Supersymmetric Lorentz Chern-Simons terms coupled to supergravity

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We present supersymmetric Lorentz Chern-Simons terms coupled to anti–de Sitter supergravity in three dimensions with an arbitrary number ($\aleph_0$) of supersymmetries. As an application to higher dimensions, we present analogous supersymmetric Lorentz Chern-Simons terms coupled to $N = 1$ supergravity in 11 dimensions.

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I. INTRODUCTION

There have been many developments recently in three-dimensional (3D) topological massive gravity with cosmological constant [1]. They include the black hole exact solution [2], and the inclusion of a Chern-Simons (CS) term [3], in order to overcome the old obstruction without the CS term [4]. Furthermore, interesting features such as Witten-Nester energy [5] in topological massive $N = 1$ supergravity have also been presented [6].

These developments indicate that there are many non-trivial aspects related to massive supergravity in three dimensions, both with simple ($N = 1$) and extended ($N \geq 2$) local supersymmetries, yet to be discovered in future studies. Motivated by this viewpoint, we study in this paper a supersymmetric Lorentz connection CS Lagrangian coupled to ${}^\forall N$ ($\aleph_0$) extended massive supergravity in three dimensions. Despite the arbitrary number of supersymmetries, we can show the action invariance, as well as the closure of supersymmetry.

As a natural application of this result, we also consider 11D supergravity with $N = 1$ supersymmetry [7], to which an 11D analog of the supersymmetric Lorentz CS term is coupled. In this case also, despite the tight system of the maximal $N = 1$ supergravity in 11 dimensions, we can show the invariance of the action of the new system, in addition to the closure of supersymmetry.

In the context of M-theory [8], there have been formulations of 11D supergravity in terms of M-algebra spanned by the $OSp(32|1)$ generators $(P_m, J_{mn}, Q_\alpha, Z_{m,n}, Z_{m_1\cdots m_5})$ [9]. Our approach in 11 dimensions is different in the sense that we do not introduce new generators like $Z_{m_1\cdots m_5}$ of M-algebra, but it is within the conventional 11D supergravity [7] with $N = 1$ local super Poincaré algebra (P_m, J_{mn}, Q_α). Our formulation in 11 dimensions is also different from the so-called CS supergravity [10], because our formulation is based on the usual Poincaré supergravity, instead of the supergroups in [10].

Our formulation in 11 dimensions is similar to a previous trial in [11]. However, there are two crucial differences from the latter: (i) We introduce a new field $\omega_\mu{}^{mn}$ as an *independent* field, and separate it from the original Cremmer-Julia-Scherk (CJS) Lagrangian [7]. (ii) The modification of the Lorentz connection field equation was not taken into account in [11], which was not a consistent treatment, because if higher-derivative curvature terms are present in the Lagrangian, the original *algebraic* field equation for $\omega_\mu{}^{mn}$ is *no longer algebraic*. In contrast, we analyze in our formulation the whole system consistently, relying on the so-called supersymmetric Palatini identity [12], as a generalization of the analogous identity in three or four dimensions [13,14].

Even though our result here gives only *pure* Lorentz CS theory in 11 dimensions, it has been shown that higher-dimensional pure CS theories have dynamical degrees of freedom [15], in contrast to the 3D case, where there is none. From this viewpoint, the construction of *pure* Lorentz CS theory in higher dimensions $D \geq 5$ has non-trivial significance for future applications.

II. $\aleph_0$ SUPERSYMMETRIC LORENTZ CS TERM IN THREE DIMENSIONS

We start with the result by Achucarro-Townsend [16], in which $OSp(p|2) \otimes OSp(q|2)$ symmetry with (p, q)-type anti–de Sitter (AdS) supergravity is realized. In our present paper, we concentrate on the $q = 0$ case. However, the generalization to $q \neq 0$ case is straightforward. Most importantly, the number of supersymmetries N is arbitrary, i.e., we are dealing with $\aleph_0$ supergravity.

Therefore, the pure AdS $\aleph_0$ massive supergravity Lagrangian is[1]

$$\begin{aligned}\mathcal{L}_{\aleph_0} = &+\frac{1}{4}eR(\hat{\omega}) - \frac{1}{4}\epsilon^{\mu\nu\rho}[\bar{\psi}_\mu{}^i\mathcal{R}_{\nu\rho}{}^i(\hat{\omega}, A)] \\ &+\frac{1}{2}g\epsilon^{\mu\nu\rho}\Big(F_{\mu\nu}{}^{ij}A_\rho{}^{ij} - \frac{2}{3}A_\mu{}^{ij}A_\nu{}^{jk}A_\rho{}^{ki}\Big) \\ &+\frac{1}{32}g^2e - \frac{1}{16}ge(\bar{\psi}_\mu{}^i\gamma^{\mu\nu}\psi_\nu{}^i), \end{aligned} \tag{2.1}$$

where $i, j, \ldots = 1, 2, \ldots, N$ is for ${}^\forall N$ supersymmetries

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[1]Our metric in this section is $(\eta_{mn}) = \mathrm{diag}(-, +, +)$.

with $SO(N)$ local gauge symmetry [16]. The $A_\mu{}^{ij}$ is the gauge field for $SO(N)$ with the antisymmetric indices ij.

The action $I_{\aleph_0} \equiv \int d^3 \mathcal{L}_{\aleph_0}$ is invariant under $^\forall N$ local supersymmetry [16]

$$\delta e_\mu{}^m = +(\bar\epsilon^i \gamma^m \psi_\mu{}^i), \tag{2.2a}$$

$$\begin{aligned}\delta \psi_\mu{}^i &= +\partial_\mu \epsilon^i + \frac{1}{4}\hat\omega_\mu{}^{rs}(\gamma_{rs}\epsilon^i) + gA_\mu{}^{ij}\epsilon^j - \frac{1}{8}g(\gamma_\mu \epsilon^i) \\ &= +D_\mu(\hat\omega)\epsilon^i + gA_\mu{}^{ij}\epsilon^j - \frac{1}{8}g(\gamma_\mu\epsilon^i) \\ &= +\mathcal{D}_\mu(\hat\omega, A)\epsilon^i - \frac{1}{8}g(\gamma_\mu \epsilon^i),\end{aligned} \tag{2.2b}$$

$$\delta_Q A_\mu{}^{ij} = -\frac{1}{2}(\bar\epsilon^{[i}\psi_\mu{}^{j]}). \tag{2.2c}$$

The gravitino field strength $\mathcal{R}_{\mu\nu}{}^i$ is defined by

$$\mathcal{R}_{\mu\nu}{}^i(\hat\omega, A) \equiv +\mathcal{D}_\mu(\hat\omega, A)\psi_\nu{}^i - \mathcal{D}_\nu(\hat\omega, A)\psi_\mu{}^i. \tag{2.3}$$

The supercovariant $\hat\omega_\mu{}^{rs}$ is defined as usual by

$$\begin{aligned}\hat\omega_{mrs} &\equiv +\frac{1}{2}(\hat C_{mrs} - \hat C_{msr} - \hat C_{rsm}), \\ C_{\mu\nu}{}^m &\equiv +2\partial_{[\mu}e_{\nu]}{}^m - (\bar\psi_\mu{}^i \gamma^m \psi_\nu{}^i).\end{aligned} \tag{2.4}$$

We are so far adopting the so-called second-order formalism [14] in which the Lorentz connection $\hat\omega_\mu{}^{mn}$ is *not* an independent variable, but is expressed in terms of $e_\mu{}^m$ and $\psi_\mu{}^i$. Our next step is to rewrite the Lagrangian (2.1) in terms of the Lorentz connection $\omega_\mu{}^{mn}$ as a new independent field variable. This is because we are going to introduce a new multiplet $(\omega_\mu{}^{rs}, \lambda^{rs})$ of Lorentz connection. This is also slightly different from going from the second-order formalism [17] to first-order formalism [18], because, as we will see in the final result, the supersymmetry transformation rule for $\omega_\mu{}^{mn}$ is different from that in the usual first-order formalism [18].

One way of doing this is to rely on the so-called supersymmetric Palatini identity, originally developed in four dimensions [13], but it is also valid in three dimensions. Its explicit form in three dimensions is

$$\begin{aligned}&+\frac{1}{4}eR(\omega+\tau) - \frac{1}{4}[\bar\psi_\mu{}^i\gamma^{\mu\nu\rho}\mathcal{R}_{\nu\rho}{}^i(\omega+\tau)] \\ &\equiv +\frac{1}{4}eR(\omega) - \frac{1}{4}[\bar\psi_\mu{}^i\gamma^{\mu\nu\rho}\mathcal{R}_{\nu\rho}{}^i(\omega)] \\ &\quad -\frac{1}{4}e\tau_{\mu\nu\rho}\tau^{\nu\rho\mu} - \frac{1}{4}e(\tau_\mu)^2 + \partial_\mu(eW^\mu),\end{aligned} \tag{2.5}$$

where the last term is a total divergence, and τ_{mrs} is arbitrary, as long as $\tau_{mrs} = -\tau_{msr}$.

Equation (2.5) is actually a rewriting of the original Palatini identity in three dimensions with $\hat\omega_\mu{}^{mn}$ analogous to the 4D case [13], into an alternative form with an independent field $\omega_\mu{}^{mn}$. This is possible, due to another identity relating a Lagrangian with $\hat\omega_\mu{}^{mn}(e, \psi)$ to an alternative Lagrangian with an independent $\omega_\mu{}^{mn}$. The nonsupersymmetric case of such an identity is given by Eq. (7) in [19]. In any case, Eq. (2.5) can be directly confirmed.

We can therefore choose

$$\tau_{mrs} = \hat\omega_{mrs} - \omega_{mrs}, \tag{2.6}$$

to get the identity

$$\begin{aligned}&+\frac{1}{4}eR(\hat\omega) - \frac{1}{4}\epsilon^{\mu\nu\rho}[\bar\psi_\mu{}^i\mathcal{R}_{\nu\rho}{}^i(\hat\omega, A)] \\ &\equiv +\frac{1}{4}eR(\omega) - \frac{1}{4}\epsilon^{\mu\nu\rho}[\bar\psi_\mu{}^i\mathcal{R}_{\nu\rho}{}^i(\omega, A)] + \frac{1}{16}e(\hat T_{\mu\nu}{}^m)^2 \\ &\quad -\frac{1}{8}e\hat T_{\rho\sigma\tau}\hat T^{\sigma\tau\rho} - \frac{1}{4}e(\hat T_\mu)^2 + \partial_\mu(eW^\mu).\end{aligned} \tag{2.7}$$

The $\hat T_{\mu\nu}{}^m$ is the supercovariant torsion tensor defined by

$$\hat T_{\mu\nu}{}^m \equiv 2(\partial_{[\mu}e_{\nu]}{}^m + \omega_{[\mu}{}^{mt}e_{\nu]t}) - (\bar\psi_\mu{}^i\gamma^m\psi_\nu{}^i), \tag{2.8}$$

where $\omega_\mu{}^{rs}$ is an independent field. The $\hat T_{mrs}$ is also related to τ, ω, and $\hat\omega$ by

$$\begin{aligned}\tau_{mrs} &= -\hat K_{mrs} \equiv +\frac{1}{2}(\hat T_{mrs} - \hat T_{msr} - \hat T_{rsm}), \\ \hat\omega_\mu{}^{rs} &= \omega_\mu{}^{rs} - \hat K_\mu{}^{rs},\end{aligned} \tag{2.9}$$

where $\hat K_{mrs}$ is supercovariant contorsion tensor. Because of the independent $\omega_\mu{}^{rs}$, the old *on-shell* equality, such as $\hat T_{\mu\nu}{}^m = 0$ is *no* longer valid. Equation (2.8) is manifestly covariant under local Lorentz symmetry in terms of Riemann-Cartan geometry.

It is not a hard task to show that the right-hand side (RHS) of (2.7) does *not* have the field $\omega_\mu{}^{mn}$ effectively, i.e., $\delta[(\text{RHS})\text{ of }(2.7)]/\delta\omega_\mu{}^{mn} \equiv 0$. One way to see this is that the left-hand side (LHS) of (2.7) is explicitly only in terms of $e_\mu{}^m$ and $\psi_\mu{}^i$, and there is no involvement of $\omega_\mu{}^{mn}$. This can be directly confirmed by taking the direct variation of the RHS by $\omega_\mu{}^{rs}$.

Once this feature is understood, we can rewrite the Lagrangian (2.1) in terms of $R(\omega)$, $\mathcal{R}_{\mu\nu}{}^i(\omega, A)$, and $\hat T_{\mu\nu}{}^m$, using (2.7) as

$$\begin{aligned}\tilde{\mathcal{L}}_{\aleph_0} &= +\frac{1}{4}eR(\omega) - \frac{1}{4}\epsilon^{\mu\nu\rho}[\bar\psi_\mu{}^i\mathcal{R}_{\nu\rho}{}^i(\omega, A)] \\ &\quad + \frac{1}{16}e(\hat T_{\mu\nu}{}^m)^2 - \frac{1}{8}e\hat T_{\rho\sigma\tau}\hat T^{\sigma\tau\rho} - \frac{1}{4}e(\hat T_\mu)^2 \\ &\quad + \frac{1}{2}g\epsilon^{\mu\nu\rho}\left(F_{\mu\nu}{}^{ij}A_\rho{}^{ij} - \frac{2}{3}A_\mu{}^{ij}A_\nu{}^{jk}A_\rho{}^{ki}\right) \\ &\quad + \frac{1}{32}g^2 e - \frac{1}{16}ge(\bar\psi_\mu{}^i\gamma^{\mu\nu}\psi_\nu{}^i),\end{aligned} \tag{2.10}$$

up to a total divergence. We repeat the fact that the Lorentz connection $\omega_\mu{}^{mn}$ is effectively *not* involved in $\tilde{\mathcal{L}}_{\aleph_0}$. We use the *tilde* on $\tilde{\mathcal{L}}_{\aleph_0}$ in (2.10), distinguished from $\mathcal{L}_{\aleph_0}$, due to a significant difference between them, even though they agree up to a total divergence, We can now introduce an independent Lorentz connection multiplet $(\omega_\mu{}^{rs}, \lambda^{rs})$, and add a supersymmetric CS Lagrangian $\mathcal{L}_{R\omega}$ to $\tilde{\mathcal{L}}_{\aleph_0}$:

$$\mathcal{L}_{R\omega} = +\frac{1}{8}\mu^{-1}\epsilon^{\mu\nu\rho}\Big(R_{\mu\nu}{}^{mn}\omega_{\rho mn} - \frac{2}{3}\omega_{\mu r}{}^s\omega_{\nu s}{}^t\omega_{\rho t}{}^r\Big) - \frac{1}{2}\mu^{-1}e(\bar{\lambda}^{mni}\lambda_{mn}{}^i). \tag{2.11}$$

Our total action $I_{3D} \equiv \tilde{I}_{\aleph_0} + I_{R\omega}$ with $\tilde{I}_{\aleph_0} \equiv \int d^3x \tilde{\mathcal{L}}_{\aleph_0}$ and $I_{R\omega} \equiv \int d^3x \mathcal{L}_{R\omega}$ is invariant under $\aleph_0$ local supersymmetry

$$\delta e_\mu{}^m = +(\bar{\epsilon}^i\gamma^m\psi_\mu), \tag{2.12a}$$

$$\begin{aligned}\delta\psi_\mu{}^i &= +\partial_\mu\epsilon^i + \frac{1}{4}\omega_\mu{}^{rs}(\gamma_{rs}\epsilon^i) - \frac{1}{4}\hat{K}_\mu{}^{rs}(\gamma_{rs}\epsilon^i) + gA_\mu{}^{ij}\epsilon^j - \frac{1}{8}g(\gamma_\mu\epsilon^i)\\ &\equiv +D_\mu(\omega - \hat{K})\epsilon^i + gA_\mu{}^{ij}\epsilon^j - \frac{1}{8}g(\gamma_\mu\epsilon^i)\\ &\equiv +\mathcal{D}_\mu(\omega - \hat{K}, A)\epsilon^i - \frac{1}{8}g(\gamma_\mu\epsilon^i),\end{aligned} \tag{2.12b}$$

$$\delta_Q\omega_\mu{}^{mn} = +(\bar{\epsilon}^i\gamma_\mu\lambda^{mni}), \tag{2.12c}$$

$$\delta_Q\lambda^{mni} = -\frac{1}{4}(\gamma^{\mu\nu}\epsilon^i)\hat{R}_{\mu\nu}{}^{mn} - \lambda^{mn[i|}(\bar{\epsilon}^j\gamma^\mu\psi_\mu{}^{|j]}) + \frac{1}{2}(\gamma^\mu\lambda^{mnj})(\bar{\epsilon}^{[i}\psi_\mu{}^{j]}), \tag{2.12d}$$

where we have used the feature that $\hat{\omega}_\mu{}^{mn} = \omega_\mu{}^{mn} - \hat{K}_\mu{}^{mn}$. This rearrangement is needed to make the expression in (2.12b) manifestly covariant in terms of the independent $\omega_\mu{}^{rs}$ field.

The confirmation of the superinvariance $\delta_Q I_{3D} = 0$ is straightforward, because the $\omega_\mu{}^{mn}$ field is *not* effectively involved in $\tilde{\mathcal{L}}_{\aleph_0}$, namely, $\delta\tilde{\mathcal{L}}_{CJS}/\delta\omega_\mu{}^{mn} \equiv 0$ thanks to the supersymmetric Palatini identity (2.5). Therefore, the only contribution of $\delta_Q\omega_\mu{}^{mn}$ to $\delta_Q I_{3D}$ is from $\mathcal{L}_{CS}$. Also, the dreibein is not involved in the CS term in $\mathcal{L}_{R\omega}$, except for the λ^2 term. We also see that the peculiar $\psi\lambda$ terms in (2.12d) are needed to cancel the like terms arising from $\delta_Q e_\mu{}^m$ in the λ^2 term in $\mathcal{L}_{R\omega}$.

As we have briefly mentioned before, the transformation of $\omega_\mu{}^{mn}$ in (2.12c) is different from that in the so-called first-order formalism [14,18]. The reason is that in our formulation, $\omega_\mu{}^{mn}$ is *not* involved explicitly in the Lagrangian $\tilde{\mathcal{L}}_{\aleph_0}$, which is a rewriting of the second-order formalism Lagrangian [17]. This rewriting in turn has been done by the Palatini identity, based on the relationship $\hat{\omega}_\mu{}^{mn} = \omega_\mu{}^{mn} - \hat{K}_\mu{}^{mn}$.

In the usual formulation of supergravity, the presence of the bare ψ-dependent such as those in $\delta_Q\lambda^{mn}$ of (2.12d) is problematic, because they create the derivative terms $D_\mu\epsilon$ in the closure of supersymmetry on λ^{mn}. In our system, however, this does not pose any problem, because the λ-field equation is simply $\lambda^{mni} \doteq 0$,[2] and any term with $\psi_\mu{}^i$ always contains λ^{mni} which is vanishing on-shell.

III. SUPERSYMMETRIC LORENTZ CS TERM COUPLED TO 11D SUPERGRAVITY

Encouraged by the 3D result, we can consider the coupling of supersymmetric CS term to $N = 1$ supergravity in 11 dimensions by Cremmer-Julia-Scherk [7].

The original CJS Lagrangian [7] is equivalent to[3]

$$\begin{aligned}\mathcal{L}_{CJS} = &-\frac{1}{4}eR(\tilde{\omega}) - \frac{i}{2}e\Big[\bar{\psi}_\mu\gamma^{\mu\nu\rho}D_\nu\Big(\frac{\tilde{\omega}+\hat{\omega}}{2}\Big)\psi_\rho\Big]\\ &- \frac{1}{48}e(F_{\mu\nu\rho\sigma})^2 + \frac{1}{192}(\bar{\psi}_\mu\gamma^{[\mu|}\gamma^{\rho\sigma\tau\lambda}\gamma^{|\nu]}\psi_\nu)\\ &\times(F_{\rho\sigma\tau\lambda} + \hat{F}_{\rho\sigma\tau\lambda})\\ &+ \frac{2}{(144)^2}\epsilon^{\mu\nu\rho\sigma\tau\lambda\phi\chi\psi\omega\upsilon}F_{\mu\nu\rho\sigma}F_{\tau\lambda\phi\chi}A_{\psi\omega\upsilon},\end{aligned} \tag{3.1}$$

where

$$\tilde{\omega}_\mu{}^{mn} = \hat{\omega}_\mu{}^{mn} - \frac{i}{4}(\bar{\psi}_\rho\gamma_\mu{}^{mn\rho\sigma}\psi_\sigma) \equiv \hat{\omega}_\mu{}^{mn} + K_\mu{}^{mn}(5), \tag{3.2a}$$

$$\begin{aligned}\hat{\omega}_{mrs} &\equiv +\frac{1}{2}(\hat{C}_{mrs} - \hat{C}_{msr} - \hat{C}_{rsm}),\\ \hat{C}_{\mu\nu}{}^m &\equiv 2\delta_{[\mu}e_{\nu]}{}^m + i(\bar{\psi}_\mu\gamma^m\psi_n).\end{aligned} \tag{3.2b}$$

Especially, $\tilde{\omega}_{\mu rs}$ is the Lorentz connection obtained by the first-order formalism field equation from the CJS Lagrangian [7]. The ψ^2 term $K(5)$ with five γ's arises in general space-time dimensions $D \geq 5$.

As in three dimensions, when introducing a Lorentz connection multiplet, we need to rewrite $\mathcal{L}_{CJS}$ in terms of Riemann-Cartan geometry tensors, with $\omega_\mu{}^{mn}$ as an independent field. This is not too difficult, if we use the 11D analog of supersymmetric Palatini identity [12,14][4]

$$\begin{aligned}&-\frac{1}{4}eR(\omega+\tau) - \frac{i}{2}e\Big[\bar{\psi}_\mu\gamma^{\mu\nu\rho}D_\nu\Big(\omega + \tau - \frac{1}{2}K(5)\Big)\psi_\rho\Big]\\ &\equiv -\frac{1}{4}eR(\omega) - \frac{i}{2}e[\bar{\psi}_\mu\gamma^{\mu\nu\rho}D_\nu(\omega)\psi_\rho] + \frac{1}{4}e\tau_{mnr}\tau^{nrm}\\ &\quad + \frac{1}{4}e(\tau_m)^2 + \frac{1}{4}e[K_{mrs}(5)]^2 + \frac{1}{4}eK_{mrs}(1)K^{mrs}(5)\\ &\quad + \partial_\mu(eW^\mu).\end{aligned} \tag{3.3}$$

[2]We use the symbol $\doteq$ for a field equation, distinguished from algebraic equality.

[3]In this section, we are using the same signature $(\eta_{mn}) = \mathrm{diag}(+,-,-)$ as [7], due to the popularity of the Lagrangian in [7]. This causes the presence of imaginary units compared with the previous section.

[4]The 11D version of supersymmetric Palatini identity had been already mentioned in [14], but it is not as explicit as the following.

Here $K(1)$ is a combination of ψ bilinears:

$$K_{mrs}(1) \equiv -\frac{i}{2}[(\bar{\psi}_m \gamma_r \psi_s) - (\bar{\psi}_m \gamma_s \psi_r) + (\bar{\psi}_r \gamma_m \psi_s)]. \tag{3.4}$$

Similarly to the 3D case, we can choose $\tau_{mrs} = \tilde{\omega}_{mrs} - \omega_{mrs}$. By this choice of τ, the Hilbert action becomes $-(1/4)eR(\tilde{\omega})$, while the gravitino-kinetic term will contain $(\tilde{\omega} + \hat{\omega})/2$ as its Lorentz connection term, as in the corresponding terms in $\mathcal{L}_{\rm CJS}$ in (3.1).

We can now give the total action $I_{\rm 11D} \equiv \tilde{I}_{\rm CJS} + I_{R^5\omega} \equiv \int d^{11}x(\tilde{\mathcal{L}}_{\rm CJS} + \mathcal{L}_{R^5\omega})$, where

$$\begin{aligned}\tilde{\mathcal{L}}_{\rm CJS} = & -\frac{1}{4}eR(\omega) - \frac{i}{2}e[\bar{\psi}_\mu \gamma^{\mu\nu\rho} D_\nu(\omega)\psi_\rho] \\ & -\frac{1}{48}e(F_{\mu\nu\rho\sigma})^2 - \frac{1}{16}e(\hat{T}_{\mu\nu}{}^m)^2 + \frac{1}{8}e\hat{T}_{\mu\nu\rho}\hat{T}^{\nu\rho\mu} \\ & + \frac{1}{4}e(\hat{T}_\mu)^2 + \frac{1}{192}e(\bar{\psi}_\mu \gamma^{[\mu|}\gamma^{\rho\sigma\tau\lambda}\gamma^{|\nu]}\psi_\nu) \\ & \times(F_{\rho\sigma\tau\lambda} + \hat{F}_{\rho\sigma\tau\lambda}) + \frac{2}{(144)^2}\epsilon^{\mu\nu\rho\sigma\tau\phi\chi\psi\omega\upsilon}F_{\mu\nu\rho\sigma} \\ & \times F_{\tau\lambda\phi\chi}A_{\psi\omega\upsilon} + \frac{1}{4}e\hat{T}_{mrs}K^{mrs}(5) \\ & + \frac{1}{4}eK_{mrs}(5)[2K^{mrs}(5) + K^{mrs}(1)],\end{aligned} \tag{3.5}$$

up to a total divergence with $\hat{T}_{\mu\nu}{}^m \equiv +2D_{[\mu}e_{\nu]}{}^m + i(\bar{\psi}_\mu\gamma^m\psi_\nu)$. The $\mathcal{L}_{R^5\omega}$ is the Lorentz CS term

$$\begin{aligned}\mathcal{L}_{R^5\omega} = & \frac{1}{6}\epsilon^{\mu_1\cdots\mu_{11}}[R_{\mu_1\mu_2}\dots R_{\mu_9\mu_{10}}\omega_{\mu_{11}} + \cdots \\ & + \alpha_r R_{\mu_1\mu_2}\dots R_{\mu_{2r-1}\mu_{2r}}\omega_{\mu_{2r+1}}\dots\omega_{\mu_{11}} + \cdots \\ & + \alpha_0\omega_{\mu_1}\dots\omega_{\mu_{11}}]_m{}^m + \frac{1}{2}e(\bar{\lambda}^{mn}\lambda_{mn})\end{aligned} \tag{3.6a}$$

$$= -\frac{1}{12}\int_0^1 dy \check{\epsilon}^{\check{\mu}_1\dots\check{\mu}_{12}}(\check{R}_{\check{\mu}_1\check{\mu}_2}\dots\check{R}_{\check{m}_{11}\check{\mu}_{12}})_{\check{m}}{}^{\check{m}} + \frac{1}{2}e(\bar{\lambda}^{mn}\lambda_{mn}), \tag{3.6b}$$

where we use the *tilde* for $\tilde{\mathcal{L}}_{\rm CSJ}$ for the same reason as in the 3D case. Also similarly to the 3D case, our $\omega_\mu{}^{mn}$ is an independent field, so that equations such as $\hat{T}_{\mu\nu}{}^m = -2K_{\mu\nu}{}^m(5)$ in the original system [7] are *no* longer valid.

The constants $\alpha_0, \dots, \alpha_4$ in (3.6a) are the coefficients for the nonleading terms covariantizing the whole Lorentz CS form. There are in total six terms of the forms $R^5\omega$, $\alpha_4R^4\omega^3$, $\alpha_3R^3\omega^5$, $\alpha_2R^2\omega^7$, $\alpha_1R\omega^9$, and $\alpha_0\omega^{11}$. These terms are completed in (3.6b) in terms of the so-called "Vainberg variable" [20,21][5] to be explained below. The product of R's in (3.6) is understood, e.g., as $(R_{\mu_1\mu_2}R_{\mu_3\mu_4})_m{}^n \equiv R_{\mu_1\mu_2 m}{}^r R_{\mu_3\mu_4 r}{}^n$ or $(R_{\mu_1\mu_2}R_{\mu_3\mu_4}R_{\mu_5\mu_6})_m{}^n \equiv R_{\mu_1\mu_2 m}{}^r R_{\mu_3\mu_4 r}{}^s R_{\mu_3\mu_4 s}{}^n$.

All the "checked" quantities and indices, such as $\check{R}_{\check{\mu}\check{\nu}}{}^{\check{m}\check{n}}$ refer to the enlarged twelve dimensions with the coordinates $(\check{x}^{\check{\mu}}) \equiv (x^\mu, y)$ in the Vainberg construction [21]. The totally antisymmetric constant $\check{\epsilon}^{\check{\mu}_1\cdots\check{\mu}_{12}}$ in twelve dimensions is defined by $\check{\epsilon}^{\mu_1\dots\mu_{11}y} \equiv \epsilon^{\mu_1\dots\mu_{11}}$. The Vainberg construction [21] enables us to construct a Lagrangian out of a given field equation $F[\varphi] \equiv \delta\mathcal{L}[\varphi]/\delta\varphi \doteq 0$ of an arbitrary field φ, by introducing a new coordinate y in the integration [21]

$$\mathcal{L}[\varphi] = \int_0^1 dy \check{F}[\check{\varphi}]\partial_y\check{\varphi}, \tag{3.7}$$

where the checked field $\check{\varphi}(\check{x}) \equiv \check{\varphi}(x, y)$ is defined by [21]

$$\check{\varphi}(x, 1) = \varphi(x), \qquad \check{\varphi}(x, 0) = 0. \tag{3.8}$$

The validity of (3.7) is easily confirmed, by the fact that its RHS is rewritten as $\int_0^1 dy\partial_y\check{\mathcal{L}}[\check{\varphi}]$. Applying this to the ω-field equation

$$\epsilon^{\mu\nu_1\dots\nu_{10}}(R_{\nu_1\nu_2}\dots R_{\nu_9\nu_{10}})^{mn} \doteq 0, \tag{3.9}$$

we get the Lagrangian for the Lorentz CS [20,23] with the manifest $SO(1, 10)$ invariance

$$\begin{aligned}&\int_0^1 dy\epsilon^{\nu_1\dots\nu_{10}\mu}(\check{R}_{\mu_1\mu_2}\dots\check{R}_{\mu_9\mu_{10}}\partial_y\check{\omega}_\mu)_m{}^m \\ &= -\frac{1}{12}\int_0^1 dy\check{\epsilon}^{\check{\mu}_1\dots\check{\mu}_{12}}(\check{R}_{\check{\mu}_1\check{\mu}_2}\dots\check{R}_{\check{\mu}_{11}\check{\mu}_{12}})_{\check{m}}{}^{\check{m}},\end{aligned} \tag{3.10}$$

where we have introduced the generalized curvature tensor in the "extended" twelve-dimensional space-time with the coordinates $(\check{x}^{\check{\mu}}) \equiv (x^\mu, y)$. In particular,

$$\begin{aligned}\check{R}_{y\mu}{}^{mn} &\equiv \check{\partial}_u\check{\omega}_\mu{}^{mn} - \check{\partial}_\mu\check{\omega}_y{}^{mn} + \check{\omega}_y{}^{mr}\check{\omega}_{\mu r}{}^n - \check{\omega}_\mu{}^{mr}\check{\omega}_{yr}{}^n \\ &= \partial_y\check{\omega}_\mu{}^{mn},\end{aligned} \tag{3.11}$$

where we required $\check{\omega}_y{}^{mn} = 0$. Relevantly, we require $\check{\omega}_\mu{}^{ym} = 0$, so that the trace over the local Lorentz indices ${}_{\check{m}}{}^{\check{m}}$ in (3.10) is equivalent to ${}_m{}^m$ within 11 dimensions with $SO(1, 10)$ symmetry. We thus get the twelve-dimensional covariant expression for the CS term (3.6b), and the domain of the twelve-dimensional integration should coincide with $\int d^{11}x\int_0^1 dy$. The ω^{11} term is sometimes important, when we use the solution $R_{\mu\nu}{}^{mn} \doteq 0$, which has *pure gauge* $\omega_\mu{}^{mn}$ field. In 11 dimensions, however, the set of solutions to (3.9) is much wider than that of $R_{\mu\nu}{}^{mn} \doteq 0$, so other solutions than a pure gauge solution are allowed. For example, the solution $R_{\mu\nu}{}^{rs} \doteq ce_{[\mu}{}^r e_{\nu]}{}^s$ with a constant c is allowed, even though this is *not* a supersymmetric solution.

The supersymmetry transformation rule leaving the total action $I_{\rm 11D}$ invariant is

[5]This integration formula is equivalent to the so-called "transgression form" in some references [22].

$$\delta_Q e_\mu{}^m = -i(\bar{\epsilon}\gamma^m \psi_\mu), \tag{3.12a}$$

$$\delta_Q \psi_\mu = +D_\mu(\omega - \hat{K})\epsilon + \frac{i}{144}(\gamma_\mu{}^{\nu\rho\sigma\tau}\hat{F}_{\nu\rho\sigma\tau} - 8\gamma^{\rho\sigma\tau}\hat{F}_{\mu\rho\sigma\tau})\epsilon, \tag{3.12b}$$

$$\delta_Q A_{\mu\nu\rho} = +\frac{3}{2}(\bar{\epsilon}\gamma_{[\mu\nu}\psi_{\rho]}), \tag{3.12c}$$

$$\delta_Q \omega_\mu{}^{mn} = +i(\bar{\epsilon}\gamma_\mu\lambda^{mn}), \tag{3.12d}$$

$$\delta_Q \lambda^{mn} = +ie^{-1}\epsilon^{\mu\nu_1\ldots\nu_{10}}(\gamma_\mu\epsilon)(R_{\nu_1\nu_2}\ldots R_{\nu_9\nu_{10}})^{mn} + \frac{i}{2}(\bar{\epsilon}\gamma^\mu\psi_\mu)\lambda^{mn}. \tag{3.12e}$$

The confirmation of the superinvariance $\delta_Q I_{11\text{D}} = 0$ is straightforward, because of the important features $\delta\tilde{\mathcal{L}}_{\text{CJS}}/\delta\omega_\mu{}^{mn} = 0$ and $\mathcal{L}_{R^5\omega}$ do not contain $e_\mu{}^m$ *except* the λ^2 term. The closure of supersymmetry on the fields does not pose any problem, for a reason similar to the case of $\aleph_0$ supergravity in three dimensions.

We can further generalize the field equation (3.9). For example, we can think of

$$\epsilon^{\nu_1\ldots\nu_{10}\mu}(R_{\nu_1\nu_2}\ldots R_{\nu_5\nu_6})^{mn}(R_{\nu_7\nu_8}R_{\nu_9\nu_{10}})^r{}_r \doteq 0. \tag{3.13}$$

The LHS is *not* identically zero. More generally, we can have the generalized $R\omega$ term

$$\mathcal{L}^{\text{gen}}_{R^5\omega} \equiv \int_0^1 dy\check{\epsilon}^{\check{\mu}_1\ldots\check{\mu}_{12}}\check{C}_{\check{m}_1\ldots\check{m}_{12}}(\check{R}_{\check{\mu}_1\check{\mu}_2}{}^{\check{m}_1\check{m}_2}\ldots\check{R}_{\check{\mu}_{11}\check{\mu}_{12}}{}^{\check{m}_{11}\check{m}_{12}}) + \frac{1}{2}e(\bar{\lambda}^{mn}\lambda_{mn}), \tag{3.14}$$

replacing (3.6b). The $\check{C}_{\check{m}_1\ldots\check{m}_{11}}$ is a generalized $SO(1, 11)$-invariant constant tensor, e.g., the combination of Kronecker's deltas corresponding to (3.13). Accordingly, $\delta_Q\lambda$ in (3.12e) is replaced by

$$\delta_Q\lambda^{mn} = +i(\gamma_\mu\epsilon)\left(\frac{\delta\mathcal{L}^{\text{gen}}_{R^5\omega}}{\delta\omega_{\mu mn}}\right) + \frac{i}{2}(\bar{\epsilon}\gamma^\mu\psi_\mu)\lambda^{mn}, \tag{3.15}$$

while (3.12a) through (3.12d) stay the same.

IV. CONCLUDING REMARKS

In this paper, we have presented a supersymmetric CS term in three dimensions coupled to $\aleph_0$ supergravity. Because of the feature of the gaugino field equation, there arises no problem with the on-shell closure of supersymmetry. The fundamental technique is the supersymmetric Palatini identity (2.5) with an *independent* $\omega_\mu{}^{mn}$ as a rewriting of the original form [13,14], leading us to the manifestly Lorentz-invariant Lagrangian.

Based on this encouraging result, we have applied the same technique to 11D supergravity with $N = 1$ [7]. Despite the technical complication with the contorsion $K(5)$ with five γ's, the basic structure of supersymmetric CS terms (3.6) and (3.14) is valid also in 11 dimensions. This is because the Palatini identity in lower dimensions [13,14] can be generalized to 11 dimensions [12], leading us to the manifestly Lorentz-invariant Lagrangians with the *independent* $\omega_\mu{}^{mn}$.

According to common wisdom, 11D supergravity theory is so tight that we cannot modify the original Lagrangian [7], unless it is related to M-theory [8] corrections or something related. In our present paper, we have a counterexample against this notion, i.e., supersymmetric Lorentz CS term (3.6) can be added to the original CJS Lagrangian [7] with $N = 1$ local supersymmetry. The first example of CS term (3.6) is further generalized by the $SO(1, 11)$ constant tensor $\check{C}_{\check{m}_1\ldots\check{m}_{12}}$ in (3.14).

Needless to say, our methodology given in this paper is universally applicable to supergravity theory in odd dimensions in $3 \leq D \leq 11$, where Hilbert actions are nontrivial.

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