Erratum: Higher-order spin effects in the dynamics of compact binaries. II. Radiation field [Phys. Rev. D 74, 104034 (2006)]

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The spin effects in the quadrupole moment I_{ij} have been incorrectly computed at 2.5PN order. We find that the mistake was a consequence of (i) having overlooked the contribution of the spin terms coming from the monopolar part of the stress-energy tensor (Eq. (2.2) of Ref. [1], referred to as paper I henceforth) when computing the first term under the integral in Eq. (4.1a), (ii) having not taken into account the spin terms produced by the time differentiation of the nonspin parts of Σ and Σ_i when computing the second and fourth terms in Eq. (4.1a), and (iii) having used by mistake a non–up-to-date file containing in principle the sum of all compact support terms entering the right-hand side of Eq. (4.1a). As a consequence the expressions of the energy flux $\mathcal{F}$, the time derivative of the orbital frequency $\dot{\omega}$ and the phase ϕ are also modified.

The equation (5.3) for the quadrupole moment should be replaced by

$$\underset{\mathrm{S}}{I_{ij}} = \frac{\nu}{c^3}\left\{\frac{8}{3}x^{\langle i}(\mathbf{v}\times\mathbf{S})^{j\rangle} - \frac{4}{3}v^{\langle i}(\mathbf{x}\times\mathbf{S})^{j\rangle} + \frac{8}{3}\frac{\delta m}{m}x^{\langle i}(\mathbf{v}\times\mathbf{\Sigma})^{j\rangle} - \frac{4}{3}\frac{\delta m}{m}v^{\langle i}(\mathbf{x}\times\mathbf{\Sigma})^{j\rangle}\right\} + \frac{\nu}{c^5}\left\{\left[\frac{5}{3}+\frac{2}{7}\nu\right]\frac{Gm}{r^3}\frac{\delta m}{m}\right.$$
$$\times (xv)x^{\langle i}(\mathbf{x}\times\mathbf{\Sigma})^{j\rangle} + \left(\left[\frac{7}{3}+4\nu\right]\frac{Gm}{r} + \left[\frac{26}{21}-\frac{116}{21}\nu\right]v^2\right)\frac{\delta m}{m}x^{\langle i}(\mathbf{v}\times\mathbf{\Sigma})^{j\rangle} + \left[\frac{31}{21}+\frac{19}{21}\nu\right]\frac{Gm}{r^3}(xv)x^{\langle i}(\mathbf{x}\times\mathbf{S})^{j\rangle}$$
$$+ \left(\left[\frac{25}{7}+\frac{55}{21}\nu\right]\frac{Gm}{r} + \left[\frac{26}{21}-\frac{26}{7}\nu\right]v^2\right)x^{\langle i}(\mathbf{v}\times\mathbf{S})^{j\rangle} + \left(\left[-4-\frac{2}{7}\nu\right]\frac{Gm}{r} + \left[-\frac{6}{7}+\frac{64}{21}\nu\right]v^2\right)\frac{\delta m}{m}v^{\langle i}(\mathbf{x}\times\mathbf{\Sigma})^{j\rangle}$$
$$+ \left[\frac{10}{21}-\frac{8}{21}\nu\right]\frac{\delta m}{m}(xv)v^{\langle i}(\mathbf{v}\times\mathbf{\Sigma})^{j\rangle} + \left(\left[-\frac{26}{3}-\frac{2}{3}\nu\right]\frac{Gm}{r} + \left[-\frac{6}{7}+\frac{18}{7}\nu\right]v^2\right)v^{\langle i}(\mathbf{x}\times\mathbf{S})^{j\rangle} + \left[\frac{10}{21}-\frac{10}{7}\nu\right]$$
$$\times (xv)v^{\langle i}(\mathbf{v}\times\mathbf{S})^{j\rangle} + \left(\left[\frac{52}{21}-\frac{10}{7}\nu\right](S,x,v) + \left[\frac{62}{21}-\frac{18}{7}\nu\right]\frac{\delta m}{m}(\Sigma,x,v)\right)\frac{Gm}{r^3}x^{\langle i}x^{j\rangle} + \left(\left[-\frac{5}{21}+\frac{5}{7}\nu\right](S,x,v)\right.$$
$$\left. + \left[-\frac{5}{21}-\frac{4}{7}\nu\right]\frac{\delta m}{m}(\Sigma,x,v)\right)v^{\langle i}v^{j\rangle} + \left(\left[-\frac{8}{3}+\frac{16}{3}\nu\right](xS) + \left[-\frac{8}{3}+\frac{8}{3}\nu\right]\frac{\delta m}{m}(x\Sigma)\right)\frac{Gm}{r^3}x^{\langle i}(\mathbf{x}\times\mathbf{v})^{j\rangle}$$
$$\left. + \left(\left[\frac{4}{3}-4\nu\right](vS) + \left[\frac{4}{3}-\frac{8}{3}\nu\right]\frac{\delta m}{m}(v\Sigma)\right)v^{\langle i}(\mathbf{x}\times\mathbf{v})^{j\rangle}\right\} + \mathcal{O}\left(\frac{1}{c^7}\right). \quad (1)$$

The equations (5.12), (6.4), (6.5), and (7.11) for the energy fluxes should be replaced by

$$\underset{\mathrm{S}}{f_{2.5\,\mathrm{PN}}} = \frac{(S,n,v)}{mr}\left[(nv)^4\left(-\frac{2244}{7}+\frac{3144}{7}\nu\right) + \frac{G^2m^2}{r^2}\left(\frac{972}{7}+\frac{166}{7}\nu\right) + \frac{Gm}{r}(nv)^2\left(-\frac{2866}{7}+\frac{170}{7}\nu\right)\right.$$
$$\left. + (nv)^2v^2\left(\frac{3519}{7}-\frac{5004}{7}\nu\right) + \frac{Gm}{r}v^2\left(\frac{3504}{7}-20\nu\right) + v^4\left(-\frac{1207}{7}+\frac{1810}{7}\nu\right)\right]$$
$$+ \frac{(\Sigma,n,v)}{mr}\left[(nv)^4\left(-\frac{7941}{28}+\frac{2676}{7}\nu\right) + \frac{G^2m^2}{r^2}\left(-\frac{109}{7}+18\nu\right) + \frac{Gm}{r}(nv)^2\left(-\frac{6613}{14}+\frac{1031}{7}\nu\right)\right.$$
$$\left. + (nv)^2v^2\left(\frac{2364}{7}-\frac{3621}{7}\nu\right) + \frac{Gm}{r}v^2\left(\frac{4785}{14}-65\nu\right) + v^4\left(-\frac{2603}{28}+\frac{1040}{7}\nu\right)\right]\frac{\delta m}{m}, \quad (2)$$

$$\mathcal{F} = \frac{32}{5}\frac{c^5}{G}\gamma^5\nu^2\left\{1 + \gamma\left(-\frac{2927}{336}-\frac{5}{4}\nu\right) + 4\pi\gamma^{3/2} + \gamma^2\left(\frac{293\,383}{9072}+\frac{380}{9}\nu\right) + \pi\gamma^{5/2}\left(-\frac{25\,663}{672}-\frac{125}{8}\nu\right) + \frac{\gamma^{3/2}}{Gm^2}\right.$$
$$\left.\times\left[-\frac{37}{3}S_\ell - \frac{25}{4}\frac{\delta m}{m}\Sigma_\ell\right] + \frac{\gamma^{5/2}}{Gm^2}\left[\left(\frac{17\,897}{168}+23\nu\right)S_\ell + \left(\frac{6253}{112}+\frac{277}{24}\nu\right)\frac{\delta m}{m}\Sigma_\ell\right] + \mathcal{O}\left(\frac{1}{c^6}\right)\right\}, \quad (3)$$

$$\mathcal{F} = \frac{32}{5}\frac{c^5}{G}x^5\nu^2\left\{1 + x\left(-\frac{1247}{336}-\frac{35}{12}\nu\right) + 4\pi x^{3/2} + x^2\left(-\frac{44\,711}{9072}+\frac{9271}{504}\nu+\frac{65}{18}\nu^2\right) + \pi x^{5/2}\left(-\frac{8191}{672}-\frac{583}{24}\nu\right)\right.$$
$$\left. + \frac{x^{3/2}}{Gm^2}\left[-4S_\ell - \frac{5}{4}\frac{\delta m}{m}\Sigma_\ell\right] + \frac{x^{5/2}}{Gm^2}\left[\left(-\frac{23}{4}+\frac{245}{9}\nu\right)S_\ell + \left(-\frac{33}{16}+\frac{37}{4}\nu\right)\frac{\delta m}{m}\Sigma_\ell\right] + \mathcal{O}\left(\frac{1}{c^6}\right)\right\}, \quad (4)$$

and

$$\underset{\mathrm{S}}{\mathcal{F}} = \frac{32}{5}\frac{c^5}{G}x^5\nu^2\left\{1 + \frac{x^{3/2}}{Gm^2}\left[-4S_\ell^{\mathrm{c}} - \frac{5}{4}\frac{\delta m}{m}\Sigma_\ell^{\mathrm{c}}\right] + \frac{x^{5/2}}{Gm^2}\left[\left(-\frac{9}{2} + \frac{272}{9}\nu\right)S_\ell^{\mathrm{c}} + \left(-\frac{13}{16} + \frac{43}{4}\nu\right)\frac{\delta m}{m}\Sigma_\ell^{\mathrm{c}}\right] + \mathcal{O}\left(\frac{1}{c^6}\right)\right\}. \quad (5)$$

These results for the flux agree in the test-mass limit $\nu \to 0$ with the black-hole perturbation calculation of Tagoshi *et al.* [2] [see Eq. (G19) there]. Following Ref. [3] one should express the time derivative of the orbital frequency and the phase in terms of the constant-magnitude spins S_ℓ^{c} and Σ_ℓ^{c}. Eqs. (7.12), (7.13), and (7.14), and Eqs. (1), (2), and (3) in Ref. [3] should be replaced by

$$\frac{\dot{\omega}}{\omega^2} = \frac{96}{5}\nu x^{5/2}\left\{1 + x\left(-\frac{743}{336} - \frac{11}{4}\nu\right) + 4\pi x^{3/2} + x^2\left(\frac{34\,103}{18\,144} + \frac{13\,661}{2016}\nu + \frac{59}{18}\nu^2\right) + \pi x^{5/2}\left(-\frac{4159}{672} - \frac{189}{8}\nu\right)\right.$$
$$\left. + \frac{x^{3/2}}{Gm^2}\left[-\frac{47}{3}S_\ell^{\mathrm{c}} - \frac{25}{4}\frac{\delta m}{m}\Sigma_\ell^{\mathrm{c}}\right] + \frac{x^{5/2}}{Gm^2}\left[\left(-\frac{5861}{144} + \frac{1001}{12}\nu\right)S_\ell^{\mathrm{c}} + \left(-\frac{809}{84} + \frac{281}{8}\nu\right)\frac{\delta m}{m}\Sigma_\ell^{\mathrm{c}}\right]\right\}, \quad (6)$$

$$\phi = \phi_0 - \frac{1}{32\nu}\left\{x^{-5/2} + x^{-3/2}\left(\frac{3715}{1008} + \frac{55}{12}\nu\right) + \frac{x^{-1}}{Gm^2}\left(\frac{235}{6}S_\ell^{\mathrm{c}} + \frac{125}{8}\frac{\delta m}{m}\Sigma_\ell^{\mathrm{c}}\right) - 10\pi x^{-1} + x^{-1/2}\left(\frac{15\,293\,365}{1\,016\,064} + \frac{27\,145}{1008}\nu\right.\right.$$
$$\left.\left. + \frac{3085}{144}\nu^2\right) + \pi \ln x\left(\frac{38\,645}{1344} - \frac{65}{16}\nu\right) + \frac{\ln x}{Gm^2}\left[\left(-\frac{554\,345}{2016} - \frac{55}{8}\nu\right)S_\ell^{\mathrm{c}} + \left(-\frac{41\,745}{448} + \frac{15}{8}\nu\right)\frac{\delta m}{m}\Sigma_\ell^{\mathrm{c}}\right]\right\}, \quad (7)$$

and

$$\underset{\mathrm{S}}{\phi} = -\frac{1}{32\nu}\sum_{i=1,2}\chi_i^{\mathrm{c}}\kappa_i^{\mathrm{c}}\left\{\left(\frac{565}{24}\frac{m_i^2}{m^2} + \frac{125}{8}\nu\right)x^{-1} + \left[\left(-\frac{732\,985}{4032} - \frac{35}{4}\nu\right)\frac{m_i^2}{m^2} + \left(-\frac{41\,745}{448} + \frac{15}{8}\nu\right)\nu\right]\ln x\right\}. \quad (8)$$

The new results for the accumulated number of GW cycles showed in Tables II and III are given below. In addition, the term $-V\partial_t^2 V$ in Eq. (4.3) should be removed, and the term $\varepsilon_{lmn}S_1^m\partial_{2k}\partial_{1kn}Y_{j\rangle k}$ in Eq. (4.7b) should read $\varepsilon_{lmn}S_1^m\partial_{2p}\partial_{1pn}Y_{j\rangle k}$. No other results in the paper or in paper I are modified. We thank P. Ajith for having also noticed (independently of us) the previous disagreement in the test-mass limit with the perturbative calculation of Ref. [2].

	$(10 + 1.4)M_\odot$	$(10 + 10)M_\odot$	$(1.4 + 1.4)M_\odot$
Newtonian	3577	601	16034
1PN	+213	+59.3	+441
1.5PN	$-181 + 114\kappa_1^{\mathrm{c}}\chi_1^{\mathrm{c}} + 11.8\kappa_2^{\mathrm{c}}\chi_2^{\mathrm{c}}$	$-51.4 + 16.0\kappa_1^{\mathrm{c}}\chi_1^{\mathrm{c}} + 16.0\kappa_2^{\mathrm{c}}\chi_2^{\mathrm{c}}$	$-211 + 65.7\kappa_1^{\mathrm{c}}\chi_1^{\mathrm{c}} + 65.7\kappa_2^{\mathrm{c}}\chi_2^{\mathrm{c}}$
2PN	$+9.8 - 4.4\kappa_1^{\mathrm{c}}\kappa_2^{\mathrm{c}}\chi_1^{\mathrm{c}}\chi_2^{\mathrm{c}} + 1.5\xi^{\mathrm{c}}\chi_1^{\mathrm{c}}\chi_2^{\mathrm{c}}$	$+4.1 - 3.3\kappa_1^{\mathrm{c}}\kappa_2^{\mathrm{c}}\chi_1^{\mathrm{c}}\chi_2^{\mathrm{c}} + 1.1\xi^{\mathrm{c}}\chi_1^{\mathrm{c}}\chi_2^{\mathrm{c}}$	$+9.9 - 8.0\kappa_1^{\mathrm{c}}\kappa_2^{\mathrm{c}}\chi_1^{\mathrm{c}}\chi_2^{\mathrm{c}} + 2.8\xi^{\mathrm{c}}\chi_1^{\mathrm{c}}\chi_2^{\mathrm{c}}$
2.5PN	$-20 + 33.9\kappa_1^{\mathrm{c}}\chi_1^{\mathrm{c}} + 2.9\kappa_2^{\mathrm{c}}\chi_2^{\mathrm{c}}$	$-7.1 + 5.7\kappa_1^{\mathrm{c}}\chi_1^{\mathrm{c}} + 5.7\kappa_2^{\mathrm{c}}\chi_2^{\mathrm{c}}$	$-11.7 + 9.3\kappa_1^{\mathrm{c}}\chi_1^{\mathrm{c}} + 9.3\kappa_2^{\mathrm{c}}\chi_2^{\mathrm{c}}$
3PN	+2.3	+2.2	+2.6
3.5PN	−1.8	−0.8	−0.9

	$(10^6 + 10^6)M_\odot$	$(10^6 + 10^5)M_\odot$	$(10^5 + 10^5)M_\odot$
Newtonian	2267	4985	9570
1PN	+134	+281	+323
1.5PN	$-92.4 + 28.8\kappa_1^{\mathrm{c}}\chi_1^{\mathrm{c}} + 28.8\kappa_2^{\mathrm{c}}\chi_2^{\mathrm{c}}$	$-243 + 161\kappa_1^{\mathrm{c}}\chi_1^{\mathrm{c}} + 11.5\kappa_2^{\mathrm{c}}\chi_2^{\mathrm{c}}$	$-170 + 53\kappa_1^{\mathrm{c}}\chi_1^{\mathrm{c}} + 53\kappa_2^{\mathrm{c}}\chi_2^{\mathrm{c}}$
2PN	$+6.0 - 4.8\kappa_1^{\mathrm{c}}\kappa_2^{\mathrm{c}}\chi_1^{\mathrm{c}}\chi_2^{\mathrm{c}} + 1.7\xi^{\mathrm{c}}\chi_1^{\mathrm{c}}\chi_2^{\mathrm{c}}$	$+12.5 - 4.4\kappa_1^{\mathrm{c}}\kappa_2^{\mathrm{c}}\chi_1^{\mathrm{c}}\chi_2^{\mathrm{c}} + 1.5\xi^{\mathrm{c}}\chi_1^{\mathrm{c}}\chi_2^{\mathrm{c}}$	$+8.7 - 7.1\kappa_1^{\mathrm{c}}\kappa_2^{\mathrm{c}}\chi_1^{\mathrm{c}}\chi_2^{\mathrm{c}} + 2.4\xi^{\mathrm{c}}\chi_1^{\mathrm{c}}\chi_2^{\mathrm{c}}$
2.5PN	$-9.0 + 7.1\kappa_1^{\mathrm{c}}\chi_1^{\mathrm{c}} + 7.1\kappa_2^{\mathrm{c}}\chi_2^{\mathrm{c}}$	$-26.5 + 47.0\kappa_1^{\mathrm{c}}\chi_1^{\mathrm{c}} + 2.7\kappa_2^{\mathrm{c}}\chi_2^{\mathrm{c}}$	$-11.0 + 8.7\kappa_1^{\mathrm{c}}\chi_1^{\mathrm{c}} + 8.7\kappa_2^{\mathrm{c}}\chi_2^{\mathrm{c}}$
3PN	+2.3	+2.3	+2.5
3.5PN	−0.9	−2.3	−0.9

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