keV sterile neutrino dark matter in gauge extensions of the standard model

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(Received 19 January 2010; published 22 April 2010)

It is known, that a keV scale sterile neutrino is a good warm dark matter candidate. We study how this possibility could be realized in the context of gauge extensions of the standard model. The naïve expectation leads to large thermal overproduction of sterile neutrinos in this setup. However, we find that it is possible to use out-of-equilibrium decay of the other right-handed neutrinos of the model to dilute the present density of the keV sterile neutrinos and achieve the observed dark matter density. We present the universal requirements that should be satisfied by the gauge extensions of the standard model, containing right-handed neutrinos, to be viable models of warm dark matter, and provide a simple example in the context of the left-right symmetric model.

DOI: 10.1103/PhysRevD.81.085032 PACS numbers: 14.60.St, 12.60.−i, 14.60.Pq, 95.35.+d

I. INTRODUCTION

Dark matter (DM) is one of the experimentally observed indications of physics beyond the standard model (SM). A wide variety of astrophysical and cosmological observations confirm that $\Omega_{\rm DM} \simeq 0.2$ part of the total energy density of the Universe is composed of some form of nonbaryonic matter which interacts very weakly [1]. The most common particle physics explanation comes in the form of weakly interacting massive particles, which are heavy and weakly interacting thermal relics, leading to cold dark matter. Another common candidate for the cold dark matter is the axion, which is light, but due to a specific generation mechanism it has an extremely small temperature [2]. Hot dark matter has high velocities and a large free streaming length and it contradicts the experiment, because it prevents the formation of the observed small scale structures in the Universe. The intermediate situation, warm dark matter (WDM) is, however, less explored. It may even provide a solution to some of the problems of the DM simulations, reducing the number of Dwarf satellite galaxies, or smoothing the cusps in the DM halos.

A natural candidate commonly considered for WDM is a light sterile neutrino [3].[1] A simple realization is the νMSM [6,7], where only three singlet fermions, which have Majorana masses and Dirac mixing with ordinary (active) neutrinos, are added to the standard model. Then, the mass of one sterile neutrino can be chosen in the range of several keV and with very small mixing with the active neutrinos, it will provide a particle with the lifetime exceeding the age of the Universe, which can be the WDM candidate. The virtue and at the same time the problem of the model is, that the sterile neutrino with such a small mixing (the only interaction of this particle is via the Yukawa couplings) never enters into the thermal equilibrium, and it can be produced only by some nonthermal mechanism. If this were not true and the neutrino reached thermal equilibrium at some moment in the early universe, then without any additional mechanism, the thermal relics with mass of over about 90 eV would overclose the Universe. At the same time, one needs knowledge of the physics before the beginning of thermal evolution of the Universe in order to calculate unambiguously the abundance of sterile neutrinos in the νMSM (see Refs. [8–10]).

The possibility analyzed in this article is opposite to the νMSM. We assume, that there is some additional (gauge) scale between the electroweak and Planck scales, and that the sterile neutrinos are charged under these additional gauge transformations.

It turns out that it is possible to reconcile the thermal overproduction of the DM with the observations. To do this, the abundance of the sterile neutrino should be diluted *after* it drops out of the thermal equilibrium. This happens if some long-lived particle decays while being out of thermal equilibrium after the DM sterile neutrino freeze-out. This effectively reduces the amount of the DM sterile neutrino relative to the overall energy balance of the Universe; see Fig. 1. It is also easy to find a candidate for this long-lived heavy particle—another (heavier) sterile neutrino in the model. We formulate the requirements on the properties of the DM sterile neutrino and the out-of-equilibrium decaying particle to make the model consistent with existing observations and bounds. This generic analysis, important for all possible models of this type, is made in the Sec. II. In the end of this section, all the requirements are summarized.

There exist other ways to avoid the overproduction of the DM sterile neutrino in the analyzed class of the models, which we will only mention here. One possibility is realized if all the new gauge interactions are at the grand

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[1]Note that WDM can also be many other particles, like a gravitino or even heavy particles; see [4,5].

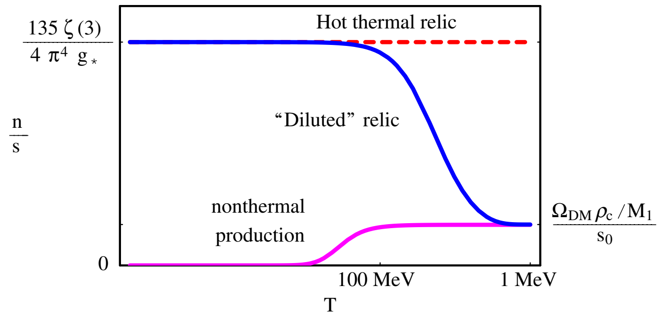

FIG. 1 (color online). Schematic evolution of the light relic abundance in the Universe. The dashed line is a thermal relic decoupled while being relativistic (hot thermal relic), leading to the overclosure of the Universe. The blue decreasing line is the same hot thermal relic, but with the abundance diluted by rapid expansion of the Universe (entropy production), leading to correct DM abundance. The lowest (magenta) line depicts the evolution of the nonthermally produced particle with zero primordial abundance.

unified theory (GUT) scale, while the reheating after inflation leads to temperatures below the GUT scale. This situation is similar to the νMSM, because the sterile neutrinos do not reach thermal equilibrium. Another possibility requires large (of the order of thousand) number of degrees of freedom at the moment of the sterile neutrino freeze-out, which does not seem natural.

In Secs. III and IV, we analyze the possibility to realize these constraints in the simplest models. We then use other sterile neutrinos to dilute the density of the DM sterile neutrino. In Sec. III, we show that it is impossible for the same right-handed neutrinos to be involved in the DM abundance dilution and at the same time to give the masses to the active neutrinos via a type I seesaw like mechanism. The reason for this is that the mixing angles (or, equivalently, Yukawa coupling constants) are extremely small for the sterile neutrinos. This would lead to masses of the active neutrinos smaller, than the minimal ones, allowed by the neutrino oscillation observations.

In Sec. IV, we provide a working example, where the active neutrino masses are generated by a type II seesaw from the scalar sector of a left-right (LR) symmetric model, and sterile neutrinos have very small mixing angles with the active neutrino sector.

The appendices are devoted to the calculation of the total decay width of the sterile neutrinos in the model (Appendix A), radiative decay width (Appendix B), and to the description of the useful parametrization of the neutrino mass matrix (Appendix C).

II. COSMOLOGICAL REQUIREMENTS AND CONSTRAINTS FROM EXPERIMENTS

In this section, we introduce the generic framework we will work with and discuss the various constraints and bounds resulting from cosmological considerations and various experimental results. Note that these constraints are rather general and apply to most variations of the specified model.

A. Assumptions and definitions

In the following, we will assume the existence of right-handed (sterile) neutrinos N_{IR}. These sterile neutrinos are not charged under the SM gauge group, but they could be charged under the gauge transformations of an extended model (ultimately, emerging in the breaking chain of some GUT model). Though for most of the statements in this article the precise details of this gauge interaction are not important, we will use a specific LR symmetric extension of the SM and stick to it to obtain definite numbers. This specific model (see e.g. Ref. [11] for a detailed review) with the gauge group $SU(3)\times SU(2)_L\times SU(2)_R\times U(1)_{B-L}$ appears as a subgroup of many GUT theories.

In this model, we have the interaction with the gauge bosons of the form

$$-\mathcal{L}_{CC}=\frac{g}{\sqrt{2}}\sum_a(W_L^\mu\bar{l}_{aL}\gamma_\mu\nu_{aL}+W_R^\mu\bar{l}_{aL}\gamma_\mu N_{aR})+\text{H.c.},\qquad(1)$$

where W_L is the SM W boson, W_R is the corresponding right-handed boson from $SU(2)_R$, and l_a are the charged SM leptons. The neutrino mass matrix appears from the vacuum expectation values of various Higgs bosons in the model. Up to the Sec. IV, we will not be interested in the details of this, and will just write the general mass matrix as

$$\mathcal{L}_{\text{mass}}=-\frac{1}{2}(\overline{\tilde{\nu}^c_{aL}},\overline{\tilde{N}_{aR}})\begin{pmatrix}M_L & m_D\\ m_D^T & M_R\end{pmatrix}\begin{pmatrix}\tilde{\nu}_{aL}\\ \tilde{N}^c_{aR}\end{pmatrix}+\text{H.c.}\qquad(2)$$

Note that a tilde over the neutrinos indicates that they are written in the *flavor* basis. In the following, we will also assume that the mass matrices obey in some sense the relations $M_R > m_D > M_L$ such that we can use seesaw-type formulas. Thus, the rotation to the mass basis has the form

$$\begin{pmatrix}\tilde{\nu}_{aL}\\ \tilde{N}^c_{aR}\end{pmatrix}\simeq\begin{pmatrix}1 & (M_R^{-1}m_D^T)^\dagger\\ -M_R^{-1}m_D^T & 1\end{pmatrix}\begin{pmatrix}U & 0\\ 0 & V_R\end{pmatrix}\begin{pmatrix}\nu_{iL}\\ N^c_{IR}\end{pmatrix},\qquad(3)$$

where U is the standard Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix and where V_R describes the mixing in the right-handed sector

$$M_L-m_DM_R^{-1}m_D^T=U^\star\cdot\text{diag}(m_1,m_2,m_3)\cdot U^\dagger,\qquad(4)$$

$$M_R=V_R^\star\cdot\text{diag}(M_1,M_2,M_3)\cdot V_R^\dagger,\qquad(5)$$

with m_i being the active neutrino masses and M_I the sterile neutrino masses. Note, that if $M_L=0$, then Eq. (4) is the usual seesaw formula.

For the analysis of the sterile neutrino decay, when the oscillations of the active neutrinos are not important, while the masses of the charged leptons are, it is helpful to make the described rotation only partially—without the PMNS rotation by the matrix U. Then, we get the mixing angles between the mass states of the sterile neutrinos and SM flavors

$$\theta_{aI} \equiv \frac{(m_D V_R)_{aI}}{M_I}, \tag{6}$$

and also

$$\theta_I^2 \equiv \sum_{a=e,\mu,\tau} |\theta_{aI}|^2. \tag{7}$$

These squared mixing angles describe the overall strength of interaction (decay) of sterile neutrinos with the SM particles.

Before moving on to the analysis of the cosmological properties of sterile neutrinos, let us note an additional possible complication. Specifically, the W_L and W_R bosons in Eq. (1) may not coincide with the mass eigenstates, W_1 and W_2 with masses M_W and M, respectively, but be slightly mixed

$$\begin{aligned} W_L &= \cos\zeta W_1 + \sin\zeta W_2, \\ W_R &= -\sin\zeta W_1 + \cos\zeta W_2. \end{aligned} \tag{8}$$

Normally this can be neglected, but it may give significant contribution to the radiative decay of the DM sterile neutrinos, analyzed in Sec. II F.

B. Temperature of freeze-out

Let us now calculate the moment of decoupling of the neutrinos N_1 in the early universe. We will denote values corresponding to this moment by the subscript "f." As far as the DM sterile neutrino is relatively light and the freeze-out happens while it is still relativistic, the calculation is analogous to those for the usual active neutrinos [12]. The only difference is that the annihilation cross section is suppressed by the larger mass M of the right-handed gauge boson W_R, compared to the SM W boson mass M_W,

$$\sigma_{N_1N_1} \approx \sigma_{\nu\bar{\nu}}\left(\frac{M_W}{M}\right)^4 \sim G_F^2 E^2 \left(\frac{M_W}{M}\right)^4. \tag{9}$$

Here, $\sigma_{\nu\bar{\nu}}$ is the SM neutrino annihilation cross section, $G_F = 1.166 \times 10^{-5}\ \mathrm{GeV}^{-2}$ is the Fermi constant and E is the energy of the colliding neutrinos. Requiring the equality of the mean free path and the Hubble scale, we get for the freeze-out temperature

$$T_\mathrm{f} \sim g_{*\mathrm{f}}^{1/6}\left(\frac{M}{M_W}\right)^{4/3}(1 \div 2)\ \mathrm{MeV}, \tag{10}$$

where $g_{*\mathrm{f}}$ is the effective number of degrees of freedom immediately after freeze-out (at least 10.75 for SM content if freeze-out happened below 100 MeV).

We see that for the not very large scale M, the sterile neutrino N_1 decouples at a rather low temperature. Thus, it normally is in thermal equilibrium at the early stages of the Universe evolution, making it a thermal relic. This will be the possibility which we peruse in the current study. In this case, calculation of the present day density of the sterile neutrinos is insensitive to the history of the Universe before T_f.

Note, however, that if the reheating temperature after inflation is lower than Eq. (10), the neutrinos never enter the thermal equilibrium. In this case, additional assumptions about the initial abundance of the sterile neutrinos are necessary to predict their current density, and the generation mechanism is very different form the analyzed here (see, e.g. Refs. [13–16]). Such a situation can be realized for a very low reheating temperature (see e.g. [17]), or naturally if the right-handed scale is the GUT scale, $M \sim M_\mathrm{GUT}$, leading to $T_\mathrm{f} \sim M_\mathrm{GUT}$, and the reheating after inflation reached slightly lower temperatures. Another way to implement this situation is perused in the νMSM [6,7], where *no new physics* is present up to Planck scale, leading to N_1 never entering the thermal equilibrium.

C. Abundance of N_1 at present time

The number to entropy density ratio of the sterile neutrino (two fermionic degrees of freedom) after freeze-out is given by

$$\left.\frac{n_{N_1}}{s}\right|_\mathrm{f} = \frac{1}{g_{*\mathrm{f}}}\frac{135\zeta(3)}{4\pi^4}. \tag{11}$$

While the Universe expands slowly with all the processes approaching thermal equilibrium, both the number density and entropy density scale are inversely proportional to the volume of the Universe, and this ratio remains constant. If nonequilibrium processes happen during expansion (for example an intermediate matter dominated stage caused by out-of-equilibrium decay of a heavy species), additional entropy release is possible, which we will take into account by the factor S:

$$\frac{n_{N_1}(t_0)}{n_{N_1}(t_\mathrm{f})} = \left(\frac{a(t_\mathrm{f})}{a(t_0)}\right)^3 = \frac{s(t_0)}{s(t_\mathrm{f})}\frac{1}{S}. \tag{12}$$

Let us calculate the contribution of N_1 to the present energy density. Rescaling the number to entropy density ratio at present moment by this factor, as compared to the freeze-out moment, we get for the sterile neutrino contribution to the energy density of the Universe Ω_N

$$\begin{aligned} \frac{\Omega_N}{\Omega_\mathrm{DM}} &= \left(\left.\frac{n_{N_1}}{s}\right|_\mathrm{f}\right)\frac{1}{S}M_1\frac{s_0}{\Omega_\mathrm{DM}\rho_c} \\ &\simeq \frac{1}{S}\left(\frac{10.75}{g_{*\mathrm{f}}}\right)\left(\frac{M_1}{1\ \mathrm{keV}}\right) \times 100, \end{aligned} \tag{13}$$

where $\Omega_\mathrm{DM} = 0.105h^{-2}$ is the DM density, $s_0 = 2889.2\ \mathrm{cm}^{-3}$ is the present day entropy density, and $\rho_c = 1.05368 \times 10^{-5}h^2\ \mathrm{GeV\,cm}^{-3}$ is the critical density of the

Universe. The observational requirement is $\Omega_N/\Omega_{\rm DM} \le 1$ with equality being the nicest choice (all DM is made out of N_1) and inequality opting for multispecies DM.

Let us analyze Eq. (13) further. Without entropy release ($S = 1$), the Universe is overclosed, unless the neutrino is very light, which corresponds to the hot dark matter case, excluded by the structure formation in the Universe. Models with the number of degrees of freedom at freeze-out $g_{*{\rm f}}$ of order 1000 seem rather unnatural and will not be considered. The only opportunity is thus the entropy release after freeze-out of N_1,

$$S \simeq 100\left(\frac{10.75}{g_{*{\rm f}}}\right)\left(\frac{M_1}{1\ {\rm keV}}\right). \tag{14}$$

Having this entropy release after N_1 decoupling will lead to the observed DM abundance today. In the following, we will analyze possibilities of generation of this large amount in the model.[2]

C. Mass bounds

The mass of the DM particle can not be too light, or the observed structure in the Universe would have been erased by a too hot DM. The simplest and most robust bound can be obtained from the phase space density arguments. The phase space density of a collisionless DM can only become smaller during the evolution of the Universe, as an effect of coarse-graining. Comparison of the primordial phase space density, which is calculated using the initial DM particle distribution function and the maximal modern one, derived from the observation of the Dwarf spheroidal galaxies [4,18] gives the lower bound

$$M_1 > 1\text{–}2\ {\rm keV}. \tag{15}$$

Another important bound is the Lyman-α (Ly-α) bound [19,20]. This bound constrains the velocity distribution of the DM particles from the effect of their free streaming on the formation of the structure on the scales, probed by the Ly-α forest. It should be noted that to convert this constraint into a bound for the mass of the DM particle, one needs to take into account the initial velocity distribution of the particles. In our case it takes the form of a usual thermal distribution, but with the temperature lowered by the dilution factor $S^{-1/3}$. This corresponds to the *thermal relic* case in Ref. [19], and not to the case of the nonresonantly produced sterile neutrinos, denoted $m_{\rm NRP}$ in Ref. [19]. Thus, the result of Ref. [19] should be rescaled as

$$M_1 > \frac{T}{T_\nu} m_{\rm NRP} \simeq 1.6\ {\rm keV}, \tag{16}$$

where T is the present temperature of the DM neutrino diluted with the entropy factor (14), T_ν is the temperature of the usual relic neutrinos, $m_{\rm NRP} = 8$ keV [19], and the ratio of the temperatures $(T/T_\nu)^3 = \Omega_{\rm DM} h^2 (94\ {\rm eV}/M_1)$ is obtained from the requirement of the observed $\Omega_{\rm DM}$.

E. Generation of entropy

The entropy (14) can be generated by some heavy long-lived particle which goes out of the thermal equilibrium at some moment after DM neutrino freeze-out $t_{\rm f}$, and it decays after becoming nonrelativistic and dominating the Universe expansion. The obvious candidates for such particles are the two remaining heavier neutrinos (though other candidates are possible and can be analyzed in a similar way). Let us assume for simplicity that only one of these two neutrinos is responsible for entropy generation, and we denote it by N_2. Then, according to Refs. [12,21], the entropy release is

$$S \simeq \left(1 + 2.95\left(\frac{2\pi^2 \bar{g}_*}{45}\right)^{1/3} \frac{(rM_2)^{4/3}}{(M_{\rm Pl}\Gamma)^{2/3}}\right)^{3/4}, \tag{17}$$

where M_2 is the mass of N_2, $r = n_{N_2}/s$ is the initial abundance of the N_2 particles after decoupling (or, probably more precise, before they start to drive the matter dominated intermediate stage of the Universe expansion), and $\bar{g}_*$ is the properly averaged effective number of degrees of freedom during the N_2 decay. The ratio r is maximal when the particle decouples when it is still relativistic, and is equal to

$$r \equiv \frac{n_{N_2}}{s} = \frac{g_N}{2}\frac{135\zeta(3)}{4\pi^4 g_*}, \tag{18}$$

where $g_N = 2$ is the number of degrees of freedom for N_2, and g_* is taken at the N_2 freeze-out.

If the entropy generation is large, we can neglect the 1 in Eq. (17) and get

$$S \simeq 0.76 \frac{g_N}{2} \frac{\bar{g}_*^{1/4} M_2}{g_* \sqrt{\Gamma M_{\rm Pl}}}. \tag{19}$$

By combining Eqs. (14) and (19), we obtain

$$\Gamma \simeq 0.50 \times 10^{-6} \frac{g_N^2}{4} \frac{g_{*{\rm f}}^2}{g_*^2} \bar{g}_*^{1/2} \frac{M_2^2}{M_{\rm Pl}} \left(\frac{1\ {\rm keV}}{M_1}\right)^2. \tag{20}$$

Note that for our case, the freeze-out temperatures of the DM sterile neutrino and of the entropy generating one coincide, so $g_* = g_{*{\rm f}}$. If Eq. (20) is satisfied, then we have proper DM abundance in the present Universe.

However, Eq. (20) is not the only requirement for the lifetime of the heavier sterile neutrino for the realistic model. Entropy generation should finish before the big bang nucleosynthesis (BBN), i.e. N_2 should decay before it. According to Refs. [22–24], the temperature after the decay of the sterile neutrino N_2 should be greater than $0.7 \div 4$ MeV in order not to spoil BBN predictions. This temperature is approximately equal to (see Ref. [21])

[2]The exact value of the required entropy release S may be slightly different if, for example, some amount of DM sterile neutrino was generated nonthermally after the freeze-out. In the examples analyzed in the paper, this effect is negligible.

$$T_r \simeq \frac{1}{2}\left(\frac{2\pi^2 \bar{g}_*}{45}\right)^{-1/4} \sqrt{\Gamma M_{\rm Pl}}, \tag{21}$$

leading to the bound on the N_2 lifetime which should be shorter than approximately $0.1 \div 2$s. The neutrino with such a lifetime can produce enough entropy, satisfying Eq. (20) only if it is sufficiently heavy,

$$M_2 > \left(\frac{M_1}{1\ \text{keV}}\right)(1.7 \div 10)\ \text{GeV}. \tag{22}$$

Finally, as far as we were assuming that the sterile neutrino N_2 decoupled while still relativistic (otherwise the entropy generation is much less efficient), we should require $T_f > M_2$. This, using Eq. (10), is translated into a bound for the scale of the right-handed bosons,

$$M > \frac{1}{g_{*f}^{1/8}}\left(\frac{M_2}{\text{GeV}}\right)^{3/4}(10 \div 16)\ \text{TeV}. \tag{23}$$

Thus, on the one hand the sufficient entropy generation requires a long-lived neutrino, but on the other hand, the requirement of the successful BBN limits its lifetime from above, leading to the lower bounds on its mass and on the mass scale of the additional gauge interactions.

F. X-ray observations

A sterile neutrino in the considered class of the models is unstable, so it provides a *decaying* DM. Through its mixing, it decays via the neutral current into three active neutrinos. To lead to a successful DM scenario, the lifetime of the unstable neutrino N_1 should be greater than the age of the Universe $\tau_u \sim 10^{17}$ sec, which constrains its total decay width. However, one obtains significantly stronger restrictions resulting from a subdominant decay channel—the radiative decay $N_1 \to \nu\gamma$, induced at the one loop level (Fig. 2). This process produces a narrow line in the x-ray spectrum of astrophysical objects [3,25]. In the context of νMSM, the only source of this decay is via the active-sterile neutrino mixing θ_1^2, and the recent x-ray observations bound it from above. A very rough bound, which will be enough for our purposes, is given in Ref. [13][3]

$$\theta_1^2 \lesssim 1.8 \times 10^{-5}\left(\frac{1\ \text{keV}}{M_1}\right)^5. \tag{24}$$

This bound corresponds to the following upper bound on the radiative decay width

$$\Gamma_{N_1\to\gamma\nu} \lesssim 9.9 \times 10^{-27}\ \text{sec}^{-1}. \tag{25}$$

We also note that there are bounds resulting from supernova cooling. They are also much weaker than the diffuse x-ray background limits (24) for all possible neutrino masses M_1 [3].

[3]We must note that careful analysis gives a stronger (in some regions of masses by an order of magnitude) bound. See the detailed discussion in Sec. 5.1.2 of [13,26–32]. For our purposes, this approximate (weak) bound is sufficient.

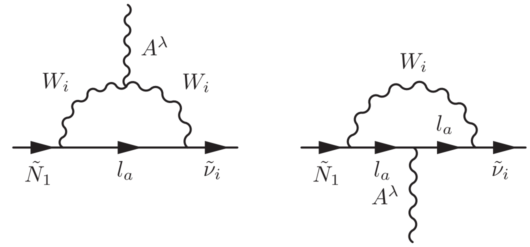

FIG. 2. Unitary-gauge diagrams contributing to the radiative neutrino decay with charged leptons propagating in the loop.

In the LR symmetric model, the x-ray bound (25) leads not only to the bound on the mixing angle (24), but also bounds the properties of the bosonic sector of the theory. The reason is the possible mixing of the right W_R gauge bosons with the SM W_L ones. Without mixing, the contribution of the W_R bosons to the process $N \to \gamma\nu$ is additionally suppressed by the ratio of the left and right gauge boson masses $(M_W/M)^4$, and can be safely neglected. With the mixing, however, the chiral structure of the diagram changes, and the contribution can be enhanced by the factor $(m_l/M_1)^2$, where m_l is the mass of the charged lepton running in the loop.

We calculate the total decay width for $N_1 \to \gamma\nu$, summed over the active neutrino flavors, following Refs. [33,34] (for details, see Appendix B). Supposing from the very beginning that the right-handed scale is much larger than the left one, $M \gg M_W$, neglecting the active neutrino masses and assuming small gauge boson mixing, we get

$$\Gamma_{N_1\to\gamma\nu} \simeq \frac{G_F^2 \alpha M_1^3}{64\pi^4} \sum_{a=e,\mu,\tau} \left| 4m_{l_a}(V_R)_{a1} \cdot \zeta - \frac{3}{2}\theta_{a1} M_1 \right|^2. \tag{26}$$

Here, α is the fine-structure constant, and m_{l_a} is the mass of the charged lepton propagating in the loop. The second term in the amplitude is proportional to the mass of the sterile neutrino M_1, while the first term to the mass of the charged intermediate lepton m_{l_a}. This can be understood from the following consideration. Because the photon has spin one, there must be a chirality flip on the fermionic line. If, in flavor basis, there is a W_R-W_L mixing, we have the chirality flip (mass insertion) on the internal line of the charged fermion, which produces a term proportional to m_{l_a}. Otherwise, the chirality flip happens on one of the outer lines of the diagram, with a term proportional to the Majorana mass of the incoming sterile neutrino.

If the gauge boson mixing is absent, $\zeta = 0$, only the second term contributes and we obtain the usual result

$$\Gamma_{N_1\to\gamma\nu}|_{\zeta=0} \simeq \frac{9G_F^2 \alpha M_1^5}{256\pi^4} \times \theta_1^2. \tag{27}$$

If the mixing ζ is present, then, barring the unlikely cancellation between the two terms in Eq. (26), we can constrain ζ using the x-ray bound (25)

$$\zeta^2 \lesssim 9 \times 10^{-19} \frac{m_{l_\tau}^2}{\sum_{a=e,\mu,\tau} |m_{l_a}(V_R)_{a1}|^2} \left(\frac{\text{keV}}{M_1}\right)^3. \quad (28)$$

Thus, the mixing angle of the W bosons must be vanishingly small.

We would like to note that in a LR symmetric model with the Higgs sector as described in Sec. IV, the W_L-W_R mixing angle ζ is given by (see Ref. [11], and references therein)

$$\tan(2\zeta) \simeq -\frac{2\kappa_1\kappa_2}{v_R^2}, \quad (29)$$

where κ_1 and κ_2 are bidoublet vacuum expectation values (VEVs) and v_R is the VEV of the right Higgs triplet. Therefore Eq. (28) restricts the ratio $\kappa_1\kappa_2/v_R^2$.

G. Summary of constraints

Let us summarize this section. A theory where the DM sterile neutrino was in thermal equilibrium at some moment during the evolution of the Universe should satisfy the following set of constraints:

(i) From X/γ-ray observations, we have the model independent upper limit on the radiative decay width of the DM neutrino N_1 [see Eq. (25)]:

$$\Gamma_{N\to\gamma\nu} \lesssim 9.9 \times 10^{-27}\ \text{sec}^{-1}. \quad (30)$$

Note that this is a conservative value; c.f. footnote [3]. This limit translates to the limit on the sterile-active neutrino mixing angle Eq. (24) and to the bound on the mixing between the left and right gauge bosons Eq. (28).

(ii) From the structure formation requirements (Ly-α bound), the mass of the sterile neutrino is constrained in the same way as the mass of a thermal relic, i.e.

$$M_1 \gtrsim 1.6\ \text{keV}. \quad (31)$$

(iii) The right abundance of the sterile neutrino can be then achieved by an out-of-equilibrium decay of a long-lived heavy particle. We will use another sterile neutrino of the model, N_2, for this purpose, but most considerations here can be also applied to another long-lived particle present in the theory.

To provide proper the entropy dilution, Eq. (14), N_2 should decouple while relativistic and has decay width

$$\Gamma \simeq 0.50 \times 10^{-6} \frac{g_N^2}{4} \frac{g_{*\text{f}}^2}{g_*^2} \bar{g}_*^{1/2} \frac{M_2^2}{M_{\text{Pl}}} \left(\frac{1\ \text{keV}}{M_1}\right)^2. \quad (32)$$

(iv) At the same time, the heavy neutrino N_2 should decay before BBN, which bounds its lifetime to be shorter than approximately $0.1 \div 2$ s. Then, the proper entropy can be generated only if its mass is larger than

$$M_2 > \left(\frac{M_1}{1\ \text{keV}}\right)(1.7 \div 10)\ \text{GeV}. \quad (33)$$

(v) The entropy is effectively generated by out-of-equilibrium decay (see Sec. II E), if the particle decoupled while still relativistic. If this particle is one of the sterile neutrinos, then its decoupling happens at temperature (10), and it leads to the bound on the right-handed gauge boson mass

$$M > \frac{1}{g_{*\text{f}}^{1/8}} \left(\frac{M_2}{1\ \text{GeV}}\right)^{3/4} (10 \div 16)\ \text{TeV}. \quad (34)$$

Note that this is the only requirement which changes in the case of entropy generated by some other particle instead of the heavy sterile neutrino.

III. MODELS WITH LOW SCALE TYPE I SEE-SAW

Let us start from the analysis of the models where the active neutrino masses are generated by a "type I" seesaw formula. This means that $M_L = 0$ in the neutrino mass matrix (2). The mixing angles (7) are bounded from above by the requirements on the decay width of the sterile neutrinos—by the x-ray observations for the DM neutrino angle θ_1 [see Eq. (24)], and by the long enough lifetime of the entropy generating neutrino angle θ_2 (additional generation of the entropy by the third neutrino does not change conclusions). A convenient way to parametrize the Dirac mass matrix m_D, separating parameters in the active and sterile neutrino sectors, is provided by the Casas and Ibarra parametrization [35] reviewed in Appendix C. Using Eq. (C9) for the Dirac masses,[4] we get

$$\theta_I^2 = \frac{[\sqrt{M_R} R^T m_\nu^{\text{diag}} R^\star \sqrt{M_R}]_{II}}{M_I^2}, \quad (35)$$

with

$$m_\nu^{\text{diag}} = \text{diag}(m_1, m_2, m_3). \quad (36)$$

Here, R is a complex orthogonal matrix, describing the details of the mixing between the sterile and active sectors, and it can be parametrized by three complex angles ω_{12}, ω_{13}, and ω_{23} as in Eq. (C8). Let us check whether we can

[4] As far as we are using in this section the basis with diagonal M_R, the right-handed mixing matrix is trivial, $V_R = I$.

satisfy the bounds on the mixing angles if the active masses m_i are consistent with the observed neutrino oscillation mass differences, summarized below. The current best-fit and 3σ ranges are (see Ref. [36])

$$\Delta m^2_{\rm sol} = (7.65^{+0.69}_{-0.6}) \times 10^{-5}\ {\rm eV}^2, \tag{37a}$$

$$\Delta m^2_{\rm atm} = (2.4^{+0.35}_{-0.33}) \times 10^{-3}\ {\rm eV}^2. \tag{37b}$$

In the following discussion, we will for convenience order the active neutrino masses as $m_1 < m_2 < m_3$. From Eq. (35), we get for the first two sterile neutrinos

$$M_1\theta_1^2 = m_3|\sin\omega_{13}|^2 + m_2|\cos\omega_{13}|^2|\sin\omega_{12}|^2 + m_1|\cos\omega_{13}|^2|\cos\omega_{12}|^2, \tag{38a}$$

$$M_2\theta_2^2 = m_3|\cos\omega_{13}|^2|\sin\omega_{23}|^2 + m_2|\cos\omega_{23}\cos\omega_{12} - \sin\omega_{23}\sin\omega_{13}\sin\omega_{12}|^2 + m_1|\cos\omega_{23}\sin\omega_{12} + \sin\omega_{23}\sin\omega_{13}\cos\omega_{12}|^2. \tag{38b}$$

Note that as far as we ordered the active neutrino masses, if we change m_1 to zero, and replace m_3 by m_2, the right-hand sides of Eqs. in (38) can only become smaller. We can also confine ourselves to the real values of the mixing angles, as far as the sine and cosine absolute values only become larger for complex angles, and the inequality $|z - w| \geq ||z| - |w||$ is used to transform the square of the difference of the angles in Eq. (38b). Thus, the following inequalities should be satisfied:

$$M_1\theta_1^2 \geq m_2\{\sin^2\omega_{13} + \cos^2\omega_{13}\sin^2\omega_{12}\}, \tag{39a}$$

$$M_2\theta_2^2 \geq m_2\{\cos^2\omega_{13}\sin^2\omega_{23} + (|\cos\omega_{23}||\cos\omega_{12}| - |\sin\omega_{23}||\sin\omega_{13}||\sin\omega_{12}|)^2\}. \tag{39b}$$

The minimum of the sum of the right-hand sides is m_2, and therefore the following very simple inequality always holds:

$$M_1\theta_1^2 + M_2\theta_2^2 \geq m_2 \geq \Delta m_{\rm sol}. \tag{40}$$

The second inequality is trivially fulfilled, since in all possible mass hierarchies the mass of the second (in mass) active neutrino is larger than $\Delta m_{\rm sol}$. The meaning of the inequality (40) is very simple—one cannot generate active neutrino masses with type I seesaw formula without sufficient mixings between the active and sterile neutrino sectors. Note in passing that the cancellation is possible in another direction—one can have very small active neutrino masses and large active-sterile mixings.

Now, we are ready to compare the requirement from the observed active neutrino masses, Eq. (40), and the DM bounds on the mixings. The angle θ_2 can be bound from the width required to generate sufficient entropy, Eq. (20). Estimating the width of the heavy neutrino as (see Appendix A)

$$\Gamma_{N_2} \geq \frac{G_F^2 M_2^5}{192\pi^3}\cdot\theta_2^2, \tag{41}$$

we have

$$M_2\theta_2^2 \lesssim 1.8\times 10^{-3}\bar{g}_*^{1/2}\left(\frac{\rm GeV}{M_2}\right)^2\left(\frac{\rm keV}{M_1}\right)^2. \tag{42}$$

It can be clearly seen that for all possible masses M_1 and M_2, this is much smaller than $\Delta m_{\rm sol}$.

The contribution of the DM sterile neutrino itself can be larger. From Eq. (24), we have

$$M_1\theta_1^2 \lesssim 1.8\times 10^{-2}\left(\frac{1\ {\rm keV}}{M_1}\right)^4. \tag{43}$$

Together with the Ly-α bound on the WDM mass, Eq. (16), this contribution again violates Eq. (40). Thus, we conclude that the small mixing angles, required by the proper DM abundance and good DM properties in the model, prevent generation of the observed active neutrino masses by the type I seesaw formula.[5]

IV. TYPE II SEESAW—WORKING EXAMPLE

In the previous section, we have seen that if one of the not DM-like sterile neutrinos is responsible for entropy production, it is impossible to get the observed active neutrino masses with the type I like seesaw. Here, we will present a working example of the sterile neutrino DM in the framework of a LR symmetric model, where the active neutrino masses are generated by the contribution of the type II seesaw.

We will continue to work in the framework of the $SU(3)\times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ model, sketched in Sec. II A. Here, we will concentrate on a properly LR symmetric model, where the left- and right-handed leptons are treated symmetrically. One has the usual SM doublets ψ_L^i; $i = 1, 2, 3$, and in addition 3 right-handed neutrinos which form together with the 3 charged right-handed leptons the $SU(2)_R$ doublets ψ_R^i. The Higgs sector consists of one $SU(2)_L$ triplet, one $SU(2)_R$ triplet, and one bidoublet. In such a model, the mass matrix for the neutrinos has the pattern

$$\mathcal{M} = \begin{pmatrix} f_L v_L & yv \\ y^T v & f_R v_R \end{pmatrix} = \begin{pmatrix} M_L & m_D \\ m_D^T & M_R \end{pmatrix}, \tag{44}$$

where the Majorana blocks on the diagonal come from the coupling of $\psi_L^{i\,T}\mathcal{C}\psi_L^j$ and $\psi_R^{i\,T}\mathcal{C}\psi_R^j$ with the triplets $\Delta_{L,R}$, respectively, and the Dirac-type ones from the coupling of $\bar{\psi}_L^i\psi_R^j$ with the bidoublet ϕ and its complex conjugate $\tilde{\phi} = \tau_2\phi^\star\tau_2$. The VEVs of the neutral components in $\Delta_{L,R}$ are called $v_{L,R}$; whereas, the SM scale $v = \sqrt{\kappa_1^2+\kappa_2^2} =$ 174 GeV is a combination of the bidoublet VEVs κ_1 and κ_2. These VEVs are related by the expression

[5] Note, however, that without the Ly-α bound it would have been possible for very light WDM, with $M_1 < 1.2$ keV.

$$x \equiv \frac{v_L v_R}{v^2}, \tag{45}$$

where x is a function of the parameters in the Higgs potential, which is naturally of order one (for more details, see e.g. Ref. [11]).

In the following, we postulate *exact* discrete LR symmetry. In general, it can be realized in two different ways: as a $\mathcal{C}$ conjugation or as a parity symmetry. In the former case, it is required that $f_L = f_R$, $y = y^T$, and this is what we use.

With such a model, it is now possible to satisfy all the requirements from Sec. II. Let us consider the type II seesaw formula following from block diagonalization of Eq. (44) and the assumption $\mathcal{O}(M_R) \gg \mathcal{O}(m_D) \gg \mathcal{O}(M_L)$:

$$m_\nu = v_L f_L - \frac{v^2}{v_R} y f_R^{-1} y^T. \tag{46}$$

After applying the conditions of discrete left-right symmetry, $y = y^T$ and $f \equiv f_L = f_R$, one arrives at

$$m_\nu = v_L f - \frac{v^2}{v_R} y f^{-1} y. \tag{47}$$

To simplify the calculations, we further assume for illustration that the Dirac-Yukawa y is proportional to the triplet Yukawa f, i.e. $y = pf$, where p is a number. Equation (47) then goes into

$$m_\nu = \left(v_L - \frac{v^2 p^2}{v_R}\right) f. \tag{48}$$

In this case, all Yukawas are diagonalized by the same transformation—the transformation which brings m_ν into diagonal form, i.e. the PMNS matrix. The ratios of the eigenvalues of the matrices on both sides of the equality are then the same

$$\frac{m_1}{m_2} = \frac{f_1}{f_2} = \frac{M_1}{M_2}. \tag{49}$$

Thus, the mass spectrum of the sterile neutrinos [or, specifically, the BBN requirement (22)] leads to the same hierarchical active neutrino spectrum

$$\frac{m_1}{m_2} \lesssim 5.9 \times 10^{-7}. \tag{50}$$

This implies that the lightest active neutrino should be very light, and we can have either normal or inverse hierarchy. For definiteness, we will use the normal hierarchy for our example, though the inverse one works equally well (one should only take into account that in the latter case the $M_2 \simeq M_3$, $\Gamma_2 \simeq \Gamma_3$ and both N_1 and N_2 generate the same amount of entropy). As far as the active neutrino mass hierarchy is fixed, we have $m_2 \simeq \Delta m_{\rm sol}$ and $m_3 \simeq \Delta m_{\rm atm}$. We can then get the mass for the third sterile neutrino from

$$M_3 = \frac{m_3}{m_2} M_2. \tag{51}$$

The active-sterile mixing angles in the case of proportional Yukawa constants are all the same and equal to

$$\theta_1^2 = \theta_2^2 = \theta_3^2 = \frac{v^2 p^2}{v_R^2}, \tag{52}$$

while the mixing angles for individual flavors are proportional to the PMNS matrix

$$\theta_{\alpha I} = (U^\star)_{\alpha I} \frac{vp}{v_R}. \tag{53}$$

Thus the decay width Γ_2 is proportional to θ_2^2 (see Appendix A). The value of θ_2^2 is then defined from the requirement of the sufficient entropy production, Eq. (20), and depends only on M_1 and M_2.

At this moment the only free parameter left is the VEV ratio x, and everything can be expressed via x, M_1, M_2, and $m_2 \simeq \Delta m_{\rm sol}$, $m_3 \simeq \Delta m_{\rm atm}$. From Eqs. (48) and (45), we get

$$v_R = \sqrt{\frac{v^2 x}{\frac{m_2}{M_2} + \theta_2^2}}. \tag{54}$$

The VEV of the left-handed triplet Δ_L is then given by

$$v_L = \sqrt{v^2 x \left(\frac{m_2}{M_2} + \theta_2^2\right)}. \tag{55}$$

Together with Eq. (54), Eq. (52) determines the proportionality constant p:

$$p = \sqrt{\theta_2^2 v_R^2 / v^2}. \tag{56}$$

The full mass matrix (44) is then given by

$$\mathcal{M} = \begin{pmatrix} U^\star & 0 \\ 0 & U^\star \end{pmatrix} \begin{pmatrix} M_L^{\rm diag} & m_D^{\rm diag} \\ m_D^{\rm diag} & M_R^{\rm diag} \end{pmatrix} \begin{pmatrix} U^\dagger & 0 \\ 0 & U^\dagger \end{pmatrix}, \tag{57}$$

where

$$m_D^{\rm diag} = p \frac{v}{v_L} M_L^{\rm diag} = p \frac{v}{v_R} M_R^{\rm diag}, \tag{58a}$$

$$M_L^{\rm diag} = \frac{v_L}{v_R} M_R^{\rm diag}, \tag{58b}$$

and

$$M_R^{\rm diag} = {\rm diag}(M_1, M_2, M_3). \tag{59}$$

Let us fix now the input values. It was mentioned before that $x \sim \mathcal{O}(1)$ is natural in the LR symmetric model, therefore we simply choose $x = 1$. For the masses of the DM and the entropy producing sterile neutrinos, we take the smallest possible ones (see Sec. II):

$$M_1 = 1.6 \text{ keV}, \tag{60a}$$

$$M_2 = 2.7 \text{ GeV}. \tag{60b}$$

With this input, we obtain

$$
\begin{aligned}
m_1 &= 5.2 \times 10^{-9}\ \text{eV},\\
m_2 &\simeq \sqrt{\Delta m^2_{\text{sol}}} = 8.7 \times 10^{-3}\ \text{eV},\\
m_3 &\simeq \sqrt{\Delta m^2_{\text{atm}}} = 4.9 \times 10^{-2}\ \text{eV},\\
M_3 &= 15.1\ \text{GeV},\\
\theta_1^2 &= \theta_2^2 = \theta_3^2 = 2.3 \times 10^{-15},\\
v_R &= 9.67 \times 10^4\ \text{TeV},\\
v_L &= 313\ \text{keV},\\
p &= 0.027.
\end{aligned} \tag{61}
$$

We also plot the values of θ_1^2 and v_R for several M_1 and $M_2 \gtrsim 2.4$ GeV in the Figs. 3 and 4. Because of the smallness of θ_2^2 compared to m_2/M_2 and its suppression with M_1^2 or M_2^3 ([see Eq. (42)], v_R given by Eq. (54) is effectively independent of θ_2^2. Therefore, the curve of v_R has only a very weak M_1 dependence. However, for bigger M_1 one has to account for the BBN bound (22) on M_2.

One can check that none of the bounds, summarized in Sec. II G is violated. Also the mixing angle θ_1^2 corresponding to our DM neutrino is much lower than its upper bound, Eq. (24). However, we should also choose the Higgs potential to have very small mixing between the left and right gauge bosons [see Eqs. (28) and (29)].

The right-handed scale v_R is large, and the additional gauge and Higgs bosons are not observable (they all have masses $\propto v_R$). The famous ρ parameter

$$\rho \equiv \frac{M_W^2}{M_Z^2 \cos^2\theta_W} = 1, \tag{62}$$

which is equal to 1 at tree level in SM also gets a negligible correction which is equal to

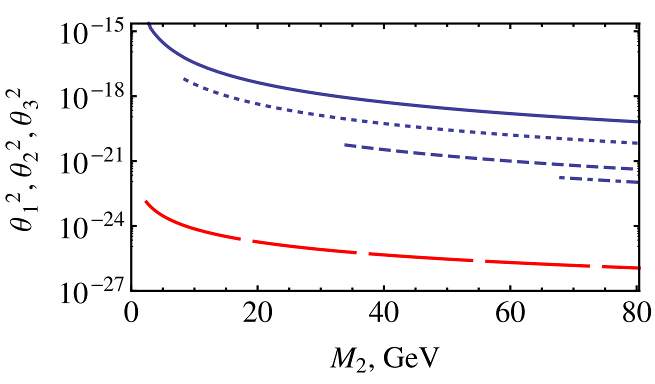

FIG. 3 (color online). θ_2^2 as a function of M_2. $M_1 = 1.6$ keV (continuous); $M_1 = 5$ keV (dotted); $M_1 = 20$ keV (dashed) and $M_1 = 40$ keV (dashed-dotted). It is accounted for the lower bound, Eq. (22). The long-dashed (red) line shows the ratio $(M_W/M)^4$ and illustrates that processes mediated by W_R bosons can be neglected in the decay rate of N_2.

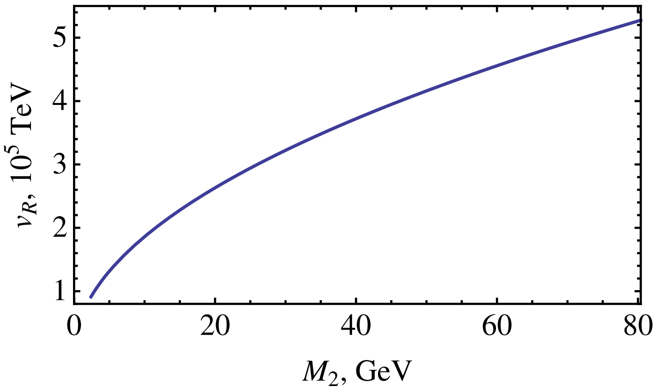

FIG. 4 (color online). v_R as a function of M_2. The dependence on M_1 is very weak.

$$\rho = \frac{v^2 + 2|v_L|^2}{v^2 + 4|v_L|^2} \tag{63}$$

in the LR symmetric model [37]. For the small v_L of the order of MeV, the deviations are well below the current experimentally allowed deviation of the order $\mathcal{O}(10^{-4})$ [38].

V. CONCLUSIONS

In this paper, we analyzed the possibility to have a keV scale sterile neutrino warm dark matter in gauge extensions of the standard model. We found that it is possible to circumvent the naïve expectation of significant overproduction of dark matter in case of a light particle (sterile neutrino) decoupling from the thermal equilibrium while still relativistic. The possible ways out include a low reheating temperature (so that the thermal equilibrium is never reached by the would be DM sterile neutrino), (very) large number of degrees of freedom in the early universe at the DM neutrino freeze-out, or subsequent dilution of its density by the out-of-equilibrium decay of a massive particle (another sterile neutrino). We further analyze this last possibility as being the most natural[6] and formulate a set of requirements for this scenario. In short, these requirements bound the mass of the DM sterile neutrino from below from structure formation considerations, limit its mixing angle with active neutrinos and constrain mixing between the SM (left) and additional (right) gauge bosons from the radiative decay of the DM sterile neutrino, fix the lifetime of the heavier sterile neutrino from the requirement of the dilution of the DM abundance down to the observed value, and finally constrain the mass of these heavier sterile neutrinos from the big bang nucleosynthesis considerations.

[6]Another natural possibility is achieved in the νMSM model [6,7], where the sterile neutrinos are the only extension of the SM, and then the keV sterile neutrino does not enter thermal equilibrium up to Planck scale temperatures.

We demonstrated in this scenario that the type I low scale seesaw mechanism of generating masses for the active neutrino can not lead to sufficient dilution of the DM abundance. At the same time, we provide a working example, where the active neutrinos are generated by a type II style seesaw in the context of an exactly LR symmetric theory. The provided general constraints and observations can serve as a basis for the search of a grand unified theory with WDM sterile neutrinos.

ACKNOWLEDGMENTS

F. B. thanks M. Shaposhnikov and S. Sibiryakov for helpful comments and O. Ruchayskiy for discussion on the Lyman-α bound. This work has been supported in part by the DFG Grant No. SFB-TR27 "Neutrinos and Beyond."

APPENDIX A: DECAY WIDTHS OF A STERILE NEUTRINO

In the mass range 2.7 GeV $\leq M_2 < M_W$, a sterile neutrino N_2 dominantly decays into leptons and spectator quarks. The corresponding partial widths can be calculated in the νMSM, because additional boson interactions, which usually appear in more complicated models, are of high scale—of order $\mathcal{O}(M)$—compared to that of the electroweak scale. To make use of the νMSM results, we have to compare, to be on the safe side, the suppression factors in the νMSM $\sim\mathcal{O}(|\theta_i|^2)$ with that of additional interactions $\sim\mathcal{O}(M_W/M)^4$ which appear in the models we are interested in. Furthermore, the mixing of the new bosons should be small compared to $\mathcal{O}(|\theta_i|)$; otherwise, there could be significant contributions from processes where new bosons mix with the SM ones (see, for example, Appendix B). These effects are neglected in the following calculations. One can see that for most practical purposes it is the case, as far as the bound on the gauge boson mixings (28) is much stronger then those for the active-sterile neutrino mixings.

Moreover, these additional contributions do not affect the conclusions in the main part of the article. Really, in Sec. III additional interactions can only result in a stronger bound, and therefore the conclusion remains the same. In the type II seesaw model discussed in Sec. IV, we need the exact value of the width. However, if we assume no mixing of the W bosons ($\zeta = 0$), the contributions of the W_R boson mediated processes are in the considered mass range negligible small (cf. Figure 3).

In the following, we give all relevant formulas for the decay rates (at tree level) of a sterile neutrino N_2 with a mass M_2 above the BBN bound 2.7 GeV [cf. (22)] and below the SM W-boson mass $M_W \simeq 80$ GeV [39]:

$$\Gamma_1\Big(N_2 \to \sum_{\alpha,\beta} \nu_\alpha \bar{\nu}_\beta \nu_\beta\Big) = \frac{G_F^2 M_2^5}{192\pi^3} \cdot \sum_\alpha |\theta_{2\alpha}|^2, \tag{A1a}$$

$$\Gamma_2(N_2 \to l^-_{\alpha\neq\beta} l^+_\beta \nu_\beta) = \frac{G_F^2 M_2^5}{192\pi^3} \cdot |\theta_{2\alpha}|^2 (1 - 8x_l^2 + 8x_l^6 - x_l^8 - 12x_l^4 \log x_l^2), \qquad x_l = \frac{\max[M_{l_\alpha}, M_{l_\beta}]}{M_2}, \tag{A1b}$$

$$\begin{aligned}\Gamma_3(N_2 \to \nu_\alpha l^+_\beta l^-_\beta) = \frac{G_F^2 M_2^5}{192\pi^3} \cdot |\theta_{2\alpha}|^2 \cdot [(C_1 \cdot (1 - \delta_{\alpha\beta}) + C_3 \cdot \delta_{\alpha\beta})((1 - 14x_l^2 - 2x_l^4 - 12x_l^6)\sqrt{1 - 4x_l^2} \\ + 12x_l^4(x_l^4 - 1)L) + 4(C_2 \cdot (1 - \delta_{\alpha\beta}) + C_4 \cdot \delta_{\alpha\beta})(x_l^2(2 + 10x_l^2 - 12x_l^4)\sqrt{1 - 4x_l^2} \\ + 6x_l^4(1 - 2x_l^2 + 2x_l^4)L)],\end{aligned} \tag{A1c}$$

with

$$L = \log\left[\frac{1 - 3x_l^2 - (1 - x_l^2)\sqrt{1 - 4x_l^2}}{x_l^2(1 + \sqrt{1 - 4x_l^2})}\right], \qquad x_l \equiv \frac{M_l}{M_2},$$

and

$$\begin{aligned}C_1 &= \tfrac{1}{4}(1 - 4\sin^2\theta_w + 8\sin^4\theta_w),\\ C_2 &= \tfrac{1}{2}\sin^2\theta_w(2\sin^2\theta_w - 1),\\ C_3 &= \tfrac{1}{4}(1 + 4\sin^2\theta_w + 8\sin^4\theta_w),\\ C_4 &= \tfrac{1}{2}\sin^2\theta_w(2\sin^2\theta_w + 1).\end{aligned}$$

The formulas for the decay modes into quarks are presented below. In the range 2.7 GeV $\leq M_2 < M_W$, it is sufficient to use the free quark approximation for the decay products. We give these formulas in the approximation where M_2 is much heavier than the decay product masses (unlike above [Eqs. (A1)] for the lepton decays). The corrections are important at the threshold, when new decay channels open. However, at high mass M_2, this introduces a rather small relative error, because the number of open channels into light particles is significant and provides the main part of the decay width. The exact analysis would smooth the discontinuities of the decay width at the mass thresholds (Fig. 5):

$$\Gamma_4(N_2 \to l^-_\alpha U\bar{D}) = \frac{G_F^2 M_2^5}{192\pi^3} \cdot 3 \cdot |V_{UD}|^2 \cdot |\theta_{2\alpha}|^2, \tag{A2a}$$

$$\Gamma_5(N_2 \to \nu_\alpha q\bar{q}) = \frac{G_F^2 M_2^5}{192\pi^3} \cdot 3 \cdot \Xi^q \cdot |\theta_{2\alpha}|^2, \tag{A2b}$$

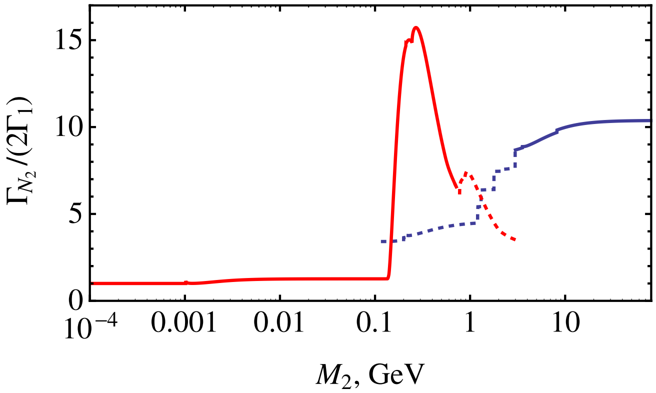

FIG. 5 (color online). The ratio $\Gamma_{N_2}/2\Gamma_1$ calculated in the specific LR model of Sec. IV. We used Eq. (A1) together with the formulas for the two-body decays into mesons [39] for the left (red) curve and Eq. (A1) together with Eq. (A2) for the right (blue) curve. The dotted curves correspond to the region, where both approximations are not entirely reliable.

with

$$\Xi^q = (g_L^\nu)^2 \cdot ((g_L^q)^2 + (g_R^q)^2).$$

The factor 3 is the color factor and V_{UD} are the Cabibbo-Kobayashi-Maskawa–matrix elements. The coupling constants g_L and g_R correspond to the coupling of the Z boson to left- or right-handed particles, respectively. For a fermion f with weak isospin component I_3^f and charge q_f, one has

$$g_L^f = I_3^f - q_f \sin^2\theta_W, \tag{A3a}$$

$$g_R^f = -q_f \sin^2\theta_W. \tag{A3b}$$

Table I gives the required values of the charges. In the mass range of M_2 mentioned above, the Majorana neutrino total decay rate Γ_{N_2} is a sum of all rates presented above multiplied by a factor of 2, which accounts for charge-conjugated decay modes.

When M_2 exceeds the SM Z-boson mass $M_Z = 91$ GeV and if contributions of new interactions are negligible, the sterile neutrino predominantly decays into a SM gauge boson and a lepton. Then its total decay width is given by

$$\Gamma_{N_2} = 2\frac{G_F M_2^3}{8\sqrt{2}\pi}\left[\left(1+\frac{2M_W^2}{M_2^2}\right)\left(1-\frac{M_W^2}{M_2^2}\right)^2 + \frac{1}{2}\left(1+\frac{2M_Z^2}{M_2^2}\right)\left(1-\frac{M_Z^2}{M_2^2}\right)^2\right]\cdot\sum_\alpha|\theta_{2\alpha}|^2. \tag{A4}$$

TABLE I. Coupling constants.

fermions	g_L^q	g_R^q
ν_e, ν_μ, ν_τ	$g_L^\nu = \frac{1}{2}$	$g_R^\nu = 0$
$U = u, c, t$	$g_L^U = \frac{1}{2} - \frac{2}{3}s_W^2$	$g_R^U = -\frac{2}{3}s_W^2$
$D = d, s, b$	$g_L^D = -\frac{1}{2} + \frac{1}{3}s_W^2$	$g_R^D = \frac{1}{3}s_W^2$

In between, where $M_W \le M_2 < M_Z$, one can approximate the width by the first term in Eq. (A4).

Below $M_2 \sim 2$ GeV, it is important to consider mesons instead of quarks as final states for the sterile neutrino decay. Therefore, instead of the three-body decay modes into spectator quarks (A2), one has to use the corresponding two-body ones into mesons; see [39] for the decay width formulas. In our case, this mass range of M_2 is forbidden by the BBN bound (22) and therefore we do not list them here. Nevertheless, to get a feeling of the behavior of the total width Γ_{N_2}, we show in Fig. 5 the ratio $\Gamma_{N_2}/2\Gamma_1$ calculated in the specific model described in Sec. IV, using both the free quark and chiral meson approximations. It is clearly seen that the transition between two approximations happens around 1 GeV. Because of Eq. (53), the total decay width is proportional to θ_2^2 and therefore the plotted ratio is independent of this quantity. In more general models, Eq. (53) will no longer be valid. However, as one recognizes by considering the formulas together with the definition of θ_2 [cf. Equation (7)], there will be no significant difference, especially for heavy masses M_2, so that Fig. 5 can be used as a good estimate in such models. Note that in the region $M_2 \sim \mathcal{O}(1)$ GeV (dotted in Fig. 5), decays into spectator quarks more and more replace decays into mesons and therefore one has to carefully reanalyze the given formulas, if one is interested in this mass range.

APPENDIX B: RADIATIVE DECAY WIDTH

Here, we give some details of calculation of the width for the radiative decay $N_1 \to \gamma\nu_i$ shown in Fig. 2. We will follow Ref. [33], where general formulas for this type of process are given. In our case, N_1 denotes a heavy sterile neutrino with mass M_1, ν_i one of the active neutrinos with mass m_i, and γ a photon. The neutrinos are considered as Majorana particles.

The amplitude for such a decay is $e\epsilon_\mu^\star(q)\mathcal{M}^\mu$, where e is the electric charge of the positron and $\epsilon_\mu^\star(q)$ the polarization vector of the outgoing photon. The Ward identity for the electromagnetic current implies that $q_\mu\mathcal{M}^\mu$ must be zero; therefore $\mathcal{M}^\mu$ must have the form

$$\mathcal{M}^\mu = \bar{u}_i[i\sigma^{\mu\nu}q_\nu(\sigma_L L + \sigma_R R)]u_1, \tag{B1}$$

where $\sigma^{\mu\nu} = (i/2)[\gamma^\mu, \gamma^\nu]$, $L = (1-\gamma_5)/2$, and $R = (1+\gamma_5)/2$ are the projectors of chirality. σ_L and σ_R are numerical coefficients with dimension of inverse mass. The partial decay width for $N_1 \to \nu_i\gamma$ is then given by

$$\Gamma_{N_1\to\gamma\nu_i} = \frac{(M_1^2 - m_i^2)^3}{16\pi M_1^3}(|\sigma_L|^2 + |\sigma_R|^2). \tag{B2}$$

By comparing the Lagrange term for the charged current (1) combined with Eq. (8) and the transformation rule which diagonalizes the neutrino mass matrix in Eq. (2)

$$\begin{pmatrix} \tilde{\nu}_{aL} \\ \tilde{N}^c_{aR} \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} \nu_{iL} \\ N^c_{IR} \end{pmatrix}, \tag{B3}$$

with that given in Chapter 5 of [33], we can easily calculate the coefficients σ_L and σ_R. Supposing from the very beginning, that the right-handed scale is much larger than the left one, $M \gg M_W \simeq 80.4$ GeV, and neglecting the active neutrino masses, we get[7]

$$i\sigma_R = \frac{g^2 e}{32 M_W^2 \pi^2} \times \sum_{a=e,\mu,\tau} \{\cos\zeta \sin\zeta A^\star_{ai} D^\star_{a1} m_{l_a} \mathcal{F}(r_a) + \cos^2\zeta A^\star_{ai} B_{a1} M_1 F(r_a)\}, \tag{B4a}$$

$$i\sigma_L = \frac{g^2 e}{32 M_W^2 \pi^2} \times \sum_{a=e,\mu,\tau} \{\cos\zeta \sin\zeta C_{ai} B_{a1} m_{l_a} \mathcal{F}(r_a)\}, \tag{B4b}$$

where $F(r_a)$ and $\mathcal{F}(r_a)$ are functions of $r_a \equiv m^2_{l_a}/M_W^2$. In our case, we have in good approximation $F(r_a) \simeq -3/2$ and $\mathcal{F}(r_a) \simeq 4$. The exact expressions for these functions were calculated by us and do agree with that given in Ref. [40].

Because of the Majorana nature of our ingoing and outgoing neutrinos, we also have to add the contribution of the complex conjugated process to our amplitude. This is easily obtained out of Eq. (B4) by putting in the substitutions

$$\begin{gathered} A, B \to A^\star, B^\star \quad \text{and} \quad C, D \to C^\star, D^\star, \\ \gamma_5 \to -\gamma_5 \Rightarrow L, R \to R, L, \end{gathered} \tag{B5}$$

and an overall negative sign coming from the photon vertex. After adding the derived σ_L and σ_R, it is easy to see that $|\sigma_L|^2 = |\sigma_R|^2$, where

$$|\sigma_L|^2 = \left(\frac{g^2 e}{32 M_W^2 \pi^2}\right)^2 \times \left| 4\cos\zeta \sin\zeta \sum_{a=e,\mu,\tau} (A_{ai} D_{a1} - C_{ai} B_{a1}) m_{l_a} - \frac{3}{2}\cos^2\zeta \left(\sum_{a=e,\mu,\tau} A_{ai} B^\star_{a1}\right) M_1 \right|^2. \tag{B6}$$

By putting this into Eq. (B2), we obtain

$$\Gamma_{N_1 \to \gamma\nu_i} \simeq \frac{G_F^2 \alpha M_1^3}{64\pi^4} \times \left| 4\cos\zeta \sin\zeta \sum_{a=e,\mu,\tau} (A_{ai} D_{a1} - C_{ai} B_{a1}) m_{l_a} - \frac{3}{2}\cos^2\zeta \left(\sum_{a=e,\mu,\tau} A_{ai} B^\star_{a1}\right) M_1 \right|^2. \tag{B7}$$

Here, G_F is the Fermi constant, α is the fine-structure constant, and m_{l_a} is the mass of the charged lepton propagating in the loop.

The total width of the radiative decay is given by

$$\Gamma_{N_1 \to \gamma\nu} = \sum_{i=1}^{3} \Gamma_{N_1 \to \gamma\nu_i}. \tag{B8}$$

In a model where a seesaw mechanism of type I or type II is responsible for the small active neutrino masses, the transformation (B3) is given by Eq. (3). Putting this into our formulas, we get out of Eq. (B7) the expression (26).

APPENDIX C: CASAS-IBARRA PARAMETRIZATION

In this part of the Appendix, we describe the approach of parametrizing the Dirac-Yukawa matrix, which was proposed by Casas and Ibarra [35]. Here, we want to give a short review of the generalised version which also applies to the type II seesaw mechanism [41].

Let us consider a Majorana mass matrix with the pattern

$$\begin{pmatrix} M_L & m_D \\ m_D^T & M_R \end{pmatrix} = \begin{pmatrix} f_L v_L & y v \\ y^T v & f_R v_R \end{pmatrix}. \tag{C1}$$

The type II seesaw formula can be written in the form

$$m_\nu - M_L = -m_D M_R^{-1} m_D^T, \tag{C2}$$

where m_ν is the active neutrino mass matrix. Let us define the symmetric and in general complex 3×3 matrix

$$X_\nu \equiv m_\nu - M_L. \tag{C3}$$

This matrix and M_R can be diagonalized by unitary transformations:

$$X_\nu = V_\nu^\star X_\nu^{\text{diag}} V_\nu^\dagger = [V_\nu^\star (X_\nu^{\text{diag}})^{1/2}][V_\nu^\star (X_\nu^{\text{diag}})^{1/2}]^T, \tag{C4a}$$

$$M_R = V_R^\star M_R^{\text{diag}} V_R^\dagger. \tag{C4b}$$

Multiplying Eq. (C2) by $[V_\nu^\star (X_\nu^{\text{diag}})^{1/2}]^{-1}$ from the left and by $\{[V_\nu^\star (X_\nu^{\text{diag}})^{1/2}]^T\}^{-1}$ from the right and using Eq. (C4), we find

$$I = RR^T, \tag{C5}$$

with

$$R = \pm i (X_\nu^{\text{diag}})^{-1/2} V_\nu^T m_D V_R (M_R^{\text{diag}})^{-1/2}. \tag{C6}$$

Equations (C5) and (C6) mean that the type II seesaw relation requires R to be a complex orthogonal matrix,

[7]Note, that our results do not coincide with the formulas in [40]. This is because of a mistake in the second term of the third line of Eq. (10) in [40]. The correct labeling of the transformation matrices should be $P_{aB} Q_{aA}$ instead of $P_{aA} Q_{aB}$. In our notations, where a sterile neutrino (with mass eigenstate index 1) decays through the radiative process into an active neutrino (with mass eigenstate index i), the expression $P_{aB} Q_{aA}$ translates into $B_{a1} C_{ai}$ which is contained in Eq. (B4b).

but otherwise does not constrain it. In this way we obtain for the Dirac-type Yukawa coupling in the basis where M_R is diagonal

$$m_D = vy = \pm i V_\nu^\star \sqrt{X_\nu^{\rm diag}} R \sqrt{M_R^{\rm diag}} V_R^\dagger, \qquad (C7)$$

where R is an arbitrary complex orthogonal matrix. It can be parametrized as

$$R = \pm R_{12} R_{13} R_{23}, \qquad (C8)$$

where R_{ij} is the matrix of rotation by a complex angle ω_{ij} in the ij plane. This is the so-called Casas-Ibarra parametrization of the Dirac-Yukawa [35]. Note that this parametrization has its origin in the difference of the number of high energy and low energy parameters. There are less low energy parameters, because the high energy ones are integrated out. The latter cannot influence the low energy theory, and therefore can be parametrized arbitrarily.

The formula for the type I seesaw can easily derived out of (C7). Because of $M_L = 0$, X_ν corresponds in this case to the active neutrino mass matrix m_ν. Thus the basis transformation matrix V_ν in Eq. (C4a) is the PMNS matrix U, and we arrive at

$$m_D = vy = \pm i U^\star \sqrt{m_\nu^{\rm diag}} R \sqrt{M_R^{\rm diag}} V_R^\dagger. \qquad (C9)$$

In chapter III, we make use of this parametrization.

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