$(L_e - L_\mu - L_\tau)$ discrete symmetry for heavy right-handed neutrinos and degenerate leptogenesis

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The degenerate leptogenesis is studied when the degeneracy in two of the heavy right-handed neutrinos [the third one is irrelevant if $\mu - \tau$ symmetry is assumed] is due to $\bar{L} \equiv (L_e - L_\mu - L_\tau)$ discrete symmetry. It is shown that a sizable leptogenesis asymmetry ($\varepsilon \geq 10^{-6}$) is possible. The level of degeneracy required also predicts the Majorana phase needed for the asymmetry and this prediction is testable since it is the same phase, which appears in the double β decay. Implications of nonzero reactor angle θ_{13} are discussed. It is shown that the contribution from $\sin^2\theta_{13}$ to the leptogenesis asymmetry parameter may even dominate. An accurate measurement of $\sin^2\theta_{13}$ would have important implications for the mass degeneracy of heavy right-handed neutrinos.

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I. INTRODUCTION

The purpose of this paper is to study degenerate [i.e. when two of the three right-handed neutrinos are (nearly) degenerate] leptogenesis in a seesaw mechanism where the mass matrix for right-handed neutrinos has $\mu - \tau$ symmetry and the degeneracy is the result of $\bar{L} \equiv (L_e - L_\mu - L_\tau)$ discrete symmetry. This is studied in a generic seesaw gauge model [1], in which in addition to the usual fermions and $SU_L(2)$ Higgs doublets, there are three $SU_L(2)$—singlet right-handed neutrinos $N_R^i (i = e, \mu, \tau)$ with $\mu - \tau$ symmetry and two Higgs with quantum numbers given below:

$$\begin{aligned} &L_e: (2, -1, 0), \qquad \phi^{(1)}: (2, -1, 0), \qquad N_R^e: (1, -1, 1) \\ &e_R: (1, -2, 0) \\ &L_{\mu-\tau}: (2, 0, -1), \qquad \phi^{(2)}: (2, 0, -1), \qquad N_R^{\mu,\tau}: (1, 1, -1) \\ &\mu_R, \tau_R: (1, 0, -2) \\ &\Sigma: (1, 0, 0) \\ &\Sigma': (1, 2, -2), \end{aligned} \tag{1}$$

where the numbers in the parentheses, respectively, correspond to $SU_L(2)$ and $U_i(1)$ quantum numbers. It is important to remark that as a result of $\mu - \tau$ symmetry the leptogenesis asymmetry parameter is proportional to $\Delta m^2_{\rm sol}$ [2,3] rather than $\Delta m^2_{\rm atm}$, and in general an unknown Majorana phase which, however, also appears in the neutrinoless double β decay. This was studied for the hierarchal ($M_2 \gg M_1$) leptogenesis. Now a study is made for degenerate leptogenesis when the degeneracy is the result of $\bar{L}$ discrete symmetry for the right-handed heavy neutrinos sector. This degeneracy is protected by the symmetry (although global) and as such would be softly broken. It is shown that a sizable lepton asymmetry ($\varepsilon \gtrsim 10^{-6}$) is possible. The level of degeneracy needed for this to occur also predicts the Majorana phase needed for the asymmetry. This is the distinguishing feature of the model considered. In general, the asymmetry parameter is proportional to the product of degeneracy parameter $(\frac{\Delta M}{M})^{-1}$ and a CP-violating phase; the fixation of this product to get a sizable leptogenesis parameter does not necessarily predict one from the other. This is because they do not get related, in contrast to the model considered here; see, for example, [4,5]. Since the phase involved is the same which occurs in the neutrinoless double β decay, this prediction is testable. Further the effect of a nonzero reactor angle θ_{13} on leptogenesis is considered in some detail. It is shown that the contribution from $\sin^2\theta_{13}$ to the leptogenesis asymmetry parameter may even dominate. As such an accurate measurement of $\sin^2\theta_{13}$ would have important implications for the mass spectrum of heavy right-handed neutrinos, particularly for $\frac{M_2 - M_1}{M_2 + M_1}$.

The Yukawa couplings of neutrinos with Higgs, using $\mu - \tau$ symmetry for right-handed neutrinos only, is given by

$$\begin{aligned} \mathcal{L}_Y = {}& g_{11}\bar{L}_e e_R \tilde{\phi}^{(1)} + [g_{22}\bar{L}_\mu \mu_R + g_{23}\bar{L}_\mu \tau_R + g_{32}\bar{L}_\tau \mu_R + g_{33}\bar{L}_\tau \tau_R]\tilde{\phi}^{(2)} + hc + h_{11}\bar{L}_e N_e \phi^{(2)} \\ &+ [h_{22}\bar{L}_\mu (N_\mu + N_\tau) + h_{32}\bar{L}_\tau (N_\mu + N_\tau)]\phi^{(1)} + hc + f_{11} N_e^T C N_e \Sigma' + f_{12} N_e^T C (N_\mu + N_\tau)\Sigma + hc \\ &+ [f_{22}(N_\mu^T C N_\mu + N_\tau^T C N_\tau) + f_{23}(N_\mu^T C N_\tau + N_\tau^T C N_\mu)]\bar{\Sigma}', \end{aligned} \tag{2}$$

where

$$\tilde{\phi} = -i\tau_2\phi^*, \qquad \phi = \begin{pmatrix} \phi^0 \\ -\phi^- \end{pmatrix}, \qquad \tilde{\phi} = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}.$$

One can also write Yukawa couplings of quarks with Higgs doublets, in the same fashion as for the charged lepton as follows:

$$L_Y = G_{11}\bar{L}_u u_R \phi^{(1)} + G_{22}\bar{L}_c c_R \varphi^{(2)} + G_{33}\bar{L}_t t_R \phi^{(2)} + \tilde{G}_{11}\bar{L}_u d_R \tilde{\phi}^{(1)} + (\tilde{G}_{22}\bar{L}_c s_R + \tilde{G}_{23}\bar{L}_c b_R + \tilde{G}_{32}\bar{L}_t s_R + \tilde{G}_{33}\bar{L}_t b_R)\tilde{\phi}^{(2)}. \quad (3)$$

A remark about Yukawa couplings is in order. The model contains two Higgs doublets $\phi^{(1)}$ and $\phi^{(2)}$, the former is coupled to the first generation while the latter to second and third generations. Then the quantum number given in Eq. (1) dictates the couplings as in Eqs. (2) and (3). Except for heavy $SU(2)$ singlet right-handed neutrinos, $2 \leftrightarrow 3$ symmetry is not the symmetry of the Lagrangian in Eq. (2). The Yukawa couplings with quarks will not be considered further as they are not relevant for what follows. In general, the Yukawa couplings used above are complex. It is convenient to introduce Yukawa coupling matrices Y_l and Y_D:

$$Y_l = \begin{pmatrix} g_{11} & 0 & 0 \\ 0 & g_{22} & g_{23} \\ 0 & g_{32} & g_{33} \end{pmatrix}, \qquad Y_D = \begin{pmatrix} h_{11} & 0 & 0 \\ 0 & h_{22} & h_{23} \\ 0 & h_{32} & h_{33} \end{pmatrix}. \quad (4)$$

Then the charged lepton and Dirac neutrino mass matrices are

$$M_l = \begin{pmatrix} g_{11}v_1 & 0 & 0 \\ 0 & g_{22}v_2 & g_{23}v_2 \\ 0 & g_{32}v_2 & g_{33}v_2 \end{pmatrix} \quad (5)$$

$$m_D = \begin{pmatrix} h_{11}v_2 & 0 & 0 \\ 0 & h_{22}v_1 & h_{22}v_1 \\ 0 & h_{32}v_1 & h_{32}v_1 \end{pmatrix} \quad (6)$$

while M_R is

$$M_R = \begin{pmatrix} f_{11}\Lambda' & f_{12}\Lambda & f_{12}\Lambda \\ f_{12}\Lambda & f_{22}\Lambda' & f_{23}\Lambda' \\ f_{12}\Lambda & f_{23}\Lambda' & f_{22}\Lambda' \end{pmatrix}, \quad (7)$$

where $\langle\phi_{1,2}\rangle = v_{1,2}$, $\langle\Sigma\rangle = \Lambda$, $\langle\Sigma'\rangle = \Lambda'$. It is convenient to have a basis in which Y_l and M_R are simultaneously diagonal:

$$Y_l \to \hat{Y}_l = U_L^{-1} Y_l U_E. \quad (8)$$

Correspondingly (for left-handed doublets L_i and right-handed singlets E_{iR}),

$$L_i \to U_L L_i, E_{iR} \to U_R E_{iR}, \quad (9)$$

where $i = e, \mu, \tau$ is the flavor index.

It is pertinent to remark that, by imposing the $\mu - \tau$ symmetry at the Lagrangian level only on the $SU(2)$ singlet right-handed neutrinos, a well-known problem [6,7] for simultaneous imposing of $\mu - \tau$ symmetry on the left-handed charged leptons and the left-handed neutrinos is avoided. For example, in the basis where charged leptons are diagonal, this would imply $m_\mu = m_\tau$. $\nu_\mu - \nu_\tau$ symmetry can, however be imposed on m_D independent of the $\mu - \tau$ symmetry for the right-handed neutrinos, giving $h_{22} = h_{33}$ implying in turn the maximum atmosphere mixing angle and $\theta_{13} = 0$. We would not impose this symmetry exactly and in fact consider the consequence of its breaking, in particular $\theta_{13} \neq 0$, which has important implications for the leptogenesis asymmetry parameter. It may, in fact, dominate depending on the value of $\sin^2\theta_{13}$. Thus an accurate measurement $\sin^2\theta_{13}$ would be of great interest.

II. MASS MATRICES IN SEESAW MECHANISM WITH $\bar{L}$ DISCRETE SYMMETRY

As is well known, M_R as given in Eq. (7) is diagonalized by a mixing matrix with $\sin^2\theta'_{23} = \frac{1}{2}$ and $\theta'_{13} = 0$, i.e. by

$$V = \begin{pmatrix} \cos\theta'_{12} & \sin\theta'_{12} & 0 \\ -\frac{\sin\theta'_{12}}{\sqrt{2}} & \frac{\cos\theta'_{12}}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{\sin\theta'_{12}}{\sqrt{2}} & \frac{\cos\theta'_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} P(\gamma), \quad (10)$$

where $P(\gamma)$ is the diagonal phase matrix (consisting of nontrivial Majorana phases γ_1, γ_2, and γ_3). Thus,

$$V^T M_R V = \hat{M}_R = \mathrm{diag}(\hat{M}_1, \hat{M}_2, \hat{M}_3) \quad (11)$$

with $\hat{M}_i = M_i e^{2i\gamma_i}$, $i = 1, 2, 3$ and

$$\tan 2\theta'_{12} = \frac{2\sqrt{2} f_{12}\Lambda}{(f_{22} + f_{23} - f_{11})\Lambda'} \quad (12)$$

$$\hat{M}_3 = M_3 e^{2i\gamma_3} = [f_{22} - f_{23}]\Lambda' e^{2i\gamma_3}. \quad (13)$$

Then the effective Majorana mass matrix for the light neutrinos is

$$M_\nu = \hat{m}_D \hat{M}_R^{-1} \hat{m}_D^T, \quad (14)$$

where $\hat{m}_D$ is the Dirac matrix in

$$\begin{pmatrix} \bar{N}_1 & \bar{N}_2 & \bar{N}_3 \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

basis:

$$\hat{m}_D = m_D V^*$$

and the corresponding Yukawa matrix is

$$\hat{Y}_D = Y_D V^*. \quad (15)$$

Before proceeding further, let me display the Higgs potential for ϕ fields:

$$\begin{aligned} V_H = {} & \mu_1^2|\phi_1|^2 + \mu_2|\phi_2|^2 + \mu_3(\phi_1^\dagger\phi_2 + \phi_2^\dagger\phi_1) \\ & + \lambda_1(\phi_1^\dagger\phi_1)^2 + \lambda_2(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_2 + \phi_2^\dagger\phi_1)^2 \\ & - \lambda_4(\phi_1^\dagger\phi_2 - \phi_2^\dagger\phi_1)^2 + \lambda_5(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2). \end{aligned} \quad (16)$$

When the symmetry is broken

$$\tilde{\phi}_i = \begin{pmatrix} H_i^+ \\ v_i + h_i + ia_i \end{pmatrix}, \qquad i = 1, 2. \quad (17)$$

Because of the presence of the terms λ_3 and λ_4 in Eq. (16), when the symmetry is broken, each pair of Higgs particles (h_1, h_2), (a_1, a_2) and (H_1^+, H_2^+) mixes. Further, when the resulting mass matrices are diagonalized, one of the charged Higgs and one of the neutral Higgs acquire zero masses and as such are eaten up by W^+ and Z^0 to give them masses. As a result, one has four massive physical Higgs particles, one charged H^+ and three neutral H, h, and A^0. Because of the presence of the Majorana mass term

$$H_M = N^T C M_R N + \text{H.c.} \quad (18)$$

providing explicit breaking of family lepton number, flavor changing interactions can arise due to radiative corrections and are controlled by elements of M_R and have been shown to be calculable and finite [8]. At one loop level such corrections arise due to charged Higgs and have been shown [8] to be highly suppressed. It may be remarked here that the Higgs potential for Σ fields can be included but even after breaking of the symmetry there is no mixing between Σ and ϕ fields. Σ' gives mass to one of the neutral gauge bosons and makes it super heavy. Σ is not coupled to gauge bosons. But both Σ and Σ' give mass to right-handed neutrinos.

We now apply $\bar{L}$ discrete symmetry on the purely heavy right-handed neutrino part of the Lagrangian (2), i.e.,

$$N_i \to e^{i\xi L} N_i \quad (19)$$

which leaves only the f_{12} term invariant so that

$$f_{11} = 0 = f_{22} = f_{33}. \quad (20)$$

As a result

$$\hat{M}_1 = M e^{2i\gamma_1}, \qquad \hat{M}_2 = -M e^{2i\gamma_2} = M e^{2i\gamma_2'}, \quad (21)$$

where $M = \sqrt{2}|f_{12}\Lambda|$, $\gamma_2' = \gamma_2 + \frac{\pi}{2}$ so that the minus sign in Eq. (21) has been absorbed in the redefinition of the Majorana phase γ_2. Further $\theta_{12}' = \pm\frac{\pi}{4}$. To break the $\bar{L}$ discrete symmetry so as to obtain near degeneracy of M_1 and M_2, we assume $|f_{22}\Lambda'|, |f_{23}\Lambda'|, |f_{11}\Lambda'| \leq |f_{12}\Lambda|$, and $f_{22} \simeq f_{23}$ so that the third right-handed neutrino becomes sterile with $M_3 \simeq$ a few eV. Then it is easy to see that $[\Delta M = \frac{M_2 - M_1}{2}, M = \frac{M_2 + M_1}{2}]$

$$\frac{\Delta M}{M} = |\eta|, \qquad \tan\theta_{12}' = \pm 1 - \eta', \quad (22)$$

where

$$\eta = \frac{(f_{22} + f_{23} + f_{11})\Lambda'}{2\sqrt{2} f_{12}\Lambda}, \qquad \eta' = \frac{(f_{22} + f_{23} - f_{11})\Lambda'}{2\sqrt{2} f_{12}\Lambda}. \quad (23)$$

The degree of degeneracy needed for providing sizable asymmetry (see Sec. III) requires $|\eta| \simeq 10^{-3}$ and correspondingly η' is also of the same order. A remark about the sterile neutrino would be in order. Even if $\bar{L}$ discrete symmetry is broken, the sterile neutrino cannot mix with any of the active neutrinos unless $\mu - \tau$ symmetry is also broken for right-handed neutrinos [1]. Even then the primordial nucleosynthesis bound on the active member of neutrino at $t \sim 1s$: $N_\nu < 3.1$ implies that the oscillation of active neutrinos into the sterile one should obey the bound $\delta m^2 \sin^2 2\theta \leq 1.6 \times 10^{-6}$ eV^2 which excludes the $\nu_\mu \to \nu_s$ and $\nu_e \to \nu_s$ oscillations and as such do not effect the atmospheric and solar neutrino solutions [9].

The effective Majorana mass matrix for light neutrinos, given in Eq. (14), is

$$M_\nu = \hat{m}_D \hat{M}_R^{-1} \hat{m}_D^T = \hat{A}, \quad (24)$$

where $\hat{A}$ is a 3×3 matrix with matrix elements

$$\begin{aligned} a_{11} &= h_{11}^2 v_2^2 A \\ \sqrt{2} a_{12} &= h_{11}(2h_{22}) v_1 v_2 B \\ \sqrt{2} a_{13} &= h_{11}(2h_{32}) v_1 v_2 B \\ a_{22} &= \tfrac{1}{2}(4h_{22}^2 v_1^2) C \\ a_{23} &= \tfrac{1}{2}(2h_{22})(2h_{32}) v_1^2 C \\ a_{33} &= \tfrac{1}{2}(4h_{32}^2) v_1^2 C. \end{aligned} \quad (25)$$

Here

$$\begin{aligned} A &= \frac{e^{-i(\gamma_1 + \gamma_2')}}{M}\left\{\cos\frac{\Delta\gamma}{2} - i\frac{\Delta M}{M}\sin\frac{\Delta\gamma}{2}\right\} = C \\ B &= -\frac{e^{-i(\gamma_1 + \gamma_2')}}{M} c's' \left\{\frac{\Delta M}{M}\cos\frac{\Delta\gamma}{2} - i\sin\frac{\Delta\gamma}{2}\right\}, \end{aligned} \quad (26)$$

where

$$\Delta\gamma = 2(\gamma_1 - \gamma_2') \qquad c's' = \pm\tfrac{1}{2}. \quad (27)$$

If one assumes $\nu_\mu \to \nu_\tau$ symmetry for the M_ν, then $h_{22} = h_{32}$ would imply, as is well known, maximal $\theta_{23} = \pm\pi/4$, $\theta_{13} = 0$, and $m_3 = 0$. If θ_{23} is not exactly maximal, or $\theta_{13} \neq 0$, then $\nu_\mu \to \nu_\tau$ symmetry for left-handed neutrinos is broken but m_3 is still zero since the second and third columns of m_D given in Eq. (6) are identical [6]. However, present experiments indicate that the breaking has to be small. Thus defining $h_\pm = \frac{h_{22} \pm h_{33}}{2}$, where $|\frac{h_+}{h_-}| \ll 1$, we have, neglecting $|\frac{h_+}{h_-}|^2$,

$$\sqrt{2}(a_{12}-a_{13}) = (h_{11}v_2)(2h_+v_1)\Big[2\frac{h_-}{h_+}\Big]B$$
$$(a_{22}-a_{23}) = \frac{1}{2}(4h_+^2v_1^2)\Big[4\frac{h_-}{h_+}\Big]C \tag{28}$$
$$a_{23} = \frac{1}{2}(4h_+^2v_1^2)C.$$

To quantify the breaking and in order not to introduce too many parameters, we assume the maximal atmospheric angle, but $\theta_{13}\neq 0$. The M_ν as given in Eq. (24) can be diagonalized with the matrix [6]

$$U = \begin{pmatrix} c & s & s_2 \\ -\frac{s-s_2}{\sqrt{2}} & \frac{c+s_2}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{s+s_2}{\sqrt{2}} & \frac{c-s_2}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}\times \mathrm{diag}(e^{i\beta_1}, e^{i\beta_2}, e^{i\beta_3}), \tag{29}$$

where $c=\cos\theta_{12}$, $s=\sin\theta_{12}$, θ_{12} is solar mixing angle and $s_2=\sin\theta_{13}$, θ_{13} is the reactor angle. Finally, then the elements of M_ν are

$$\begin{aligned}
a_{11} &= e^{-i(\beta_1+\beta_2)}m\Big\{\cos\frac{\Delta}{2}\Big(1-\frac{\Delta m}{m}\cos2\theta_{12}\Big) - i\sin\frac{\Delta}{2}\Big(\cos2\theta_{12}-\frac{\Delta m}{m}\Big)\Big\}\\
\sqrt{2}a_{12(13)} &= e^{-i(\beta_1+\beta_2)}m\Big\{\sin2\theta_{12}\Big[\frac{\Delta m}{m}\cos\frac{\Delta}{2}+i\sin\frac{\Delta}{2}\Big] \pm s_2\Big(\Big(1-\cos2\theta_{12}\frac{\Delta m}{m}\Big)\cos\frac{\Delta}{2}+i\Big(\frac{\Delta m}{m}-\cos2\theta_{12}\Big)\sin\frac{\Delta}{2}\Big)\Big\}\\
2a_{22(33)} &= e^{-i(\beta_1+\beta_2)}m\Big\{\cos\frac{\Delta}{2}\Big(1+\frac{\Delta m}{m}\cos2\theta_{12}\Big) + i\sin\frac{\Delta}{2}\Big(\frac{\Delta m}{m}+\cos2\theta_{12}\Big) \pm 2s_2\sin2\theta_{12}\Big(\frac{\Delta m}{m}\cos\frac{\Delta}{2}+i\sin\frac{\Delta}{2}\Big)\Big\}\\
a_{23} &= e^{-i(\beta_1+\beta_2)}m\Big\{\cos\frac{\Delta}{2}\Big(1+\frac{\Delta m}{m}\cos2\theta_{12}\Big) + i\sin\frac{\Delta}{2}\Big(\frac{\Delta m}{m}+\cos2\theta_{12}\Big)\Big\}\\
\Delta &= 2(\beta_1-\beta_2)\\
m &= \frac{m_2+m_1}{2}, \qquad \Delta m = \frac{m_2-m_1}{2}.
\end{aligned} \tag{30}$$

Some combinations of the above parameters are needed to calculate the asymmetry parameter in leptogenesis, which are now summarized. Calculation of $\Im[2a_{12}a_{13}a_{11}^*2a_{23}^*]$ from Eqs. (25) and (30) and equating them gives

$$c'^2s'^2\sin[2(\gamma_1-\gamma_2')]\frac{M_2^2-M_1^2}{M_1^3M_2^3} = -\frac{1}{|h_{11}v_2|^4|2h_{22}v_1|^2|2h_{32}v_2|^2}m_1m_2 \times\Big[c^2s^2(m_2^2-m_1^2)+\frac{1}{2}s_2^2\cos2\theta_{12}\Big(\begin{matrix} m_1^2+m_2^2 \\ -8m_1m_2c^2s^2\sin^2\frac{\Delta}{2}-2c^2s^2(m_1-m_2)^2\end{matrix}\Big)\Big]\sin\Delta. \tag{31}$$

Further calculating $|2a_{11}a_{23}-2a_{12}a_{13}|$ from the same equations and equating them gives

$$m_1m_2[1+O(s_2^2)] = \frac{1}{M_1M_2}[|h_{11}v_2|^2|2h_{22}v_1||h_{32}v_2|]. \tag{32}$$

Another useful relation is obtained by calculating $|2a_{12}a_{13}|$ from Eqs. (25) and (30) and equating them:

$$\begin{aligned}
&|c^2s^2((m_2^2+m_1^2)\cos\Delta-2m_1m_2)+i\sin\Delta(c^2s^2(m_2^2-m_1^2)+s_2^2\cos2\theta_{12}(m_2^2+m_1^2))|\\
&= c'^2s'^2\frac{m_1m_2}{M_1M_2}[(M_2-M_1)^2+4M_1M_2\sin^2(\gamma_1-\gamma_2')],
\end{aligned} \tag{33}$$

where terms of order s_2^2 compared to 1 and $(c^2s^2)/(1-2c^2s^2)$ and of order $s_2^2(m_2^2-m_1^2)$ compared to $(m_2^2+m_1^2)$ have been neglected and Eq. (32) has been used. This gives, on neglecting terms of order $s_2^4(\frac{\Delta m}{m})^2$, s_2^8, and $s_2^6(\frac{\Delta m}{m})$,

$$\begin{aligned}
&\Big\{c^2s^2\Big[(m_2-m_1)^2+4m_1m_2\sin^2\frac{\Delta}{2}\Big]+\frac{\cos2\theta_{12}}{c^2s^2}\frac{m_2^2+m_1^2}{2m_1m_2}s_2^2[2c^2s^2(m_2^2-m_1^2)+s_2^2\cos2\theta_{12}(m_2^2+m_1^2)]\Big\}\\
&= c'^2s'^2\frac{m_1m_2}{M_1M_2}[(M_2-M_1)^2+4M_1M_2\sin^2(\gamma_1-\gamma_2')].
\end{aligned} \tag{34}$$

In the present case, when $c'^2s'^2=\frac{1}{4}$, the relations (31) [on using Eqs. (32) and (34)] become $[\Delta_\gamma = 2(\gamma_1-\gamma_2')]$

$$\sin\Delta_\gamma = -\sin^2 2\theta_{12}\frac{\Delta m/m}{\Delta M/M}\sin\Delta\left[\left(1+\frac{1}{\sin^2 2\theta_{12}}\left(1-\sin^2 2\theta_{12}\sin^2\frac{\Delta}{2}\right)\right)\cos 2\theta_{12} r'\right] \tag{35}$$

$$\sin^2 2\theta_{12}\left[\sin^2\frac{\Delta}{2}+(\Delta m/m)^2\left(1+8\frac{\cos 2\theta_{12}}{\sin^4 2\theta_{12}}r'(\sin^2 2\theta_{12}+r'\cos 2\theta_{12})\right)\right]=\left[(\Delta M/M)^2+\sin^2\frac{\Delta_\gamma}{2}\right], \tag{36}$$

where

$$r' = s_2^2/(\Delta m/m) = \sin^2\theta_{13}/(\Delta m/m).$$

Now the Yukawa couplings $|h_{11}v_2|$, $|2h_+v_1|$, and $|\frac{h_-}{h_+}|$ can be evaluated. From Eqs. (25) and (28)

$$|h_{11}v_2|^2 = \frac{|a_{11}|}{|C|}, \qquad |2h_+v_1|^2 = 2\frac{|a_{23}|}{|C|}$$
$$|2h_+||2h_-|v_1^2 = \frac{|a_{22}-a_{33}|}{2|C|}. \tag{37}$$

From Eqs. (30)

$$|a_{11}| = m\left[1-\sin^2 2\theta_{12}\sin^2\left(\frac{\Delta}{2}\right)-2\cos 2\theta_{12}\frac{\Delta m}{m}+O\left(\left(\frac{\Delta m}{m}\right)^2\right)\right]^{1/2}$$
$$|a_{23}| = \frac{m}{2}\left[1-\sin^2 2\theta_{12}\sin^2\left(\frac{\Delta}{2}\right)+2\cos 2\theta_{12}\frac{\Delta m}{m}+O\left(\left(\frac{\Delta m}{m}\right)^2\right)\right]^{1/2}$$
$$|a_{22}-a_{33}| = 2ms_2\sin 2\theta_{12}\left[\sin^2\left(\frac{\Delta}{2}\right)+\left(\frac{\Delta m}{m}\right)^2\right]^{1/2} \tag{38}$$

while from Eq. (26)

$$|C| = \frac{1}{M}\left[\cos^2\frac{\Delta_\gamma}{2}+\left(\frac{\Delta M}{M}\right)^2\sin^2\frac{\Delta_\gamma}{2}\right]^{1/2} \simeq \frac{1}{M}\left[1-\sin^2\frac{\Delta_\gamma}{2}\right] \tag{39}$$

neglecting $(\frac{\Delta M}{M})^2$ compared to 1, which on using Eq. (36) becomes

$$|C| \simeq \frac{1}{M}\left[1-\sin^2 2\theta_{12}\sin^2\left(\frac{\Delta}{2}\right)+O\left(\left(\frac{\Delta M}{M}\right)^2\right)+O\left(\left(\frac{\Delta m}{m}\right)^2\right)+O(s_2^2)\right]^{1/2}. \tag{40}$$

Thus,

$$|h_{11}v_2|^2 = Mm\left[1-2\cos 2\theta_{12}\frac{\Delta m}{m}-\sin^2 2\theta_{12}\sin^2\frac{\Delta}{2}\right]^{1/2}\times\left[1-\sin^2 2\theta_{12}\sin^2\frac{\Delta}{2}\right]^{-1/2}$$
$$|2h_+v_1|^2 = Mm\left[1+2\cos 2\theta_{12}\frac{\Delta m}{m}-\sin^2 2\theta_{12}\sin^2\frac{\Delta}{2}\right]^{1/2}\times\left[1-\sin^2 2\theta_{12}\sin^2\frac{\Delta}{2}\right]^{-1/2}$$
$$|2h_-v_1|^2 = Mms_2^2\sin^2 2\theta_{12}\left[\sin^2\frac{\Delta}{2}+\left(\frac{\Delta m}{m}\right)^2\right]\times\left(1-\sin^2 2\theta_{12}\sin^2\frac{\Delta}{2}\right)^{-1}\left[1+O\left(\frac{\Delta m}{m}\right)\right]. \tag{41}$$

III. LEPTOGENESIS

As is well known [3,4,10,11], the leptogenesis asymmetry is given by [11]

$$\epsilon_i = \frac{1}{8\pi}\sum_{k\neq i}\frac{1}{v_i^2 R_{ii}}\,\mathrm{Im}\left[(R_{ik})^2 f\left(\frac{M_k^2}{M_i^2}\right)\right], \tag{42}$$

where M_i denotes the heavy Majorana neutrino masses, R_{ij} are defined by

$$R = \hat{m}_D^\dagger \hat{m}_D = V^T m_D^\dagger m_D V^*. \tag{43}$$

The loop function $f(x)$ containing vertex and self-energy corrections is

$$f(x) = \sqrt{x}\left(\frac{2-x}{1-x}-(1+x)\ln\frac{1+x}{x}\right).$$

Now $(|v_1|^2+|v_2|^2) = (174\ \mathrm{GeV})^2 = |v|^2$. One may take $|v_1|^2 = |v_2|^2 = \frac{1}{2}v^2$, so that

$$\epsilon_1 = \frac{1}{8\pi}f\left(\frac{M_2^2}{M_1^2}\right)\frac{1}{v_1^2 R_{11}}\,\mathrm{Im}[(R_{12})^2]. \tag{44}$$

Using the constraint [2,3]

$$R_{11} < 4.3\times 10^{-7} v_1^2, \tag{45}$$

obtained from out of equilibrium decay of $M_1 \simeq 10^{10}$ GeV, one finally obtains the lower limit on ϵ_1:

$$\epsilon_1 = \frac{1}{8\pi}f(x)\frac{2.3\times 10^6}{v_1^4}\{\Im[(R_{12})^2]\}, \tag{46}$$

where $x = \frac{M_2^2}{M_1^2}$, and for $M_2 \simeq M_1$,

$$f(x) = -\frac{M}{4\Delta M}. \tag{47}$$

Now $\Im[(R_{12})^2]$ as calculated from Eq. (43) is given by [1]

$$\Im[(R_{12})^2] = c'^2 s'^2\Big(|h_{11}v_2|^2 - \frac{1}{2}|2h_{12}v_1|^2 + |2h_{32}v_1|^2\Big)^2 \sin\Delta_\gamma = \frac{1}{4}\{|h_{11}v_2|^2 - |2h_+v_1|^2 - |2h_-v_1|^2\}^2 \sin\Delta_\gamma$$
$$= M^2m^2\cos^2 2\theta_{12}\Big(\frac{\Delta m}{m}\Big)^2\Big\{1 + \frac{1}{2}\frac{\sin^2\theta_{13}}{(\Delta m/m)}\frac{\sin^2 2\theta_{12}}{\cos 2\theta_{12}}\Big(\sin^2\frac{\Delta}{2} + \Big(\frac{\Delta m}{m}\Big)^2\Big)\Big\}^2 \frac{1}{[1 - \sin^2 2\theta_{12}\sin^2\Delta/2]^2}\sin\Delta_\gamma. \tag{48}$$

Using Eqs. (35) and (46)–(48) along with

$$\cos 2\theta_{12} = \tfrac{1}{3}, \qquad \sin^2 2\theta_{12} = \tfrac{8}{9} \tag{49}$$

$$\epsilon \simeq (6\times 10^2)\frac{M^2}{v_1^4}\Delta m^2_{\rm solar}\sin\Delta\Big(\frac{\frac{\Delta m}{m}}{\frac{\Delta M}{M}}\Big)^2\frac{1}{[1-\frac{8}{9}\sin^2\frac{\Delta}{2}]^2}\Big\{1 + \frac{4}{3}r'\Big(\sin^2\frac{\Delta}{2} + \Big(\frac{\Delta m}{m}\Big)^2\Big)\Big\}^2\Big[1 + \frac{3}{8}\Big(1 - \frac{8}{9}\sin^2\frac{\Delta}{2}\Big)r'\Big] \tag{50}$$

$$\simeq 2\times 10^{-8}\Big(\frac{M}{10^{10}\ {\rm GeV}}\Big)^2\frac{\Delta m^2_{\rm solar}}{7.6\times 10^{-5}\ {\rm eV}^2}\Big(\frac{174\ {\rm GeV}}{v}\Big)^4\sin\Delta\Big(\frac{\frac{\Delta m}{m}}{\frac{\Delta M}{M}}\Big)^2$$
$$\times\frac{1}{[1-\frac{8}{9}\sin^2\frac{\Delta}{2}]^2}\Big\{1 + \frac{4}{3}r'\Big(\sin^2\frac{\Delta}{2} + \Big(\frac{\Delta m}{m}\Big)^2\Big)\Big\}^2\Big[1 + \frac{3}{8}\Big(1 - \frac{8}{9}\sin^2\frac{\Delta}{2}\Big)r'\Big] \tag{51}$$

with $r' = \sin^2\theta_{13}/(\Delta m/m)$, $v_1^2 = \frac{1}{2}v^2 = \frac{1}{2}(174\ {\rm GeV})^2$, $4m\Delta m = \Delta m^2_{\rm solar}$, and $\Delta m^2_{\rm solar} = 7.6\times 10^{-5}\ {\rm eV}^2$. Using the neutrino oscillation data [12]

$$m \simeq (\Delta m^2_{\rm atm})^{1/2} = 4.9\times 10^{-2}\ {\rm eV} \tag{52}$$

$$\frac{\Delta m}{m} = \frac{1}{4}\frac{\Delta m^2_{\rm solar}}{\Delta m^2_{\rm atm}} = 0.8\times 10^{-2} \tag{53}$$

$$\sin^2\theta_{13} \le 4.6\times 10^{-2}(0.016\pm 0.010) \tag{54}$$

giving

$$r' \le 5.75(2\pm 1.25).$$

It can be seen from Eq. (51) that the contribution from $\sin^2\theta_{13}$ may dominate. The Majorana phase Δ which is the same as would appear in neutrinoless double β decay [cf. first of Eq. (34)] can be fixed from Eqs. (35) and (36). This gives

$$x[(1-x) + C] = r^2[(1-x)(x - \tfrac{1}{9})(1 + \tfrac{3}{8}r'x)^2],$$

where

$$x = 1 - \frac{8}{9}\sin^2\frac{\Delta}{2}$$
$$C = \Big(\frac{\Delta M}{M}\Big)^2\Big[\frac{8}{9}r^2A - 1\Big]$$
$$A = 1 + 3r'B, \qquad 1\le A\le 55$$
$$B = 1 + \frac{3}{8}r', \qquad 1\le B\le 3$$

for $r' \le 5.75$. It is clear that $\frac{1}{9}\le x < 1$. C is negligibly small except for $x\to 1$. In that case we have:

Solution I

$$\sin^2\frac{\Delta}{2} = \frac{9}{8}\Big(\frac{\Delta M}{M}\Big)^2\frac{\frac{8}{9}r^2A - 1}{\frac{8}{9}r^2B^2 - 1}$$
$$\sin\Delta = \frac{3}{\sqrt{2}}\Big(\frac{\frac{8}{9}r^2A - 1}{\frac{8}{9}r^2B^2 - 1}\Big)^{1/2}\frac{\Delta M}{M}. \tag{55}$$

In this case, leptogenesis asymmetry ϵ given in Eq. (50) gives

$$\epsilon \simeq 3\times 10^{-10}r\Big(\frac{\frac{8}{9}r^2A - 1}{\frac{8}{9}r^2B^2 - 1}\Big)^{1/2}B. \tag{56}$$

Thus, ϵ is of the right order of magnitude (10^{-6}–10^{-5}) for $1\le\sqrt{A}\le 7.4$ provided that $r\simeq 4\times 10^3$; i.e., $\frac{\Delta M}{M}\simeq 2\times 10^{-6}$.

Solution II

There is another solution, for which for $r' = 0$, $\sin^2\frac{\Delta}{2} = 1 - \frac{1}{8}\frac{1}{r^2-1}$, i.e. near maximal value as $(r^2-1) > 1$. For $r'\neq 0$, such a solution is modified to

$$\sin^2\frac{\Delta}{2} = 1 - \frac{1}{8}\frac{1}{D^2r^2 - 1}$$
$$\sin\Delta = \frac{1}{\sqrt{2}}\frac{1}{rD}\Big(1 - \frac{1}{r^2D^2}\Big)^{-1/2}\Big[1 - \frac{1}{8D^2r^2(1 - \frac{1}{D^2r^2})}\Big]^{1/2}, \tag{57}$$

where $D = 1 + r'/24$. In this case

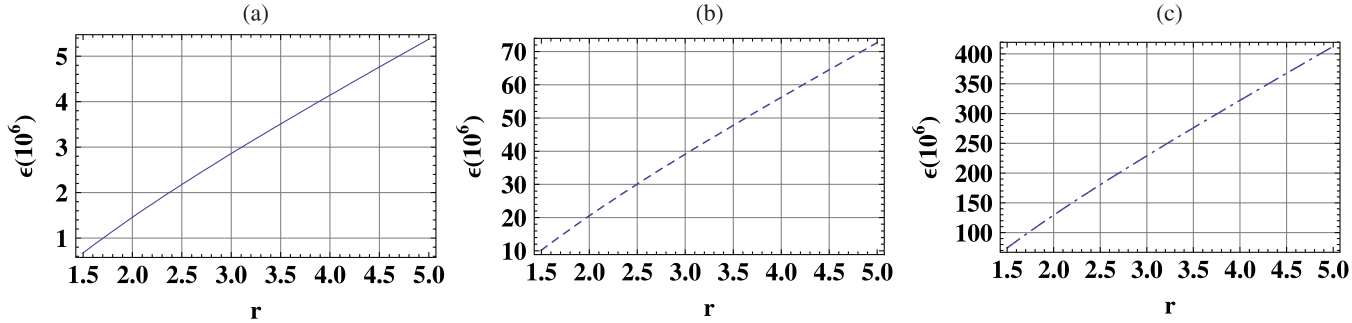

FIG. 1 (color online). The asymmetry $\epsilon \times 10^6$ as a function of r for different values of r'. The labels (a), (b), and (c) correspond to $r' = 0$, 2, and 5.75, respectively.

$$\epsilon \simeq 2 \times 10^{-8} \frac{81}{\sqrt{2}} r f(r) \left[1 + \frac{4}{3}\left(1 - \frac{1}{8}\frac{1}{D^2 r^2 - 1}\right) r'\right]^2 \times \left[1 + \frac{1}{24D}\frac{1}{D^2 r^2 - 1} r'\right],$$

where

$$f(r) = \left(1 - \frac{1}{8(D^2 r^2 - 1)}\right)^{1/2} \left(1 - \frac{1}{D^2 r^2}\right)^{3/2}. \tag{58}$$

The asymmetry ϵ and $\sin^2\Delta/2$ are plotted as a function of r for $r' = 0$, 2, and 5.75 in Figs. 1(a)–1(c) and 2, respectively.

One can see from the plot that (i) $\sin^2\Delta/2$ is near maximal (≥ 0.90) for any $r \geq 1.5$ and r'. (ii) For any given value of r, $\sin^2\theta_{13}$ gives the dominant contribution to ϵ, e.g. for $r = 2$, ϵ is 1.5×10^{-6} ($r' = 0$), 2.0×10^{-5} ($r' = 2$), and 1.3×10^{-4} ($r' = 5.75$). (iii) An accurate measurement of r' will have important implications for r and in turn for $\frac{\Delta M}{M}$. The value $r \simeq 2$ implies $\frac{\Delta M}{M} \simeq 4 \times 10^{-3}$ which gives the degeneracy required for heavy right-handed neutrinos. It is important to note that CP violation responsible for the generation of the baryogenesis parameter through leptogenesis comes entirely from Majorana phase Δ which is now predicted to be negligible for the first solution and for the second solution $\sin^2\frac{\Delta}{2} \succeq 0.95$. This can in principle be tested in neutrinoless double β decay, where the effective electron neutrinos mass is given by

$$\begin{aligned} m_{ee} &= |a| \\ &\simeq m\left[1 - \sin^2 2\theta_1 \sin^2\frac{\Delta}{2}\right]^{1/2} \\ &\simeq 4.3 \times 10^{-2}\ \text{eV}, \quad \text{for solution I} \qquad (59) \\ &\simeq 1.7 \times 10^{-2}\ \text{eV}, \quad \text{for solution II.} \qquad (60) \end{aligned}$$

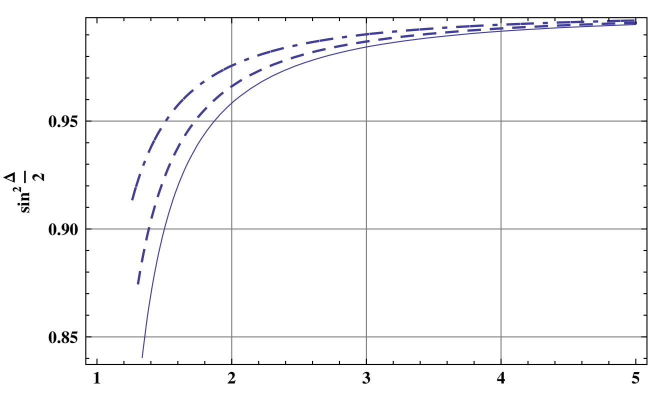

FIG. 2 (color online). $\sin^2\Delta/2$ as a function of r for different values of r'. The solid line corresponds to $r' = 0$, the dashed line is for $r' = 2$, and the double-dash-dotted line is for $r' = 5.75$.

IV. CONCLUSION

By considering a simple generic seesaw model gauge model with $\mu - \tau$ symmetry for the heavy right-handed neutrinos, degenerate leptogenesis has been studied, where the exact degeneracy is due to $\bar{L}$ discrete symmetry for the heavy right-handed neutrinos. When this degeneracy is slightly broken, an adequate lepton asymmetry ($\epsilon \simeq 10^{-6}$–10^{-5}) can be obtained. The level of degeneracy required in one case is $\frac{\Delta M}{M} \simeq 10^{-6}$, much smaller than $\frac{\Delta m}{m} \simeq 8 \times 10^{-3}$ obtained from neutrino oscillations and in the second case is $\frac{\Delta M}{M} \simeq 4 \times 10^{-3}$, which is of the same order as $\frac{\Delta m}{m} \simeq 8 \times 10^{-3}$. This in turn predicts the Majorana phase responsible for the lepton asymmetry. Since the same phase appears in neutrinoless double β decay, it can in principle be tested. Further $\sin^2\theta_{13}$ has important implications for the leptogenesis asymmetry parameter ϵ and degree of degeneracy $\frac{\Delta M}{M}$ needed. Thus, an accurate measurement of $\sin^2\theta_{13}$ will be of great interest.

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