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Potentials for soft-wall AdS/QCD

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Soft-wall models in AdS/QCD generally have dilaton and scalar fields that vary with the fifth-dimension coordinate. These fields can be parametrized to yield hadron mass spectra with linear radial trajectories and to incorporate spontaneous breaking of chiral symmetry. We show how to construct scalar potentials which lead to such solutions.

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I. INTRODUCTION

The correspondence between certain gauge theories in four dimensions and gravitational theories in higher dimensions has proven to be a very fruitful one. It allows for the calculation of physical observables in a strongly coupled gauge theory, where perturbation theory is inapplicable, by performing a corresponding calculation in a classical gravitational theory. This general gauge/gravity duality was originally inspired by the anti–de Sitter/conformal field theory (AdS/CFT) correspondence, and as such the metric usually takes the form of anti–de Sitter space in five dimensions [1–4]. QCD, the gauge theory of the strong interactions, is strongly coupled at low energies. Heretofore, if one wanted to understand the structure of hadrons, then the only reliable means was via numerical computations in lattice gauge theory. Aside from lattice gauge theory, one must resort to models. The discovery of a gravitational dual to QCD would be a fantastic achievement. Unfortunately, the gravitational dual is not known, nor is it known whether one exists even in principle. Deriving such a duality from a fundamental theory, such as string theory, is referred to as the top down approach. The bottom up approach assumes that such a dual exists, and it models QCD by an effective five-dimensional gravity theory. One would like to incorporate the essential features of QCD into such a model. Such effective five-dimensional models are generically referred to as AdS/QCD, and allow for the computation of physical quantities in QCD such as mass spectra [5–11], form factors [12–16], and thermodynamic observables [17–24].

The first model to be constructed via the bottom up approach is referred to as the hard-wall model [6,25,26]; it simply places an infrared cutoff on the fifth-dimensional coordinate. This cutoff breaks the conformal symmetry by hand, and allows for the introduction of the QCD scale. The soft-wall model improves upon the hard wall, both physically and mathematically, by using a dilaton field to suppress the infrared contributions in a softer and continuous fashion [7]. This model is a mathematical improvement on the hard wall, because the geometry is everywhere continuous and thus avoids discontinuities and/or singularities which may exist in a hard-wall setup. The soft-wall model is a physical improvement because the dilaton field is parametrized as a function of the fifth-dimensional coordinate in order to provide linear radial trajectories for the meson masses. There have been numerous improvements and variations on these models, some of which attempt to incorporate confinement [27] and chiral symmetry breaking [28]. In the latter case, a scalar field which is dual to the condensation of the quark bilinear operator in QCD is included. As such, the behavior of this scalar field in the extra-dimensional geometry corresponds to the properties of chiral symmetry in the gauge theory. In an AdS/QCD model, this scalar field can be parametrized to incorporate both spontaneous and explicit breaking of chiral symmetry.

As mentioned above, in bottom up models the dilaton and scalar fields are parametrized to reproduce certain important features of QCD, namely, confinement and spontaneous and explicit chiral symmetry breaking. In almost all cases, these background fields are imposed by hand, and are not derived as the solution to any equations of motion. There are (at least) two reasons that a well-defined action is desirable within AdS/QCD models. First, one needs a proper set of background equations in order to add perturbations to the geometry which can give access to transport coefficients in the dual field theory. Second, a well-defined action may provide insight as to how such a model could arise from a top down approach. In order to meet these goals, one must know the potential which gives rise to such backgrounds. Some recent work on this subject was done in [29], where the authors explore the correspondence between the potential and the running coupling in the dual field theory.

A notable case where the potential was determined is the dynamical soft-wall model of [30]. There, an auxiliary scalar field is introduced in addition to the dilaton, and a scalar potential is derived which has the soft-wall model as a solution to the equations of motion. Throughout the aforementioned work, some attempt is made to identify this auxiliary scalar field with a closed string tachyon field in string theory. Instead of this identification, one can promote the scalar field to a matrix valued field and identify it as dual to the quark bilinear operator $\bar{q}q$ which is responsible for chiral symmetry breaking. However, [30] and the majority of other AdS/QCD models, such as those in [7,12,31], have the (light) quark condensate proportional to the (light) quark mass, $\Sigma \propto m_q$. This is in contrast to QCD, where the condensate has a nonvanishing limiting value as $m_q \to 0$. An exception is the phenomenological model of [28] which parameterizes the fields in such a way as to allow m_q and Σ to be varied independently.

In this paper we attempt to bridge the gap between the dynamical model [30] and phenomenological models of chiral symmetry breaking, such as [28], by demonstrating how to compute a potential given a phenomenological parametrization of the dilaton and/or scalar fields. This methodology is an improvement upon [30] in two respects. First, it is more general in that the potential can be found for many reasonable parametrizations of the dilaton and scalar fields. Second, even in the case considered in [30], our resulting potential is both simpler and less constrained, as discussed in Sec. IV.

Our paper is organized as follows. In Sec. II we identify the basic ingredients of the AdS/QCD model and elements of the AdS/CFT dictionary which we will use. In Sec. III we give the relevant Einstein equations, and equations for the background fields which generate the desired metric. In Sec. IV we find the scalar potential when the fields have power-law profiles. In Sec. V we show how to find the potentials for models wherein the fields are allowed to have more complicated profiles. In Sec. VI we provide concrete examples of some parametrizations which lead to analytic potentials. In Sec. VII we discuss what happens if the potential is restricted to a form which has only quadratic and quartic terms in the scalar field. We conclude the paper in Sec. VIII.

II. INGREDIENTS OF THE MODEL

We are interested in dynamically generating phenomenological AdS/QCD backgrounds. Following [30], we assume that the matter which supports the metric is a set of two scalar fields: ϕ and χ. These scalar fields interact through a scalar potential $V(\phi, \chi)$; it will be our goal to determine suitable potentials which lead to phenomenologically desirable backgrounds.

We would like to have a background which has the following characteristics:

(1) The metric in the string frame should be exactly five-dimensional anti–de Sitter space.

$$ds^2_{\text{string}} = \frac{L^2}{z^2}[-dt^2 + dx_i dx^i + dz^2]. \tag{1}$$

Here L denotes the AdS curvature radius, the index i runs over the three spatial dimensions, and z denotes the extra (fifth) dimension.

(2) At asymptotically large z the dilaton should behave as $\phi(z) \sim z^2$.

(3) The mass of the dilaton should take a value such that the dual operator in the 4D field theory has a physically relevant mass dimension.

Points 1 and 2 are necessary ingredients for a soft-wall model. Such a background leads to linear radial trajectories in the resulting meson spectrum ($m_n^2 \sim n$), in agreement with the data [7]. Point 3 is related to the mass of the dilaton. The relationship between the field's mass and the dimension Δ of the dual field theory operator is given by the AdS/CFT dictionary [1–4].

$$m^2L^2 = \Delta(\Delta - 4). \tag{2}$$

We will show that we are able to keep this parameter arbitrary in many of our solutions, but of course we want the corresponding gauge theory operator dimension to be physically relevant, and the mass of the dilaton should not violate the Breitenlohner-Freedman stability bound [32] which states that $m_\phi^2 L^2 \geq -4$.

The above points are the only necessary requirements for a soft-wall type of model. If, in addition to this, we would like to associate the scalar field χ with the operator responsible for chiral symmetry breaking, $\bar{q}q$, we require the following:

(4) The mass of the scalar field should be $m_\chi^2 = -3/L^2$.

(5) At small z the scalar field should behave as $\chi(z) = Az + Bz^3 + \cdots$.

(6) At asymptotically large z the scalar field should behave as $\chi(z) \sim z$.

Because the dimension of the operator $\bar{q}q$ is 3, we require that $m_\chi^2 = -3/L^2$. If the scalar field χ is dual to the operator responsible for chiral symmetry breaking, then in the UV ($z \to 0$) regime it should have a profile which can be expanded as in item 5 above with the coefficients A and B proportional to the quark mass and the chiral condensate, respectively (cf. [4,6,28]). Often in this work we will consider the chiral limit of zero quark mass; in this case the coefficient A vanishes, and chiral symmetry is broken spontaneously. It was also argued in [33–35] that the scalar field χ should behave linearly at large z so as to realize the nonrestoration of chiral symmetry at large n, again in agreement with the linear radial trajectories present in the data.[1] Note that to properly be dual to this operator, the field χ should be charged under the bulk chiral symmetry, meaning it should be complex and matrix valued. We have introduced a single real scalar field for simplicity, but we will discuss promotion of this field to a complex, matrix valued one in Sec. VII.

Before proceeding, let us make some comments on the relation of our approach to that of standard AdS/CFT. In

[1] It is important to note there is not a consensus in the literature regarding chiral symmetry restoration in highly excited hadrons. For an opposing viewpoint, see [36]. The requirement of chiral symmetry restoration at large n would require different field profiles at large z. We will not address such a model within the context of this work.

the usual AdS/CFT approach, a conformal field theory (e.g. $\mathcal{N} = 4$ supersymmetric Yang-Mills theory) is deformed by introducing operators into the field theory Lagrangian. From the gravity point of view, one takes an existing bulk geometry (e.g. AdS_5) and introduces fields *into* this background. Near the UV boundary, these fields act as free, noninteracting fields on AdS_5. As one moves into the interior of the AdS space, the geometry will no longer be AdS_5 due to the backreaction induced by the new bulk fields.

In this work, we compute all backreaction by introducing the fields into the Lagrangian and then solving the equations of motion for all values of the radial coordinate, including the interior of the AdS space. As mentioned in point 1 above, we have *chosen* the metric to be exactly AdS_5, and thus the potentials which we detail in this work lead to exactly AdS_5 metrics after all backreaction of the fields is taken into account.

Our choice of metric is made simply to reduce the amount of freedom which exists in the equations of motion; the methods which we detail here could easily be extended to metrics which are warped versions of AdS_5. Of course, one could introduce deformations into the backgrounds which we detail below in the usual AdS/CFT way; such deformations would, in general, introduce backreaction which would cause the metric to be no longer exactly AdS_5 for all values of the radial coordinate.

Now that we have detailed the set of requirements for our model and clarified our approach, let us now proceed to derive the relevant background equations and attempt to find potentials and solutions which have the characteristics we have outlined above. We will see that it is quite difficult to create a simple phenomenological model which satisfies all of the above requirements.

III. BACKGROUND EQUATIONS

In the Einstein frame, the action can be written in its canonical form

$$\mathcal{S}_E = \frac{1}{16\pi G_5}\int d^5x\sqrt{-g_E}\Big(R_E - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - V(\phi,\chi)\Big). \tag{3}$$

Here R denotes the Ricci scalar, G_5 stands for the five-dimensional gravitational constant, and the subscript E denotes the Einstein frame. Note that our conventions are such that the fields ϕ and χ are dimensionless while $V(\phi, \chi)$ has dimensions of energy squared. The energy momentum tensor which is derived from this action is

$$8\pi G_5 T_{\mu\nu} = \tfrac{1}{2}(\partial_\mu\phi\partial_\nu\phi + \partial_\mu\chi\partial_\nu\chi - g_{\mu\nu}\mathcal{L}), \tag{4}$$

$$\mathcal{L} \equiv \tfrac{1}{2}\partial_\lambda\phi\partial^\lambda\phi + \tfrac{1}{2}\partial_\lambda\chi\partial^\lambda\chi + V(\phi,\chi). \tag{5}$$

We assume that in the string frame there is a nontrivial coupling of the dilaton to the Ricci scalar. As such, the string and Einstein frame metrics are related by the conformal transformation

$$g^{\rm string}_{\mu\nu} = e^{2a\phi}g^E_{\mu\nu}, \tag{6}$$

where a is a constant which depends on the coupling of the dilaton to the Ricci scalar in the string frame action. Because the metric is static and depends only on the extra-dimensional coordinate z, we make the usual assumption that the fields themselves are only functions of this coordinate.

There are four nontrivial background equations. We will work with the following combinations, where $G_{\mu\nu}$ denotes the Einstein tensor:

$$\begin{aligned} g^{tt}G_{tt} - g^{zz}G_{zz} &= 8\pi G_5(g^{tt}T_{tt} - g^{zz}T_{zz}) \\ &= \tfrac{1}{2}g^{zz}([\phi'(z)]^2 + [\chi'(z)]^2), \end{aligned} \tag{7}$$

$$g^{tt}G_{tt} + g^{zz}G_{zz} = 8\pi G_5(g^{tt}T_{tt} + g^{zz}T_{zz}) = -V(\phi,\chi), \tag{8}$$

$$\Box\phi = \frac{\partial V}{\partial\phi}, \tag{9}$$

$$\Box\chi = \frac{\partial V}{\partial\chi}. \tag{10}$$

As usual, $\Box \equiv \nabla_\mu\nabla^\mu$ and ∇_μ denotes the covariant derivative with respect to the background metric. Using the presumed form of the metric expressed by Eqs. (1) and (6), these become

$$6a\phi''(z) + [\phi'(z)]^2(6a^2 - 1) - [\chi'(z)]^2 + \frac{12a\phi'(z)}{z} = 0, \tag{11}$$

$$\begin{aligned} &3e^{2a\phi(z)}\frac{z^2}{L^2}\Big[a\phi''(z) - 3a^2[\phi'(z)]^2 - \frac{6}{z}a\phi'(z) - \frac{4}{z^2}\Big] \\ &\quad = V(\phi(z),\chi(z)), \end{aligned} \tag{12}$$

$$\begin{aligned} &e^{2a\phi(z)}\frac{z^2}{L^2}\Big[\phi''(z) - 3a[\phi'(z)]^2 - \frac{3\phi'(z)}{z}\Big] \\ &\quad = \frac{\partial V}{\partial\phi}\Big|_{\phi=\phi(z),\chi=\chi(z)}, \end{aligned} \tag{13}$$

$$\begin{aligned} &e^{2a\phi(z)}\frac{z^2}{L^2}\Big[\chi''(z) - 3\chi'(z)\Big(a\phi'(z) + \frac{1}{z}\Big)\Big] \\ &\quad = \frac{\partial V}{\partial\chi}\Big|_{\phi=\phi(z),\chi=\chi(z)}. \end{aligned} \tag{14}$$

These equations are not all independent. Because the potential depends on z only through the fields, we have

$$\frac{d}{dz}V(\phi(z),\chi(z)) = \frac{\partial V}{\partial \phi}\phi'(z) + \frac{\partial V}{\partial \chi}\chi'(z). \quad (15)$$

This relation allows one to eliminate either (14) or (13).

So far we have kept the constant a arbitrary; its value can be fixed by examining (11). Consider this equation in the limit of large z, where the dilaton is required to be quadratic in z. Assuming

$$\phi(z) = \lambda z^2, \quad (16)$$

with λ a constant, Eq. (11) becomes

$$[\chi'(z)]^2 = 36a\lambda + 4\lambda^2 z^2(6a^2 - 1). \quad (17)$$

In the limit of large z, the constant term can be dropped. The solution to the resulting differential equation gives $\chi(z) \sim z^2$ at large z. This is in contradiction with the desired behavior detailed in Sec. II. The only way to avoid this problem is by choosing

$$a = \pm 1/\sqrt{6} \quad (18)$$

so that the z^2 term drops out in the above equation. With this choice, one is able to have the desired quadratic dilaton and linear scalar field at large z. We must choose the positive sign so as to keep χ real. Thus, we will fix $a = 1/\sqrt{6}$ for the remainder of this work. It is quite interesting that this value of a appears frequently in the literature and can arise quite naturally from noncritical string theory [10,11,30,37,38]. With this value of the a, the string frame action is

$$\mathcal{S}_{\text{string}} = \frac{1}{16\pi G_5}\int d^5x\sqrt{-g_s}e^{-2\Phi}\Big(R_s + 4\partial_\mu\Phi\partial^\mu\Phi - \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - e^{-4\Phi/3}V(\phi,\chi)\Big). \quad (19)$$

Here we are using the subscript s to distinguish the string frame from the Einstein frame. The field Φ is a scaled version of ϕ,

$$\Phi = \sqrt{\tfrac{3}{8}}\phi. \quad (20)$$

The authors of [30] considered this action based on string theory considerations. In fact, the normalization of the kinetic term for the dilaton above is exactly the same as that which often appears in a low energy effective action for string theory (cf. Sec. 3.7 of [39]). We have shown that this is the *only* possible dilaton-scalar action which is consistent with our desired behavior outlined in Sec. II. In other words, if the kinetic term in (19) were normalized differently, one could not satisfy all of the requirements in Sec. II.

IV. POWER-LAW SOLUTIONS

Let us first try to construct potentials which have solutions where χ is purely a power law in z. As such, we make the ansatz

$$\chi(z) = \chi_0 z^n. \quad (21)$$

Of course, only certain powers of z appear in the ingredients of our model outlined in Sec. II, but it is just as easy to work with a general n at this stage. Equation (11) gives the solution for ϕ,

$$\phi(z) = \frac{n\sqrt{6}}{12(1+2n)}\chi_0^2 z^{2n}. \quad (22)$$

Here we have assumed a Dirichlet boundary condition on ϕ so that $\phi(0) = 0$. With this choice, the Einstein frame metric is asymptotically anti–de Sitter.

Inserting the solutions for the fields into (12) and (13) yields the system

$$V(\phi(z),\chi(z)) = \frac{e^{2\phi/\sqrt{6}}}{L^2}\Big[-12 + \frac{(\chi_0 n z^n)^2}{2(1+2n)}(2n-7) - \frac{(\chi_0 n z^n)^4}{4(1+2n)^2}\Big], \quad (23)$$

$$\frac{\partial V}{\partial \phi}\Big|_{\phi=\phi(z),\chi=\chi(z)} = \frac{2e^{2\phi/\sqrt{6}}}{\sqrt{6}}\Big[\frac{(\chi_0 n z^n)^2}{2(1+2n)}(2n-4) - \frac{(\chi_0 n z^n)^4}{4(1+2n)^2}\Big]. \quad (24)$$

As mentioned previously, we need not consider Eq. (14) since it is not independent of the two listed above.

The challenge now is to determine the potential $V(\phi,\chi)$. The authors of [30] assume that the potential can be derived from a superpotential and use this fact to help determine V. We will take a different route. By examining the structure of the above equations we notice two facts. First, both equations contain the same exponential factor. This leads us to believe that the potential can be written as

$$V(\phi,\chi) = e^{2\phi/\sqrt{6}}\tilde{V}(\phi,\chi). \quad (25)$$

This form of the potential is also natural on the grounds that the exponential factor arises due to the transformation between the string and Einstein frames. ($\tilde{V}$ is the string frame potential.) Second, we notice that the only powers of z which appear in the above system of equations are z^{2n} and z^{4n}. Motivated by this, and the fact that we know the power-law behavior of both ϕ and χ, we make the ansatz

$$V(\phi,\chi) = \frac{e^{2\phi/\sqrt{6}}}{L^2}[c_0 + c_1\phi + c_2\chi^2 + c_3\phi^2 + c_4\chi^4 + c_5\phi\chi^2]. \quad (26)$$

The terms proportional to ϕ and χ^2 give rise to the terms containing z^{2n}, while the terms proportional to ϕ^2, χ^4, and $\phi\chi^2$ give rise to the z^{4n} terms. One should then insert this ansatz into the background equations (23) and (24), match the coefficients of z on each side of the equation, and try to solve for the coefficients $c_0 \ldots c_5$.

The resulting system of equations *does* have a solution. Even more remarkably, the solution only determines five of the six coefficients. For example, $c_1 \ldots c_6$ can be written in terms of c_3 as follows:

$$c_0 = -12, \tag{27}$$

$$c_1 = 4\sqrt{6}, \tag{28}$$

$$c_2 = \frac{n(n-4)}{2}, \tag{29}$$

$$c_4 = \frac{n^2(c_3 - 6n(1+n))}{24(1+2n)^2}, \tag{30}$$

$$c_5 = \frac{n(3n - c_3)}{\sqrt{6}(1+2n)}. \tag{31}$$

At first sight, one may be concerned about the term linear in ϕ in (26). But when one expands the Einstein frame potential, one finds

$$V(\phi, \chi) \approx \frac{-12}{L^2} + \frac{4+c_3}{L^2}\phi^2 + \frac{n(n-4)}{2L^2}\chi^2 - \left[\frac{n(4+c_3+4n-2n^2)}{\sqrt{6}(1+2n)L^2}\right]\phi\chi^2 \ldots \tag{32}$$

In fact, the value of c_1 which solves the background equations is also the value which ensures the expansion of the potential has no linear terms. This form of the potential is exactly what we would expect from the AdS/CFT correspondence. The first term is the usual cosmological constant term, which is followed by the mass terms of the two scalar fields. Evidently, we should identify

$$c_3 = \frac{(m_\phi L)^2 - 8}{2}. \tag{33}$$

Some comments are in order regarding this arbitrary mass. Astute readers will notice that an arbitrary mass term m_ϕ and a field ϕ, which behaves as z^{2n} near the AdS boundary, do not agree with the standard AdS/CFT correspondence where the fields near the boundary behave as

$$\phi(z \to 0) \sim Az^{\Delta}(1 + \ldots) + Bz^{4-\Delta}(1 + \ldots). \tag{34}$$

This discrepancy is due to two facts. First, we have two scalar fields with particular scaling $\phi \sim \chi^2$ near the boundary, and second there is an interaction term proportional to $\phi\chi^2$ in (32). Consider the background equation (13) for the field ϕ near $z = 0$ where the fields are small. The right-hand side depends on $\partial V/\partial\phi$, and using (32), we see that the leading term is $m_\phi \phi$ as expected. However, the final term in (32) contributes a term proportional to χ^2. For the solutions we consider, ϕ is proportional to χ^2 near the boundary, and thus this term cannot be neglected with respect to the term $m_\phi \phi$ in (13). This is significant because it means that the equation of motion (13) does *not* reduce to that of a free scalar field on AdS_5 near the boundary. The original papers on AdS/CFT (cf. [4]), assumed the fields in question were free (i.e. noninteracting) near the boundary, and this is the reason for the discrepancy of our results with the usual AdS/CFT ones. *If* one requires agreement with the usual AdS/CFT prescription, one can easily do so by requiring that the interaction term in (32) vanish. This leads to the requirement that

$$4 + c_3 = 2n^2 - 4n, \tag{35}$$

and hence

$$(m_\phi L)^2 = 2n(2n-4), \tag{36}$$

which is in agreement with standard AdS/CFT. For a quadratic dilaton ($n = 1$), we have $(m_\phi L)^2 = -4$, the same value found in [30]. Unfortunately, this requires an operator of dimension $\Delta_\phi = 2$ which does not correspond to any local gauge invariant operator in QCD. For the remainder of this work, we will choose to continue to keep m_ϕ arbitrary as much as possible. Because the standard AdS/CFT dictionary does not apply except for particular values of m_ϕ, field theory interpretations of the operator dual to ϕ will require a careful analysis which is beyond the scope of this work. This issue does not arise for the field χ. Regardless of the value of m_ϕ,

$$m_\chi^2 L^2 = n(n-4). \tag{37}$$

Furthermore, if we require that $m_\chi^2 L^2 = -3$ as outlined in Sec. II, the only acceptable values of n are $n = 1$ and $n = 3$. (This fact was also noted in the recent work [40].) These are exactly the two different powers of n which appear in the low and high z regions in a desired phenomenological model. Putting everything together, we can write the potential as

$$V(\phi, \chi) = \frac{e^{2\phi/\sqrt{6}}}{L^2}\left\{-12 + 4\sqrt{6}\phi + \frac{(m_\phi L)^2 - 8}{2}\phi^2 + \frac{n(n-4)}{2}\chi^2 + [8 - (m_\phi L)^2 + 6n]k\phi\chi^2 + \frac{1}{2}[(m_\phi L)^2 - 8 - 12n(1+n)]k^2\chi^4\right\}. \tag{38}$$

Here we have defined

$$k \equiv \frac{n\sqrt{6}}{12(1+2n)}. \tag{39}$$

This potential admits a solution where the string frame metric is given by (1), and the fields ϕ and χ have power-law profiles (21) and (22). For the case of $n = 1$, we have the dynamical soft-wall model with a quadratic dilaton and linear scalar field. The potential listed above is an alternative to that given in [30]. We believe that this potential is an improvement over the latter because it is simpler and because the mass of the dilaton can be chosen at will.

Of course, the potential listed above is not unique. One could add terms provided they vanish upon application of the equations of motion. If a term $\Delta V(\phi, \chi)$ is added, the above solution will still be a solution of the new potential provided that

$$\Delta V(\phi(z), \chi(z)) = 0, \tag{40}$$

$$\frac{\partial(\Delta V)}{\partial \phi}\bigg|_{\phi=\phi(z),\chi=\chi(z)} = 0, \tag{41}$$

$$\frac{\partial(\Delta V)}{\partial \chi}\bigg|_{\phi=\phi(z),\chi=\chi(z)} = 0, \tag{42}$$

where $\phi(z)$ and $\chi(z)$ are given in (21) and (22). As an example, a term proportional to

$$(\phi - k\chi^2)^l$$

will not change the equations of motion provided $l \geq 2$.

V. SOLUTIONS FOR GENERAL PARAMETRIZATIONS

We are now in a position to find potentials for more complicated field profiles, those which are not exactly power laws. As mentioned previously, at small z we would like either $\chi \sim z$ or $\chi \sim z^3$ depending on whether the quark mass is zero or not. At large z we require $\chi \sim z$; this is necessary for a quadratic dilaton (and hence linear radial trajectories) by (11).

By examining Eq. (38) for $n = 1$ and $n = 3$, one notices that the only differences between these two potentials are the coefficients of the χ^4 term and the $\phi\chi^2$ term. Motivated by this, let us make an ansatz that the potential can be written

$$V(\phi, \chi) = \frac{e^{2\phi/\sqrt{6}}}{L^2}\bigg[-12 + 4\sqrt{6}\phi + \bigg(\frac{(m_\phi L)^2 - 8}{2}\bigg)\phi^2 - \frac{3}{2}\chi^2 + f_1(\chi)\chi^4 + f_2(\chi)\phi\chi^2\bigg]. \tag{43}$$

The functions f_1 and f_2 could, in general, also depend on ϕ, but we have chosen them to only be functions of χ for simplicity. Let us also assume that χ is a monotonically increasing function of z such that when z is large, χ is large, and when z is small, χ is small. Then, to have the desired scalar field profile, we simply require that

$$f_1(\chi \to \infty) = \frac{1}{432}(L^2 m_\phi^2 - 32), \tag{44}$$

$$f_2(\chi \to \infty) = \frac{\sqrt{6}}{36}(14 - L^2 m_\phi^2), \tag{45}$$

and

$$f_1(\chi \to 0) = \frac{1}{432}(L^2 m_\phi^2 - 32), \tag{46}$$

$$f_2(\chi \to 0) = \frac{\sqrt{6}}{36}(14 - L^2 m_\phi^2), \tag{47}$$

if the quark mass is nonzero. If the quark mass is zero, we desire

$$f_1(\chi \to 0) = \frac{3}{784}(L^2 m_\phi^2 - 152), \tag{48}$$

$$f_2(\chi \to 0) = \frac{\sqrt{6}}{28}(26 - L^2 m_\phi^2), \tag{49}$$

with f_1 and f_2 smooth functions of χ. In other words, when the fields are large, the potential should take the form of (38) with $n = 1$, and when the fields are small, the potential should take the form of (38) with either $n = 3$ or $n = 1$. One cannot choose any convenient functions f_1 and f_2; they must be consistent with the equations of motion. To see this, for the moment let us assume that the function $\chi(z)$ is known and exhibits the desired low and high z behavior. Let us now determine the functions $f_1(\chi)$ and $f_2(\chi)$.

One must go back to the original background equations (11)–(14) using this potential ansatz. First, one can solve (11) for $\phi(z)$ in terms of $\chi(z)$. The solution is

$$\phi(z) = \phi_0 + \frac{\phi_1}{z} + \frac{1}{\sqrt{6}}\int_0^z \frac{dy}{y^2}\int_0^y x^2[\chi'(x)]^2 dx, \tag{50}$$

where ϕ_0 and ϕ_1 are integration constants. We would like $\phi(0) = 0$ so that the Einstein frame metric is asymptotically anti–de Sitter. In order to satisfy this boundary condition, both integration constants must vanish. For convenience, we now perform an integration by parts so that

$$\phi(z) = \frac{1}{\sqrt{6}}\bigg(\int_0^z x[\chi'(x)]^2 dx - \frac{1}{z}\int_0^z x^2[\chi'(x)]^2 dx\bigg). \tag{51}$$

Next, one can take a linear combination of (12) and (13) such that both the ϕ'' and ϕ'^2 terms are eliminated. This equation is

$$\phi'(z)z + \alpha\phi(z) - \frac{\chi^2(z)}{3}f_2(\chi(z)) = 0, \tag{52}$$

where

$$\alpha \equiv \tfrac{1}{3}[8 - (m_\phi L)^2]. \tag{53}$$

Substituting in the solution for $\phi(z)$ here, one can solve for f_2:

$$f_2(\chi(z))\chi^2(z) = \frac{3}{\sqrt{6}}\bigg[\frac{(1-\alpha)}{z}\int_0^z x^2[\chi'(x)]^2 dx + \alpha\int_0^z x[\chi'(x)]^2 dx\bigg]. \tag{54}$$

Finally, one has to go back to either (12) or (13) and solve for $f_1(\chi)$. First, let us define some convenient notation,

$$I(z) \equiv \int_0^z x[\chi'(x)]^2 dx. \tag{55}$$

Then one finds the following solution for $f_1(\chi)$.

$$f_1(\chi(z))\chi^4(z) = \frac{z^2}{2}[\chi'(z)]^2 + \frac{3}{2}\chi^2(z) + \frac{1}{6(1-\alpha)} f_2^2(\chi(z))\chi^4(z) - \frac{I(z)}{2}\left[(m_\phi L)^2 + 3\alpha + \frac{2}{\sqrt{6}(1-\alpha)} f_2(\chi(z))\chi^2(z) - \frac{\alpha}{2(1-\alpha)} I(z)\right]. \tag{56}$$

Now we see how the functions f_1 and f_2 are correlated with the solution $\chi(z)$.

Of course we are not finished, because the potential should be a function of the fields ϕ, χ only and should not depend explicitly on the coordinates. The expressions for f_1 and f_2 above need to be rephrased so that they only depend on the field χ. To achieve this, instead of parametrizing $\chi(z)$, we should specify the inverse relationship $z(\chi)$. As before, we assume that z is a monotonically increasing function of χ. For the correct asymptotic behavior, we require

$$z(\chi \to \infty) \sim \chi \tag{57}$$

and

$$z(\chi \to 0) \sim \chi, \tag{58}$$

if the quark mass is nonzero, and

$$z(\chi \to 0) \sim \chi^{1/3}, \tag{59}$$

if the quark mass is zero. It is now a simple matter to transform the potential using the relation

$$\chi'(z) = \frac{1}{z'(\chi)}. \tag{60}$$

For example,

$$I(\chi) = \int_{\chi(0)}^{\chi(z)} z(\chi)\frac{1}{[z'(\chi)]^2}\frac{dz}{d\chi}d\chi = \int_0^\chi \frac{z(\chi)}{z'(\chi)}d\chi. \tag{61}$$

This allows us to compute the potential as a function of the fields only. For convenience, define

$$\xi_1(\chi) \equiv \frac{\alpha}{1-\alpha}\int_0^\chi \frac{z(\chi')}{z'(\chi')}d\chi', \tag{62}$$

$$\xi_2(\chi) \equiv \frac{1}{z(\chi)}\int_0^\chi \frac{z(\chi')^2}{z'(\chi')}d\chi'. \tag{63}$$

Then the solutions for ϕ and the potential are

$$\phi(\chi) = \frac{1}{\sqrt{6}}\left[\frac{1-\alpha}{\alpha}\xi_1(\chi) - \xi_2(\chi)\right], \tag{64}$$

$$\chi^2 f_2(\chi) = \frac{3}{\sqrt{6}}(1-\alpha)[\xi_1(\chi) + \xi_2(\chi)], \tag{65}$$

$$\chi^4 f_1(\chi) = \frac{1}{2}\left(\frac{z(\chi)}{z'(\chi)}\right)^2 + \frac{3}{2}\chi^2 + \frac{1-\alpha}{4\alpha}\{(\alpha-1) \times [\xi_1(\chi) + \xi_2(\chi)]^2 + \xi_2^2(\chi) - 16\xi_1(\chi)\}. \tag{66}$$

In all,

$$V(\phi,\chi) = \frac{e^{2\phi/\sqrt{6}}}{L^2}\left\{-12 + \frac{\sqrt{6}}{2}\phi[8 + (1-\alpha) \times (\xi_1(\chi) + \xi_2(\chi))] - \frac{3\alpha}{2}\phi^2 + \frac{1}{2}\left(\frac{z(\chi)}{z'(\chi)}\right)^2 + \frac{\alpha-1}{4\alpha}[(1-\alpha)[\xi_1(\chi)+\xi_2(\chi)]^2 - \xi_2^2(\chi) + 16\xi_1(\chi)]\right\}. \tag{67}$$

This set of equations is one of the central results of this work. Given a phenomenological parametrization $z(\chi)$, one needs to do two integrals to determine ξ_1 and ξ_2, after which point the potential which gives rise to the desired solution can be determined. In addition, one can see how the solution for ϕ must be correlated with such a parametrization from (64).

The simplicity of the potential is dependent on the simplicity of the ξ functions, which are in turn related to the parametrization $z(\chi)$. We are free to choose $z(\chi)$ at will. Unfortunately, there is no guarantee that the potential has an analytic form, as the integrals (62) and (63) cannot always be done in closed form. In the following section, we will give a few examples of parametrizations $z(\chi)$ which lead to analytic potentials.

VI. EXAMPLES

A simple parametrization $z(\chi)$ which has the correct behavior when the quark mass is zero is

$$z(\chi) = \gamma\left[\chi + \left(\frac{\chi}{\beta^2}\right)^{1/3}\right], \tag{68}$$

where β is a positive dimensionless constant and γ is a positive constant with dimension of length. Such a parametrization allows for an analytic potential. The relevant functions which appear in the potential are

$$\xi_1(\chi) = \frac{\alpha}{18\beta^2(\alpha - 1)}[6y^{2/3} - 9y^{4/3} - 9y^2 - 2\ln(1 + 3y^{2/3})], \tag{69}$$

$$\xi_2(\chi) = \frac{4}{27\beta^2(y^{1/3} + y)}\left[y^{1/3} - y + \frac{27}{15}y^{5/3} + \frac{135}{28}y^{7/3} + \frac{27}{12}y^3 - \frac{1}{\sqrt{3}}\arctan(\sqrt{3}y^{1/3})\right], \tag{70}$$

where $y \equiv \beta\chi$. In fact, the above parametrization can be generalized to

$$z(\chi) = \gamma\left(\frac{\chi}{\beta^2}\right)^{1/3}[1 + (\beta\chi)^{2/3n}]^n. \tag{71}$$

Both of the integrals involved in the computation of ξ_1 and ξ_2 can be done analytically without the use of special functions if n is an integer (though n must be positive in order for the field χ to have the correct asymptotic behavior). The complexity of ξ_1 and ξ_2 appears to increase with increasing n, so we have quoted the simplest example above with $n = 1$.

Another parametrization for zero quark mass which leads to the desired behavior is

$$z(\chi) = \frac{\gamma\chi^{1/3}}{\beta^{2/3}\{1 + (\beta\chi)^{1/3}[\arctan((\beta\chi)^{1/3}) - \pi/2]\}}. \tag{72}$$

This parametrization was found by examining the behavior of the integral appearing in $\xi_2(\chi)$. The integrand must behave as $\chi^{4/3}$ for small χ, and as χ^2 for large χ. Solving the differential equation

$$\frac{z^2(\chi)}{z'(\chi)} = 3\frac{\gamma}{\beta^2}[(\beta\chi)^{4/3} + (\beta\chi)^2] \tag{73}$$

leads to the above parametrization. With this parametrization, the solutions for ξ_1 and ξ_2 are

$$\xi_1(\chi) = \frac{\alpha}{7\beta^2(1 - \alpha)}\left[-y^{2/3} + \frac{1}{2}y^{4/3} + \frac{61}{6}y^2 + 7y^{8/3} + (7y^3 + 9y^{7/3})\left(\arctan(y^{1/3}) - \frac{\pi}{2}\right) + \ln(1 + y^{2/3})\right], \tag{74}$$

$$\xi_2(\chi) = \frac{9y^2 + 7y^{8/3}}{7\beta^2}\left[1 + y^{1/3}\left(\arctan(y^{1/3}) - \frac{\pi}{2}\right)\right], \tag{75}$$

where again $y = \beta\chi$. This can be generalized to other potentials by solving the differential equation

$$\frac{z^2(\chi)}{z'(\chi)} = \frac{\gamma}{\beta^3}\frac{d}{d\chi}\left[(\beta\chi)^{7/3}\left(\frac{9}{7} + (\beta\chi)^{2/3n}\right)^n\right], \tag{76}$$

and requiring z to have the correct asymptotic behavior. This method again leads to an analytic potential for n being an integer, though the simplest result is that given above with $n = 1$.

It is tempting to try to use functions such as the exponential and hyperbolic tangent to parametrize $z(\chi)$. However, we have not found any applicable parametrization using these functions where *both* of the relevant integrals appearing in ξ_1 and ξ_2 have an analytic solution.

VII. POTENTIALS QUADRATIC AND QUARTIC IN THE SCALAR FIELD

As mentioned in the Introduction, in order for the field χ to be dual to the operator $\bar{q}q$ it should be a complex, matrix valued field. So far, we have only considered χ to be a real scalar field. In order to address this difficulty, one can simply promote the field with the replacement

$$\tfrac{1}{2}\chi^2 \to \chi^\dagger\chi, \tag{77}$$

where the extra factor of $1/2$ is introduced to give a canonical action for a complex scalar field. However, one will notice that all of the potentials which were constructed in the previous section contain fractional powers of χ, and hence such a replacement is less than desirable. If the action is a function of χ^2 only, then such a promotion is possible. Below, we will discuss two methods which allow one to construct potentials which depend only on χ^2.

The first method we use to determine a potential which is a function of χ^2 only is to modify the ansatz (43) so that the functions f_1 and f_2 are functions of ϕ only. More generally, let us assume

$$V(\phi, \chi) = \frac{e^{2\phi/\sqrt{6}}}{L^2}[F_0(\phi) + F_2(\phi)\chi^2 + F_4(\phi)\chi^4], \tag{78}$$

with the F functions to be determined. This strategy involves parametrizing $z(\phi)$ instead of $z(\chi)$, but the general steps involved are similar to the cases already discussed. The challenge is to find a parametrization with the desired behavior that leads to an analytic expression for $\chi(\phi)$. The latter can be determined from (11) to be

$$\chi(\phi) = \pm 6^{1/4}\int_0^\phi\left[\frac{d}{d\phi}\ln\left(\frac{z(\phi)^2}{z'(\phi)}\right)\right]^{1/2}d\phi. \tag{79}$$

In practice, it often is easiest to start somewhere in the middle by parametrizing some intermediate quantity such as $\chi'(\phi)$. It turns out that a parametrization of the form

$$[\chi'(\phi)]^2 = \frac{\sqrt{6}}{\phi}\left(\frac{\frac{7}{6} + \frac{3}{2}\beta\phi^n}{1 + \beta\phi^n}\right) \tag{80}$$

has the desired asymptotic behavior and leads to analytic solutions for both $\chi(\phi)$ and $z(\phi)$. In general, these solutions involve hypergeometric functions. A notable special case is if $n = 1/2$. Then the solutions are

$$z(\phi) = \frac{\gamma\phi^{1/6}}{6[(\beta\sqrt{\phi})^{1/3} - (1+\beta\sqrt{\phi})^{1/3}]}, \tag{81}$$

$$\chi(\phi) = \frac{2^{3/4}}{3^{5/4}\beta}\bigg(G(\phi) - G(0) + \ln\bigg[\frac{8+3\sqrt{7}}{8+9\beta\sqrt{\phi}+G(\phi)}\bigg]\bigg), \tag{82}$$

with

$$G(\phi) = 3\sqrt{(1+\beta\sqrt{\phi})(7+9\beta\sqrt{\phi})}. \tag{83}$$

For this parametrization define

$$R(\phi) = \frac{\sqrt{6}}{2}\frac{z(\phi)}{z'(\phi)}, \tag{84}$$

and make use of the calculus relations

$$\phi''(z) = -z''(\phi)/[z'(\phi)]^3, \tag{85}$$

$$\chi'(z) = \chi'(\phi)/z'(\phi), \tag{86}$$

$$\chi''(z) = \frac{\chi''(\phi)}{[z'(\phi)]^2} - \frac{\chi'(\phi)z''(\phi)}{[z'(\phi)]^3}. \tag{87}$$

Then the background equations (12) and (14) can be written

$$F_0(\phi) + F_2(\phi)\chi^2(\phi) + F_4(\phi)\chi^4(\phi) = -12 - 8R(\phi) - R^2(\phi) + \tfrac{1}{3}R^2(\phi)[\chi'(\phi)]^2, \tag{88}$$

$$2F_2(\phi)\chi(\phi) + 4F_4\chi^3(\phi) = \frac{2}{3}\chi''(\phi)R^2(\phi) - \frac{\sqrt{6}R(\phi)\chi'(\phi)}{3}\bigg\{5 + R(\phi) - \frac{1}{3}R(\phi)[\chi'(\phi)]^2\bigg\}. \tag{89}$$

These equations can be solved to determine the F functions, and hence the potential in terms of the known functions $\chi(\phi)$ and $R(\phi)$. Notice that there are only two equations for the three F functions; we have some freedom to choose one of the F functions at will. One possible choice is $F_0 = -12 + 4\sqrt{6}\phi - \frac{3}{2}\alpha\phi^2$, which is the same as the form in the ansatz (43). The resulting solutions for F_2 and F_4 are quite complicated. In addition to this fact, the leading term in the small field expansion of F_4 is $\phi^{-5/6}$, which is in contradiction with a well-defined conformal limit: $V(\phi \to 0, \chi \to 0) = -12/L^2$.

On the basis of algebraic simplicity, an alternative is to choose F_4 to be constant. It is then straightforward to solve for F_0 and F_2. For small values of ϕ, these functions have the expansions

$$F_0(\phi) = -12 + \frac{\sqrt{6}}{\beta^2}x[4 + 3x^{1/6} + \mathcal{O}(x^{1/3})], \tag{90}$$

$$F_2(\phi) = -\tfrac{3}{2} - \tfrac{9}{2}x^{1/6} + \mathcal{O}(x^{1/3}), \tag{91}$$

with $x \equiv \beta^2\phi$. This solution appears to be consistent; however, it suffers from the drawback that the potential is quite complicated, and that the small field expansion of F_0 contains fractional powers of ϕ starting with $\phi^{7/6}$. Thus, in this solution the mass of the dilaton is not well defined. This solution adheres to all of the ingredients we set out in Sec. II except for point 3.

We now detail a second way to determine a potential which is a function of χ^2 and is thus a good candidate for a dynamical model of chiral symmetry breaking. As above, we will sacrifice point 3 in the ingredients of the model by choosing a nonstandard dilaton mass.

One may have noticed that the general potential (67) simplifies greatly in the special case $\alpha = 1$. This corresponds to $m_\phi^2 L^2 = 5$. If one naively uses the AdS/CFT dictionary to compute the dimension of the corresponding operator in this case, one finds $\Delta_\phi = 5$, which is nonrenormalizable. We are unsure whether it is simply coincidence that this choice simplifies the potential greatly, or whether there is some physics hidden here. Such a choice is certainly nonstandard, though it could be acceptable within the context of effective field theories. However, it should be emphasized that in light of the discussion in Sec. IV, any use of the standard AdS/CFT dictionary in regards to the field ϕ should be approached with caution due to the fact that it does not reduce to a free field near the AdS boundary.

With the choice of $(m_\phi L)^2 = 5$, the potential (61) becomes[2]

$$V(\phi,\chi) = \frac{e^{2\phi/\sqrt{6}}}{L^2}\bigg\{-12 + 4\sqrt{6}\phi - \frac{3}{2}\phi^2 + \frac{\sqrt{6}}{2}\phi I(\chi) - \frac{1}{4}I(\chi)^2 - 4I(\chi) + \frac{1}{2}[I'(\chi)]^2\bigg\}, \tag{92}$$

where $I(\chi)$ is defined in (61). Notice that only one of the integrals appears, and all reference to the function ξ_2 has disappeared in the potential. At this point we can simply parametrize $I(\chi)$ to fit our needs. Any parametrization of I will do, provided that for small or large χ the function behaves as

$$I(\chi) \to \frac{n}{2}\chi^2, \tag{93}$$

where n is the desired power of the scalar field in this regime [i.e. $\chi(z) \sim z^n$]. For example, if one chooses

$$I(\chi) = \frac{\chi^2}{2}\bigg(\frac{3 + (\beta\chi)^2}{1 + (\beta\chi)^2}\bigg), \tag{94}$$

with β a constant, one gets the desired asymptotic behavior

[2] One should take care when making this simplification due to the presence of $(\alpha - 1)$ in the definition of the function ξ_1.

for zero quark mass. All factors of χ^2 can be promoted to $\chi^\dagger\chi$ without any problem. Another such parametrization is

$$I(\chi) = \frac{\chi^2}{2} + \frac{1}{\beta^2}\ln[1 + \beta^2\chi^2]. \tag{95}$$

The solutions for the fields $\phi(z)$ and $\chi(z)$ could then be found numerically from (61) and (64).

VIII. CONCLUSION

In this paper we addressed the problem of constructing models of AdS/QCD. Specifically, we showed how to construct a potential for dilaton and scalar fields that leads to an AdS/QCD model with many essential features of QCD. Given a suitable parametrization $z(\chi)$ or $z(\phi)$, we show how to construct a potential $V(\phi, \chi)$ which has a solution with the desired properties; as such, the main results of this paper are (64) and (67). Linear radial trajectories, conformal symmetry breaking, and both spontaneous and explicit chiral symmetry breaking can be incorporated. The desired ingredients of the model as detailed in Sec. II are numerous, and the fact that such a solution can be dynamically generated at all within such a simple setup is somewhat surprising. It is especially interesting that the mass of the dilaton can be kept arbitrary throughout much of the analysis. However, as discussed in Sec. IV, traditional field theory interpretations of dual operators via the standard AdS/CFT dictionary are questionable, except for particular values of the dilaton mass.

Explicit examples of potentials were given, at least for the case when the light quark mass is zero, although this was only for purposes of illustration and not a limitation in principle. We have also not discussed the stability of the examples we provide, as that analysis would be beyond the scope of this work. Before calculating physical observables within the context of such a solution, its stability would have to be checked. In order to incorporate chiral symmetry breaking, the scalar field we introduce into our action needs to be promoted to a matrix valued field, and this requirement complicates the analysis. However, we find that a nonstandard dilaton mass choice greatly simplifies the results.

One might have expected to find simple expressions for the potential since the Lagrangian for QCD is so simple to write down. However, there is no reason for this to be so and, unfortunately, the analytic expressions in our illustrative examples are not so simple, although the general structure is. It remains a challenge to find a simple expression for the potential that leads to the desired properties for the dilaton and scalar fields. Nevertheless, the potentials given here can be useful in the context of hadronic structure. We also believe that the methods we have outlined here may be useful for the determination of potentials in other bottom up AdS/QCD models, and at finite temperature.

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