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Gauge symmetry breaking in ten-dimensional Yang-Mills theory dynamically compactified on S^6

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We study fluctuation modes in ten-dimensional Yang-Mills theory with a higher derivative term for the gauge field. We consider the ten-dimensional space-time to be a product of a four-dimensional space-time and six-dimensional sphere which exhibits dynamical compactification. Because of the isometry on S^6, there are flat directions corresponding to the Nambu-Goldstone zero modes in the effective theory on the solution. The zero modes are absorbed into gauge fields and form massive vector fields as a consequence of the Higgs-Kibble mechanism. The mass of the vector fields is proportional to the inverse of the radius of the sphere and larger than the mass scale set by the radius because of the higher derivative term.

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I. INTRODUCTION

Extra dimensions and their compactification have been attracting attention [1] for several decades. Among many scenarios of compactification of extra dimensions, Cremmer and Scherk suggested an interesting idea that compactification may occur if a topologically nontrivial gauge configuration exists in compactified space [2]. An example is given by the 't Hooft-Polyakov monopole [3] on S^2. Recently, in [4], some of us have studied a scenario of dynamical compactification and inflation in ten-dimensional Einstein-Yang-Mills theory with the SO(6) gauge group, using the Cremmer-Scherk gauge configuration on S^6 [5,6]. We have added a higher derivative coupling term,[1] originally introduced by Tchrakian [8], in order to ensure the nonexistence of tachyonic modes on the Cremmer-Scherk configuration because of the Bogomol'nyi equation.

In this paper we study the issue of the stability and fluctuations of the Cremmer-Scherk configuration in this scenario. We will work on the space-time given by the product of a four-dimensional space-time and a six-dimensional sphere, where the radius of the sphere shrinks to a constant value in the limit $t \to +\infty$. We assume that the four-dimensional space-time can be treated as $\{(t, \mathcal{N}_t)\}_{t\in\mathbb{R}}$, where $\mathcal{N}_t$ is diffeomorphic to a three-dimensional manifold $\mathcal{N}$.[2]

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[1]Such a quartic term is known to appear in the low-energy effective theory of quantum electrodynamics, too [7].

[2]For instance, four-dimensional Friedmann-Lemaitre-Robertson-Walker space-time satisfies the condition.

We first show the absence of tachyonic modes in general background gauge fields which satisfy the Bogomol'nyi equation discussed in [4]. In general, the Cremmer-Scherk configuration is obtained by the identification of a compact direction and an internal (gauge) direction as an extension of the 't Hooft-Polyakov monopole. By rotating this identification, with the rotation depending on $\mathbb{R}^{1,3}$, we obtain massless fluctuation modes which can be regarded as Nambu-Goldstone bosons [9]. Then we show that these massless modes are actually absorbed into gauge fields to form massive vector (Proca) fields by the Higgs-Kibble mechanism [10]. Since the number of Nambu-Goldstone modes, 15, coincides with the dimension of SO(6), we find that the gauge symmetry SO(6) is completely broken. By scaling fields for normalization of the coefficient of their kinetic terms, we obtain the mass proportional to the inverse of the radius of the compact space. We thus conclude that there are neither tachyonic nor massless modes in the physical spectrum around the background and that the configuration is stable. Similar mechanisms for generating masses in compactifications have been discussed in the literature. Scherk and Schwarz introduced mass by using a generalized dimensional reduction or a twisted boundary condition of fields [11]. Horvath *et al.* considered a system coupled with Higgs fields to obtain the mass of gauge fields [12]. Manton showed that the components of gauge fields along extra dimensions provide Higgs fields in the four-dimensional effective theory without additional scalar fields [13,14]. Hosotani found a mechanism for obtaining Higgs fields belonging to representations different from the adjoint representation and endowing mass to fermions on orbifolds [15].

This paper is organized as follows: in Sec. II we will study the general treatment of the fluctuation of gauge fields about some classical solution. In the effective theory on the classical background, the fluctuation of the ten-dimensional vector field splits into two parts, a four-dimensional vector field and six scalar fields. We will see that the lowest Kaluza-Klein mode of the four-dimensional vector field gets mass because of the gauge configuration. In Sec. III we review the Cremmer-Scherk gauge configuration and Tchrakian's self-duality equation. In Sec. IV we show the Higgs mechanism during the dynamical compactification. By considering rotated identification between the internal (gauge) space and the compact space (S^6), we explicitly show that the Nambu-Goldstone bosons form the Proca fields with vector fields. In Sec. V we summarize this article. In the Appendix we prove the nonexistence of covariantly constant functions belonging to the adjoint representation on the Cremmer-Scherk configuration.

II. FLUCTUATIONS AROUND SOLUTIONS OF THE BOGOMOL'NYI EQUATION

Let us consider the SO(6) Yang-Mills theory on the ten-dimensional curved space-time, which is a direct product of a four-dimensional space-time $\mathcal{M}$ and a six-dimensional sphere S^6, whose radius varies. We assume that the four-dimensional space-time $\mathcal{M}$ is the form of $\mathcal{M} = \{(t, \mathcal{N}_t)\}_{t\in\mathbb{R}}$, where $\mathcal{N}_t$ is diffeomorphic to a three-dimensional space $\mathcal{N}$, and that any tangent vectors on the time slice $\mathcal{N}_t$ are spacelike. The metric on the ten-dimensional space is given as

$$\begin{aligned} ds^2 &= g_{\mu\nu}(x)dx^\mu dx^\nu + g_{ij}(x,y)dy^i dy^j, \\ g_{ij}(x,y) &= R_2(x)^2 \frac{\delta_{ij}}{(1+|y|^2/4)^2}. \end{aligned} \tag{1}$$

The indices are μ, $\nu = 0, 7, 8, 9$ and i, $j = 1, \cdots, 6$. x^μ represent four-dimensional coordinates along the four-dimensional space-time $\mathcal{M}$, and y^i represent six-dimensional coordinates along the sphere. We denote $X^M \equiv (x^\mu, y^i)$ for the coordinates of total space-time, and their indices are represented by capital letters. $R_2(x)$ is the radius of the six-dimensional sphere and depends on the four-dimensional coordinates x. We consider only the case where $R_2(x)$ converges to a nonzero constant value in the limit $t \to +\infty$. We start from the following action,

$$S := \frac{1}{16}\int \mathrm{Tr}\{-F\wedge *F + \alpha^2(F\wedge F)\wedge *(F\wedge F)\}, \tag{2}$$

where F is the field strength two-form and α is the quartic coupling constant. The second term quartic in F is the term introduced by Tchrakian [8], which we call the Tchrakian term. This action is quadratic in the time derivative acting on the gauge field, $\partial A/\partial t$. We consider the gauge potential one-form A taking a value in so(6). In our notation the generators of so(6) are represented with spinor indices, but one should not confuse them with spinor fields. In order to define the product of the field strength two-form, we have to indicate the representation matrix of gauge fields. Let us use the Clifford algebra $\{\gamma_a, \gamma_b\} = 2\delta_{ab}$ $(a, \ b = 1, 2, \ldots, 6)$ with respect to SO(6). Here γ_a are Hermitian 8×8 matrices. The internal indices are represented by $a, b, c, \cdots$. This is independent of the space indices along the six-dimensional sphere. Commutators $\gamma_{ab} := (1/2)\times [\gamma_a, \gamma_b]$ of γ matrices satisfy the commutation relation of so(6), and their normalization is shown as follows,

$$\begin{aligned} [\gamma_{ab}, \gamma_{cd}] &= 2(\delta_{bc}\gamma_{ad} - \delta_{bd}\gamma_{ac} - \delta_{ac}\gamma_{bd} + \delta_{ad}\gamma_{bc}), \\ \mathrm{Tr}\gamma_{ab}\gamma_{cd} &= 8(\delta_{bc}\delta_{ad} - \delta_{ac}\delta_{bd}). \end{aligned} \tag{3}$$

The anticommutation relation of these generators γ_{ab} is given by

$$\{\gamma_{ab}, \gamma_{cd}\} = 2\gamma_{abcd} + 2(\delta_{bc}\delta_{ad} - \delta_{ac}\delta_{bd}), \tag{4}$$

where we have used the following notation,

$$\gamma_{a(1)\cdots a(p)} := \frac{1}{p!}\sum_{\sigma\in S_p} \mathrm{sgn}(\sigma)\gamma_{a(\sigma(1))}\gamma_{a(\sigma(2))}\cdots\gamma_{a(\sigma(p))}, \tag{5}$$

where S_p is the symmetric group of p characters. Further, we use the matrix $\gamma_7 := -\boldsymbol{i}\gamma_{123456}$. The gauge potential can then be written as

$$A := \tfrac{1}{2}A_M^{ab}\gamma_{ab}dX^M. \tag{6}$$

The field strength is defined as $F := dA + gA\wedge A$, with gauge coupling constant g. The equation of motion of the Lagrangian (2) reads

$$D\{*F - \alpha^2(F\wedge *(F\wedge F) + (F\wedge F)\wedge F)\} = 0. \tag{7}$$

Let us suppose that $A^{(0)}$ is some solution of Eq. (7), which we fix to be our background solution. We make the assumption that $A^{(0)}$ has components only along the compact directions and depends only on y^i. Then it follows that the corresponding field strength, $F^{(0)} := dA^{(0)} + gA^{(0)}\wedge A^{(0)}$, has components only along the compact directions.

Let us consider fluctuations δA around the solution $A^{(0)}$, as

$$\begin{aligned} A &= A^{(0)} + \delta A, \qquad \delta A = v + \Phi, \\ v &= v_\mu dx^\mu, \qquad \Phi = \Phi_i dy^i. \end{aligned} \tag{8}$$

The fluctuation δA is divided into two parts, v and Φ. v is a one-form whose components are nonzero only along the four-dimensional space-time, while Φ has components only along the six-dimensional sphere. The coefficients depend on x and y, $v_\mu := v_\mu(x,y)$, $\Phi_i := \Phi_i(x,y)$. Our objective is then to obtain the four-dimensional effective theory for these fluctuations. The fluctuation v is a vector field, and the fluctuations $\Phi_i (i = 1, 2, \ldots, 6)$ are six scalar fields under the general coordinate transformation of the four-dimensional space-time. Each v_μ or Φ_i belongs to the adjoint representation of the gauge group SO(6), and $(\Phi_1^{ab}, \Phi_2^{ab}, \ldots, \Phi_6^{ab})$ transform as vectors under the rotation of the six-dimensional space. Let us decompose the field strengths in terms of v and Φ,

$$\begin{aligned} F &= F^{(0)} + d\delta A + g(A^{(0)} \wedge \delta A + \delta A \wedge A^{(0)}) + g\delta A \wedge \delta A \\ &= F^{(0)} + d_{(4)}v + d_{(4)}\Phi + d_{(6)}v + d_{(6)}\Phi \\ &\quad + g(A^{(0)} \wedge v + v \wedge A^{(0)}) + g(A^{(0)} \wedge \Phi + \Phi \wedge A^{(0)}) \\ &\quad + g(v \wedge v + v \wedge \Phi + \Phi \wedge v + \Phi \wedge \Phi), \end{aligned} \tag{9}$$

where the exterior derivative d is decomposed into two parts, $d = d_{(4)} + d_{(6)}$. These operators are defined as

$$d_{(4)} := dx^\mu \frac{\partial}{\partial x^\mu}, \qquad d_{(6)} := dy^i \frac{\partial}{\partial y^i}. \tag{10}$$

Let us gather terms as

$$\begin{aligned} W &:= d_{(4)}v + gv \wedge v, \\ D_v^{(4)}\Phi &:= d_{(4)}\Phi + g(v \wedge \Phi + \Phi \wedge v), \\ D_0^{(6)}v &:= d_{(6)}v + g(A^{(0)} \wedge v + v \wedge A^{(0)}), \\ D_0^{(6)}\Phi &:= d_{(6)}\Phi + g(A^{(0)} \wedge \Phi + \Phi \wedge A^{(0)}), \end{aligned} \tag{11}$$

where W is a field-strength-like quantity of the vector field v in four-dimensional space-time. Φ becomes a scalar field which belongs to the adjoint representation of SO(6) in four-dimensional low-energy effective theory. $D_v^{(4)}$ represents the covariant exterior derivative with the vector field v on the four-dimensional space-time, while $D_0^{(6)}$ represents the covariant exterior derivative with classical gauge configuration $A^{(0)}$, with derivatives along the six-dimensional sphere. By using these definitions the field strength F is written as

$$\begin{aligned} F &= F^{(0)} + W + D_v^{(4)}\Phi + D_0^{(6)}v + D_0^{(6)}\Phi + g(\Phi \wedge \Phi) \\ &= F_{(c)} + F_{(nc)}, \\ F_{(c)} &:= F^{(0)} + D_0^{(6)}\Phi + g(\Phi \wedge \Phi), \\ F_{(nc)} &:= W + D_v^{(4)}\Phi + D_0^{(6)}v. \end{aligned} \tag{12}$$

We have separated the field strength into two parts according to its indices. $F_{(c)}$ has components only along compact directions, while $F_{(nc)}$ has components along four-dimensional space-time. The Yang-Mills part of the action is written as

$$\mathrm{Tr}\, F \wedge *F = \mathrm{Tr}(F_{(c)} \wedge *F_{(c)} + F_{(nc)} \wedge *F_{(nc)}). \tag{13}$$

There are no terms obtained by the contraction of $F_{(c)}$ and $F_{(nc)}$. Similarly, the higher derivative coupling term is decomposed as

$$\begin{aligned} \mathrm{Tr}(F \wedge F) \wedge *(F \wedge F) = \mathrm{Tr}[&(F_{(c)} \wedge F_{(c)}) \wedge *(F_{(c)} \wedge F_{(c)}) \\ &+ (F_{(c)} \wedge F_{(nc)} + F_{(nc)} \wedge F_{(c)}) \\ &\wedge *(F_{(c)} \wedge F_{(nc)} + F_{(nc)} \wedge F_{(c)}) \\ &+ (F_{(nc)} \wedge F_{(nc)}) \wedge *(F_{(nc)} \wedge F_{(nc)})]. \end{aligned} \tag{14}$$

We now decompose the action S given in Eq. (2) as

$$\begin{aligned} S &:= S_{(c)} + S_{(nc)}, \\ S_{(c)} &:= \frac{1}{16} \int \mathrm{Tr}[-F_{(c)} \wedge *F_{(c)} \\ &\quad + \alpha^2 (F_{(c)} \wedge F_{(c)}) \wedge *(F_{(c)} \wedge F_{(c)})], \\ S_{(nc)} &= \frac{1}{16} \int \mathrm{Tr}[-F_{(nc)} \wedge *F_{(nc)} + \alpha^2 (F_{(c)} \wedge F_{(nc)} \\ &\quad + F_{(nc)} \wedge F_{(c)}) \wedge *(F_{(c)} \wedge F_{(nc)} + F_{(nc)} \wedge F_{(c)}) \\ &\quad + \alpha^2 (F_{(nc)} \wedge F_{(nc)}) \wedge *(F_{(nc)} \wedge F_{(nc)})]. \end{aligned} \tag{15}$$

It was shown in [4] that there are no tachyonic modes obtained from Φ in $S_{(c)}$ if the solution $A^{(0)}$ is a solution of the Bogomol'nyi equation given in Eqs. (27) and (28) below. Now we focus only on the v fluctuations, and we need to check only the remaining part $S_{(nc)}$. In order to study the mass spectrum, we need only quadratic terms in $S_{(nc)}$, and all such terms are included in the following,

$$\begin{aligned} S_{(nc)}|_{\mathrm{quad}} = \frac{1}{16} \int \mathrm{Tr}[&-F_{(nc)} \wedge *F_{(nc)} + \alpha^2 (F^{(0)} \wedge F_{(nc)} \\ &+ F_{(nc)} \wedge F^{(0)}) \wedge *(F^{(0)} \wedge F_{(nc)} + F_{(nc)} \wedge F^{(0)})]. \end{aligned} \tag{16}$$

Let us decompose $F_{(nc)}$ as

$$F_{(nc)} = W + F_{(m)}, \qquad F_{(m)} := D_v^{(4)}\Phi + D_0^{(6)}v, \tag{17}$$

where W has two four-dimensional indices and $F_{(m)}$ has one four-dimensional index and one six-dimensional index. Therefore, in this part, $S_{(nc)}|_{\mathrm{quad}}$, of this action, there are no terms obtained by the contraction of W and $F_{(m)}$:

$$\begin{aligned} S_{(nc)}|_{\mathrm{quad}} = \frac{1}{16} \int \mathrm{Tr}[&-W \wedge *W + \alpha^2 (F^{(0)} \wedge W \\ &+ W \wedge F^{(0)}) \wedge *(F^{(0)} \wedge W + W \wedge F^{(0)})] \\ + \frac{1}{16} \int \mathrm{Tr}[&-F_{(m)} \wedge *F_{(m)} + \alpha^2 (F^{(0)} \wedge F_{(m)} \\ &+ F_{(m)} \wedge F^{(0)}) \wedge *(F^{(0)} \wedge F_{(m)} + F_{(m)} \wedge F^{(0)})]. \end{aligned} \tag{18}$$

From the second integral of Eq. (18) we obtain the mass term in four-dimensional effective theory of v as

$$S_{(\mathrm{nc})}|_{\text{mass of } v} = \frac{1}{16}\int \mathrm{Tr}[-D_0^{(6)} v \wedge *D_0^{(6)} v + \alpha^2\{F^{(0)}, D_0^{(6)} v\} \wedge *\{F^{(0)}, D_0^{(6)} v\}]. \quad (19)$$

This gives a mass matrix defined as eigenvalues of a second-order differential operator on S^6. In order to show that the square of the mass is positive, let us do the following: suppose that ω and λ are functions or 0-forms on S^6 taking values in the Lie algebra so(6). The inner product is defined as

$$\langle \omega, \lambda\rangle_6 := -\frac{1}{8}\int_{S^6} \mathrm{Tr}\, \omega \wedge *_6 \lambda = -\frac{1}{8}\int_{S^6} d^{(6)} v\, \mathrm{Tr}\, \omega\lambda. \quad (20)$$

This gives a positive-definite norm. Any arbitrary function f globally defined on S^6 has a finite norm. $L^2(S^6) \otimes \mathrm{so}(6)$ with respect to this norm is a Hilbert space and is separable. The mass matrix $\mathcal{M}$ is defined as

$$\mathcal{M}\lambda := D_0^{(6)} *_6 D_0^{(6)} \lambda - \alpha^2\{F^{(0)}, D_0^{(6)} *_6 \{F^{(0)}, D_0^{(6)} \lambda\}\}. \quad (21)$$

This matrix is a self-adjoint operator in $L^2(S^6) \otimes \mathrm{so}(6)$. By using the inner product $\langle \cdot, \cdot\rangle_6$ and the mass matrix $\mathcal{M}$, we obtain

$$\langle \omega, \mathcal{M}\omega\rangle_6 = \frac{1}{8}\int \mathrm{Tr}[-D_0^{(6)} \omega \wedge *D_0^{(6)} \omega + \alpha^2\{F^{(0)}, D_0^{(6)} \omega\} \wedge *\{F^{(0)}, D_0^{(6)} \omega\}]. \quad (22)$$

The integrand cannot be negative. Hence the operator $\mathcal{M}$ has non-negative eigenvalues. Actually, this operator can be considered as the mass matrix of v. The mass term of v is written as

$$S_{(\mathrm{nc})}|_{\text{mass of } v} = \frac{1}{2}\int d^{(4)} v \langle v_\mu, \mathcal{M} v^\mu\rangle_6. \quad (23)$$

From this we obtain that the vector field v has non-negative mass squared. This ensures that there are no tachyonic modes in the full fluctuation of the gauge field around the solution of the Bogomol'nyi equation in [4]. Apparently, solutions of the equation $D_0^{(6)} v = 0$ are massless modes if they exist, although this does not mean that only solutions of $D_0^{(6)} v = 0$ are massless modes. We show the nonexistence of covariantly constant functions on the Cremmer-Scherk configuration in the Appendix.

III. CREMMER-SCHERK GAUGE CONFIGURATION AND TCHRAKIAN'S DUALITY EQUATION

So far we have not identified the background configuration. Let us now focus on the Cremmer-Scherk gauge configuration on S^6. To describe the background solution, let us restrict our interest here only to the part $S_{(\mathrm{c})}$,

$$S_{(\mathrm{c})} = -\frac{1}{16}\int \sqrt{-g^{(4)}} d^4x \int \mathrm{Tr}\{-F_{(\mathrm{c})} \wedge *_6 F_{(\mathrm{c})} + \alpha^2(F_{(\mathrm{c})} \wedge F_{(\mathrm{c})}) \wedge *_6(F_{(\mathrm{c})} \wedge F_{(\mathrm{c})})\}, \quad (24)$$

where $F_{(\mathrm{c})}$ has components only along the compact direction, and the Hodge duals of F_0 and $F_0 \wedge F_0$ are split into the four-dimensional invariant volume form and the Hodge dual of those on S^6. This part of the action is a functional of only Φ and R_2. Let us define the following quantity,

$$M_{(\mathrm{c})}[\Phi, R_2] := \frac{1}{16}\int \mathrm{Tr}\{-F_{(\mathrm{c})} \wedge *_6 F_{(\mathrm{c})} + \alpha^2(F_{(\mathrm{c})} \wedge F_{(\mathrm{c})}) \wedge *_6(F_{(\mathrm{c})} \wedge F_{(\mathrm{c})})\}. \quad (25)$$

By using this, the $S_{(\mathrm{c})}$ part of the action is rewritten as

$$S_{(\mathrm{c})} = -\int \sqrt{-g^{(4)}} d^4x M_{(\mathrm{c})}[\Phi, R_2]. \quad (26)$$

From this expression, the term $M_{(\mathrm{c})}[\Phi, R_2]$ seems to be part of the effective action including coupling terms of R_2 and Φ.

The term $M_{(\mathrm{c})}[\Phi, R_2]$ can be treated as pseudoenergy on a space with Euclidean signature. By the Bogomol'nyi completion, it can be rewritten as

$$M_{(\mathrm{c})}[\Phi, R_2] = -\frac{1}{16}\int \mathrm{Tr}\{F_{(\mathrm{c})} \mp *_6 \boldsymbol{i}\alpha\gamma_7 F_{(\mathrm{c})} \wedge F_{(\mathrm{c})}\} \wedge *_6\{F_{(\mathrm{c})} \mp *_6 \boldsymbol{i}\alpha\gamma_7 F_{(\mathrm{c})} \wedge F_{(\mathrm{c})}\} \mp \frac{\boldsymbol{i}}{8}\alpha \int \mathrm{Tr}\gamma_7 F_{(\mathrm{c})} \wedge F_{(\mathrm{c})} \wedge F_{(\mathrm{c})}, \quad (27)$$

and the Bogomol'nyi equation is obtained as [5,6]

$$F_{(\mathrm{c})} \mp *_6 \boldsymbol{i}\alpha\gamma_7 F_{(\mathrm{c})} \wedge F_{(\mathrm{c})} = 0. \quad (28)$$

The term

$$Q := \mathrm{Tr}\gamma_7 F_{(\mathrm{c})} \wedge F_{(\mathrm{c})} \wedge F_{(\mathrm{c})} \quad (29)$$

is a total derivative and the integral over it reduces to a surface integral. The resultant of the integral gives a topological quantity, and $M_{(\mathrm{c})}[\Phi, R_2]$ is bounded from below. The solution of Eq. (28) with a minimal topological charge has been given in [5,6,8]. The minimal charge is

$$Q = \frac{96\pi^3}{g^3}. \quad (30)$$

The gauge configuration

$$A^{(0)} = \frac{1}{4gR_2}\gamma_{ab} y^a V^b, \qquad F^{(0)} = \frac{1}{4gR_2^2}\gamma_{ab} V^a \wedge V^b \quad (31)$$

satisfies the "self-duality" equation

$$F^{(0)} = *_6 \boldsymbol{i}\frac{gR_2^2}{3}\gamma_7 F^{(0)} \wedge F^{(0)}, \quad (32)$$

where R_2 is the radius of S^6 and V^i are vielbeins of S^6, given as

$$V^i = R_2 \frac{dy^i}{(1 + |y|^2/4)}. \quad (33)$$

The configuration (31) solves the Bogomol'nyi equation (28) if and only if R_2 takes a special constant value determined by the constant α [6], given as

$$\alpha = gR_2^2/3. \quad (34)$$

Let us call this special radius $L_c := \sqrt{3\alpha/g}$ the "*Bogomol'nyi radius*." In the case of $\alpha \neq gR_2^2/3$, the gauge configuration (31) does not satisfy the Bogomol'nyi equation (28) anymore.

In this article, we are considering the metric with $R_2(x) \to L_c$ in the limit $t \to +\infty$. An example of such a realization is given in [4], where the four-dimensional part $g_{\mu\nu}$ is given by the Friedmann-Lemaitre-Robertson-Walker metric. The gauge field configuration then approaches the solution of the Bogomol'nyi equation. When $R_2(x)$ is close to L_c, we can consider the perturbation of $R_2(x)$ about L_c, such that the deviation is quantified by a scalar field, $\phi_2(x)$:

$$R_2 = L_c \exp(\phi_2(x)). \quad (35)$$

L_c being a stable fixed point ensures that $\phi_2(x)$ approaches zero at all spatial points of the (1, 3) part of space-time. Note that in [4] we had considered only the case of spatially constant $\phi_2(x) \equiv \phi_2(t)$ and had shown that it approaches zero with time.

We are now ready to study the mass spectrum around the background. To this end, let us first expand $M_{(c)}$ as

$$M_{(c)}[\Phi, R_2]|_{\Phi^2} = M_{(c)}[\Phi, R_2]|^{(1)}_{\Phi^2} + M_{(c)}[\Phi, R_2]|^{(2)}_{\Phi^2} \quad (36)$$

with

$$\begin{aligned} M_{(c)}[\Phi, R_2]|^{(1)}_{\Phi^2} &= -\frac{1}{16}\int \mathrm{Tr}\{D_0^{(6)}\Phi - \boldsymbol{i} *_6 \alpha\gamma_7 \\ &\quad \times \{F^{(0)}, D_0^{(6)}\Phi\}\} \wedge *_6\{D_0^{(6)}\Phi - \boldsymbol{i} *_6 \alpha\gamma_7 \\ &\quad \times \{F^{(0)}, D_0^{(6)}\Phi\}\}, \\ M_{(c)}[\Phi, R_2]|^{(2)}_{\Phi^2} &= -\frac{\boldsymbol{i}}{16}\frac{gR_2^2}{3}(1 + e^{-4\phi_2(x)})\int \mathrm{Tr}\gamma_7 F^{(0)} \wedge F^{(0)} \\ &\quad \wedge F^{(0)} - \frac{\boldsymbol{i}g}{8}\frac{gR_2^2}{3}\int \mathrm{Tr}(1 - e^{-2\phi_2(x)})^2\gamma_7 \\ &\quad \times F^{(0)} \wedge F^{(0)} \wedge (\Phi \wedge \Phi) \\ &\quad + \text{total derivative}. \end{aligned} \quad (37)$$

The first term $M_{(c)}[\Phi, R_2]|^{(1)}_{\Phi^2}$ has only positive eigenvalues in the mass matrix, and so it gives a positive contribution to the eigenvalues of the total mass matrix. On the other hand, the second term $M_{(c)}[\Phi, R_2]|^{(2)}_{\Phi^2}$ includes terms which lower the eigenvalues of the mass matrix. This term vanishes for $\phi_2 = 0$, which is realized in dynamical compactification [4]. As we show later, there are massless modes satisfying

$$D_0^{(6)}\Phi - \boldsymbol{i} *_6 \alpha\gamma_7\{F^{(0)}, D_0^{(6)}\Phi\} = 0. \quad (39)$$

This equation is satisfied for the deformation of the gauge configuration, after which the compact part $F_{(c)}$ of the deformed configuration still satisfies the Bogomol'nyi equation. Because these zero modes are not included in $M_{(c)}[\Phi, R_2]|^{(2)}_{\Phi^2}$, as shown in Eqs. (52) and (53) below, we do not care about the tachyonic modes coming from these zero modes. Hence the eigenvalue of the lowest modes which give a nonzero contribution to $M_{(c)}[\Phi, R_2]|^{(1)}_{\Phi^2}$ gives a gap. This implies that there is a range of ϕ_2 in which the mass matrix does not have tachyonic eigenvalues.

Around this background the vector field $v_\mu(x)$ becomes massive, and the mass term is

$$S_{\text{mass}} = -\frac{16\pi^3}{15}L_c^4\int\sqrt{-g^{(4)}}d^4x\left(1 + \frac{10}{9}e^{-4\phi_2}\right)v_\mu^{ab}v^{\mu,ab}. \quad (40)$$

Since S_{mass} is quadratic in v and our interest is in getting the mass terms, we replace R_2 by L_c. This implies that the gauge symmetry is broken completely. Hence, we expect that the Higgs-Kibble mechanism occurs on this background. We show it explicitly in the next section.

IV. ROTATION OF THE CREMMER-SCHERK CONFIGURATION AND THE HIGGS MECHANISM

The gauge configuration is obtained by the identification of compact directions and internal directions. Here we consider the rotated identification. Let us use the following quantities,

$$\begin{aligned} z^a &= U^{ai}y^i, \qquad W^a = U^{ai}V^i, \\ U^{ai}U^{bi} &= \delta^{ab}, \qquad U^{ci}U^{cj} = \delta^{ij}, \end{aligned} \quad (41)$$

where the rotation matrix $U^{ai} \in \mathrm{SO}(6)$ has different types of indices. Note that this is different from gauge transformations in general.

When U^{ai} is a constant matrix the rotation can be absorbed into gauge transformations. As a first step, let us see what happens to the Cremmer-Scherk gauge configuration under such a constant rotation. W^a as well as V^i form vielbeins as follows,

$$ds^2 = \delta_{ij}V^iV^j = \delta_{ij}W^iW^j. \quad (42)$$

Therefore, the Hodge dual of the four-product of W is given as

$$*_6 W^{ijkl} = \tfrac{1}{2}\epsilon^{ijklmn}W^{mn}. \quad (43)$$

Then the gauge configuration

$$A^{(U)} := \frac{1}{4gR_2}\gamma_{ab}z^a W^b,$$
$$F^{(U)} = dA^{(U)} + gA^{(U)} \wedge A^{(U)} = \frac{1}{4gR_2^2}\gamma_{ab}W^a \wedge W^b \tag{44}$$

satisfies the self-duality equation

$$F^{(U)} = *_6 \frac{gR_2^2}{3}\boldsymbol{i}\gamma_7 F^{(U)} \wedge F^{(U)}. \tag{45}$$

Among the deviations from the Cremmer-Scherk configuration, the six-dimensional part of the full field strength, including fluctuations, with the least mass should preserve the self-duality equation in six dimensions, because such fluctuations saturate the Bogomol'nyi energy bound. Here we show that such fluctuations can be obtained by simply promoting a constant rotation U considered above to a local rotation $U(x)^{ai}$ which depends on the four-dimensional coordinates x. In this case, $U(x)^{ai}$ is not constant and the rotation cannot be absorbed into a gauge transformation. By using the quantities

$$z^a(x) = U^{ai}(x)y^i, \qquad W^a(x) = U^{ai}(x)V^i,$$
$$U^{ai}(x)U^{bi}(x) = \delta^{ab}, \qquad U^{ci}(x)U^{cj}(x) = \delta^{ij}, \tag{46}$$

we consider a gauge configuration

$$A^{(U(x))} := \frac{1}{4gR_2}\gamma_{ab}z^a(x)W^b(x) \tag{47}$$

as the Cremmer-Scherk configuration plus the fluctuations. Although it can be split into background and fluctuation parts as in Eq. (8), we keep it in the present form for our purpose. Because the derivatives $\partial/\partial x^\mu$ along the four-dimensional space-time act on $U(x)$, the quantity $F^{(U(x))}$ has components along the four-dimensional space-time, $F^{(U(x))}_{(\mathrm{nc})}$:

$$F^{(U(x))} = \frac{1}{4g}dx^\mu\left(\frac{\partial}{\partial x^\mu}U^{ac}(x)U^{bd}(x)\right)\gamma_{ab}y^c\frac{dy^d}{(1+|y|^2/4)} + \frac{1}{4gR_2^2}\gamma_{ab}W^a(x)\wedge W^b(x). \tag{48}$$

$F^{(U(x))}_{(\mathrm{nc})}$ is part of the field strength, which has four-dimensional components,

$$F^{(U(x))}_{(\mathrm{nc})} := \frac{1}{4g}dx^\mu\left(\frac{\partial}{\partial x^\mu}U^{ac}(x)U^{bd}(x)\right)\gamma_{ab}y^c\frac{dy^d}{(1+|y|^2/4)}$$
$$= \frac{1}{4gR_2}dx^\mu\alpha_\mu(x)^{ae}\gamma_{ab}z^e(x)W^b(x) + \frac{1}{4gR_2}dx^\mu\alpha_\mu(x)^{bf}\gamma_{ab}z^a(x)W^f(x), \tag{49}$$

where we have used the pullback of the Maurer-Cartan form,

$$\alpha_\mu(x)^{ab} := \left(\frac{\partial}{\partial x^\mu}U^{ac}(x)\right)(U^{-1})^{cb}(x). \tag{50}$$

The six-dimensional part $F^{(U(x))}_{(\mathrm{c})}$ defined in Eq. (12) satisfies the self-duality equation

$$F^{(U(x))}_{(\mathrm{c})} := \frac{1}{4gR_2^2}\gamma_{ab}W^a(x)\wedge W^b(x),$$
$$F^{(U(x))}_{(\mathrm{c})} = *_6\frac{gR_2^2}{3}\boldsymbol{i}\gamma_7 F^{(U(x))}_{(\mathrm{c})}\wedge F^{(U(x))}_{(\mathrm{c})}. \tag{51}$$

At each fixed point x, this configuration is the same as that in the previous section. Let us substitute this into $S_{(\mathrm{c})}$,

$$S_{(\mathrm{c})}[F^{(U(x))}_{(\mathrm{c})}] = -\frac{1}{16}\int dv^{(4)}\int \mathrm{Tr}[-F^{(U(x))}_{(\mathrm{c})}\wedge *_6 F^{(U(x))}_{(\mathrm{c})} + \alpha^2(F^{(U(x))}_{(\mathrm{c})}\wedge F^{(U(x))}_{(\mathrm{c})})\wedge *_6(F^{(U(x))}_{(\mathrm{c})}\wedge F^{(U(x))}_{(\mathrm{c})})]$$
$$= \boldsymbol{i}\frac{1}{16}\int dv^{(4)}\frac{gR_2^2}{3}(1+e^{-4\phi_2})\int \mathrm{Tr}[\gamma_7 F^{(U(x))}_{(\mathrm{c})}\wedge F^{(U(x))}_{(\mathrm{c})}\wedge F^{(U(x))}_{(\mathrm{c})}]. \tag{52}$$

Because of the self-duality property, the action becomes a total derivative as the integral over the six-dimensional sphere with the same value as that of $F^{(0)}$,

$$\int \mathrm{Tr}[\gamma_7 F^{(U(x))}_{(\mathrm{c})}\wedge F^{(U(x))}_{(\mathrm{c})}\wedge F^{(U(x))}_{(\mathrm{c})}] = \int \mathrm{Tr}[\gamma_7 F^{(0)}_1\wedge F^{(0)}_1\wedge F^{(0)}_1] = \frac{96\pi^3}{g^3}, \tag{53}$$

where we have used the fact that $U^{ab}(x)$ is a rotation matrix with unit determinant, the relation $\mathrm{Tr}\gamma_7\gamma_{ab}\gamma_{cd}\gamma_{ef}\sim\epsilon^{abcdef}$ which is an invariant tensor under rotations, and finally Eq. (30). Of course, as an integral over four-dimensional space-time, Eq. (52) is not a total derivative.

Let us define scalar fields $\pi(x)$ by $U(x)\equiv e^{\pi(x)}$. A mass term of $\pi(x)$ in the low-energy effective theory on four-dimensional space-time could be found in $S_{(\mathrm{c})}$ if it exists. Since $S_{(\mathrm{c})}$ does not depend on $U(x)$, we find that $\pi(x)$ are massless fields which are candidates for the Nambu-Goldstone bosons.

Let us expand $\alpha_\mu(x)$ with respect to small fields $\pi(x)$. The leading term is just the derivative of π,

$$\alpha_\mu(x) = \left(\frac{\partial}{\partial x^\mu}U(x)\right)U^T(x) = \left(\frac{\partial}{\partial x^\mu}e^{\pi(x)}\right)e^{-\pi(x)} = \partial_\mu\pi(x)+\cdots. \tag{54}$$

Keeping in mind the facts that the action can be divided into two parts ($S_{(\mathrm{c})}$ and $S_{(\mathrm{nc})}$) and that $S_{(\mathrm{c})}$ is independent of $U(x)$ or $\pi(x)$ as shown in Eqs. (52) and (53), we obtain

$$\begin{aligned} S[\pi(x)] &= \frac{1}{16}\int \mathrm{Tr}\{-F^{(U(x))}\wedge *F^{(U(x))} \\ &\quad + \alpha^2(F^{(U(x))}\wedge F^{(U(x))})\wedge *(F^{(U(x))}\wedge F^{(U(x))})\} \\ &= \frac{1}{16}\int \mathrm{Tr}\{-F^{(U(x))}_{(\mathrm{nc})}\wedge *F^{(U(x))}_{(\mathrm{nc})} + \alpha^2(F^{(U(x))}_{(\mathrm{nc})}\wedge F^{(U(x))}_{(\mathrm{c})} \\ &\quad + F^{(U(x))}_{(\mathrm{c})}\wedge F^{(U(x))}_{(\mathrm{nc})})\wedge *(F^{(U(x))}_{(\mathrm{nc})}\wedge F^{(U(x))}_{(\mathrm{c})} \\ &\quad + F^{(U(x))}_{(\mathrm{c})}\wedge F^{(U(x))}_{(\mathrm{nc})}) + \alpha^2(F^{(U(x))}_{(\mathrm{nc})}\wedge F^{(U(x))}_{(\mathrm{nc})}) \\ &\quad \wedge *(F^{(U(x))}_{(\mathrm{nc})}\wedge F^{(U(x))}_{(\mathrm{nc})})\} \\ &\quad + (\text{terms independent of } \pi(x)). \end{aligned} \tag{55}$$

Since in our notation gauge fields and field strengths are written in terms of the gamma matrices, let us rewrite α_μ^{ab} in terms of the gamma matrices: $\alpha_\mu^{ab}\gamma_{ab}$. By using this quantity, $F^{(U(x))}_{(\mathrm{nc})}$ can be rewritten as

$$F^{(U(x))}_{(\mathrm{nc})} = \tfrac{1}{2}[A^{(0)}, \tfrac{1}{2}\alpha_\mu^{cd}\gamma_{cd}]\wedge dx^\mu. \tag{56}$$

So far we have considered only gauge fields in the compact space $A^{(0)} + \Phi = \frac{1}{4gR_2}\gamma_{ab}z^a(x)W^b(x)$ but not the one $v_\mu(x)$ in the four-dimensional space-time, the (1, 3) part. Here we consider the total gauge field

$$A_{\mathrm{H}} := v_\mu(x)dx^\mu + \frac{1}{4gR_2}\gamma_{ab}z^a(x)W^b(x). \tag{57}$$

By a gauge transformation, the Nambu-Goldstone modes are absorbed into the vector fields v_μ as

$$v_\mu \to u_\mu := v_\mu + \frac{1}{2g}\left(\frac{1}{2}\alpha_\mu^{cd}\gamma_{cd}\right) \tag{58}$$

with

$$u_\mu^{ab} = \left\{U\left[U^{-1}v_\mu U + \frac{1}{2g}U^{-1}\frac{\partial}{\partial x^\mu}U\right]U^{-1}\right\}^{ab}. \tag{59}$$

By taking unitary gauge, we obtain

$$\mathrm{Tr}\, W\wedge *W = \mathrm{Tr} W_u \wedge *W_u, \qquad W_u := du + gu\wedge u. \tag{60}$$

u_μ^{ab} are massive Proca fields with the mass given by

$$S_{\mathrm{mass}} = -\frac{19}{9}\frac{16\pi^3}{15}L_c^4\int\sqrt{-g^{(4)}}d^4xu_\mu^{ab}u^{\mu,ab}. \tag{61}$$

This is nothing but the Higgs mechanism. Here we note that there are no cross terms between u and ϕ_2 up to quadratic order, and the whole mass matrix is block diagonal between u and ϕ_2. The number of Nambu-Goldstone fields π, 15, is the same as the dimension of Lie algebra so (6). Therefore, the gauge symmetry SO(6) is completely broken by the gauge configuration.

V. CONCLUSION

In this paper we have considered ten-dimensional Einstein-Yang-Mills theory, where the gauge field is given by the Cremmer-Scherk configuration with a higher derivative coupling on S^6. We have studied the consequences of ten-dimensional fluctuations of the gauge field on the stability of the background metric and gauge field solutions and on the gauge symmetry in the theory. The Cremmer-Scherk configuration is obtained by the identification of the compact direction and the internal (gauge) direction as an extension of the 't Hooft-Polyakov monopole [3]. By rotating the identification, we have obtained massless fluctuation modes identified as Nambu-Goldstone bosons [9]. These massless modes are absorbed into vector fields and form massive vector fields. Because there are 15 Nambu-Goldstone modes, the gauge symmetry SO(6) is completely broken and all the massless modes are absorbed into vector fields. By scaling the vector fields so that the coefficients of their kinetic terms are canonically normalized, we found that the mass is proportional to the inverse of the radius of the compact space. We conclude that there are neither tachyonic nor massless modes in the physical spectrum around the background and that the configuration is stable.

We must point out that in this paper we have not considered perturbations of gravity, and this remains as an important future problem. One possible extension of the present work is to consider other internal manifolds, such as the projective space $\mathbb{C}P^3$ instead of S^6, using a nontrivial gauge configuration given in [16].

In Ref. [14] Forgacs and Manton used an S^2 reduction with a nontrivial background gauge field to $SU(2)$ Yang-Mills instantons on $\mathbb{R}^2\times S^2$, resulting in the Abrikosov-Nielsen-Olesen vortex solution in the Abelian-Higgs model on $\mathbb{R}^2$. Our case of the S^6 compactification may relate higher dimensional solitons in pure Yang-Mills theory to topological solitons in the Yang-Mills-Higgs system in four or five dimensions [17] such as wall-vortex-monopole composites [18] or instanton-vortex composites [19].

As for higher derivative corrections to non-Abelian gauge theory, much more attention has been given, so far, to the Dirac-Born-Infeld (DBI) action. The fourth derivative term proposed for the non-Abelian DBI action, which is unknown in full order yet, is different from the Tchrakian-type term considered in this paper. However, our work relies on the existence of a Bogomol'nyi-Prasad-Sommerfield soliton on the compactified space, and the DBI action is also known to admit several solitons [20]. Therefore, a similar scenario should be applicable to the DBI action, too. Further, in [21] the possibility of embedding the self-duality equation into larger groups was considered. As an extension of this work, it would be interesting to consider similar symmetry breaking for this configuration embedded into a larger group.

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APPENDIX: NONEXISTENCE OF COVARIANTLY CONSTANT FUNCTIONS BELONGING TO THE ADJOINT REPRESENTATION ON THE CREMMER-SCHERK CONFIGURATION

In this appendix, we study covariantly constant functions on the Cremmer-Scherk gauge configuration on S^6. Let us consider the equation $D_0^{(6)}\varphi = 0$, where $\varphi = \varphi^{ab}\gamma_{ab}$ and the gauge field $A^{(0)}$ is $\frac{1}{4gR_2} y^a V^b \gamma_{ab}$. The equation $D_0^{(6)}\varphi = 0$ is written as

$$\frac{\partial}{\partial y^b}\varphi + \frac{1}{4(1 + |y|^2/4)} y^a[\gamma_{ab}, \varphi] = 0. \quad \text{(A1)}$$

Let us take the contraction of y^b and Eq. (A1). Because γ_{ab} is antisymmetric in indices a, b, the equation simply becomes

$$y^b \frac{\partial}{\partial y^b}\varphi = 0, \qquad \frac{\partial}{\partial r}\varphi = 0, \quad \text{(A2)}$$

where the radial coordinate is defined as $r = |y|$. This shows that φ does not depend on r and depends only on angular coordinates θ_i ($i = 1, 2, \ldots, 5$). The partial differential operator $\partial/\partial y^a$ is rewritten in terms of these coordinates,

$$\frac{\partial}{\partial y^a} = \hat{y}^a \frac{\partial}{\partial r} + \frac{1}{r} L_a, \quad \text{(A3)}$$

where L_a are a linear combination of $\partial/\partial\theta_i$ and do not depend on r. The unit vector $\hat{y}^a = y^a/r$ only depends on those angles. Then Eq. (A1) becomes

$$L_b\varphi + \frac{r^2}{(1 + r^2/4)} \hat{y}^a[\gamma_{ab}, \varphi] = 0. \quad \text{(A4)}$$

The first term in Eq. (A4) and $\hat{y}^a[\gamma_{ab}, \varphi]$ do not depend on r. Therefore, it turns out that φ is constant and commutes with all γ_{ab}. This means that $\varphi \equiv 0$.

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