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Dirac-Born-Infeld actions and tachyon monopoles

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We investigate magnetic monopole solutions of the non-Abelian Dirac-Born-Infeld (DBI) action describing two coincident non-BPS D9-branes in flat space. Just as in the case of kink and vortex solitonic tachyon solutions of the full DBI non-BPS actions, as previously analyzed by Sen, these monopole configurations are singular in the first instance and require regularization. We discuss a suitable non-Abelian ansatz that describes a pointlike magnetic monopole and show it solves the equations of motion to leading order in the regularization parameter. Fluctuations are studied and shown to describe a codimension three BPS D6-brane, and a formula is derived for its tension.

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I. INTRODUCTION

Tachyon condensation has been a subject of considerable investigation via the physics of non-Bogomol'nyi-Prasad-Sommerfield (BPS) D-branes (for a comprehensive review see [1]). Such tachyons arise quite naturally in the open string spectrum when one considers non-BPS D-branes in type IIA or IIB string theories. A growing body of research has developed in open string field theory (for a review see [2] or [3,4] for more recent works), boundary string field theory (BSFT) [5–8], and various effective actions around the tachyon vacuum [9–13] to demonstrate Sen's results [1] concerning the fate of the open string vacuum in the presence of tachyons.

In related developments, it was also shown that D-brane charges take values in appropriate K-theory groups of space-time. A major result is that all lower-dimensional D-branes can be considered in a unifying manner as nontrivial excitations on the appropriate configuration of higher-dimensional branes. In type IIB, it was demonstrated by Witten in [15] that all branes can be built from sufficiently many D9–anti-D9 pairs. In type IIA, Horava described how to construct BPS D$(p-2k-1)$-branes as bound states of unstable Dp-branes [16].

The mechanism of tachyon condensation into lower-dimensional BPS D-branes has been verified in some cases at the level of tachyon effective actions. In [17], Sen showed that tachyon kink solutions (that represent codimension one BPS D-branes) exist even when one considers the full nonlinear Dirac-Born-Infeld (DBI)-like action of a non-BPS D-brane in a flat background. Compared to their counterpart obtained in the truncated theories [7,18–20], these kinks are singular and require regularization. Remarkably, it was shown that in the limit where the regularization parameter is removed, the effective theory of fluctuations about the regularized tachyon kink profile, which depends only on a single spatial world-volume coordinate, are precisely those of a codimension 1 BPS D-brane and is described by a DBI action. Furthermore Sen also showed that in brane-antibrane systems, in which a single complex tachyon field is present, regularized vortex solutions to the equations of motion derived from the DBI non-BPS action exist, which naturally depend on two spatial world-volume coordinates. Analysis of the fluctuations in this case again showed that to leading order, they are those of a codimension 2 BPS D-brane as described by the appropriate full nonlinear DBI action.

In [21], we investigated the generalization of tachyon kink solutions to the case of the full nonlinear non-Abelian action of two coincident non-BPS D-branes. We showed that, in certain cases, starting with two non-BPS D9-branes, the fluctuations about the regularized non-Abelian tachyon kink profile describe a coincident pair of BPS D8-branes.

In this paper, we want to investigate codimension 3 magnetic monopole solutions, arising from the same DBI-like action of two coincident non-BPS D9-branes, which correspond to one BPS D6-brane. Monopole solutions in certain truncations of tachyon models have already been studied in [18]. In [19] the authors extended their results to include all higher derivatives using the BSFT approach and thus argued the ansatz for the tachyon monopole introduced in [18] survives higher derivative corrections. In this paper we wish to investigate magnetic monopole solutions arising from the full nonlinear non-Abelian DBI-like action, i.e., without assuming an action truncated in an expansion in derivatives of the tachyon field. From our understanding of the DBI tachyon kink and vortex solutions discussed above, we expect (and find) that such monopole solutions will again be singular in the first instance and require regularization. We find solutions that are in perfect agreement with those obtained in BSFT and so provide an independent check of the tachyon monopole ansatz first presented in [18,19].

Our starting point will be the effective description of two coincident non-BPS D9-branes proposed in [12]. This

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theory describes a non-Abelian version of the DBI action in which the tachyon field transforms in the adjoint representation of the $U(2)$ gauge symmetry of the coincident non-BPS D9-brane world-volume action. In the original construction of this action and its generalization to coincident non-BPS Dp-branes, a standard trace prescription (which we denote as Tr) was taken over the gauge indices. Another prescription, motivated by string scattering calculations (at least to low orders in α' [22,23]), is to take the symmetrized trace (which we denote by STr) over gauge indices. In both cases the expression being traced over is the same, but the STr prescription results, in general, in significantly more complicated terms in the action compared to Tr. In this paper we will adopt the STr procedure, and we will find that its implementation in the case of a tachyon monopole profile is straightforward and leads to the correct expression for the D6-brane tension.

The structure of the paper is as follows. We begin in Sec. II with a ‘t Hooft-Polyakov monopole-like ansatz for the $U(2)$ non-Abelian DBI tachyon world-volume theory and show how it leads to the correct expression for the resulting D6-brane tension, realized as a codimension 3 solution of the equations of motion, with a suitable regularization. In Sec. III a study of the fluctuation spectrum about these monopoles shows them to be precisely described by a DBI action of a single BPS D6 brane in flat space, in the limit where the regularization is removed. We end with some conclusions and speculations. Finally in the Appendix, we show how the tachyon monopole ansatz satisfies the correct Dirac quantization of magnetic charge.

II. THE ‘T HOOFT-POLYAKOV MONOPOLE AND THE DBI ACTION

We begin by reviewing an effective DBI action for the coincident non-BPS D9-brane pair [12]. This system is unstable and it contains a tachyon in its spectrum; in particular, around the maximum of the tachyon potential, the theory contains a $U(2)$ gauge field and four tachyon states represented by a 2×2 Hermitian matrix-valued scalar field transforming in the adjoint representation of the gauge group.

In this paper we are going to use the following DBI action for the two non-BPS D9-branes:

$$S_{\rm DBI} = -{\rm STr}\int d^{10}x e^{-\phi} V(T)\sqrt{-\det G_{\mu\nu}}, \tag{2.1}$$

where

$$G_{\mu\nu} = g_{\mu\nu}\mathbb{1}_2 + B_{\mu\nu}\mathbb{1}_2 + \lambda D_\mu T D_\nu T + \lambda F_{\mu\nu}, \tag{2.2}$$

where $\lambda = 2\pi\alpha'$. In Eq. (2.1), $g_{\mu\nu}$, $B_{\mu\nu}$, and ϕ are, respectively, the space-time metric, the antisymmetric Kalb-Ramond tensor, and dilaton fields, whereas $\mathbb{1}_2$ is the 2×2 unit matrix. The covariant derivative is defined to be $D_\mu T = \partial_\mu T - i[A_\mu, T]$, and the field strength takes the usual form $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]$.

For the potential, we shall assume only that a family of minima can be found by taking [up to a $SU(2)$ rotation]

$$T = \begin{pmatrix} +\infty & 0 \\ 0 & -\infty \end{pmatrix}, \tag{2.3}$$

which represent the tachyon on the first D-brane at its minimum $T_0 = +\infty$ and the tachyon on the second D-brane at its minimum $T_0 = -\infty$. We shall also assume that the potential vanishes at $T = T_0$. The monopole solution of the DBI action (2.1) corresponds in taking the tachyon and the gauge fields to depend on three world-volume coordinates x_i, with $i = 1, 2, 3$, whereas $\alpha, \beta = 0, 4, \ldots, 9$ will label the other world-volume coordinates including time.

Apart from a $U(1)$ subgroup, the effective theory of two unstable D-branes admits as a solution the ‘t Hooft-Polyakov monopole, which in the limit of zero-size core is of the form

$$T(x) = t(r)\frac{x\cdot\sigma}{r}, \qquad A_i(x) = \frac{1}{2}\epsilon_{ijk}\frac{x_j}{r^2}\sigma_k, \tag{2.4}$$

where r is the radial distance from the origin in the three transverse directions [24]. In [25] it was shown that the limit of zero-size core correctly reproduces also the Ramond-Ramond couplings of a D6-brane. It is actually more convenient to work in spherical coordinates

$$x_1 = r\cos\theta, \qquad x_2 = r\sin\theta\cos\phi, \qquad x_3 = r\sin\theta\sin\phi \tag{2.5}$$

to make use of the spherical symmetry of the solution. In these coordinates the tachyon takes the form

$$T = t(r)x_r\cdot\sigma \tag{2.6}$$

and the gauge fields

$$A_r = 0, \qquad A_\theta = -\frac{1}{2\sin\theta}x_{\phi r}\cdot\sigma, \qquad A_\phi = \frac{1}{2}\sin\theta x_{\theta r}\cdot\sigma, \tag{2.7}$$

where $x_r^i = \partial_r x^i$, $x_{\phi r}^i = \partial_r\partial_\phi x^i$, and so on. The covariant derivatives of the tachyon are

$$D_r T = t'(r)x_r\cdot\sigma, \qquad D_\theta T = D_\phi T = 0, \tag{2.8}$$

the gauge field strength

$$F_{r\theta} = F_{r\phi} = 0, \qquad F_{\theta\phi} = -\tfrac{1}{2}\sin\theta x_r\cdot\sigma. \tag{2.9}$$

Finally, the determinant becomes:

$$-\det G = (1 + \lambda D_r T D_r T)(r^4\sin^2\theta + \lambda^2 F_{\theta\phi}^2). \tag{2.10}$$

Studying solutions to the equations of motion is equivalent to finding configurations that satisfy the conservation of the energy-momentum tensor. This approach follows that used by Sen (see [1], and references therein) in his study of kink and vortex solitonic solutions to non-BPS

DBI actions. The energy-momentum tensor associated with the action (2.1) is

$$T^{\mu\nu} = -\mathrm{STr}(V(T)\sqrt{-\det G}(G^{-1})^{\mu\nu}) \tag{2.11}$$

The elements with one r component are

$$T_{rr} = -\mathrm{STr}\left[\frac{V(T)\sqrt{r^4\sin^2\theta + \lambda^2 F_{\theta\phi}^2}}{\sqrt{1+\lambda D_r T D_r T}}\right], \qquad T_{r\theta} = T_{r\phi} = 0. \tag{2.12}$$

From the previous expressions it is clear that the conservation equation for the r component reduces to $\partial_r T_{rr} = 0$. If we assume that the potential vanishes at infinity, then T_{rr} must vanish everywhere because of the conservation equation, hence T_{rr} should vanish for all r. However, for r close to the origin, the potential is finite and T_{rr} does not vanish, and so at least for small r we require $t'(r)$ to blow up. This forces us to consider a regularization of the form

$$T = \hat{t}(kr)x_r \cdot \sigma \tag{2.13}$$

such that in the $k \to \infty$ limit $t'(r)$ goes to infinity while keeping $t(r)$ fixed. In particular, in the large k limit,

$$D_r T D_r T = k^2 \hat{t}'^2 (x_r \cdot \sigma)^2, \tag{2.14}$$

and the energy-momentum tensor that goes like $T_{rr} \sim 1/k$ vanishes everywhere as required. This shows that the monopole solution is indeed a solution to the conservation equation and hence a consistent solution of the system equations of motion. Note, however, that it is only strictly a solution in the limit that $k \to \infty$ [that is, for finite k there are terms that violate conservation of energy momentum, which go like $O(1/k)$ and higher, just as in the kink and vortex cases studied previously by Sen].

Let us now calculate the tension associated with the D6-brane: the energy-momentum tensor along the directions orthogonal to the monopole is

$$T_{\alpha\beta} = -\eta_{\alpha\beta}\,\mathrm{STr}[V(T) \times \sqrt{(1+\lambda D_r T D_r T)(r^4\sin^2\theta + \lambda^2 F_{\theta\phi}^2)}], \tag{2.15}$$

which, by taking the large k limit and by performing the following coordinate transformation,

$$y = \hat{t}(kr), \qquad r \equiv \hat{r}(y) = k^{-1}\hat{t}^{-1}(y), \tag{2.16}$$

becomes, after integrating over the x_i world-volume coordinates

$$T^{\mathrm{int}}_{\alpha\beta} = -\frac{1}{2}\lambda^{3/2}\eta_{\alpha\beta} \times \mathrm{STr}\left[\int dy d(-\cos\theta)d\phi V(T(y))(x_r\cdot\sigma)^2\right]. \tag{2.17}$$

In a similar fashion to the kink and vortex calculations [17] most of the contribution to $T_{\alpha\beta}$ comes from a small region in r space centered around $\frac{1}{k}$. We can identify the tension of the D6-brane as

$$\mathcal{T}_6 = \frac{1}{2}\lambda^{3/2}\,\mathrm{STr}\int d(-\cos\theta)d\phi dy V(y)(x_r\cdot\sigma)^2. \tag{2.18}$$

The tension of the D6-brane is determined only by the tachyon potential and does not depend on the explicit form of the function $t(r)$ used in the ansatz to describe the soliton tachyon configuration. We remark, however, that the above expression is strictly true in the limit where $k \to \infty$. In this sense it follows closely similar calculations for the tension of codimension 1 and 2 D-branes described by the tachyon kink and vortex cases, respectively [17].

Now we try to evaluate the previous expression by choosing an explicit expression for the tachyon potential. One that gives a lot of quantitative agreements with string theory results is [26]

$$V(T) = \frac{\sqrt{2}\mathcal{T}_9}{\cosh(\sqrt{\pi}T)} = \sqrt{2}\mathcal{T}_9\sum_{i=0}^{\infty}\frac{E_{2i}(\sqrt{\pi}y)^{2i}(x_r\cdot\sigma)^{2i}}{(2i)!}, \tag{2.19}$$

where E_i is the ith Euler number. We see that in order to compute the tension of the D6-brane we need to evaluate

$$\mathrm{STr}[(x_r\cdot\sigma)^{2m}] = \mathrm{Tr}[(x_r\cdot\sigma)^{2m}] = 2. \tag{2.20}$$

Therefore, the tension becomes

$$\mathcal{T}_6 = \sqrt{2}\mathcal{T}_9\lambda^{3/2}4\pi\int_0^{\infty} dy\frac{1}{\cosh(\sqrt{\pi}y)} = (2\pi\sqrt{\alpha'})^3\mathcal{T}_9, \tag{2.21}$$

which correctly reproduces the D-brane tension descent relation between the $\mathcal{T}_9$ and the $\mathcal{T}_6$ tension.

III. WORLD-VOLUME ACTION ON THE MONOPOLE

This section is devoted to analyzing the world-volume fluctuations of the tachyon monopole background described in the previous section. We plan to show that the world-volume theory of the monopole condensed on a Dp-brane results in a D$(p-3)$-brane, described by an action with a $U(1)$ gauge theory. Although our analysis involves the presence of non-Abelian tachyon and gauge fields, what follows is similar to [17] because all our computations are carried out inside the STr operation, in which objects are effectively commutative. We begin by recasting the ansatz for the monopole in the following way:

$$T(\vec{x}) = f(r)x_i\sigma_i, \qquad A_i(\vec{x}) = g(r)\epsilon_{ijk}x_j\sigma_k, \tag{3.1}$$

where $g(r) = 1/(2r^2)$ and $f(r) = t(r)/r$. We make the following ansatz for the fluctuating fields:

$$\bar{T}(\vec{x},\xi) = T(\vec{x} - \vec{\phi}(\xi)) = f(\hat{r})(x_i - \phi_i(\xi))\sigma_i,$$
$$\bar{A}_i(\vec{x},\xi) = A_i(\vec{x} - \vec{\phi}(\xi)) = g(\hat{r})\epsilon_{ijk}(x_j - \phi_j(\xi))\sigma_k,$$
$$\bar{A}_\alpha(\vec{x},\xi) = -\bar{A}_i(\vec{x},\xi)\partial_\alpha\phi^i + a_\alpha(\xi)\otimes\mathbb{1}. \tag{3.2}$$

In the previous expressions, $\phi_i(\xi)$ are scalar fluctuations that depend on the world-volume coordinate of the D-brane, and we have defined

$$\hat{r}^2 = (x_i - \phi_i(\xi))(x^i - \phi^i(\xi)). \tag{3.3}$$

Using the fact that at the end we have to take the symmetrized trace, we can write $\partial_\alpha\bar{T} = -\partial_\alpha\phi^i\partial_i\bar{T}$ and $[\bar{A}_\alpha,\bar{T}] = -\partial_\alpha\phi^i[\bar{A}_i,\bar{T}]$ to obtain

$$D_\alpha\bar{T} = -D_i\bar{T}\partial_\alpha\phi^i, \tag{3.4}$$

and similarly, using the fact that $\partial_\alpha\bar{A}_j = -\partial_\alpha\phi^i\partial_i\bar{A}_j$ and defining $f_{\alpha\beta} \equiv \partial_\alpha a_\beta - \partial_\beta a_\alpha$, we have

$$F_{\alpha\beta} = \bar{F}_{ij}\partial_\alpha\phi^i\partial_\beta\phi^j + f_{\alpha\beta}\mathbb{1}, \qquad F_{\alpha j} = -\partial_\alpha\phi^i\bar{F}_{ij},$$
$$F_{i\alpha} = -\bar{F}_{ij}\partial_\alpha\phi^j, \qquad F_{ij} = \partial_i\bar{A}_j - \partial_j\bar{A}_i - i[\bar{A}_i,\bar{A}_j].$$

From these we can proceed to compute the matrix elements of our determinant. By defining

$$g_{ij} \equiv \lambda D_i\bar{T}D_j\bar{T} + \lambda\bar{F}_{ij} \tag{3.5}$$

we have

$$G_{\mu\nu} = \begin{pmatrix} G_{\alpha\beta} & G_{\alpha j} \\ G_{i\beta} & G_{ij} \end{pmatrix} = \begin{pmatrix} \eta_{\alpha\beta} + \lambda f_{\alpha\beta} + g_{ij}\partial_\alpha\phi^i\partial_\beta\phi^j & -\partial_\alpha\phi^i g_{ij} \\ -g_{ij}\partial_\beta\phi^j & \delta_{ij} + g_{ij} \end{pmatrix}.$$

Next, we introduce a new matrix $\hat{G}_{\mu\nu}$ whose elements are $\hat{G}_{\alpha\nu} \equiv G_{\alpha\nu} + \partial_\alpha\phi^i G_{i\nu}$ and $\hat{G}_{i\nu} = G_{i\nu}$, namely,

$$\hat{G}_{\mu\nu} = \begin{pmatrix} \hat{G}_{\alpha\beta} & \hat{G}_{\alpha j} \\ \hat{G}_{i\beta} & \hat{G}_{ij} \end{pmatrix} \equiv \begin{pmatrix} G_{\alpha\beta} & G_{\alpha j} \\ G_{i\beta} & G_{ij} \end{pmatrix} + \partial_\alpha\phi^i\begin{pmatrix} G_{i\beta} & G_{ij} \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} \eta_{\alpha\beta} + f_{\alpha\beta} & \partial_\alpha\phi_j \\ G_{i\beta} & G_{ij} \end{pmatrix}. \tag{3.6}$$

If we were considering matrices whose elements were commuting, then clearly $\det G_{\mu\nu} = \det\hat{G}_{\mu\nu}$ because in that case the determinant would be invariant under the addition of a multiple of a row (column) to another row (column). This property follows from the fact that if each element in a row (column) is a sum of two terms, the determinant equals the sum of the two corresponding determinants. In our case the entries of the matrix $G_{\mu\nu}$ are $su(2)$ algebra-valued elements, and therefore it is not clear *a priori* whether in this case that result should hold. However, notice that also in our case

$$\det\hat{G}_{\mu\nu} \equiv \begin{vmatrix} G_{\alpha\beta} + \partial_\alpha\phi^i G_{i\beta} & G_{\alpha j} + \partial_\alpha\phi^i G_{ij} \\ G_{i\beta} & G_{ij} \end{vmatrix} = \begin{vmatrix} G_{\alpha\beta} & G_{\alpha j} \\ G_{i\beta} & G_{ij} \end{vmatrix} + \begin{vmatrix} \partial_\alpha\phi^i G_{i\beta} & \partial_\alpha\phi^i G_{ij} \\ G_{i\beta} & G_{ij} \end{vmatrix} \tag{3.7}$$

and the latter determinant is zero because $\partial_\alpha\phi^i$, being proportional to the identity in group space, commutes with all the other elements and, therefore, $\det G_{\mu\nu} = \det\hat{G}_{\mu\nu}$. Using the same arguments, we perform a final redefinition by introducing the matrix $\tilde{G}_{\mu\nu}$ whose elements are $\tilde{G}_{\mu\beta} = \hat{G}_{\mu\beta} + \hat{G}_{\mu j}\partial_\beta\phi^j$ and $\tilde{G}_{\mu j} = \hat{G}_{\mu j}$, namely,

$$\tilde{G}_{\mu\nu} = \begin{pmatrix} \tilde{G}_{\alpha\beta} & \tilde{G}_{\alpha j} \\ \tilde{G}_{i\beta} & \tilde{G}_{ij} \end{pmatrix} \equiv \begin{pmatrix} \hat{G}_{\alpha\beta} & \hat{G}_{\alpha j} \\ \hat{G}_{i\beta} & \hat{G}_{ij} \end{pmatrix} + \begin{pmatrix} \hat{G}_{\alpha j} & 0 \\ \hat{G}_{ij} & 0 \end{pmatrix}\partial_\beta\phi^j = \begin{pmatrix} \eta_{\alpha\beta} + f_{\alpha\beta} + \partial_\alpha\phi^i\partial_\beta\phi_i & \partial_\alpha\phi_i \\ \partial_\beta\phi_i & G_{ij} \end{pmatrix}. \tag{3.8}$$

Now, we take the determinant of the previous expression. Notice that the determinant of G_{ij} is given by (2.10) upon the replacement of r by $|\vec{x} - \vec{\phi}(\xi)|$. This determinant has an explicit factor of k^2, which becomes dominant in the large k limit; hence, we can ignore the off-diagonal contributions in computing $\det\tilde{G}_{\mu\nu}$. We have

$$-\det\tilde{G}_{\mu\nu} \approx -\det G_{ij}\det\tilde{G}_{\alpha\beta}. \tag{3.9}$$

So substituting this into the action gives

$$S = -\lambda^{1/2}\,\mathrm{STr}\int d^7\xi\int drd(-\cos\theta)d\phi V(\hat{t}(kr))k\hat{t}'(kr) \times\sqrt{r^4\sin^2\theta + \lambda^2F_{\theta\phi}}\sqrt{-\det(\tilde{G}_{\alpha\beta})}. \tag{3.10}$$

Performing the coordinate transformation in (2.16) and taking the large k limit, we find

$$S = -\frac{1}{2}\lambda^{3/2}\,\mathrm{STr}\int d^7\xi\int dyd(-\cos\theta)d\phi V(y)(x_r\cdot\sigma)^2 \times\sqrt{-\det\tilde{G}_{\alpha\beta}} = -\mathcal{T}_6\int d^7\xi\sqrt{-\det\tilde{G}_{\alpha\beta}}, \tag{3.11}$$

where

$$\tilde{G}_{\alpha\beta} = \eta_{\alpha\beta} + \lambda f_{\alpha\beta} + \partial_\alpha\phi^i\partial_\beta\phi_i. \tag{3.12}$$

This we recognize as the action of a BPS D6-brane, with the correct $U(1)$ gauge theory.

IV. CONCLUSIONS

In this paper, we have investigated codimension 3 magnetic monopole solutions arising from the DBI-like action of two coincident non-BPS D9-branes. We have shown the existence of singular monopoles that require regularization in a similar fashion to the kink and vortex soliton solutions investigated by Sen in [17]. An analysis of the fluctuations shows that in the limit where the regularization is removed, we recover the correct DBI action corresponding to a single BPS D6-brane. This extends the earlier results found by using truncated DBI-like actions [18]. Our results are complementary to those presented in [19] within the BSFT framework, where the authors showed that the basic tachyon monopole ansatz survives all higher order derivative corrections. Our results put magnetic monopoles alongside kinks and vortices as the possible products of tachyon condensation occurring in the full nonlinear, non-BPS DBI actions and which yield fluctuation spectra that are described by the full DBI action corresponding to codimension 1, 2, and 3 BPS branes.

These results were obtained within the framework of the non-BPS action presented in [12]. Recently, in [14], a modified version of this action (based on the results of [27,28]) has been proposed. In this modified version, the tachyon field carries internal Pauli matrices σ_1 and σ_2 and was obtained by considering the disk level S-matrix element of one Ramond-Ramond field and three tachyon fields. In [14] the modified action was shown to be consistent with the S-matrix element of one gauge field and four tachyon fields. The modified action amounts to a multiplication of the tachyon potential $V(T_i)$ in the symmetrized trace version of the non-BPS action [12] by a factor $\sqrt{1+\frac{1}{2}[T_i,T_j][T_i,T_j]}$, where $T_i = T\sigma_i$, $i = 1, 2$. For large tachyon field values it was argued in [28] that one may compute the STr by expanding $V(T_i)$ and that such modifications resulted in effectively the potential $V(T)$ being multiplied by a factor of T^4. The resulting modified potential still vanishes as $T \to \infty$, so tachyon condensation is still expected to occur. Indeed, one might argue that since the tachyon field configurations describing kinks, vortices, and, as we have shown, monopoles, are "large" almost everywhere in the regularized theory [the tachyon field is infinite everywhere except at the maximum of $V(T)$ where it is zero, in the unregularized theory], this large T approximation is justified. Nevertheless, it would be interesting to see the details of tachyon condensation in such a modified DBI action, including an analysis of the fluctuation spectrum, and to see whether they give the same results starting with the unmodified action in [12]. A first glance shows that at the very least, the formulas for the various tensions of the codimension 1, 2, and 3 BPS branes will change in that $V(T)$ will be replaced by $V(T)T^4$.

Finally, we have only discussed tachyon condensation in flat space. When one considers curved backgrounds, there are nonvanishing Ramond-Ramond forms, and thus Wess-Zumino (WZ) terms appear in the actions of both BPS and non-BPS branes. Therefore it is natural to consider the origin of such Wess-Zumino terms when BPS D-branes emerge as a result of tachyon condensation. This has been studied some time ago in [25] in the case where a normal trace (as opposed to a symmetrized trace) prescription is taken for the WZ term in the non-BPS D-brane action. More recently [29,30] have studied higher order derivative corrections to the WZ terms in non-BPS D-brane actions via disk amplitude S-matrix calculations. It is certainly an interesting question to consider how such corrections modify the results of [25] when one considers tachyon condensation producing codimension 1, 2, and 3 BPS D-branes.

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APPENDIX: DIRAC QUANTIZATION OF MAGNETIC CHARGE

To evaluate the magnetic charge associated with the ansatz (2.4), we need to have a definition of the magnetic field. In a $U(2)$ gauge theory, there is no unambiguous definition, but in a spontaneously broken theory, with unbroken group $U(1)$, provided that the fields are close to the vacuum, a magnetic field can be defined:

$$F^{\rm EM}_{\mu\nu} = \tfrac{1}{2}F^a_{\mu\nu}\hat{T}^a, \tag{A1}$$

where $\hat{T}^a$ is a unit vector that points along the direction of the "Higgs" field (in the present case the adjoint tachyon field T^a). In particular, $\hat{T}^a = \frac{x^a}{r}$ and the physical magnetic field becomes

$$B_i = \frac{1}{2}\epsilon_{ijk}F^{\rm EM}_{jk} = \frac{1}{4}\epsilon_{ijk}F^a_{jk}\frac{x^a}{r}. \tag{A2}$$

To find the total magnetic flux that is equal to the magnetic charge m, we have to integrate the magnetic field over S^2_∞, the 2-sphere at infinity. The magnetic charge m enclosed in some Gaussian surface Σ enclosing the magnetic charge density is given by

$$m = \int_{S^2_\infty} B_i dS_i = \lim_{r\to\infty}\frac{1}{4}\int_{S^2}\epsilon_{ijk}F^a_{jk}\frac{x^a}{r}dS_i. \tag{A3}$$

Now $dS_i = \epsilon_{ijk}dx^j\wedge dx^k$, so

$$m = \lim_{r\to\infty}\frac{1}{2}\int_{S^2}F^a_{jk}\frac{x^a}{r}dx^j\wedge dx^k \tag{A4}$$

in polar coordinates, and we can write

$$dx^j\wedge dx^k = \partial_m x^j(r,\theta,\phi)\partial_n x^k(r,\theta,\phi)d\xi^m\wedge d\xi^n, \tag{A5}$$

where ξ^n, $n = 1, 2$, correspond to the coordinates θ and ϕ. We have

$$\begin{aligned} m &= \lim_{r\to\infty}\frac{1}{2}\int_{S^2}F^a_{jk}\frac{x^a}{r}\partial_m x^j(r,\theta,\phi)\partial_n x^k(r,\theta,\phi)d\xi^m\wedge d\xi^n \\ &= \lim_{r\to\infty}\int_{S^2}F^a_{\theta\phi}\frac{x^a(r,\theta,\phi)}{r}d\theta d\phi, \end{aligned} \tag{A6}$$

where the S^2 has radius r. Using the definition of $x^a(r, \theta, \phi)$ and the expressions derived before for $F^a_{\theta\phi}$, we find

$$m = -\frac{1}{2}\int_{S^2_\infty} \sin\theta d\theta d\phi = -2\pi. \tag{A7}$$

The Dirac quantization of magnetic charge requires that

$$m = \frac{2\pi n}{e} \tag{A8}$$

for a charge m magnetic monopole where e is the electric charge. From the definition of the covariant derivative of the tachyon field T^a it is clear that $e = -1$. So for an $n = +1$ magnetic monopole, the magnetic charge is

$$m = \frac{2\pi n}{e} = -2\pi. \tag{A9}$$

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