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Multidimensional world, inflation, and modern acceleration

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Starting from pure multidimensional gravity with curvature-nonlinear terms but no matter fields in the initial action, we obtain a cosmological model with two effective scalar fields related to the size of two extra factor spaces. The model includes both an early inflationary stage and that of modern accelerated expansion and satisfies the observational data. There are no small parameters; the effective inflaton mass depends on the initial conditions which explain its small value as compared to the Planck mass. At the modern stage, the size of extra dimensions slowly increases, therefore this model predicts drastic changes in the physical laws of our Universe in the remote future.

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I. INTRODUCTION

The existence of an early inflationary stage has become a conventional feature in modern descriptions of the Universe due to great success of inflationary scenarios in explaining the observational data (see, e.g., [1,2] for recent reviews). A great number of inflationary scenarios have been suggested by now, and this number is still rapidly growing. It is really difficult to single out a scenario that has been indeed realized by Nature. Another problem is related to the origin of the scalar field (or fields), the so-called inflaton(s), which are almost inevitable ingredients of such scenarios.

On the other hand, the most important set of problems in modern cosmology are related to the observed accelerated expansion of the Universe. Its most popular explanation, fitting all observational constraints, is the so-called ΛCDM model, invoking a cosmological constant Λ as a material source that causes the accelerated expansion via the Einstein equations [3]. However, the hardest problem of this model is the extremely small observed value of Λ (usually ascribed to the physical vacuum density) as compared to the Planck density, the natural vacuum energy density of quantum fields: the corresponding ratio is about 10^{-123}.

Of greatest interest are scenarios that try to jointly describe the entire history of our Universe or at least such its important stages as the early inflation and the modern acceleration. A promising approach on this trend is to use modified theories of gravity, e.g., multidimensional ones. In our view, curvature-nonlinear multidimensional gravity is a good candidate.

It has been recently argued [4–7] that multidimensional gravity with curvature-nonlinear terms in the action can be a source of a great diversity of effective theories able to address a number of important problems of modern astrophysics and cosmology using a minimal set of postulates. Among such problems one can mention the essence of dark energy, early formation of supermassive black holes (which is a necessary stage in some scenarios of cosmic structure formation), and sufficient particle production at the end of inflation. In this approach, it is supposed that essentially different classical universes emerge from space-time foam due to quantum fluctuations, so that particular values of the total space-time dimension $D > 4$ and the topological properties of space-time may vary from one space-time region to another. Different effective theories can take place even with fixed parameters of the original Lagrangian. It can be shown that this approach, without need for fields other than gravity, is able to produce such different structures as inflationary (or simply accelerating) universes, brane worlds [6], black holes etc. The role of scalar fields is played by the metric components of extra dimensions.

In the present paper, we show how pure nonlinear multidimensional gravity, without invoking any material source, makes it possible to describe, in a single scenario, an inflationary stage of the early Universe and a late accelerating stage with a sufficiently small effective cosmological constant. The model obtained agrees with the observational data.

Let us mention some other approaches to obtaining such joint scenarios. Reference [8] shows how to achieve this goal in some models of nonlocally modified gravity theo-

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ries in four dimensions; in these models, the dark energy effect is caused by a composite graviton degree of freedom. In [9], the same goal is achieved using a Yang-Mills condensate as a matter source. Reference [10] considers a relationship between hybrid inflation and dark energy; see there also numerous references on the subject.

The paper is organized as follows. In Sec. II, we describe the general formalism. Section III shows how to obtain a successful inflationary scenario in a Kaluza-Klein type model with a single extra factor space. Section IV is devoted to obtaining models with two extra factor spaces able to unify inflation and modern acceleration. Section V is a conclusion.

II. *D*-DIMENSIONAL GRAVITY

We will briefly describe a method of considering a wide classes of Lagrangians in multidimensional gravity in a Kaluza-Klein type approach, following [4,5]. Consider the action[1]

$$S = \frac{1}{2} m_D^{D-2} \int \sqrt{{}^D g}\, d^D x [F(R) + c_1 R_{AB} R^{AB} + c_2 \mathcal{K}] \quad (1)$$

in D-dimensional space-time $\mathbb{M}$ with the structure $\mathbb{M} = \mathbb{M}_0 \times \mathbb{M}_1 \times \ldots \times \mathbb{M}_n$, where $\dim \mathbb{M}_i = d_i$ and m_D is the D-dimensional Planck mass (not necessarily coinciding with the conventional Planck mass m_4), and the metric

$$ds_D^2 = g_{ab}(x) dx^a dx^b + \sum_{i=1}^{n} e^{2\beta_i(x)} g^{(i)}, \quad (2)$$

where (x) means the dependence on the first d_0 coordinates x^a; $g_{ab} = g_{ab}(x)$ is the metric in $\mathbb{M}_0$, $g^{(i)}$ are x-independent d_i-dimensional metrics of the factor spaces $\mathbb{M}_i$, $i = \overline{1, n}$. In (1), $F(R)$ is an arbitrary function of the scalar curvature R of $\mathbb{M}$; c_1 and c_2 are constants; R_{AB} and $\mathcal{K} = R_{ABCD} R^{ABCD}$ are the Ricci tensor and the Kretschmann scalar of $\mathbb{M}$, respectively; capital Latin indices cover all D coordinates, small Latin ones $(a, b, \ldots)$ the coordinates of $\mathbb{M}_0$, and $a_i, b_i, \ldots$ the coordinates of $\mathbb{M}_i$. Let us note that terms proportional to R^2 and other powers of R, $R_{AB}R^{AB}$ and the Kretschmann scalar $\mathcal{K} = R_{ABCD} R^{ABCD}$ and other high-order curvature terms appear due to quantum corrections in quantum field theory in curved space-times [11,12].

The D-dimensional Riemann tensor has the nonzero components

$$\begin{aligned}
R^{ab}{}_{cd} &= \bar{R}^{ab}{}_{cd}, \\
R^{aa_i}{}_{bb_i} &= \delta^{a_i}_{b_i} B^a_{b\,(i)}, \\
B^a_{b\,(i)} &:= e^{-\beta_i} \nabla_b (e^{\beta_i} \beta_i^{;a}), \\
R^{a_i b_i}{}_{c_i d_i} &= e^{-2\beta_i} \bar{R}^{a_i b_i}{}_{c_i d_i} + \delta^{a_i b_i}{}_{c_i d_i} \beta_{i,a} \beta_i^{;a}, \\
R^{a_i b_k}{}_{c_i d_k} &= \delta^{a_i}_{c_i} \delta^{b_k}_{d_k} \beta_{i,\mu} \beta_k^{;\mu}, \qquad i \neq k.
\end{aligned} \quad (3)$$

Here the bar marks quantities obtained from the factor space metrics g_{ab} and $g^{(i)}$ taken separately, $\beta_{,a} \equiv \partial_a \beta$, $\delta^{ab}{}_{cd} \equiv \delta^a_c \delta^b_d - \delta^a_d \delta^b_c$ and similarly for other kinds of indices.

The nonzero components of the Ricci tensor and the scalar curvature are

$$\begin{aligned}
R^b_a &= \bar{R}^b_a + \sum_i d_i B^b_{a(i)}, \\
R^{b_i}_{a_i} &= e^{-2\beta_i} \bar{R}^{b_i}_{a_i} + \delta^{b_i}_{a_i} [\Box \beta_i + \beta_{i,a} \sigma^{,a}], \\
R &= \bar{R}[g] + \sum_i e^{-2\beta} \bar{R}_i + 2\Box\sigma + (\partial\sigma)^2 + \sum_i d_i (\partial\beta_i)^2,
\end{aligned} \quad (4)$$

where $\sum_i$ means $\sum_{i=1}^n$; $\sigma := \sum_i d_i \beta_i$; $(\partial\sigma)^2 \equiv \sigma_{,a}\sigma^{,a}$ and similarly for other functions; $\Box = g^{ab}\nabla_a \nabla_b$ is the d_0-dimensional d'Alembert operator; $\bar{R}[g]$ and $\bar{R}_i$ are the Ricci scalars corresponding to g_{ab} and $g^{(i)}$, respectively. In what follows, we will assume that the factor spaces $\mathbb{M}_i$ are compact spaces of constant curvature $K_i = \pm 1$, so that, in particular, $\bar{R}_i = K_i d_i (d_i - 1)$.

A. Slow-change approximation: Reduction to lower dimensions

Let us suppose that all quantities are slowly varying, i.e., consider each derivative ∂_a (including those in the definition of $\bar{R}$) as an expression containing a small parameter ε, and neglect all quantities of orders higher than $O(\varepsilon^2)$. Then we have the following decompositions:

$$\begin{aligned}
R &= \phi + \bar{R}[g] + f_1, \\
f_1 &:= 2\Box\sigma + (\partial\sigma)^2 + \sum_i d_i (\partial\beta_i)^2; \\
F(R) &= F(\phi) + F'(\phi)(\bar{R}[g] + f_1) + O(\varepsilon^4); \\
R_{AB}R^{AB} &= \sum_i \frac{1}{d_i}\phi_i^2 + 2\sum_i d_i \phi_i [\Box\beta_i + (\partial\beta_i, \partial\sigma)] \\
&\quad + O(\varepsilon^4); \\
\mathcal{K} &= 2\sum_i \frac{\phi_i^2}{d_i(d_i - 1)} + 4\sum_i d_i \phi_i (\partial\beta_i)^2 + O(\varepsilon^4),
\end{aligned} \quad (5)$$

where

$$\phi_i := K_i m_D^2 (d_i - 1) e^{-2\beta_i}, \qquad \phi := \sum_i d_i \phi_i. \quad (6)$$

[1] Our conventions are: the metric signature $(+ - - \ldots)$; the curvature tensor $R^\sigma{}_{\mu\rho\nu} = \partial_\nu \Gamma^\sigma_{\mu\rho} - \ldots$, $R_{\mu\nu} = R^\sigma{}_{\mu\sigma\nu}$, so that the Ricci scalar $R > 0$ for de Sitter space-time and the matter-dominated cosmological epoch; the system of units $c = \hbar = 1$.

The symbol $(\partial\alpha, \partial\beta)$ means $g^{ab}\alpha_{,a}\beta_{,b}$, and $F'(\phi) = dF/d\phi$.

As a result, neglecting $o(\varepsilon^2)$ and integrating out all $\mathbb{M}_i$, we obtain the following purely gravitational action reduced to d_0 dimensions:

$$\begin{aligned} S &= \frac{1}{2}\mathcal{V} m_D^{d_0-2}\int \sqrt{g_0}d^{d_0}x\{e^{\sigma}F'(\phi)\bar{R}_0 \\ &\quad + K_J - 2V_J(\phi_i)\}, \\ K_J &= F'(\phi)e^{\sigma}f_1 + 2e^{\sigma}\sum_i d_i\phi_i[c_1\Box\beta_i \\ &\quad + c_1(\partial\beta_i, \partial\sigma) + 2c_2(\partial\beta_i)^2], \\ -2V_J(\phi_i) &= e^{\sigma}\bigg[F(\phi) + \sum_i d_i\phi_i^2\bigg(c_1 + \frac{2c_2}{d_i - 1}\bigg)\bigg], \end{aligned} \quad (7)$$

where $g_0 = |\det(g_{\mu\nu})|$ and $\mathcal{V}$ is a product of volumes of n compact d_i-dimensional spaces $\mathbb{M}_i$ of unit curvature. The expression (7) is typical of a (multi)scalar-tensor theory (STT) of gravity in a Jordan frame.

Subtracting a full divergence, we get rid of second-order derivatives in (7), and the resulting kinetic term takes the form

$$\begin{aligned} K_J &= F'e^{\sigma}\bigg[-(\partial\sigma)^2 + \sum_i d_i(\partial\beta_i)^2\bigg] - 2F''e^{\sigma}(\partial\phi, \partial\sigma) \\ &\quad + 4e^{\sigma}(c_1 + c_2)\sum_i d_i\phi_i(\partial\beta_i)^2, \end{aligned} \quad (8)$$

where $F'' = d^2F/d\phi^2$.

B. Transition to the Einstein frame

For further analysis, it is helpful to pass on to the Einstein frame using the conformal mapping

$$g_{\mu\nu} \mapsto \tilde{g}_{\mu\nu} = |f(\phi_i)|^{2/(d_0-2)}g_{\mu\nu}, \qquad f(\phi_i) = e^{\sigma}F'(\phi). \quad (9)$$

The expression with the scalar curvature in (7) transforms as follows:

$$\begin{aligned} \sqrt{g_0}e^{\sigma}\bar{R}_0 &= \sqrt{g_0}f\bar{R}_0 \\ &= (\mathrm{sign} f)\sqrt{\tilde{g}}\bigg[\tilde{R} + \frac{d_0 - 1}{d_0 - 2}\frac{(\tilde{\partial}f)^2}{f^2}\bigg] + \mathrm{div}, \end{aligned} \quad (10)$$

where the tilde marks quantities obtained from or with $\tilde{g}_{\mu\nu}$ and div denotes a full divergence which does not contribute to the field equations. The action (7) acquires the form

$$\begin{aligned} S &= \frac{1}{2}\mathcal{V} m_D^{d_0-2}\int \sqrt{\tilde{g}}d^{d_0}x\{[\mathrm{sign}F'(\phi)][\tilde{R} + K_E] \\ &\quad - 2V_E(\phi_i)\} \end{aligned} \quad (11)$$

with the kinetic and potential terms

$$\begin{aligned} K_E &= \frac{1}{d_0 - 2}\bigg(\partial\sigma + \frac{F''}{F'}\partial\phi\bigg)^2 + \bigg(\frac{F''}{F'}\bigg)^2(\partial\phi)^2 \\ &\quad + \sum_i d_i\bigg[1 + \frac{4}{F'}(c_1 + c_2)\phi_i\bigg](\partial\beta_i)^2, \end{aligned} \quad (12)$$

$$\begin{aligned} -2V_E(\phi_i) &= e^{-2\sigma/(d_0-2)}|F'|^{-d_0/d} \\ &\quad \times\bigg[F(\phi) + \sum_i d_i\phi_i^2\bigg(c_1 + \frac{2c_2}{d_i - 1}\bigg)\bigg], \end{aligned} \quad (13)$$

where the tildes are omitted though the metric $\tilde{g}_{\mu\nu}$ is used, and the indices are raised and lowered with $\tilde{g}_{\mu\nu}$. The original quantities β_i and σ are now expressed in terms of n fields ϕ_i whose number coincides with the number of extra factor spaces.

In what follows, we consider the most relevant case $d_0 = 4$.

A further interpretation of the results depends on which conformal frame is regarded physical (observational) [13,14], and this in turn depends on the manner in which fermions appear in the (so far unknown) underlying unification theory involving all interactions. We will restrict ourselves to the simplest assumption, that the Einstein frame is simultaneously the observational frame. It means, in particular, that the effective Newtonian gravitational constant G is a true constant in the course of the cosmological evolution. Moreover, we will assume for simplicity that the D-dimensional Planck mass m_D is equal to the four-dimensional Planck mass $m_4 = G^{-1/2}$; furthermore, we put $G = 1$, and numerical values of dimensionful parameters are thus expressed in Planck units.

III. A SINGLE EXTRA FACTOR SPACE AND INFLATION

Now, our program is as follows:

(i) Choose the parameters of the original action (1) to obtain a behavior of the potential (13) providing primordial inflation.

(ii) Additionally vary the parameters to satisfy the inflationary conditions conforming to observations.

(iii) Try to describe the modern acceleration stage, providing the ratio of the effective cosmological constant to the Planck density $\Lambda_{\mathrm{eff}}/m_4^4$ of the order 10^{-123}.

We begin with the case of one factor space. Then Eqs. (12) and (13) simplify to give

$$\begin{aligned} S &= \frac{\mathcal{V}}{2}\int d^4x\sqrt{\tilde{g}}(\mathrm{sign}F')L, \\ L &= \tilde{R}_4 + K_E^{(1)}(\phi)(\partial\phi)^2 - 2V_E^{(1)}(\phi), \end{aligned} \quad (14)$$

$$K_E^{(1)}(\phi) = \frac{1}{4\phi^2}\left[6\phi^2\left(\frac{F''}{F'}\right)^2 - 2d_1\phi\frac{F''}{F'} + \frac{1}{2}d_1(d_1+2)\right] + \frac{c_1+c_2}{F'\phi}, \qquad (15)$$

$$V_E^{(1)}(\phi) = -\frac{\mathrm{sign}F'}{2F'^2}\left[\frac{|\phi|}{d_1(d_1-1)}\right]^{d_1/2}\left[F(\phi) + c_V\frac{\phi^2}{d_1}\right], \quad c_V := c_1 + \frac{2c_2}{d_1-1}. \qquad (16)$$

Here we take

$$F = F(\phi) = \phi + c\phi^2 - 2\Lambda, \qquad c, \Lambda = \mathrm{const}, \qquad (17)$$

and, in accord with the definition (6), $\phi = d_1\phi_1$.

In (14)–(16) we have actually changed the sign of the Lagrangian in case $F' < 0$; to preserve the attractive nature of gravity for ordinary matter, the matter Lagrangian density should appear with an unusual sign from the beginning. As a result, the sign of the whole action of gravity and matter will be unusual, without any effect on the equations of motion; one can show that quantum transitions are then unaffected as well, see a discussion in [4].

The presence of the parameters c_1 and c_2 adds freedom in choosing the shape of the potential. The kinetic term also has a complex form which can significantly affect the field dynamics. An analysis of kinetic terms like (15) of variable sign can lead to possibilities of interest, and we hope to return to this point in our future work.

Let us employ the fact that chaotic inflation with a quadratic potential and the inflaton mass $m_\varphi \approx 10^{-6}m_4$ well conforms to the observational data. Therefore our task is simplified and reduced to finding such parameters c, c_1 and c_2 that the potential (16) near its minimum is approximated by a quadratic function with the above inflaton mass. It turns out to be possible with the following parameter values:

$$d_1 = 4; \qquad c = 2.5\times 10^4; \qquad c_1 + c_2 = 0.6;$$
$$c_{\mathrm{tot}} := \frac{c_1}{d_1} + \frac{2c_2}{d_1(d_1-1)} = -0.62; \qquad \Lambda = 0.2. \qquad (18)$$

With these parameter values, all basic requirements to inflation are satisfied. Thus, the duration of the inflationary period exceeds 60 e-folds, the temperature fluctuations are $\sim 6\times 10^{-5}$, and the spectral index is $n_s = 0.943$, within observational bounds, $n_s = 0.958 \pm 0.016$ [15]. Thus a single factor space is sufficient for obtaining a fairly good inflationary scenario.

Since the constant c has actually the dimension of length squared, it is $\sqrt{c} \sim 100$ that should be compared with the Planck length. So this model does not contain unnaturally large or small parameters.

A serious shortcoming of this model is that it is unable to solve the problem of modern acceleration, including smallness of dark energy density. Indeed, it is easy to prove that slight variations of the parameters c, c_1 and c_2 could give rise to an arbitrarily small potential value at the minimum. However, though the values of these parameters are quite moderate, they should be extremely "fine-tuned" to fit the modern value of vacuum energy density. An attempt to solve this problem in a slightly more complex model is undertaken in the next section.

IV. TWO FACTOR SPACES: INFLATION AND MODERN ACCELERATION

A. Inflation

Additional opportunities emerge if the extra space is a product of two factor spaces, $\mathbb{M}_{d_1} \times \mathbb{M}_{d_2}$ of dimensions d_1 and d_2. For further analysis, let us make the situation more specific by putting $K_1 = K_2$, $d_1 = d_2$ and choosing the function

$$F(R) = R^2. \qquad (19)$$

(Note that one of the coefficients in the initial Lagrangian can be chosen at will, e.g., equal to unity, without affecting the field equations; it simply specifies the scale for other coefficients.)

Figure 1 presents the potential of the effective scalar fields for this model with the following choice of the parameter values:

$$d_1 = d_2 = 5, \qquad c_V = -10.001,$$
$$c_1 + c_2 = 1.25\times 10^3. \qquad (20)$$

All further numerical estimates will be obtained with these values. As follows from the above-said, at low energies (as compared to the Planck scale m_D) this model is equivalent to Einstein gravity with two scalar fields. In full similarity with Sec. III, the constants c_1 and c_2 have actually the

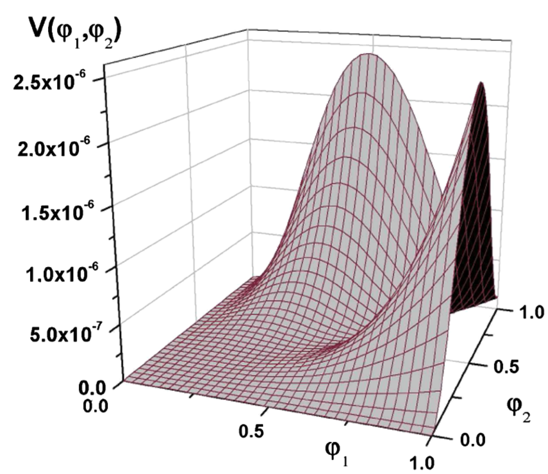

FIG. 1 (color online). Potential of the effective scalar fields for the model (1) and (19) with the parameter values given in Eq. (20).

dimension of length squared, and their square roots are not unnaturally large or small.

Note that, with the c_V value chosen, a positive potential V (hence a positive effective cosmological constant) is obtained with $K_1 = 1$, i.e., spherical extra factor spaces. For other values of c_V, e.g., $c_V > 0$ we would need hyperbolic factor spaces.

The inflationary period is characterized by moving down one of the steep slopes of the valley. The inflaton mass squared is proportional to the second-order derivative of the potential in the direction perpendicular to the valley (its bottom is located at $\phi_1 = \phi_2 = \phi_0$). It is this direction in which the field moves during inflation and oscillates during reheating at the post-inflationary stage. The specific value of ϕ_0 depends on the initial value of the inflaton field at which the classical universe was born.

Figure 2 shows the dependence of the effective inflaton mass on the parameter ϕ_0. In the framework of chaotic inflation, universes are created with different inflaton values under the horizon, leading to different values of ϕ_0 and hence different inflaton masses. This is how this model solves the problem of smallness of the inflaton mass in Planck units.

Post-inflationary particle production is a result of oscillations in the direction across the valley. The conditions suitable for our Universe correspond to the value $\phi_0 \simeq 0.5$. It is just such a value that, according to Fig. 2, the inflaton mass, related to the second-order derivative of the potential in the direction across the valley, is $\sim 10^{-6} \sim 10^{13}$ GeV, which satisfactorily explains the observational data on the CMB temperature fluctuations.

It is of interest to which extent the values of c_1 and c_2 (or, more conveniently, their combinations c_V and $c_K = c_1 + c_2$) in Eq. (20) are fine-tuned. An inspection shows that

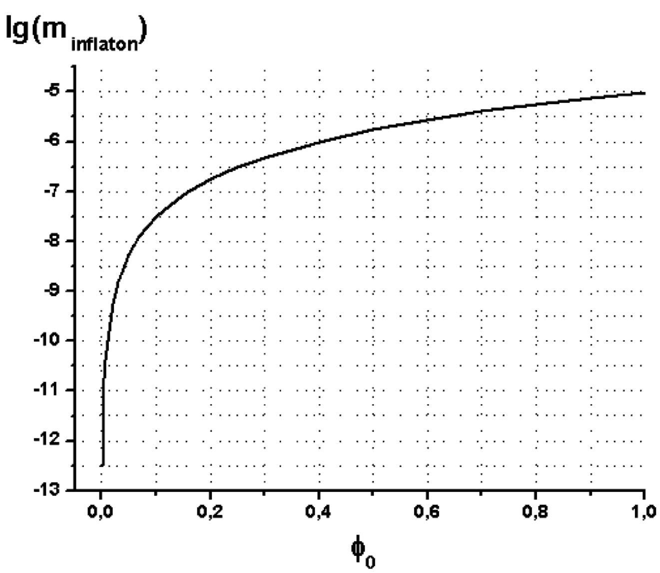

FIG. 2. Dependence of the effective inflaton mass (Planck units) on the parameter ϕ_0.

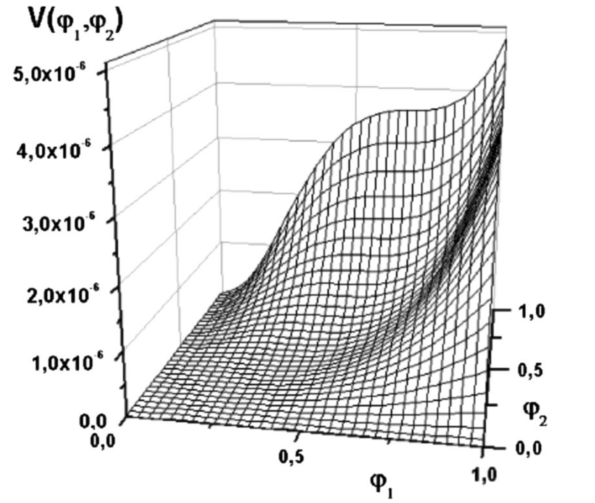

FIG. 3. Potential of the effective scalar fields for the model (1), with $d_1 = d_2 = 5$, $c_V = -10.5$, $c_1 + c_2 = 1.25 \times 10^3$.

with c_V in the range $(-10.2, -10)$ the potential provides all three necessary stages of evolution: inflation, reheating and the present expansion, in agreement with the observational data under proper initial conditions. Larger deviations destroy the valley of the potential surface thus drastically changing the whole picture. As an example, we present the potential for the same value of c_K but $c_V = -10.5$ (Fig. 3). There is no valley, hence a reheating stage is impossible.

The admissible range of c_K is wider: its value may vary by an order of magnitude with respect to the one given in (20). Within this area, the predictions are actually the same, within uncertainties in the observational data.

B. Matter dominated stage

The inflationary stage ends with rapid field oscillations across the valley in Fig. 1, on whose bottom, by our assumptions, $\phi_1 = \phi_2 = \phi/(2d_1)$. These oscillations are accompanied by effective particle production in full agreement with the standard version of chaotic inflation with a quadratic potential. In the model under discussion, the energy density of the produced particles makes the material content of the Universe and affects not only the cosmological expansion rate but also the scalar field dynamics. The latter now corresponds to slow rolling down along the bottom of the potential valley.

We assume a spatially flat cosmology in four dimensions, with the Einstein-frame metric $d\tilde{s}_4^2 = dt^2 - a^2(t)d\vec{x}^2$. So, with the choice (19), the action (11) leads to the Lagrangian

$$L_E = R_4 + K(\phi)(d\phi)^2 - 2V(\phi) \qquad (21)$$

and

$$K(\phi)(\partial\phi)^2 = K_0(\partial\phi)^2/\phi^2 = 4K_0(\partial\beta)^2,\quad 2K_0 = d_1^2 - d_1 + 3 + 4(c_1 + c_2), \tag{22}$$

$$V(\phi) = V_0|\phi|^{d_1} = V_1 e^{-2d_1\beta},\quad V_1 = -\frac{K_1}{8}\left(1 + \frac{c_V}{2d_1}\right),\quad V_0 = V_1[2d_1(d_1-1)]^{-d_1}, \tag{23}$$

where $K_1 = \mathrm{sign}\phi = \mathrm{sign}F'(\phi)$ and $\beta(t) = \beta_1(t)$ is, as before, the logarithm of the extra-dimension scale factor (which is in the present case the same for all extra dimensions), such that $d\phi/\phi = -2d\beta$, and c_V has been defined in (16). One can also notice that a usual form of the Lagrangian with a scalar field Φ and a potential V_Φ is obtained if we substitute

$$2\sqrt{K_0}\beta = \sqrt{8\pi G}\Phi, \qquad V_E = 8\pi G V_\Phi.$$

With (21), we can write two independent components of the Einstein-scalar equations for $\beta(t)$ and $a(t)$ as follows:

$$3H^2 = 2K_0\dot\beta^2 + V_1 e^{-2d_1\beta} + 8\pi\rho_m \tag{24}$$

$$2K_0[\ddot\beta + 3H\dot\beta] = d_1 V_1 e^{-2d_1\beta}, \tag{25}$$

where $H = \dot a/a$ is the Hubble parameter.

Let us begin with considering the matter dominated stage, which is the longest. The subsequent dark energy (DE) dominated stage will be discussed in the next subsection. The following simplifying assumptions will be used: (i) we neglect the pressure of matter, treating it as dust from the very beginning ($t = t_1$) thus ignoring a radiation-dominated stage; (ii) we neglect a possible direct interaction between matter and the scalar field; (iii) we neglect the scalar field contribution to the dynamics of $a(t)$ at the matter dominated stage $t_1 < t < t_2 \simeq 10^{10}$ years and, vice versa, we neglect the contribution of matter at the DE dominated stage $t > t_2$.

So, neglecting the contribution of β in Eq. (24), we obtain for times $t_1 < t < t_2$, as in the usual big bang scenario,

$$H = 2/(3t) \quad \text{at } t_1 < t < t_2. \tag{26}$$

To solve Eq. (25) numerically, we take the following initial data corresponding to the end of the post-inflationary epoch:

$$\phi_1(t_1) = \phi_2(t_1) = \frac{\phi(t_1)}{2d_1} = 0.05;\quad \frac{d\phi}{dt}(t_1) = 0 \Rightarrow e^{\beta(t_1)} = 4\sqrt{5}, \qquad \dot\beta(t_1) = 0. \tag{27}$$

The initial time t_1 is chosen to be $t_1 = 2.82 \times 10^{10}$ for definiteness.

The numerical solution of Eq. (25) then gives the following value of β at $t = t_2$:

$$e^{\beta(t_2)} = 5.48255 \times 10^{11} \simeq 5.5 \times 10^{11} \Rightarrow \phi(t_2) \simeq 1.3 \times 10^{-22}. \tag{28}$$

This value of β will be used in analyzing its dynamics at the modern stage for which the equations simplify and can be solved analytically.

C. Modern stage

The modern epoch $t > t_2$ is DE dominated. In the present approach, DE is represented by the scalar field ϕ (or equivalently β or $b = e^\beta$) with the potential (23), and the Universe dynamics is described by Eqs. (24) and (25). In (24) we now neglect the matter contribution.

It is hard to solve this set of equations exactly. However, as the ϕ field decreases (which corresponds to a growing size of the extra dimensions) along with a decreasing value of the potential (related to the effective cosmological constant), at some stage it becomes possible to treat this process as secondary slow rolling, for which the field dynamics is sufficiently simple and may be described analytically. Indeed, let us suppose

$$|\ddot\beta| \ll 3(\dot a/a)\dot\beta, \qquad K_0\dot\beta^2 \ll 3(\dot a/a)^2 \tag{29}$$

and drop the corresponding terms in Eqs. (24) and (25). Then we can express $\dot a/a$ from (24) and insert it to (25), getting

$$d_1\dot\beta e^{d_1\beta} = B_0 := \frac{d_1^2\sqrt{V_1}}{2\sqrt{3K_0}}, \tag{30}$$

whence we find the evolution law for the extra-dimension scale factor

$$e^\beta = [B_0(t - t_*)]^{1/d_1}, \tag{31}$$

where t_* is an integration constant ($t_* = t_2 - B_0^{-1}[b(t_2)]^{d_1}$). Substituting this result to (24), we find the evolution law for $a(t)$:

$$a(t) = a_*(t - t_*)^p, \qquad p := 2K_0/d_1^2, \tag{32}$$

where a_* is an integration constant.

With the parameters (20), some relevant constants are

$$V_1 = 1.25 \times 10^{-4}, \qquad 2K_0 = 5023,\quad p = \frac{5023}{25} \approx 201, \qquad B_0 \approx 3.2 \times 10^{-5}. \tag{33}$$

Equation (31) with the initial value (28) gives the present size of the extra dimensions, at $t = t_0 = 13.7 \cdot 10^9$ yr:

$$b(t_0) = 5.48259 \times 10^{11} \simeq 5.5 \times 10^{11} \approx 9 \times 10^{-22}\ \text{cm}, \tag{34}$$

well within the observational limits. From (32) we find the Hubble constant $H_0 = \dot a(t_0)/a(t_0)$ and the Hubble time $t_H = 1/H_0$:

$$H_0 \approx 1.25 \times 10^{-61}, \qquad t_H \approx 8 \times 10^{60} \approx 13.8 \times 10^9 \text{ yr}, \tag{35}$$

in agreement with observations. The potential energy density V, coinciding with the DE density,

$$V_E(\phi(t_0)) \simeq 5.1 \times 10^{-123}, \tag{36}$$

also well agrees with observations.

One can notice that in our model with $d_1 = 5$ the function (31) grows extremely slowly. The present value in (34) differs from that in (28) only in the fifth decimal digit, so that the change is actually indistinguishable. The same is true for the DE density which thus behaves like a cosmological constant. The expansion law (32) with the exponent $p = 201$ is really almost exponential, i.e., de Sitter, and the DE equation-of-state factor $w = p_{\rm DE}/\rho_{\rm DE}$ is very close to minus unity. Indeed, in the DE epoch, $a(t) \sim t^{2/(3+3w)}$, hence

$$2/(3 + 3w) = 201 \Rightarrow w \approx -0.9967.$$

Last, one can verify that this solution fairly well satisfies the slow-rolling conditions (29), which hold as long as $p \gg 1$. or, in terms of the input parameters of the theory, if $c_1 + c_2 \gg d_1^2$.

It is of interest that models of gravity (1) where $F(R)$ contains a linear term do not lead to similar attractive results in the present approach.

V. CONCLUSION

In the framework of pure curvature-nonlinear gravity with extra dimensions, it has been possible to describe (though in a rough approximation) the entire evolution of the Universe beginning with an inflationary stage and ending with the modern accelerated stage with sufficiently small dark energy density. In doing so, it has been possible to avoid unnaturally small or large parameter values in the initial Lagrangian. The small values of the inflaton mass and especially that of DE density agreeing with observations have been obtained from a Lagrangian whose dimensionless parameters differ from unity by no more than 2 orders of magnitude.

Using a single extra factor space, it appears possible to explain the emergence of an inflaton, and choosing proper values of the parameter, it is possible to fulfil all requirements applicable to inflationary models and achieve an agreement with the observational data. However, to solve the problem of small DE density, it is necessary to invoke (at least) two extra factor spaces.

The inflationary stage with an appropriate inflaton mass is again well described. Indeed, field fluctuations create universes with different initial field values. the potential in Fig. 1 (i.e., at fixed values of the initial Lagrangian parameters) has different curvatures at different points of the valley, which correspond to different inflaton masses. We live in a universe created by a suitable field fluctuation whose evolution leads to the observable inflaton mass.

As to late-time evolution, it becomes possible to obtain in a natural way a small current value of the effective potential which plays the role of DE density (effective cosmological constant), $\Lambda_{\rm eff} \sim 10^{-123} m_4^4$). The form of our late-time solution shows that the size of the extra dimensions is slowly growing in the modern epoch. In the remote future, this size, which is so far invisible for modern instruments, is to grow to such values that will lead to drastic changes in the physical laws of our Universe. Let us stress, however, that such a model is only one particular opportunity contained in our approach. There are other models where the extra dimensions are stable at late times [4] making the effective physical constants also invariable.

Our model with two factor spaces has the following advantages:

(a) Its low-energy limit represents the Hilbert-Einstein action with appropriate accuracy.
(b) It describes inflation with an inflaton mass agreeing with observations;
(c) The size of the extra dimensions $b(t)$ never exceeded the experimental threshold $\sim 10^{-17}$ cm (though should exceed it in the remote future).
(d) At the modern stage, the scalar field density (actually, the potential $V(\phi)$ in proper units) describes the modern DE density $\sim 10^{-123} m_4^4$;
(e) The DE equation-of-state parameter w satisfies the observational constraint $w < -0.8$.

This model has a somewhat unusual total dimension $D = d_0 + 2d_1 = 14$. With such a choice, it is clear that we keep aside from the ideas of string theory in this study, which makes our model less restrictive in the choice of the dimensionality. We believe that other choices of parameter sets, including dimensionality, can also lead to good potentials in the low-energy limit, and this can be a subject of future work. However, the particular values $d_1 + d_2 = 3 + 3$, leading to $D = 10$ (the "string" dimension), are probably unsuitable in our case since then $c_V = c_K = c_1 + c_2$ (see the notations above), which leaves one independent parameter instead of two thus substantially restricting the choice of effective potentials.

Since we have been working in the Einstein conformal frame, the problem of varying physical constants (above all, the effective Newtonian constant of gravity $G_{\rm eff}$) did not emerge. One should note that even remaining in the Einstein frame, we could assume $m_D \neq m_4$, which would affect the estimated boundary between the classical and quantum worlds. In a more general framework, interpreting another conformal frame (possibly but not necessarily the original Jordan frame) as the observational one, we would obtain a dependence of the constant $G_{\rm eff}$ (hence the current Planck mass $m_4 = G_{\rm eff}^{-1/2}$) on the size of extra dimensions, which in general can be not only time-dependent but also vary from point to point in space. In the cosmological context, models with variable $G_{\rm eff}$ should not only satisfy the observational bounds on the variation rate $\dot{G}_{\rm eff}/G_{\rm eff}$ ($\lesssim 10^{-13}$ according to the recent tightest constraint [16]) but also take into account the effect of $G(t)$ on stellar evolution and processes in the early Universe. (Therefore, models with self-stabilizing extra dimensions like those discussed in [4,5] can be more attractive.) In still more general models of this sort even the Planck constant $\hbar$ can be variable. A discussion of these problems is out of the scope of this paper and can be found, e.g., in [13,14,17–19].

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