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N-$\Delta(1232)$ axial form factors from weak pion production

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The $N\Delta$ axial form factors are determined from neutrino induced pion production ANL and BNL data by using a theoretical model that accounts both for background mechanisms and deuteron effects. We find violations of the off-diagonal Goldberger-Treiman relation at the level of 2σ which might have an impact in background calculations for T2K and MiniBooNE low energy neutrino oscillation precision experiments.

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I. INTRODUCTION

The $\Delta(1232)$ resonance is the lightest baryonic excitation of the nucleon. In addition, it couples very strongly to the lightest meson, the pion, and to the photon. As a consequence, the $\Delta(1232)$ is of the utmost importance in the description of a wide range of hadronic and nuclear phenomenology going from low and intermediate energy processes [1,2] to the GZK cut off of the cosmic ray flux [3,4]. On the other hand, despite its large width, it is well separated from other resonances, which facilitates its experimental investigation. In particular, the electromagnetic nucleon to $\Delta(1232)$ excitation processes, induced by electrons and photons, have been extensively studied at many experimental facilities like LEGS, BATES, ELSA, MAMI, and J-LAB. For a recent review see Ref. [5], where also many of the recent theoretical advances in the understanding of the resonance have been addressed.

There has also been a great theoretical interest in the axial nucleon Δ transition form factors. Recently, they have been studied using quark models [6], light cone QCD Sum Rules [7], lattice QCD [8] and chiral perturbation theory (χPT) [9,10]. These form factors are of topical importance in the background analysis of some of the neutrino oscillation experiments (e.g. [11]). However, their experimental knowledge is less than satisfactory. Although the feasibility of their extraction in parity-violating electron scattering has been considered [12], the best available information comes from old bubble chamber neutrino scattering experiments at ANL [13,14] and BNL [15,16]. These experiments measured pion production in deuterium at relatively low energies where the dominant contribution is given by the Δ pole (ΔP) mechanism: weak excitation of the $\Delta(1232)$ resonance and its subsequent decay into $N\pi$. Only very recently, π^0 production cross sections have been measured at low neutrino energies and with good statistics [17]. However, the target was mineral oil, which implies large and difficult to disentangle nuclear effects. Thus, these data are less well suited for the extraction of the $N\Delta$ axial form factors.

Besides the original experimental publications, there are many studies of the ANL and/or the BNL data in the literature [18–23] with different advantages and shortcomings. Some of those studies are discussed below. In this paper, we analyze the ANL and BNL data incorporating the deuteron effects, with a proper consideration of statistical and systematical uncertainties and taking advantage of several recent developments: improved vector form factors and a new model for weak pion production off the nucleon that includes background terms.

A convenient parametrization of the $W^+ n \rightarrow \Delta^+$ vertex is given in terms of eight q^2 (momentum transfer square) dependent form factors: four vector and four axial ($C^A_{3,4,5,6}$) ones. We follow the conventions and notation of Ref. [21]. Vector form factors have been determined from the analysis of photo and electro-production data. Here, we use the parametrization of Lalakulich *et al.* [24], as done in Ref. [21]. Among the axial form factors the most important contribution comes from C^A_5. The form factor C^A_6, which contribution to the differential cross section vanishes for massless leptons, can be related to C^A_5 thanks to the partial conservation of the axial current [$C^A_6(q^2) = C^A_5(q^2)\frac{M^2}{m^2_\pi - q^2}$, with m_π and M the pion and nucleon masses, respectively]. Since there are no other theoretical constraints for $C^A_{3,4,5}(q^2)$, they have to be fitted to data. Most analyses, including the ANL and BNL ones, adopt Adler's model [25] where[1] $C^A_3(q^2) = 0$ and $C^A_4(q^2) = -C^A_5(q^2)/4$. For C^A_5 several q^2 parametrizations have been used [19,22], though given the limited range of statistically significant q^2 values

[1]Setting C^A_3 to zero seems to be consistent with SU(6) symmetry [26] and recent lattice QCD results [27].

accessible in the ANL and BNL data, it should be sufficient to consider for it a dipole dependence, $C_5^A(q^2) = \frac{C_5^A(0)}{(1-q^2/M_{A\Delta}^2)^2}$, where one would expect $M_{A\Delta} \sim 0.85$–1 GeV, to guarantee an axial transition radius[2] R_A in the range of 0.7–0.8 fm, and $C_5^A(0) \sim 1.2$, which is the prediction of the off-diagonal Goldberger-Treiman relation (GTR), $C_5^A(0) = \sqrt{\frac{2}{3}} f_\pi \frac{f^*}{m_\pi} = 1.2$, with the $\pi N \Delta$ coupling $f^* = 2.2$ fixed to the Δ width and $f_\pi \sim 93$ MeV, the pion decay constant.

There is no constraint from χPT and lattice calculations are still not conclusive about the size of possible violations of the GTR. For instance, though values for $C_5^A(0)$ as low as 0.9 can be inferred in the chiral limit from the results of Ref. [27], they also predict $C_5^A(0)/(\sqrt{\frac{2}{3}} f_\pi \frac{f^*}{m_\pi})$ to be greater than one.

II. $C_5^A(q^2)$ ASSUMING ΔP DOMINANCE

Traditionally, Adler's model and the GTR have been assumed, being the $M_{A\Delta}$ axial mass adjusted in such a way that the ΔP contribution alone would lead to a reasonable description of the shape of the BNL q^2 differential $\nu_\mu p \to \mu^- p \pi^+$ cross section (see e.g. Ref. [19]). These fits also describe reasonably well the q^2 dependence of the ANL data and the BNL total cross section but overestimate the size of the ANL data by 20% near the maximum [20]. Thus, ANL data might favor $C_5^A(0)$ values smaller than the GTR prediction.

Recently, two reanalyses have been carried out trying to make compatible the GTR prediction for $C_5^A(0)$ and ANL data. In Ref. [22], $C_5^A(0)$ is kept to its GTR value and three additional parameters, which control the $C_5^A(q^2)$ fall off, are fitted to the ANL data. In fact $C_5^A(q^2 \sim 0)$ is not so relevant due to phase space, and what is actually important is the $C_5^A(q^2)$ value in the region around $-q^2 \sim 0.1\ \mathrm{GeV}^2$. Although ANL data are well reproduced, we find the outcome in [22] to be unphysical, because it provides a quite pronounced q^2-dependence that gives rise to a too large axial transition radius[3] of around 1.4 fm. Moreover, neither the fitted parameter statistical errors, nor the corresponding correlation coefficients are calculated in [22]. Undoubtedly, the fit carried out there should be quite unstable, from the statistical point of view, because of the difficulty of determining three parameters given the limited range of q^2 values covered in the ANL data set. Furthermore, the predicted cross sections turn out to be smaller than the BNL measurements [22].[4]

A second reanalysis [23] brings in the discussion two interesting points. First that both ANL and BNL data were measured in deuterium, and second, the uncertainties in the neutrino flux normalization. Deuteron structure effects in the $\nu d \to \mu^- \Delta^{++} n$ reaction, sometimes ignored, were estimated from the results of Ref. [18] to produce a reduction of the cross-section from 5–10%. In what respects to the ANL and BNL flux uncertainties, the procedure followed in [23] is not robust from the statistical point of view, since it ignores the correlations of these systematic errors.[5] Nevertheless, this latter work constitutes a clear step forward, and from a combined best fit to the ANL and BNL data, the authors of [23] find $C_5^A(0) = 1.19 \pm 0.08$ in agreement with the GTR estimate.

III. AXIAL FORM-FACTORS INCLUDING THE CHIRAL NON-RESONANT BACKGROUND

All the above mentioned determinations of $C_5^A(q^2)$ suffer from a serious theoretical limitation. Though the ΔP mechanism dominates the neutrino pion production reaction, especially in the Δ^{++} channel, there exist sizable nonresonant contributions of special relevance for low neutrino energies (below 1 GeV) of interest in T2K and MiniBooNE experiments. These background terms are totally fixed by the pattern of spontaneous chiral symmetry breaking of QCD, and are given in terms of the nucleon and pion masses, the axial charge of the nucleon, and the pion decay constant. When background terms are considered, the tension between ANL data and the GTR prediction for $C_5^A(0)$ substantially increases. Indeed, the fit carried out in [21] to the ANL data finds a value for $C_5^A(0)$ as low as 0.87 ± 0.08 with a reasonable axial transition radius of 0.75 ± 0.06 fm, and a large Gaussian correlation coefficient ($r = 0.85$), as expected from the above discussion of the results of Ref. [22].

Background terms were also considered in Refs. [30–32]. In the third reference, the chiral counting was broken

[2] It is defined from $C_5^A(q^2)/C_5^A(0) = 1 + q^2 R_A^2/6 + \mathcal{O}(q^4)$.

[3] Further details and possible repercussions in neutrino induced coherent pion production calculations are discussed in [28]. There, ANL data fits of the type proposed in [22], but including chiral nonresonant contributions are also performed, finding that then the axial transition radius becomes even larger, about 2.5 fm.

[4] In the PhD-thesis of T. Leitner, more detailed results can be found including a comparison with the BNL differential cross-section shape [29].

[5] There exist some other aspects that might require further investigation. For instance, additional parameters p_{ANL} and p_{BNL} are introduced in [23] [see the χ^2 function in Eq. (37)] to account for the flux uncertainties. At very low q^2 values, $d\sigma/dq^2$ is totally dominated by C_5^A. If we had infinitely precise statistical measurements, the fit carried out in [23] would provide a very precise determination of the ratio $C_5^A(0)/\sqrt{p}$, but not of the form factor $C_5^A(0)$. However, in such a situation, one expects to extract $C_5^A(0)$, though with an uncertainty dominated by that of the neutrino flux normalization. Besides, the fit to the BNL data uses the total cross-section data, for which the hadronic invariant mass is unconstrained, and the neutrino energy varies in the range 0.5–3 GeV. Above 1 GeV, heavier resonances than the $\Delta(1232)$, and not considered in [23], should play a role [24].

to account explicitly for ρ and $\omega-$ exchanges in the t channel, while the first two works are not consistent with the chiral counting either, since contact terms were not included. Moreover in [30,31] a rather small axial mass (~ 0.65 GeV) was used. Though the chiral counting is not respected in Ref. [32], in this approach, based on previous studies on photo [33] and electro [34] pion production off the nucleon, some meson-nucleon loops are considered. In this manner, the problem of unitarity is attacked, and for instance, a finite width for the Δ resonance is generated from a bare input. We should mention that a rather good description of the ANL data is achieved in [32], assuming the GTR at the quark level, and the pion loop dressed $C_5^A(0)$ predicted there turns out to be close to 1.

In Ref. [21], the background is calculated at leading order[6] in the chiral expansion, together with a dressed $\Delta(1232)$ term, including its physical width. Exact unitarity is not imposed in this model and that might be a problem, but at higher πN invariant masses than those considered in this work. At higher energies, the contributions of heavier baryon resonances than the $\Delta(1232)$ could be important. To properly describe their dynamics one should either include them explicitly in the model [24] (and eventually use Watson's theorem [35] to fix their relative phases) or dynamically generate these resonances by applying some unitarization procedure to the lowest order amplitudes. It is clear that the leading order chiral approach to the background terms would not be then sufficient. For the low energies that will be considered in this work, the model of Ref. [21] is still a fair approximation [36].[7]

Here, we follow the approach of Ref. [21], but implementing four major improvements: (i) We include in the fit the BNL total $\nu_\mu p \to \mu^- p \pi^+$ cross-section measurements of Ref. [15]. Since there is no cut in the outgoing pion-nucleon invariant mass in the BNL data, and in order to avoid heavier resonances from playing a significant role, we have just included the three lowest neutrino energies: 0.65, 0.9, and 1.1 GeV. We do not use the BNL measurement of the q^2-differential cross section, since it lacks an absolute normalization. (ii) We take into account deuteron effects in our theoretical calculation, (iii) we treat the uncertainties in the ANL and BNL neutrino flux normalizations as fully-correlated systematic errors, improving thus the treatment adopted in Ref. [23], and finally, (iv) in some fits, we relax the Adler's model constraints, by setting $C_{3,4}^A(q^2) = C_{3,4}^A(0)(C_5^A(q^2)/C_5^A(0))$, and explore the possibility of extracting some direct information on $C_{3,4}^A(0)$.

Let us consider first the neutrino-deuteron reaction $\nu d \to \mu^- p \pi^+ n$ measured in ANL and BNL. Owing to the inclusion of background terms, the formalism of Ref. [18], where the $p\pi^+$ pair was replaced by a Δ^{++}, cannot be used to account for deuteron corrections, and we must work with four particles in the final state. Neglecting the D-wave deuteron component and considering the neutron as a mere spectator, we find for the differential cross section on the deuteron

$$\frac{d\sigma}{dq^2 dW}\bigg|_d = \int d^3 p_d |\Psi_d(\vec{p}_d)|^2 \frac{M}{E_{p,d}} \frac{d\sigma}{dq^2 dW}\bigg|_{p\text{-off shell}}, \tag{1}$$

where $E_{p,d} = m_d - \sqrt{M^2 + \vec{p}_d^{\,2}}$, with m_d the deuteron mass, is the energy of the off-shell proton inside the deuteron which has four-momentum $p^\mu = (E_{p,d}, \vec{p}_d)$. W is the final $p\pi^+$ invariant mass. The differential cross section $d\sigma/dq^2 dW|_{p\text{-off shell}}$ is computed using the model of Ref. [21]. Finally, Ψ_d is the S-wave Paris potential deuteron wave function [39] normalized to 1.

In what respects to the neutrino flux normalization uncertainties, we consider them as sources of 20% and 10% systematic errors for the ANL and BNL experiments, respectively, (see the discussion in [23]). We have assumed that the ANL and BNL input data have independent statistical errors (σ_i) and fully-correlated systematic errors (ϵ_i), but no correlations linking the ANL and BNL sets. We end up with a 12×12 covariance matrix, C, with two diagonal blocks. The first 9×9 block is for the ANL flux averaged q^2-differential $\nu d \to \mu^- p \pi^+ n$ cross-section data (with a 1.4 GeV cut in W), while the second 3×3 block is for the BNL total cross sections mentioned above. Both blocks have the form $C_{ij} = \sigma_i^2 \delta_{ij} + \epsilon_i \epsilon_j$. The χ^2 function is constructed by using the inverse of the covariance matrix.

Results from several fits are compiled in Table I, from where we draw several conclusions. First, by comparing fit II* with Ref. [21], we deduce that the consideration of BNL data and flux uncertainties increases the value of $C_5^A(0)$ by about 9%, while it strongly reduces the statistical correlations between $C_5^A(0)$ and $M_{A\Delta}$. Second, the inclusion of background terms reduces $C_5^A(0)$ by about 13%, while deuteron effects increase it by about 5%, consistently, with the results of [21,18,23], respectively. Third, the fitted data are quite insensitive to $C_{3,4}^A(0)$, as fits V–VII results show. This is easily understood, taking for simplicity the massless lepton limit. In that case

[6]This is not totally precise, and some further improvements were considered, since experimental determinations of the form factors, fitted to the most accurate available data, were used. However, we have tested that at the low energies considered in this work, the cross-section changes induced by the form factors, entering in the nonresonant terms, are marginal.

[7]The needed relative phase between the nonresonant background and the $\Delta(1232)$ term to account for unitarity in the dominant P_{33} wave was investigated for pion production induced by real [37] and virtual photons [38] off the nucleon. The phases turn out to be reasonably small (of the order or $\pi/8$ radians at the Δ peak [37]), thanks to the use of the phenomenological Δ-resonance width. These phases induce small corrections compatible with those we expect from the next to leading terms in the chiral expansion.

TABLE I. Results from different fits to the ANL and BNL data. Deuteron effects are included in all cases except for the two first fits (marked with $*$). The nonresonant chiral background contributions are not included in fits I and III. In the $C^A_{3,4}$ columns, Ad indicates that Adler's constraints ($C^A_3 = 0$, $C^A_4 = -C^A_5/4$) are imposed. Finally, r_{ij} are Gaussian correlation coefficients between parameters i and j. For $C^A_5(q^2)$ a dipole form has been used.

	$C^A_5(0)$	$M_{A\Delta}$/GeV	$C^A_3(0)$	$C^A_4(0)$	r_{12}	r_{13}	r_{14}	r_{23}	r_{24}	r_{34}	χ^2/dof
I* (only ΔP)	1.08 ± 0.10	0.92 ± 0.06	Ad	Ad	-0.06						0.36
II*	0.95 ± 0.11	0.92 ± 0.08	Ad	Ad	-0.08						0.49
III (only ΔP)	1.13 ± 0.10	0.93 ± 0.06	Ad	Ad	-0.06						0.32
IV	1.00 ± 0.11	0.93 ± 0.07	Ad	Ad	-0.08						0.42
V	1.08 ± 0.14	0.91 ± 0.10	-1.0 ± 1.4	Ad	-0.48	-0.61		0.81			0.40
VI	1.08 ± 0.14	0.86 ± 0.15	Ad	-1.0 ± 1.3	-0.57	-0.66		0.93			0.40
VII	1.07 ± 0.15	1.0 ± 0.3	1 ± 4	-2 ± 4	-0.62	-0.45	0.30	0.89	-0.77	-0.97	0.44

$$\frac{d\sigma}{dq^2} \propto \{[C^A_5(0)]^2 + q^2 a(q^2)\}, \tag{2}$$

and $C^A_{3,4}(0)$ start contributing to $a(q^2)$, i.e. to $\mathcal{O}(q^2)$, which also gets contributions from vector form factors and terms proportional to $dC^A_5/dq^2|_{q^2=0}$. This also explains the large statistical correlations displayed in fits V–VII. Moreover, $dC^A_{3,4}/dq^2|_{q^2=0}$ appears at order $\mathcal{O}(q^4)$, which has prevented us to fitting the q^2 shape of these form factors. Fourth, fit IV is probably the most robust from the statistical point of view. In Fig. 1, we display fit IV results for the ANL and BNL $\nu d \to \mu^- p \pi^+ n$ cross sections. Looking at the central values of $C^A_5(0)$, we conclude that the violation of the off-diagonal GTR is about 15% smaller than that suggested in Ref. [21], though it is definitely greater than that claimed in [23], mostly because in this latter work background terms were not considered. However, GTR and fit IV $C^A_5(0)$ values differ in less than two sigmas, and the discrepancy is even smaller if Adler's constraints are removed. These new results are quite relevant for the neutrino induced coherent pion production process in nuclei which is much more forward peaked than the incoherent reaction. For instance, we expect the results in Ref. [40], based in the determination of $C^A_5(0)$ of Ref. [21], to underestimate cross sections by at least 30%.

By using a theoretical model that accounts both for background mechanisms and deuteron effects, we have determined the $N\Delta$ axial form factors from statistically improved fits to the combined ANL and BNL data. The inclusion of chiral background terms significantly modifies the form factors. We have found violations of the GTR at the level of 2σ, when the usual Adler's constraints are adopted. This will influence background calculations for T2K and MiniBooNE low energy neutrino precision oscillation experiments.

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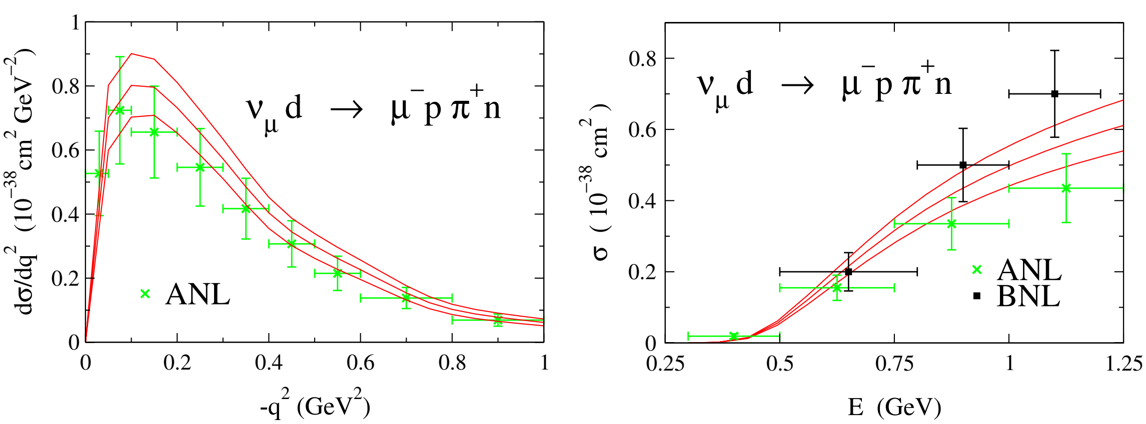

FIG. 1 (color online). Comparison of the ANL $d\sigma/dq^2$ differential (left panel) and ANL and BNL total (right panel) cross-section data with fit IV theoretical results. Theoretical 68% confidence level bands are also displayed. Data in both plots include a systematic error (20% for ANL and 10% for BNL data) added in quadrature to the statistical ones. In the left panel, both data and results include a cut $W < 1.4$ GeV.

[1] G. E. Brown and W. Weise, Phys. Rep. **22**, 279 (1975).
[2] G. Cattapan and L. S. Ferreira, Phys. Rep. **362**, 303 (2002).
[3] K. Greisen, Phys. Rev. Lett. **16**, 748 (1966).
[4] G. T. Zatsepin and V. A. Kuzmin, JETP Lett. **4**, 78 (1966).
[5] V. Pascalutsa, M. Vanderhaeghen, and S. N. Yang, Phys. Rep. **437**, 125 (2007).
[6] D. Barquilla-Cano, A. J. Buchmann, and E. Hernandez, Phys. Rev. C **75**, 065203 (2007); **77**, 019903(E) (2008).
[7] T. M. Aliev, K. Azizi, and A. Ozpineci, Nucl. Phys. **A799**, 105 (2008).
[8] C. Alexandrou *et al.*, Phys. Rev. Lett. **98**, 052003 (2007).
[9] L. S. Geng, J. Martin Camalich, L. Alvarez-Ruso, and M. J. Vicente Vacas, Phys. Rev. D **78**, 014011 (2008).
[10] M. Procura, Phys. Rev. D **78**, 094021 (2008).
[11] A. A. Aguilar-Arevalo *et al.* (MiniBooNE Collaboration), Phys. Rev. Lett. **102**, 101802 (2009).
[12] N. C. Mukhopadhyay *et al.*, Nucl. Phys. **A633**, 481 (1998).
[13] S. J. Barish *et al.*, Phys. Rev. D **19**, 2521 (1979).
[14] G. M. Radecky *et al.*, Phys. Rev. D **25**, 1161 (1982); **26**, 3297(E) (1982).
[15] T. Kitagaki *et al.*, Phys. Rev. D **34**, 2554 (1986).
[16] T. Kitagaki *et al.*, Phys. Rev. D **42**, 1331 (1990).
[17] A. A. Aguilar-Arevalo *et al.* (MiniBooNE Collaboration), Phys. Rev. D **81**, 013005 (2010).
[18] L. Alvarez-Ruso, S. K. Singh, and M. J. Vicente Vacas, Phys. Rev. C **59**, 3386 (1999).
[19] E. A. Paschos, J.-Y. Yu, and M. Sakuda, Phys. Rev. D **69**, 014013 (2004).
[20] O. Lalakulich and E. A. Paschos, Phys. Rev. D **71**, 074003 (2005).
[21] E. Hernandez, J. Nieves, and M. Valverde, Phys. Rev. D **76**, 033005 (2007).
[22] T. Leitner, O. Buss, L. Alvarez-Ruso, and U. Mosel, Phys. Rev. C **79**, 034601 (2009).
[23] K. M. Graczyk, D. Kielczewska, P. Przewlocki, and J. T. Sobczyk, Phys. Rev. D **80**, 093001 (2009).
[24] O. Lalakulich, E. A. Paschos, and G. Piranishvili, Phys. Rev. D **74**, 014009 (2006).
[25] S. L. Adler, Ann. Phys. (N.Y.) **50**, 189 (1968); J. Bijtebier, Nucl. Phys. **B21**, 158 (1970).
[26] J. Liu, N. C. Mukhopadhyay, and L. S. Zhang, Phys. Rev. C **52**, 1630 (1995).
[27] C. Alexandrou *et al.*, Phys. Rev. D **76**, 094511 (2007); **80**, 099901(E) (2009).
[28] E. Hernandez, J. Nieves, M. Valverde, and M. J. Vicente-Vacas, arXiv:0912.2631.
[29] T. Leitner (private communication); T. Leitner, Ph.D. thesis, University of Giessen.
[30] G. L. Fogli and G. Nardulli, Nucl. Phys. **B160**, 116 (1979).
[31] G. L. Fogli and G. Nardulli, Nucl. Phys. **B165**, 162 (1980).
[32] T. Sato, D. Uno, and T. S. H. Lee, Phys. Rev. C **67**, 065201 (2003).
[33] T. Sato and T. S. H. Lee, Phys. Rev. C **54**, 2660 (1996).
[34] T. Sato and T. S. H. Lee, Phys. Rev. C **63**, 055201 (2001).
[35] K. M. Watson, Phys. Rev. **95**, 228 (1954).
[36] T. E. O. Ericson and W. Weise, *Pions and Nuclei*, The International Series of Monographs on Physics Vol. 74 (Clarendon, Oxford, 1988), p. 479.
[37] R. C. Carrasco and E. Oset, Nucl. Phys. **A536**, 445 (1992).
[38] A. Gil, J. Nieves, and E. Oset, Nucl. Phys. **A627**, 543 (1997).
[39] M. Lacombe *et al.*, Phys. Lett. **101B**, 139 (1981).
[40] J. E. Amaro, E. Hernandez, J. Nieves, and M. Valverde, Phys. Rev. D **79**, 013002 (2009).