process $B\to KY(3940))\to K\omega J/\psi$, since they are both seen in $\omega J/\psi$, and the mass and width [$M = 3914.6^{+3.8}_{-3.4}(\text{stat})\pm 2.0(\text{syst})$ MeV, $\Gamma = 34^{+12}_{-8}(\text{stat})\pm 5(\text{syst})$ MeV] of $Y(3940)$ released by *BABAR* are very close to Eq. (3). Therefore, the $X(3915)$ and $Y(3940)$ might be the same state pointed out in Refs. [11–15].

In a previous work, Liu *et al.* [44,45] suggested that $Y(3940)$ is probably a molecular $D^*\bar{D}^*$ state with $J^{PC} = 0^{++}$ or $J^{PC} = 2^{++}$ in the meson-exchange model. Assuming the $D^*\bar{D}^*$ bound-state structure, Branz [46] studied the strong $Y(3940)\to\omega J/\psi$ and radiative $Y(3940)\to\gamma\gamma$ decay widths in a phenomenological Lagrangian approach. Their results are roughly compatible with the experimental data about $Y(3940)$. By the QCD sum rules, Zhang [47] also obtained the mass $M = 3.91 \pm 0.11$ GeV for molecular state $D^*\bar{D}^*$, which is consistent with $Y(3940)$ reported by *BABAR*.

The $\chi_0(2^3P_0)$ is, however, a good candidate for the $X(3915)$, due to the fact that the mass in Table I predicted by the potential model for the excited charmonium state $\chi_0(2^3P_0)$ is roughly compatible with $M = 3915 \pm 3 \pm 2$ MeV. Here, we study the strong decay of the $\chi_0(2^3P_0)$ to identify whether or not they are the same state.

The $\chi_0(2^3P_0)$ has only the strong decay channel $0^{++}\to 0^- + 0^-$ allowed by phase space. First of all, using the Numerov algorithm method, we also solve the Schrödinger equation using the same potential and parameters as in Ref. [5] to get the mass and wave function of $\chi_0(2^3P_0)$. Then, $R_A = 2.3$–$2.5\ \text{GeV}^{-1}$ is obtained by fitting Eq. (16)

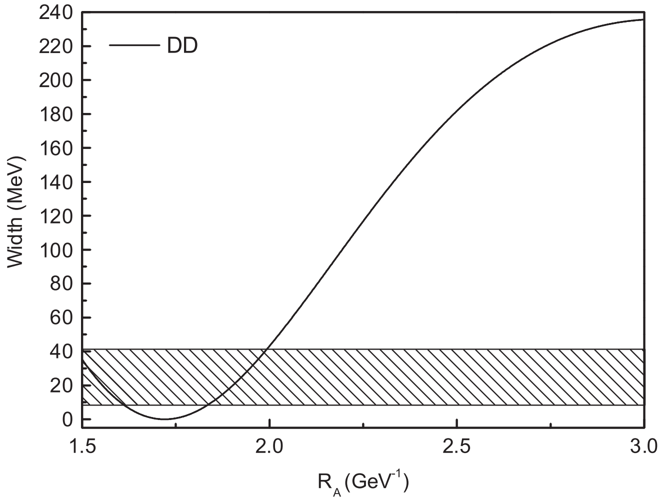

FIG. 8. The possible strong decay of the $\chi_0(2^3P_0)$.

to this wave function. The total width, which is presented in Fig. 8, ranges from 132 to 187 MeV with $R_A = 2.3$–$2.5\ \mathrm{GeV}^{-1}$. It is much larger than the $\Gamma = 17 \pm 10 \pm 3$ MeV reported by Ref. [11]. Therefore, the $X(3915)$ is unlikely to be the charmonium state $\chi_0(2^3P_0)$, although the mass is compatible. The conclusion is different from the result of Ref. [16] because different values of R_A are used. As we have stated before, the strong decay width is sensitive to the wave function of the meson (the value of R_A). To have a consistent calculation, one should get the wave function of the meson from the mass calculation. Of course, the mass calculation is more or less model dependent. Here a well-established model, the color screening potential model, is used. In order to understand the internal structure of $X(3915)$, further study is needed. To check our calculations, we also calculate the decay width of $X(4350)$ as a P-wave charmonium state $\chi_2(3^3P_2)$, The value of R_A that we obtained is in the region 2.5–2.98 GeV^{-1}, which is the same as the value taken in Ref. [16]. We also obtained the same width as Ref. [16].

V. SUMMARY

In summary, we have discussed the possible interpretations of the $X(4160)$ observed by the Belle Collaboration in $e^+e^- \to J/\psi + X(4160)$ followed by $X(4160) \to D^*\bar{D}^*$. We have also studied the newest state $X(3915)$ observed by Belle in the process $\gamma\gamma \to J/\psi\omega$ [11].

In the quark models, the masses of the charmonium states—$\chi_0(3^3P_0)$, $\chi_1(3^3P_1)$, $\eta_{c2}(2^1D_2)$—are all around 4156 MeV. By taking the effect of the virtual mesons loop [41] into account, the $\eta_c(4^1S_0)$ may also have mass around 4156 MeV. All four states have charge parity $C = +$, which is compatible with the $X(4160)$ observed by Belle.

For the strong decay of the $\chi_0(3^3P_0)$, the dominant strong decay is $\chi_0(3^3P_0) \to D\bar{D}$, while $\chi_0(3^3P_0) \to D^*\bar{D}^*$ contributes to the total width only a little in the reasonable R in the SHO. This is in contrast to the experimental result. Thus the excited charmonium state $\chi_0(3^3P_0)$ disfavors the $X(4160)$.

The $\eta_c(4^1S_0)$ cannot decay into $D\bar{D}$ and may have a high production rate [8] in the $e^+e^- \to J/\psi + \eta_c(4S)$ process by analogy with $e^+e^- \to J/\psi + \eta_c(1S) \times (\eta_c(2S)\chi_{c0}(1P))$. However, the total width in the present work is lower than the experimental data of the $X(4160)$.

The main strong decay channel of the $\chi_1(3^3P_1)$ is $D\bar{D}^*$, while $D^*\bar{D}^*$ is only a few MeV. It is inconsistent with the results of Belle. Therefore, taking the $\chi_1(3^3P_1)$ as an assignment for the $X(4160)$ is impossible.

The $\eta_{c2}(2^1D_2)$ cannot decay to $D\bar{D}$, which cannot be seen in the experiment either. The total width of the $\eta_{c2}(2^1D_2)$ matches well with the data of the $X(4160)$ in our calculation. So, the $\eta_{c2}(2^1D_2)$ is a good candidate for the $X(4160)$, as both the mass and the strong decay are compatible with the results observed by Belle, although the excited charmonium state $\eta_c(4^1S_0)$ cannot be ruled out as an assignment for the $X(4160)$.

We also give the ratio of $\frac{\mathcal{B}(\eta_{c2}(2^1D_2)\to D\bar{D}^*)}{\mathcal{B}(\eta_{c2}(2^1D_2))\to D^*\bar{D}^*)}$, which is independent of the parameter γ in the $^3P_0$ model. The numerical result is somewhat larger than the experimental data. Therefore, we suggest that Belle, *BABAR*, and other experimental collaborations measure it to confirm this state.

By assuming the $X(3915)$ is the $\chi_0(2^3P_0)$, the strong decay of the state is calculated. From our numerical results, the partial width of the $X(3915)$ to $\gamma\gamma$ or $\omega J/\psi$ is too large; we think this assumption is unacceptable. Yuan [13] also believes that it is very unlikely to be a charmonium state. Thus, it is necessary to do more studies to understand the properties of the $X(3915)$.

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APPENDIX

The spatial overlap $I^{M_{L_A},m}_{M_{L_B},M_{L_C}}(\mathbf{P}, m_1, m_2, m_3)$ is simplified as $I^{n'm'}(\mathbf{P})$ in the present work due to $M_{L_B} = M_{L_C} = 0$. According to Eq. (10), the concrete calculations of the integration are trivial after choosing the direction of $\mathbf{P}$ along the z axis [35]. In Table III we list all expressions of $I^{\pm}$, I^{00} used.

In the case of $2P \to 1S + 1S$,

$$I^{\pm} = I^{1-1} = I^{-11} = i\frac{\sqrt{6}}{\sqrt{5}\pi^{5/4}\Delta^7}(R_A^{5/2}R_B^{3/2}R_C^{3/2})\exp\left(-\frac{1}{2}\zeta^2\mathbf{P}^2\right)(10R_A^2 + \Delta^2(-5 + 2\mathbf{P}^2R_A^2(1+\lambda)^2)),$$

$$I^{00} = -i\frac{\sqrt{6}}{\sqrt{5}\pi^{5/4}\Delta^7}(R_A^{5/2}R_B^{3/2}R_C^{3/2})\exp\left(-\frac{1}{2}\zeta^2\mathbf{P}^2\right)(10R_A^2 + \Delta^2(-5 + \mathbf{P}^2(1+\lambda) \times(-5\Delta^2\lambda + 2R_A^2(3 + \lambda(8 + \Delta^2\mathbf{P}^2(1+\lambda)^2)))))). \quad \text{(A1)}$$

For $2D \to 1S + 1S$,

$$I^{\pm} = I^{1-1} = I^{-11} = \frac{2\sqrt{3}}{\sqrt{7}\pi^{5/4}\Delta^7}(R_A^{7/2}R_B^{3/2}R_C^{3/2})\exp\left(-\frac{1}{2}\zeta^2\mathbf{P}^2\right)\mathbf{P}(1+\lambda)(14R_A^2 + \Delta^2(-7 + 2\mathbf{P}^2R_A^2(1+\lambda)^2)),$$

$$I^{00} = -\frac{2}{\sqrt{7}\pi^{5/4}\Delta^7}(R_A^{7/2}R_B^{3/2}R_C^{3/2})\exp\left(-\frac{1}{2}\zeta^2\mathbf{P}^2\right)\mathbf{P}(1+\lambda)(28R_A^2 + \Delta^2(-14 + \mathbf{P}^2(1+\lambda) \times(-7\Delta^2\lambda + 2R_A^2(4 + \lambda(11 + \Delta^2\mathbf{P}^2(1+\lambda)^2)))))). \quad \text{(A2)}$$

For $3P \to 1S + 1S$,

$$I^{\pm} = I^{1-1} = I^{-11} = i\frac{\sqrt{3}}{\sqrt{70}\pi^{5/4}\Delta^9}(R_A^{5/2}R_B^{3/2}R_C^{3/2})\exp\left(-\frac{1}{2}\zeta^2\mathbf{P}^2\right)(140R_A^4 + 28\Delta^2R_A^2(-5 + 2\mathbf{P}^2R_A^2(1+\lambda)^2) + \Delta^4(35 - 28\mathbf{P}^2R_A^2(1+\lambda)^2 + 4\mathbf{P}^4R_A^4(1+\lambda)^4)),$$

$$I^{00} = -i\frac{2\sqrt{6}}{\sqrt{35}\pi^{5/4}\Delta^9}(R_A^{5/2}R_B^{3/2}R_C^{3/2})\exp\left(-\frac{1}{2}\zeta^2\mathbf{P}^2\right)\Big(35R_A^4 + \frac{1}{4}\Delta^6\mathbf{P}^2\lambda(1+\lambda)(35 - 28\mathbf{P}^2R_A^2(1+\lambda)^2 + 4\mathbf{P}^4R_A^4(1+\lambda)^4) + 7\Delta^2R_A^2(-5 + \mathbf{P}^2R_A^2(1+\lambda)(6+11\lambda)) + \frac{1}{4}\Delta^4(35 - 28\mathbf{P}^2R_A^2(1+\lambda)(3+8\lambda) + 4\mathbf{P}^4R_A^4(1+\lambda)^3(5+19\lambda))\Big). \quad \text{(A3)}$$

For $4S \to 1S + 1S$,

$$I^{00} = \frac{1}{2\sqrt{120}\pi^{5/4}\Delta^9}(R_A^{3/2}R_B^{3/2}R_C^{3/2})\exp\left(-\frac{1}{2}\zeta^2\mathbf{P}^2\right)\mathbf{P}(840R_A^6(2+3\lambda) + \Delta^6\lambda(-105 + 210\mathbf{P}^2R_A^2(1+\lambda)^2 - 84\mathbf{P}^4R_A^4(1+\lambda)^4 + 8\mathbf{P}^6R_A^6(1+\lambda)^6) + 6\Delta^4R_A^2(70 + 175\lambda - 28\mathbf{P}^2R_A^2(1+\lambda)^2(2+7\lambda) + 4\mathbf{P}^4R_A^4(1+\lambda)^4(2+9\lambda)) + 84\Delta^2R_A^4(-5(4+7\lambda) + 2\mathbf{P}^2R_A^2(1+\lambda)^2(4+9\lambda))). \quad \text{(A4)}$$

Here, the parameters Δ, ζ, and η in Eqs. (A1)–(A4) are defined as

$$\Delta^2 = R_A^2 + R_B^2 + R_C^2, \qquad \lambda = -\frac{R_A^2 + \xi_1R_B^2 + \xi_2R_C^2}{R_A^2 + R_B^2 + R_C^2}, \qquad \zeta^2 = R_A^2 + \xi_1^2R_B^2 + \xi_2^2R_C^2 - \frac{(R_A^2 + \xi_1R_B^2 + \xi_2R_C^2)^2}{R_A^2 + R_B^2 + R_C^2},$$

with

$$\xi_1 = \frac{m_3}{m_3 + m_1}, \qquad \xi_2 = \frac{m_3}{m_3 + m_2}.$$

Here, m_1, m_2, and m_3 denote the mass of the quark inside the parent meson and created from vacuum, respectively.

[1] E. Eichten, K. Gottfried, T. Kinoshita, K.D. Lane, and T.M. Yan, Phys. Rev. D **17**, 3090 (1978).
[2] E.J. Eichten, K. Lane, and C. Quigg, Phys. Rev. D **73**, 014014 (2006).
[3] T. Barnes and E.S. Swanson, Phys. Rev. C **77**, 055206 (2008).
[4] B-Q. Li, C. Meng, and K-T. Chao, Phys. Rev. D **80**, 014012 (2009).
[5] B-Q. Li, and K-T. Chao, Phys. Rev. D **79**, 094004 (2009).
[6] M. Suzuki, Phys. Rev. D **72**, 114013 (2005).
[7] P. Pakhlov *et al.* (Belle Collaboration), Phys. Rev. Lett. **100**, 202001 (2008).
[8] K.T. Chao, Phys. Lett. B **661**, 348 (2008).
[9] K-Y. Liu, Z-G. He, and K-T. Chao, Phys. Rev. D **77**, 014002 (2008).
[10] R. Molina and E. Oset, Phys. Rev. D **80**, 114013 (2009).

[11] S. Uehara (Belle Collaboration), Phys. Rev. Lett. **104**, 092001 (2010).
[12] S. L. Olsen, arXiv:0909.2713.
[13] C. Z. Yuan (BES and Belle Collaborations), arXiv:0910.3138.
[14] A. Zupanc (Belle Collaboration), arXiv:0910.3404.
[15] S. Gordfrey, arXiv:0910.3409.
[16] X. Liu, Z-G. Luo, and Z-F. Sun, Phys. Rev. Lett. **104**, 122001 (2010).
[17] T. Barnes, S. Godfrey, and E. S. Swanson, Phys. Rev. D **72**, 054026 (2005).
[18] L. Micu, Nucl. Phys. **B10**, 521 (1969).
[19] A. Le Yaouanc, L. Oliver, O. Pene, and J-C. Raynal, Phys. Rev. D **8**, 2223 (1973); **9**, 1415 (1974); **11**, 1272 (1975).
[20] W. Roberts and B. Silvestr-Brac, Few-Body Syst. **11**, 171 (1992).
[21] E. S. Ackleh, T. Barnes, and E. S. Swanson, Phys. Rev. D **54**, 6811 (1996).
[22] T. Barnes, S. Godfrey, and E. S. Swanson, Phys. Rev. D **72**, 054026 (2005).
[23] J. Lu, W. Z. Deng, X. L. Chen, and S. L. Zhu, Phys. Rev. D **73**, 054012 (2006); B. Zhang, X. Liu, W. Z. Deng, and S. L. Zhu, Eur. Phys. J. C **50**, 617 (2007); C. Chen, X. L. Chen, X. Liu, W. Z. Deng, and S. L. Zhu, Phys. Rev. D **75**, 094017 (2007).
[24] Z-G. Luo, X-L. Chen, and X. Liu, Phys. Rev. D **79**, 074020 (2009); Z-F. Sun and X. Liu, Phys. Rev. D **80**, 074037 (2009).
[25] F. E. Close and E. S. Swanson, Phys. Rev. D **72**, 094004 (2005); F. E. Close, C. E. Thomas, O. Lakhina, and E. S. Swanson, Phys. Lett. B **647**, 159 (2007); O. Lakhina and E. S. Swanson, Phys. Lett. B **650**, 159 (2007).
[26] S. Capstick and N. Isgur, Phys. Rev. D **34**, 2809 (1986); S. Capstick and W. Roberts, Phys. Rev. D **49**, 4570 (1994).
[27] P. Geiger and E. S. Swanson, Phys. Rev. D **50**, 6855 (1994).
[28] H. G. Blundell and S. Godfrey, Phys. Rev. D **53**, 3700 (1996); H. G. Blundell, S. Godfrey, and B. Phelps, Phys. Rev. D **53**, 3712 (1996).
[29] R. Kokoski and N. Isgur, Phys. Rev. D **35**, 907 (1987).
[30] T. Barnes, F. E. Close, P. R. Page, and E. S. Swanson, Phys. Rev. D **55**, 4157 (1997).
[31] T. Barnes, N. Black, and P. R. Page, Phys. Rev. D **68**, 054014 (2003).
[32] L. Burakovsky and P. R. Page, Phys. Rev. D **62**, 014011 (2000).
[33] D-M. Li and B. Ma, Phys. Rev. D **77**, 074004 (2008); **77**, 094021 (2008); D-M. Li and S. Zhou, Phys. Rev. D **79**, 014014 (2009); **81**, 014021 (2010).
[34] C. Hayne and N. Isgur, Phys. Rev. D **25**, 1944 (1982).
[35] M. Jacob and G. C. Wick, Ann. Phys. (Leipzig) **7**, 404 (1959); Ann. Phys. (N.Y.)**281**, 774 (2000).
[36] S. Godfrey and R. Kokoski, Phys. Rev. D **43**, 1679 (1991).
[37] R. Kokoski and N. Isgur, Phys. Rev. D **35**, 907 (1987).
[38] A. Le Yaouanc, L. Oliver, O. Pene, and J. C. Raynal, Phys. Lett. **72B**, 57 (1977).
[39] C. Amsler *et al.* (Particle Data Group), Phys. Lett. B **667**, 1 (2008).
[40] S. E. Koonin and D. C. Meredith, *Computational Physics* (Addison-Wesley, New York, 1990).
[41] Y. C. Yang, Z. R. Xia, and J. L. Ping (unpublished).
[42] S.-K. Cho *et al.* (Belle Collaboration), Phys. Rev. Lett. **94**, 182002 (2005).
[43] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. Lett. **101**, 082001 (2008).
[44] X. Liu, Z. G. Luo, Y. R. Liu, and S. L. Zhu, Eur. Phys. J. C **61**, 411 (2009).
[45] X. Liu and S. L. Zhu, Phys. Rev. D **80**, 017502 (2009).
[46] T. Branz, T. Gutsche, and V. E. Lyubovitskij, Phys. Rev. D **80**, 054019 (2009).
[47] J-R. Zhang and M-Q. Huang, Phys. Rev. D **80**, 056004 (2009).