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# Cosmology and the noncommutative approach to the standard model

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We study cosmological consequences of the noncommutative approach to the standard model of particle physics. Neglecting the nonminimal coupling of the Higgs field to the curvature, noncommutative corrections to Einstein's equations are present only for inhomogeneous and anisotropic space-times. Considering the nonminimal coupling however, corrections are obtained even for background cosmologies. Links with dilatonic gravity as well as chameleon cosmology are briefly discussed, and potential experimental consequences are mentioned.

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## I. INTRODUCTION

Theoretical early universe cosmology is gaining a constantly increasing interest from the scientific community. The predictions of the theoretical models can now be compared with a plethora of astrophysical data, in particular, the measurements of the cosmic microwave background temperature anisotropies, all having a surprising good accuracy. Moreover, present high energy experiments, in particular, the Large Hadron Collider, will test some of the theoretical pillars of the cosmological models. Despite this golden era of cosmology, a number of questions, such as the explanation of space-time dimensionality [1], the origin of dark energy [2] and dark matter [3], the search for the natural and well-motivated successful inflationary model, are still awaiting for a definite answer.

The main theoretical approaches upon which the cosmological models have been built are either string theory or quantum gravity. Here we will consider another one, which up to now has, rather surprisingly, gained only a limited interest, namely, noncommutative geometry (NCG) [4,5]. More precisely, we will study cosmological consequences of the NCG approach to the standard model (SM) [6], which remains to the best of our knowledge the particle physics model at present. The NCG approach leads to all the detailed structure of the SM, as well as several physical predictions at the unification scale.

In this paper, after a brief review on the noncommutative spectral action, we discuss some of its early universe cosmological consequences and their potential link to dilatonic gravity and chameleon models.

## II. NONCOMMUTATIVE GEOMETRY APPROACH

The NCG approach to the unification of all fundamental interactions including gravity is based on three *Ansätze*:

(I) Slightly below Planck energy, space-time becomes the product of a four-dimensional smooth compact Riemannian manifold $\mathcal{M}$ by a finite noncommutative space $F$. The geometry is therefore the tensor product of an internal geometry for the SM and a continuous geometry for space-time. One has to distinguish between the metric (or spectral) dimension, given by the behavior of the eigenvalues of the Dirac operator, and the KO dimension, an algebraic dimension based on $K$ theory. The relevant Dirac operator for space-time is the ordinary Dirac operator on curved space-time; thus the metric dimension is equal to 4. The internal Dirac operator consists of the fermionic mass matrix, which has a finite number of eigenvalues; thus the internal metric dimension is zero. As a result, the metric dimension of the product geometry is 4, the same as the ordinary space-time manifold.

To resolve the fermion doubling problem, by projecting out the unphysical degrees of freedom resting in the internal space, the real structure of the finite geometry $F$ must be such that its KO dimension is equal to 6 [7]. Thus, the KO dimension of the product space $\mathcal{M} \times F$ is equal to $10 \sim 2$ modulo 8. Notice that unlike earlier particle physics models based on NCG, in the approach [6] followed here the KO dimension (which is equal to 6 modulo 8) of the internal space is different than its metric dimension (which is equal to zero).

A noncommutative geometry is given by a representation of spectral nature. More precisely, $F = (\mathcal{A}, \mathcal{H}, D)$ is a spectral triple, given by an involutive algebra $\mathcal{A}$ of operators in Hilbert space $\mathcal{H}$, playing the role of the algebra of coordinates, and a linear self-adjoint ($D = D^\dagger$) operator $D$ in $\mathcal{H}$, playing the role of the inverse of the line element. The choice of Hilbert space $\mathcal{H}$ is irrelevant here, since all separable infinite-dimensional Hilbert spaces are isomorphic. The operator $D$ is such that all commutators $[D, a]$ are bounded for $a \in \mathcal{A}$. Except for finite dimensional cases, $D$ is in general not a bounded operator; hence it is only defined on a dense domain.

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The classic geodesic formula of Riemannian geometry:

$$d(x, y) = \inf \int_\gamma ds, \tag{1}$$

where the infimum is taken over all paths from $x$ to $y$, giving the distance $d(x, y)$ between two points $x$, $y$, is replaced in NCG by

$$d(x, y) = \sup\{|f(x) - f(y)| : f \in \mathcal{A}, \|[\mathcal{D}, f]\| \leq 1\}, \tag{2}$$

with $D$ the inverse of the line element $ds$. Another significance of $D$ is that its homotopy class represents the $K$-homology[1] fundamental class of the space under consideration.

The choice of the finite dimensional involutive algebra consists of the main input for the model. The hypothesis that space-time is the product of a continuous manifold $M$ by a discrete space $F$ is the easiest generalization of a commutative space. This is a strong assumption that is expected to break in the Planck era.

(II) The algebra constructed in this product space-time is then assumed to be in the *symplectic-unitary* case [8]. This choice restricts the algebra $\mathcal{A}$ to the form $\mathcal{A} = M_a(\mathbb{H}) \oplus M_k(\mathbb{C})$, with $k = 2a$; $\mathbb{H}$ is the algebra of quaternions. The first possible value for the even number $k$ is 2, corresponding to a Hilbert space of four fermions; it is ruled out from the existence of quarks. The second one, $k = 4$, leads to the correct number of $k^2 = 16$ fermions in each of the three generations. Notice that considering three generations is a physical input in NCG [8]. The involutive algebra $\mathcal{A}$ corresponds to a given space in the same way as in the classical duality between *space* and *algebra* in algebraic geometry.

(III) The Dirac operator connects the two pieces of the product geometry nontrivially. The action $S$, called the *spectral action* functional, depends only on the spectrum of the Dirac operator; it is of the form $\mathrm{Tr}(f(D/\Lambda))$, with $\Lambda$ giving the energy scale and $f$ being a test function, whose choice plays only a small role. The spectral action functional $\mathrm{Tr}(f(D/\Lambda))$ accounts only for the bosonic term; the fermionic term can be included by adding $(1/2)\langle J\psi, D\psi\rangle$. When the spectral action $S$ is expanded in inverse powers of $\Lambda$, it depends only on three first momenta $f_k = \int_0^\infty f(v)v^{k-1}dv$ for $k > 0$, and on the Taylor expansion of $f$ at 0. One of the consequences is that some of the fermions can acquire Majorana masses, realizing the seesaw mechanism.

The full Lagrangian of the SM, minimally coupled to gravity, is obtained [6] as the asymptotic expansion of the spectral action for the product space-time. For our purposes here, namely, extracting early universe cosmological consequences of the noncommutative spectral action approach, we are only interested in the gravitational and Higgs part of the action, namely

$$\mathcal{S}_{\rm grav}^{\rm Lorentzian} = \int\Big(\frac{1}{2\kappa_0^2}R + \frac{1}{2}\alpha_0 C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} + \tau_0 R^\star R^\star - \xi_0 R|\mathbf{H}|^2\Big)\sqrt{-g}d^4x; \tag{3}$$

$\mathbf{H}$ is a rescaling $\mathbf{H} = (\sqrt{af_0}/\pi)\phi$ of the Higgs field $\phi$ to normalize the kinetic energy. The momentum $f_0 = f(0)$ is physically related to the coupling constants at unification. The coefficient $a$, that enters the Higgs field redefinition, is given by

$$a = \mathrm{Tr}(Y^\star_{(\uparrow 1)}Y_{(\uparrow 1)} + Y^\star_{(\downarrow 1)}Y_{(\downarrow 1)} + 3(Y^\star_{(\uparrow 3)}Y_{(\uparrow 3)} + Y^\star_{(\downarrow 3)}Y_{(\downarrow 3)})), \tag{4}$$

where the $Y$'s are used to classify the action of the Dirac operator and give the fermion and lepton masses, as well as lepton mixing, in this asymptotic version of the spectral action. The $Y$'s matrices are only relevant for the coupling of the Higgs field with fermions through the dimensionless matrices $\pi/\sqrt{af_0}Y_x$ with $x \in \{(\uparrow\downarrow, j)\}$. Thus, $a$ has the physical dimension of a (mass)$^2$.

The coupling constants in Eq. (3) are

$$\frac{1}{\kappa_0^2} = \frac{96f_2\Lambda^2 - f_0c^2}{12\pi^2}, \qquad \alpha_0 = -\frac{3f_0}{5\pi^2}, \qquad \tau_0 = \frac{11f_0}{60\pi^2}, \qquad \xi_0 = \frac{1}{12}, \tag{5}$$

where $\Lambda$ is an energy scale about which the asymptotic expansion is performed and $c$ is expressed in terms of $Y_R$ which gives the Majorana mass matrix, $c = \mathrm{Tr}(Y_R^\star Y_R)$. The scale $\Lambda$ is fixed by the unification scale of the coupling constants of the standard model. Let us emphasize that the spectral action, Eq. (3), has to be seen as a boundary condition at unification scale. Therefore, Eq. (5) above fixes the coupling constants at unification scale; extrapolations to lower energies are possible using renormalization group analysis. It is therefore evident that this noncommutative spectral action approach is appropriate for early universe cosmology (i.e., at energies close to unification).

Several key points should be noted: First, the noncommutative geometry procedure outlined above is entirely classical; it simply provides an elegant way in which the standard model of particle physics can be produced from purely (noncommutative) geometric information. Second, the action given in Eq. (3) has been Wick rotated from the Euclidean action which is produced from noncommutative geometry.[2] The formal justification of this has yet to be

[1] $K$ homology is the homological version of $K$ theory.

[2] To use the formalism of spectral triples in noncommutative geometry, it is convenient to work with Euclidean rather that Lorentzian signature. One can go to Euclidean signature by performing a Wick rotation to imaginary time. In the Euclidean action functional for gravity, the kinetic terms must have the correct sign so that the functional is bounded below. Since such positivity is spoiled by the scalar Weyl mode, one must show that all other terms get a positive sign [5].

shown. Third, at present the entries in the Dirac operator that produce Eq. (4) are inputs to the theory. The hope is that by varying with respect to them, the values that correspond to the standard model will be dynamically chosen. Despite these issues, it remains striking that by removing the assumption that space-time is commutative in the simplest possible way (the space-time is a product of a commutative manifold $M$ and a discrete, internal, noncommutative manifold $F$), one recovers general relativity coupled to the entire standard model with no additional particles and the correct couplings.

The only nonstandard elements of the asymptotic expansion of the noncommutative geometry action are the presence of the additional terms given in Eq. (3). The purpose of this paper is indeed to investigate some cosmological consequences of these terms.

## III. COSMOLOGICAL CONSEQUENCES

Let us study the gravitational part of the spectral action Eq. (3). The first two terms give the Riemannian curvature with a contribution from the Weyl curvature, where the second term is the action for conformal gravity [9]. Notice that the presence of the Einstein-Hilbert term (and of the cosmological constant, which we neglect here) explicitly breaks conformal invariance. The third term is a topological term integrating to the Euler characteristic of the manifold:

$$R^\star R^\star = \tfrac{1}{4}\epsilon^{\mu\nu\rho\sigma}\epsilon_{\alpha\beta\gamma\delta}R^{\alpha\beta}_{\mu\nu}R^{\gamma\delta}_{\rho\sigma},$$

and hence is nondynamical. Finally, the fourth term is the scalar mass term.

The equations of motion arising from Eq. (3) read [9]

$$R^{\mu\nu} - \tfrac{1}{2}g^{\mu\nu}R - \alpha_0\kappa_0^2\delta(\Lambda)[2C^{\mu\lambda\nu\kappa}{}_{;\lambda;\kappa} - C^{\mu\lambda\nu\kappa}R_{\lambda\kappa}] = \kappa_0^2\delta(\Lambda)T^{\mu\nu}_{\mathrm{matter}}, \tag{6}$$

where

$$\delta(\Lambda) \equiv [1 - 2\kappa_0^2\xi_0|\mathbf{H}|^2]^{-1}.$$

In what follows, we study the above equations of motion first neglecting the nonminimal coupling between the geometry and the Higgs field and then including it.

### A. Neglecting the Higgs field term

Neglecting the nonminimal coupling between the geometry and the Higgs field, i.e., setting $\phi = 0$ in Eq. (6), leads to

$$R^{\mu\nu} - \tfrac{1}{2}g^{\mu\nu}R - \alpha_0\kappa_0^2[2C^{\mu\lambda\nu\kappa}_{;\lambda;\kappa} - C^{\mu\lambda\nu\kappa}R_{\lambda\kappa}] = \kappa_0^2 T^{\mu\nu}_{\mathrm{matter}}. \tag{7}$$

We are interested in the cosmology associated with these equations of motion. For a Friedmann-Lemaître-Roberston-Walker (FLRW) space-time, the Weyl tensor vanishes. Hence, the noncommutative geometry corrections to the Einstein equation, Eq. (7), vanish.

For scalar perturbations around a FLRW metric, in the conformal Newtonian (also called longitudinal) gauge, the metric reads

$$g_{\mu\nu} = \mathrm{diag}(\{a(t)\}^2[-(1+\Psi(x)), (1-\Phi(x)), (1-\Phi(x)), (1-\Phi(x))]), \tag{8}$$

where $t$ is conformal time and $(x, y, z)$ are Cartesian space coordinates; the scale factor is denoted by $a$. The $\Phi$, $\Psi$ are the gauge invariant *Bardeen potentials* [10]; the $\Psi$ is the analog of the Newtonian potential.

The (0, 0) component of Eq. (7) leads, for the metric specified by Eq. (8), to the modified Friedmann equation:

$$-3\left(\frac{\dot a}{a}\right)^2 - \nabla^2\Phi(x) + 3\left(\frac{\dot a}{a}\right)\dot\Phi(x)(x) - \frac{\alpha_0\kappa_0^2}{3a^2}\nabla^4[\Phi(x)+\Psi(x)] + \mathcal{O}(\Phi^2, \Psi^2, \ldots) = \kappa_0^2 T_{00}, \tag{9}$$

where the overdot denotes derivative with respect to conformal time $t$ and $\nabla_i = (\partial_x, \partial_y, \partial_x)$.

As expected, in an exactly homogeneous and isotropic space-time, i.e., $\Psi(x) = \Phi(x) = 0$, the modified Friedmann equation reduces to its standard form. Naively, one may have expected this result, since in a spatially homogeneous space-time the spatial points are equivalent and hence any noncommutative effects might be expected to vanish. This however is not the case here, since the noncommutativity of the theory is incorporated in the internal manifold $F$ and our space-time is a (smooth) commutative four-dimensional manifold. Despite this, noncommutative corrections to the standard cosmological models do not occur at the level of an FLRW background. Notice that in the case of an FLRW model, one can explicitly calculate the topological term $\int R^\star R^\star\sqrt{g}d^4x$, appearing in Eq. (3), and show that it is indeed nondynamical.

Considering the scalar perturbations, the noncommutative geometry corrections are in second and fourth order in spatial derivatives, which can be neglected in most cosmological situations.

Of the remaining equations of motion, given in Eq. (6), the most interesting ones are those coming from the off-diagonal terms, namely

$$-\partial_i\left[\dot\Phi(x) + \frac{\dot a}{a}\Psi(x)\right] - \frac{\alpha_0\kappa_0^2}{6a^2}\nabla^2[\partial_i(\dot\Psi(x)+\dot\Phi(x))] = \kappa_0^2 T_{0i}, \tag{10}$$

and

$$\frac{1}{2}\partial_i\partial_j[\Psi(x)-\Phi(x)] + \frac{\alpha_0\kappa_0^2}{12a^2}\left[3\frac{\ddot{a}}{a}-6\left(\frac{\dot{a}}{a}\right)^2-3\partial_t^2+\nabla^2\right] \times[\partial_i\partial_j(\Psi(x)+\Phi(x))]=\kappa_0^2T_{ij} \quad \text{with} \quad i\neq j. \tag{11}$$

Equation (11) is particularly interesting: it shows that matter with zero anisotropic stresses no longer implies equality of the Bardeen potentials (i.e., the condition $\Psi = \Phi$ does not hold). Let us emphasize that in the absence of noncommutative effects (i.e., for standard scalar perturbations in a FLRW background), the Bardeen potentials turn out to be equal, if shear-free matter fields are considered.

The above calculation can be also performed in the synchronous gauge,[3] for which the total (i.e., background + perturbed) metric can be written as

$$g_{\mu\nu}=\mathrm{diag}(\{a(t)\}^2[-1,(\delta_{ij}+h_{ij}(x))]), \tag{12}$$

leading to the modified Friedmann equation:

$$-3\left(\frac{\dot{a}}{a}\right)^2+\frac{1}{2}\left[4\left(\frac{\dot{a}}{a}\right)\dot{h}+2\ddot{h}-\nabla^2h+\nabla_i\nabla_jh^{ij}\right] -\frac{\alpha_0\kappa_0^2}{6a^2}[\partial_t^2(\nabla^2h-3\nabla_i\nabla_jh^{ij})+\nabla^2(\nabla_i\nabla_jh^{ij})-\nabla^4h] +\mathcal{O}(h^2)=\kappa_0^2T_{00}, \tag{13}$$

where $h\equiv h_i^i$ is the trace of $h_{ij}$.

The remaining gauge is removed by choosing $\nabla_ih^{ij}=0$, for which Eq. (13) reduces to

$$-3\left(\frac{\dot{a}}{a}\right)^2+2\left(\frac{\dot{a}}{a}\right)\dot{h}+\ddot{h}-\frac{1}{2}\nabla^2h-\frac{\alpha_0\kappa_0}{6a^2}\nabla^2[\partial_t^2-\nabla^2]h +\mathcal{O}(h^2)=\kappa_0^2T_{00}. \tag{14}$$

Thus, the traceless part of the perturbed metric $h_{ij}$, i.e., gravitational waves (which are in addition transverse), do not enter into the Friedmann equation, even in the presence of noncommutative geometry corrections.

The remaining equations obtained from Eq. (7) are rather involved, however for perturbations around a Minkowski background [i.e., $a(t)=1$, $\dot{a}=0$], there is a significant simplification due to the fact that

$$C^{\mu\lambda\nu\kappa}R_{\lambda\kappa}\sim\mathcal{O}(\{h_{ij}\}^2), \tag{15}$$

where $\{h_{ij}\}$ indicates all terms that are first order in the perturbation. For this situation, the transverse, traceless part of $h_{ij}$ obeys the following equations (where without loss of generality we have taken $h_{ij}$ to be transverse to the $z$ direction, and we have again used the gauge condition $\nabla_ih^{ij}=0$):

$$[1+\alpha_0\kappa_0^2(-\partial_t^2+\partial_z^2)](-\partial_t^2+\partial_z^2)h_+=0, \tag{16}$$

$$[1+\alpha_0\kappa_0^2(-\partial_t^2+\partial_z^2)](-\partial_t^2+\partial_z^2)h_\times=0, \tag{17}$$

where $h_+$ and $h_\times$ are the two independent polarizations of the gravitational waves, i.e.,

$$h_{ij}=\begin{pmatrix} h_+ & h_\times & 0\\ h_\times & -h_+ & 0\\ 0 & 0 & 0\end{pmatrix}. \tag{18}$$

The right-hand side of Eqs. (16) and (17) vanish because we are considering gravitational waves propagating against a Minkowski background, for which $T_{\mu\nu}=0$.

It is clear from Eqs. (16) and (17) that the solutions to the general relativistic equation for the components of the perturbations (produced here by setting $\alpha_0=0$) remain solutions, i.e., one finds that perturbations satisfying,

$$(-\partial_t^2+\partial_z^2)h_+=0 \quad \text{and} \quad (-\partial_t^2+\partial_z^2)h_\times=0, \tag{19}$$

are solutions to the equations of motion [Eq. (7)].

Thus, the propagation of standard gravitational waves is unaffected by the presence of noncommutative geometry effects (at least for gravitational waves propagating in Minkowski space-time). However, there are additional solutions to Eqs. (16) and (17), which correspond to *gravitational radiation*, that are not present in standard general relativity. A detailed investigation of this phenomenon is performed in Ref. [11].

In order for the corrections to Einstein's equations to be apparent at leading order (i.e., at the level of the background), we need to consider anisotropic models. As an example, we calculate the modified Friedmann equation for the Bianchi type-V model, for which the space-time metric, in Cartesian coordinates, reads

$$g_{\mu\nu}=\mathrm{diag}[-1,\{a_1(t)\}^2e^{-2nz},\{a_2(t)\}^2e^{-2nz},\{a_3(t)\}^2], \tag{20}$$

where $a(t)$, $b(t)$, and $c(t)$ are, in general, arbitrary functions and $n$ is an integer.

Defining $A_i(t)=\ln a_i(t)$ with $i=1,2,3$, the modified Friedmann equation reads

[3]It corresponds to having only two nonzero perturbation variables; the other two being zero imply that the *threading* of space-time into lines (corresponding to fixed space coordinates) consists of geodesics and the *slicing* into hypersurfaces (corresponding to fixed time) is orthogonal to them. There is a whole class of gauges with this property.

$$\kappa_0^2 T_{00} = -\dot{A}_3(\dot{A}_1 + \dot{A}_2) - n^2 e^{-2A_3}(\dot{A}_1\dot{A}_2 - 3) + \frac{8\alpha_0\kappa_0^2 n^2}{3} e^{-2A_3}[5(\dot{A}_1)^2 + 5(\dot{A}_2)^2 - (\dot{A}_3)^2 - \dot{A}_1\dot{A}_2 - \dot{A}_2\dot{A}_3 - \dot{A}_3\dot{A}_1 - \ddot{A}_1 - \ddot{A}_2 - \ddot{A}_3 + 3] - \frac{4\alpha_0\kappa_0^2}{3}\sum_i\Big\{\dot{A}_1\dot{A}_2\dot{A}_3\dot{A}_i + \dot{A}_i\dot{A}_{i+1}((\dot{A}_i - \dot{A}_{i+1})^2 - \dot{A}_i\dot{A}_{i+1}) + (\ddot{A}_i + (\dot{A}_i)^2)\Big[-\ddot{A}_i - (\dot{A}_i)^2 + \frac{1}{2}(\ddot{A}_{i+1} + \ddot{A}_{i+2}) + \frac{1}{2}((\dot{A}_{i+1})^2 + (\dot{A}_{i+2})^2)\Big] + [\dddot{A}_i + 3\dot{A}_i\ddot{A}_i - (\ddot{A}_i + (\dot{A}_i)^2)(\dot{A}_i - \dot{A}_{i+1} - \dot{A}_{i+2})][2\dot{A}_i - \dot{A}_{i+1} - \dot{A}_{i+2}]\Big\}, \tag{21}$$

where all indices are understood to be taken modulo 3 and the $t$ dependence of the terms has been suppressed. Clearly, all terms containing $\alpha_0$, in Eq. (21) above, are the modifications to the standard result. By studying the case of the Bianchi type V model, we can immediately identify the noncommutative geometry effects in other cases of the cosmological model. More precisely, Eq. (21) reduces to

(i) Bianchi type I for $n = 0$;
(ii) FLRW for $a(t) = b(t) = c(t)$ and $n = 0$;
(iii) Kasner metric[4] for $a(t) = t^A$, $b(t) = t^B$, $c(t) = t^C$, and $n = 0$, where $A$, $B$, and $C$ are constants.

For the Bianchi type-V metric, with

$$a(t) = t^{\tilde{a}_1}, \qquad b(t) = t^{\tilde{a}_2}, \qquad c(t) = t^{\tilde{a}_3},$$

where $\tilde{a}_i$ are constants as in the Kasner metric but $n \neq 0$, the modified Friedmann equation becomes:

$$\kappa_0^2 T_{00} = -\tilde{a}_3(\tilde{a}_1 + \tilde{a}_2)t^{-2} - n^2 t^{-2(\tilde{a}_3+1)}(\tilde{a}_1\tilde{a}_2 - 3) + \frac{8\alpha_0\kappa_0^2 n^2}{3} t^{-2(\tilde{a}_3+1)}[5(\tilde{a}_1)^2 + 5(\tilde{a}_2)^2 - (\tilde{a}_3)^2 - \tilde{a}_1\tilde{a}_2 - \tilde{a}_2\tilde{a}_3 - \tilde{a}_3\tilde{a}_1 + \tilde{a}_1 + \tilde{a}_2 + \tilde{a}_3 + 3] - \frac{4\alpha_0\kappa_0^2}{3} t^{-4}\sum_i \tilde{a}_i\Big\{\tilde{a}_1\tilde{a}_2\tilde{a}_3 + \tilde{a}_{i+1}((\tilde{a}_i - \tilde{a}_{i+1})^2 - \tilde{a}_i\tilde{a}_{i+1}) + (\tilde{a}_i - 1)\Big[\frac{1}{2}\tilde{a}_{i+1}(\tilde{a}_{i+1} - 1) + \frac{1}{2}\tilde{a}_{i+2}(\tilde{a}_{i+2} - 1) - \tilde{a}_i(\tilde{a}_i - 1)\Big] + [((\tilde{a}_i)^2 + 2) - 3\tilde{a}_i + (1 - \tilde{a}_i)(\tilde{a}_i - \tilde{a}_{i+1} - \tilde{a}_{i+2})][2\tilde{a}_i - \tilde{a}_{i+1} - \tilde{a}_{i+2}]\Big\}. \tag{22}$$

Since the term in braces occurs at a higher order than the terms coming from the Einstein-Hilbert action (at least for $\tilde{a}_3 < 1$), it becomes negligible at late times.

For the Kasner metric we know that $n = 0$ and hence the only correction to the standard Friedmann equation is the term in braces. However, for the inhomogeneous case ($n \neq 0$) there is an additional term that occurs at the same order as the inhomogeneous part of the standard Friedmann equation, i.e., at order $t^{-2(\tilde{a}_3+1)}$.

More generally, from Eq. (21) the correction terms come in two types. The first one contains the terms in braces in Eq. (21), which are fourth order in time derivatives. Hence for the slowly varying functions, usually used in cosmology, they can be taken to be small corrections. The second type, which is the third term in Eq. (21), occurs at the same order as the standard Einstein-Hilbert terms. However, it is proportional to $n^2$ and hence vanishes for homogeneous versions of Bianchi type V. Thus, although anisotropic cosmologies do contain corrections due to the additional NCG terms in the action, they are typically of higher order. Inhomogeneous models do contain correction terms that appear on the same footing as the original (commutative) terms.

[4] A subclass of the Bianchi type-I metrics.

### B. Nonminimal coupling of the Higgs field to curvature

Up to now, we have neglected the nonminimal coupling of the Higgs field to the curvature. This is likely to be a good approximation for late time cosmology, since we expect the Higgs field to be very small. However, at energies approaching the Higgs scale this additional term needs to be included. From Eq. (6) it is immediately apparent that for $|\mathbf{H}| \neq 0$, the effects of the NCG corrections to Einstein's equations are enhanced. In particular, for $|\mathbf{H}| \to \sqrt{6}/\kappa_0$ the correction term entirely dominates, provided the Weyl curvature term is nonzero, and the equations of motion tend to

$$2C^{\mu\lambda\nu\kappa}_{;\lambda;\kappa} - C^{\mu\lambda\nu\kappa}R_{\lambda\kappa} = -\frac{1}{\alpha_0}T^{\mu\nu}_{\text{matter}}, \tag{23}$$

which is precisely the equations of motion for conformal gravity [9], albeit with a modified gravitational constant.

As we have previously shown, the corrections to Einstein's equations are present only in inhomogeneous and anisotropic space-times. For $|\mathbf{H}| \neq 0$ however, there are corrections even for background cosmologies. To understand the effects of these corrections it is sufficient to neglect the conformal term in Eq. (6), i.e., setting $\alpha_0 = 0$. In this case, the equations of motion become

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = \kappa_0^2 \bigg[ \frac{1}{1 - \kappa_0^2 |\mathbf{H}|^2/6} \bigg] T^{\mu\nu}_{\text{matter}}. \tag{24}$$

Hence, the effect of a nonzero $|\mathbf{H}|$ field is to create an effective gravitational constant.

An alternative viewpoint is to consider the effect of this term on the equations of motion for the Higgs field in some, constant, gravitational field. The action for the pure Higgs fields reads [6]

$$\mathcal{L}_{|\mathbf{H}|} = -\frac{R}{12} |\mathbf{H}|^2 + \frac{1}{2} |D^\alpha \mathbf{H}| |D^\beta \mathbf{H}| g_{\alpha\beta} - \mu_0 |\mathbf{H}|^2 + \lambda_0 |\mathbf{H}|^4; \tag{25}$$

$D^\alpha$ is the covariant derivative. Thus, for constant curvature, the self-interaction of the Higgs field is increased, namely

$$-\mu_0 |\mathbf{H}|^2 \to -\bigg(\mu_0 + \frac{R}{12}\bigg) |\mathbf{H}|^2. \tag{26}$$

Hence, for static geometries, the nonminimal coupling of the Higgs field to the curvature increases the Higgs mass. This has potential consequences both for terrestrial experiments and for late time cosmology, since the curvature of an asymptotically de Sitter universe would increase the effective mass of the Higgs field, although in both cases the effect is likely to be minimal.

## IV. DISCUSSION

After having discussed some cosmological consequences of the noncommutative geometry spectral action, let us briefly mention some links to dilatonic gravity and chameleon cosmology, in the presence of the nonminimal coupling of the Higgs field to the background geometry.

Redefine the Higgs field $\mathbf{H}$ by

$$\tilde{\phi} = -\ln(|\mathbf{H}|/(2\sqrt{3})),$$

and thus rewrite Eq. (25) in the form of four-dimensional dilatonic gravity as

$$\mathcal{L}_{\tilde{\phi}} = e^{-2\tilde{\phi}} [-R + 6 D^\alpha \tilde{\phi} D^\beta \tilde{\phi} g_{\alpha\beta} - 12(\mu_0 - 12 \lambda_0 e^{-2\tilde{\phi}})], \tag{27}$$

providing a link to compactified string models.

In chameleon models [12], a scalar field is taken to have a nonminimal coupling to the standard matter content (thus evading solar system tests of general relativity). In the NCG spectral action studied here, we have a scalar field (the Higgs) that has a nonzero coupling to the background geometry. If we are in a regime where the equations of motion are well approximated by Einstein's equations, then the background geometry will be given (approximately) by the standard matter, making the mass and dynamics of the Higgs field explicitly dependent of the local matter content. A more detailed study of this link to chameleon models is left as a future work [13].

The noncommutative geometry spectral action gives an elegant mathematical formulation of the standard model of elementary particle physics, compatible with all known phenomenology of the standard model. In addition, it provides a natural setup to study early universe cosmology.[5]

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[5]After completion of this work, cosmological studies have been also performed in Refs. [14,15].

[1] R. H. Brandenberger and C. Vafa, Nucl. Phys. **B316**, 391 (1989); R. Durrer, M. Kunz, and M. Sakellariadou, Phys. Lett. B **614**, 125 (2005); A. Karch and L. Randall, Phys. Rev. Lett. **95**, 161601 (2005); W. Nelson and M. Sakellariadou, Phys. Lett. B **674**, 210 (2009).
[2] E. J. Copeland, M. Sami, and S. Tsujikawa, Int. J. Mod. Phys. D **15**, 1753 (2006).
[3] L. Bergstrom, Rep. Prog. Phys. **63**, 793 (2000).
[4] A. Connes, *Noncommutative Geometry* (Academic Press, New York, 1994).
[5] A. Connes and M. Marcolli, *Noncommutative Geometry, Quantum Fields and Motives* (Hindustan Book Agency, India, 2008).
[6] A. H. Chamseddine, A. Connes, and M. Marcolli, Adv. Theor. Math. Phys. **11**, 991 (2007).
[7] J. Barrett, J. Math. Phys. (N.Y.) **48**, 012303 (2007); A. Connes, J. High Energy Phys. 11 (2006) 081.
[8] A. H. Chamseddine and A. Connes, Phys. Rev. Lett. **99**, 191601 (2007).
[9] P. Mannheim, Prog. Part. Nucl. Phys. **56**, 340 (2006).

[10] V. F. Mukhanov, H. A. Feldman, and R. H. Brandenberger, Phys. Rep. **215**, 203 (1992).
[11] W. Nelson and M. Sakellariadou (unpublished).
[12] J. Khoury and A. Weltman, Phys. Rev. D **69**, 044026 (2004).
[13] W. Nelson and M. Sakellariadou (unpublished).
[14] W. Nelson and M. Sakellariadou, Phys. Lett. B **680**, 263 (2009).
[15] M. Marcolli and E. Pierpaoli, arXiv:0908.3683.