PHYSICAL REVIEW D **81,** 076010 (2010)

Probing *CPT* violation in *B* systems

Anirban Kundu,[1,*] Soumitra Nandi,[2,†] and Sunando Kumar Patra[1,‡]

[1]*Department of Physics, University of Calcutta, 92, Acharya Prafulla Chandra Road, Kolkata 700 009, India*

[2]*Dipartimento Fisica Teorica, Università di Torino & INFN Torino, I-10125, Torino, Italy*

(Received 10 February 2010; published 26 April 2010)

We discuss how a possible violation of the combined symmetry *CPT* in the *B* meson system can be investigated at the LHC. We show how a tagged and an untagged analysis of the decay modes of both B_d and B_s mesons can lead not only to a possible detection of a *CPT*-violating new physics but also to an understanding of its precise nature. The implication of *CPT* violation to a large mixing phase in the B_s system is also discussed.

DOI: 10.1103/PhysRevD.81.076010 PACS numbers: 11.30.Er, 14.40.Nd

I. INTRODUCTION

The combined symmetry *CPT* is supposed to be an exact symmetry of any local axiomatic quantum field theory. This is indeed supported by the experiments: all possible tests so far [1] have yielded negative results, consistent with no *CPT* violation. Why then should we be interested in the possibility of *CPT* violation in the *B* system? There are three main reasons: first, any symmetry which is supposed to be exact ought to be questioned and investigated, and we may get a surprise, just like the discovery of *CP* violation; second, it is not obvious that *CPT* will still be an exact symmetry in the bound state of quarks and antiquarks, where the asymptotic states are not uniquely defined [2]; third, there may be some nonlocal and nonrenormalizable string-theoretic effects at the Planck scale which have a ramification at the weak scale through the effective Hamiltonian [3]. Moreover, this effect can very well be flavor sensitive, and so the constraints obtained from the *K* system [4] may not be applicable to the *B* systems. A comprehensive study of *CPT* violation in the neutral *K* meson system, with a formulation that is closely analogous to that in the *B* system, may be found in [5].

There are already some investigations on *CPT* violation in *B* systems. Datta *et al.* [6] have shown how *CPT* violation can lead to a significant lifetime difference $\Delta\Gamma/\Gamma$ in the generic M^0-$\bar{M}^0$ system, where $M^0 = K^0$, B^0, or B_s. It was discussed in [7] how direct *CP* asymmetries and semileptonic decays can act as a probe of *CPT* violation. Signatures of *CPT* violation in non-*CP* eigenstate channels was discussed in [8]. The role of dilepton asymmetry as a test of *CPT* violation and possible discrimination from $\Delta B = -\Delta Q$ processes were investigated in [9]. The *BABAR* experiment at SLAC has tried to look for *CPT* violation in the diurnal variations of *CP*-violating observables and set some limits [10].

Right now, there is no signature of *CPT* violation, or for that matter any type of new physics, in the width difference of $B^0 - \bar{B}^0$ and decay channels of B_d.[1] The width difference for the B_d system, $\Delta\Gamma_d$, is too small yet to be measured experimentally, and the bound is compatible with the standard model (SM). On the other hand, it is expected that the width difference $\Delta\Gamma_s$ would be significant for the B_s system, but at the same time we know that the theoretical uncertainties are quite significant [11]. If there is some new physics (NP) that does not contribute to the absorptive part of the $B_s^0 - \bar{B}_s^0$ box, the width difference can only go down [12], while there are models where this conclusion may not be true [13]. To add to this murky situation, the *CP*-violating phase β_s, which is expected to be very small from the Cabibbo-Kobayashi-Maskawa (CKM) paradigm, has been measured [14] to be large, compatible with the SM expectations only at the 2.1σ level. The situation is interesting: there is a hint of some NP, but we are yet to be certain of its exact nature, not to mention whether it exists at all.

In this situation, let us try to see what we can expect at the LHC, where the B_s meson, along with the B_d, will be copiously produced. We are helped by the fact that the time resolution in ATLAS and CMS are of the order of 40 fs, so one can track the time evolution of even the rapidly oscillating B_s. Thus, we expect excellent tagged and untagged measurements of both B_d and B_s mesons. It is best to focus upon the single-amplitude observables: $B_d \to J/\psi K_S$ and $B_s \to J/\psi\phi$ or $B_s \to D_s^+ D_s^-$.[2] For the $J/\psi\phi$ mode, one has to perform the angular analysis and untangle the *CP*-even and *CP*-odd channels.

[1]We use B^0 and $\bar{B}^0$ to indicate the flavor eigenstates, B_d as a generic symbol for both of them, and similarly for B_s. The symbol B_q will mean either a B_d or a B_s.

[2]They are not strictly single channel as there is a penguin process whose dominant part has the same phase as the leading Cabibbo-allowed tree process, but on the other hand these channels are easy to measure, and the penguin pollution is quite small and well under control.

*akphy@caluniv.ac.in
†nandi@to.infn.it
‡sunandoraja@gmail.com

In this paper, we will discuss how one can detect the presence of a CPT violating new physics from the tagged and untagged measurements of the decay. We will confine our discussion to the case where CPT violation is small compared to the SM amplitude, just to make the results more transparent. The conclusions do not change qualitatively if the CPT violation is large, which, we must say, is a far-off possibility based on the data from the other experiments [10]. We will also show how the nature of the CPT violating term can be probed through these measurements.

In Sec. II, we mention the relevant expressions, and introduce CPT violation, with relevant expressions, in Sec. III. The analysis for both B_d and B_s systems is performed in Sec. IV, while we summarize and conclude in Sec. V.

II. BASIC FORMALISM

Let us introduce CPT violation in the Hamiltonian matrix through the parameter δ which can potentially be complex:

$$\delta = \frac{H_{22} - H_{11}}{\sqrt{H_{12}H_{21}}}, \tag{1}$$

so that

$$\mathcal{M} = \left[\begin{pmatrix} M_0 - \delta' & M_{12} \\ M_{12}^* & M_0 + \delta' \end{pmatrix} - \frac{i}{2}\begin{pmatrix} \Gamma_0 & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma_0 \end{pmatrix}\right], \tag{2}$$

where δ' is defined by

$$\delta = \frac{2\delta'}{\sqrt{H_{12}H_{21}}}. \tag{3}$$

Solving the eigenvalue equation of $\mathcal{M}$, we get

$$\lambda = \left(M_0 - \frac{i}{2}\Gamma_0\right) \pm H_{12}\alpha y \qquad \text{or}$$
$$\lambda = \left[H_{11} + H_{12}\alpha\left(y + \frac{\delta}{2}\right)\right], \qquad \left[H_{22} - H_{12}\alpha\left(y + \frac{\delta}{2}\right)\right], \tag{4}$$

where $y = \sqrt{1 + \frac{\delta^2}{4}}$ and $\alpha = \sqrt{H_{21}/H_{12}}$.

Hence, corresponding eigenvectors or the mass eigenstates are defined as

$$|B_H\rangle = p_1|B^0\rangle + q_1|\bar{B}^0\rangle, \qquad |B_L\rangle = p_2|B^0\rangle - q_2|\bar{B}^0\rangle. \tag{5}$$

The normalization satisfies

$$|p_1|^2 + |q_1|^2 = |p_2|^2 + |q_2|^2 = 1. \tag{6}$$

Let us define

$$\eta_1 = \frac{q_1}{p_1} = \left(y + \frac{\delta}{2}\right)\alpha; \qquad \eta_2 = \frac{q_2}{p_2} = \left(y - \frac{\delta}{2}\right)\alpha; \qquad \omega = \frac{\eta_1}{\eta_2}. \tag{7}$$

The convention of [10] leads to $z_0 = \delta/2$, where z_0 is a measure of CPT violation as used in [10]. The limits imply that $|z_0| \ll 1$. Even if the origin of CPT violation is something different, it is not unrealistic to assume $|\delta| \ll 1$.

One could even relax the assumption of $H_{21} = H_{12}^*$. However, there are two points that one must note. First, the effect of expressing $H_{12} = h_{12} + \bar{\delta}$, $H_{21} = h_{12}^* - \bar{\delta}$ appears as $\bar{\delta}^2$ in the expression of δ in Eq. (1), and can be neglected if we assume $\bar{\delta}$ to be small. The second point, which is more important, is that CPT conservation constrains only the diagonal elements and puts no constraint whatsoever on the off-diagonal elements. It has been shown in [5,7] that $H_{12} \neq H_{21}^*$ leads to T violation, and only $H_{11} \neq H_{22}$ leads to unambiguous CPT violation. Thus, we will focus on the parametrization used in Eqs. (1) and (2) to discuss the effects of CPT violation.

The time-dependent flavor eigenstates are given by

$$\begin{aligned} |B_q(t)\rangle &= f_+(t)|B_q\rangle + \eta_1 f_-(t)|\bar{B}_q\rangle \\ |\bar{B}_q(t)\rangle &= \frac{f_-(t)}{\eta_2}|B_q\rangle + \bar{f}_+(t)|\bar{B}_q\rangle, \end{aligned} \tag{8}$$

where

$$\begin{aligned} f_-(t) &= \frac{1}{(1+\omega)}(e^{-i\lambda_1 t} - e^{-i\lambda_2 t}), \\ f_+(t) &= \frac{1}{(1+\omega)}(e^{-i\lambda_1 t} + \omega e^{-i\lambda_2 t}), \\ \bar{f}_+(t) &= \frac{1}{(1+\omega)}(\omega e^{-i\lambda_1 t} + e^{-i\lambda_2 t}). \end{aligned} \tag{9}$$

So, the decay rate of the meson B_q at time t to a CP eigenstate f is given by

$$\begin{aligned} \Gamma(B_q(t) \to f_{CP}) &= [|f_+(t)|^2 + |\xi_{f_1}|^2|f_-(t)|^2 \\ &\quad + 2\,\mathrm{Re}(\xi_{f_1} f_-(t) f_+^*(t))]|A_f|^2, \\ \Gamma(\bar{B}_q(t) \to f_{CP}) &= [|f_-(t)|^2 + |\xi_{f_2}|^2|\bar{f}_+(t)|^2 \\ &\quad + 2\,\mathrm{Re}(\xi_{f_2}\bar{f}_+(t) f_-^*(t))]\left|\frac{A_f}{\eta_2}\right|^2, \end{aligned} \tag{10}$$

where

$$A_f = \langle f|H|B_q\rangle, \qquad \bar{A}_f = \langle f|H|\bar{B}_q\rangle. \tag{11}$$

Also,

$$\xi_{f_1} = \eta_1\frac{\bar{A}_f}{A_f}, \qquad \xi_{f_2} = \eta_2\frac{\bar{A}_f}{A_f}. \tag{12}$$

In the SM, both are equal and $\xi_{f_1} = \xi_{f_2} = \xi_f$. For single-channel processes, $|\xi_f| = 1$.

Now using Eqs. (7) and (9), one gets

$$
\begin{aligned}
|f_-(t)|^2 &= \frac{2e^{-\Gamma t}}{|1+\omega|^2}\left[\cosh\left(\frac{\Delta\Gamma t}{2}\right) - \cos(\Delta m t)\right],\\
|f_+(t)|^2 &= \frac{e^{-\Gamma t}}{|1+\omega|^2}\left[\cosh\left(\frac{\Delta\Gamma t}{2}\right)(1+|\omega|^2) + \sinh\left(\frac{\Delta\Gamma t}{2}\right)(1-|\omega|^2) + 2\,\mathrm{Re}(\omega)\cos(\Delta m t) - 2\,\mathrm{Im}(\omega)\sin(\Delta m t)\right],\\
|\bar{f}_+(t)|^2 &= \frac{e^{-\Gamma t}}{|1+\omega|^2}\left[\cosh\left(\frac{\Delta\Gamma t}{2}\right)(1+|\omega|^2) - \sinh\left(\frac{\Delta\Gamma t}{2}\right)(1-|\omega|^2) + 2\,\mathrm{Re}(\omega)\cos(\Delta m t) + 2\,\mathrm{Im}(\omega)\sin(\Delta m t)\right],\\
f_+^*(t)f_-(t) &= \frac{e^{-\Gamma t}}{|1+\omega|^2}\left[\cosh\left(\frac{\Delta\Gamma t}{2}\right)(1-\omega^*) - \sinh\left(\frac{\Delta\Gamma t}{2}\right)(1+\omega^*) + \cos(\Delta m t)(-1+\omega^*) - i\sin(\Delta m t)(1+\omega^*)\right],\\
\bar{f}_+(t)f_-^*(t) &= \frac{e^{-\Gamma t}}{|1+\omega|^2}\left[\cosh\left(\frac{\Delta\Gamma t}{2}\right)(\omega-1) - \sinh\left(\frac{\Delta\Gamma t}{2}\right)(1+\omega) + \cos(\Delta m t)(1-\omega) - i\sin(\Delta m t)(1+\omega)\right].
\end{aligned}
\tag{13}
$$

Here, Δm and $\Delta\Gamma$ are defined through

$$\lambda_1 - \lambda_2 = \Delta m + \frac{i}{2}\Delta\Gamma, \tag{14}$$

with

$$\lambda_{(1,2)} = m_{(1,2)} - \frac{i}{2}\Gamma_{(1,2)}, \qquad \Delta m = m_1 - m_2, \qquad \Delta\Gamma = \Gamma_2 - \Gamma_1. \tag{15}$$

III. INTRODUCING *CPT* VIOLATION

If we consider a time-independent *CPT* violation so that δ is a constant, there are only two unknowns in the picture: $\mathrm{Re}(\delta)$ and $\mathrm{Im}(\delta)$, over those in the SM. We will try to see how one can extract information about them. For our analysis, let us take δ to be complex; it will be straightforward to go to the simpler limiting cases where δ is purely real or imaginary. For example, if the width difference $\Delta\Gamma$ is much smaller than Δm, the model of [10] makes δ completely real.

When B_q and $\bar{B}_q$ are produced in equal numbers, the untagged decay rate can be defined as

$$\Gamma_U[f,t] = \Gamma(B_q(t)\to f) + \Gamma(\bar{B}_q(t)\to f). \tag{16}$$

Using the above expression, one could define the branching fraction as

$$\mathrm{Br}[f] = \frac{1}{2}\int_0^\infty dt\Gamma[f,t]. \tag{17}$$

The above equation is useful to fix the overall normalization.

We assume $\delta \ll 1$ and expand any function $f(\delta)$ using Taylor series expansion and drop all the terms $\mathcal{O}(\delta^n)$ for $n > 2$. From Eqs. (10), (13), and (16), we will get the untagged decay rate for the decay $B_q \to f$,

$$
\begin{aligned}
\Gamma_U[f,t] = |A_f|^2 e^{-\Gamma_q t}\Bigg[&\left\{(1+|\xi_f|^2)\left(1+\frac{(\mathrm{Im}(\delta))^2}{4}\right) - \mathrm{Im}(\delta)\mathrm{Im}(\xi_f)\right\}\cosh\left(\frac{\Delta\Gamma_q t}{2}\right)\\
&- \left\{(1+|\xi_f|^2)\frac{(\mathrm{Im}(\delta))^2}{4} - \mathrm{Im}(\delta)\mathrm{Im}(\xi_f)\right\}\cos(\Delta m_q t) + \left\{2\,\mathrm{Re}(\xi_f) - \frac{1}{2}(1-|\xi_f|^2)\mathrm{Re}(\delta)\right.\\
&\left.- \frac{1}{4}\,\mathrm{Re}(\xi_f)((\mathrm{Re}(\delta))^2 - (\mathrm{Im}(\delta))^2)\right\}\sinh\left(\frac{\Delta\Gamma_q t}{2}\right) - \frac{1}{2}\,\mathrm{Im}(\delta)\{(1-|\xi_f|^2) + \mathrm{Re}(\delta)\mathrm{Re}(\xi_f)\}\sin(\Delta m_q t)\Bigg].
\end{aligned}
\tag{18}
$$

Thus, for the B_s system, where the hyperbolic functions are not negligible, we get (keeping up to first order of terms in $\Delta\Gamma_s$)

$$\begin{aligned}\mathrm{Br}[f] &= \frac{1}{2}\int_0^\infty dt\Gamma[f,t]\\ &= \frac{|A_f|^2}{2}\bigg[\frac{1}{\Gamma_s}\bigg\{(1+|\xi_f|^2)\bigg(1+\frac{(\mathrm{Im}(\delta))^2}{4}\bigg) - \mathrm{Im}(\delta)\mathrm{Im}(\xi_f)\bigg\} - \frac{\Gamma_s}{(\Delta m)^2+(\Gamma_s)^2}\bigg\{(1+|\xi_f|^2)\frac{(\mathrm{Im}(\delta))^2}{4} - \mathrm{Im}(\delta)\mathrm{Im}(\xi_f)\bigg\}\\ &\quad + \frac{\Delta\Gamma_s}{2(\Gamma_s)^2}\bigg\{2\,\mathrm{Re}(\xi_f) - \frac{1}{2}(1-|\xi_f|^2)\mathrm{Re}(\delta) - \frac{1}{4}\,\mathrm{Re}(\xi_f)((\mathrm{Re}(\delta))^2 - (\mathrm{Im}(\delta))^2)\bigg\}\\ &\quad - \frac{1}{2}\,\mathrm{Im}(\delta)\{(1-|\xi_f|^2) + \mathrm{Re}(\delta)\mathrm{Re}(\xi_f)\}\frac{\Delta m}{(\Delta m)^2+(\Gamma_s)^2}\bigg] \qquad (19)\end{aligned}$$

Theoretically, one can obtain the coefficients of the trigonometric and the hyperbolic terms by fitting the untagged decay rate. In actual cases this is a difficult task. However, there is one other observable which may help us. Before we go to that, let us note that the above expression is further simplified in the following four cases:

(i) For the B_d system: We can neglect $\Delta\Gamma_d$ so that the cosh term is unity and the sinh term is zero. Thus, there are only two time-dependent terms, $\cos(\Delta m t)$ and $\sin(\Delta m t)$, and the fitting is easier. Note that $\Delta\Gamma_d$ is measured to be small, so we need not consider the case where it is enhanced to a significant value because of the *CPT* violation. In fact, if δ is small, $\Delta\Gamma_d$ is bound to be that coming from the SM, as the correction is proportional only to δ^2 and higher.

(ii) For one-amplitude processes: We can put $|\xi_f| = 1$, and only one of $\mathrm{Re}(\xi_f)$ and $\mathrm{Im}(\xi_f)$ remains a free parameter.[3]

(iii) For δ being either purely real or purely imaginary: The expressions are straightforward. For example, if δ is purely real, there is no trigonometric dependence on the untagged rate.

(iv) Finally, for $|\delta| \ll 1$: We can neglect terms higher than linear in either $\mathrm{Re}(\delta)$ or $\mathrm{Im}(\delta)$ in Eq. (19). This is expected to be the case according to [10]. For example, the expression for the branching fraction for a one-amplitude process simplifies to

$$\begin{aligned}\mathrm{Br}[f] = \frac{|A_f|^2}{2}\bigg[&\frac{1}{\Gamma_s}\{2 - \mathrm{Im}(\delta)\mathrm{Im}(\xi_f)\}\\ &+ \frac{\Gamma_s}{(\Delta m)^2+(\Gamma_s)^2}\mathrm{Im}(\delta)\mathrm{Im}(\xi_f)\\ &+ \frac{\Delta\Gamma_s}{(\Gamma_s)^2}\,\mathrm{Re}(\xi_f)\bigg]. \qquad (20)\end{aligned}$$

One can also tag the B mesons and define a tagged decay rate asymmetry:

$$\begin{aligned}\Gamma_T[f,t] &= \Gamma(B_q(t)\to f) - \Gamma(\bar{B}_q(t)\to f)\\ &= |A_f|^2 e^{-\Gamma_q t}\bigg[\bigg\{(1-|\xi_f|^2)\frac{(\mathrm{Re}(\delta))^2}{4} - \mathrm{Re}(\delta)\mathrm{Re}(\xi_f)\bigg\}\cosh\bigg(\frac{\Delta\Gamma_q t}{2}\bigg) + \bigg\{(1-|\xi_f|^2)\bigg(1-\frac{(\mathrm{Re}(\delta))^2}{4}\bigg) + \mathrm{Re}(\delta)\mathrm{Re}(\xi_f)\bigg\}\\ &\quad \times\cos(\Delta m_q t) - \frac{1}{2}\,\mathrm{Re}(\delta)\{(1+|\xi_f|^2) - \mathrm{Im}(\delta)\mathrm{Im}(\xi_f)\}\sinh\bigg(\frac{\Delta\Gamma_q t}{2}\bigg)\\ &\quad + \bigg\{2\,\mathrm{Im}(\xi_f) - \frac{1}{2}\,\mathrm{Im}(\delta)(1+|\xi_f|^2) - \frac{1}{4}\,\mathrm{Im}(\xi_f)((\mathrm{Re}(\delta))^2 - (\mathrm{Im}(\delta))^2)\bigg\}\sin(\Delta m_q t)\bigg]. \qquad (21)\end{aligned}$$

Note that (i) for $\mathrm{Re}(\delta) = \mathrm{Im}(\delta) = 0$, this reverts back to the SM expression, as it should, and (ii) if $|\delta| \ll 1$ and $\Delta\Gamma/\Gamma \ll 1$ as in the B_d system, the tagged rate can measure both $\mathrm{Re}(\delta)$ and $\mathrm{Im}(\delta)$.

For one-amplitude processes with $|\delta| \ll 1$, one may write a simplified expression:

$$\begin{aligned}\Gamma_U[f,t] &= |A_f|^2 e^{-\Gamma_q t}\bigg[(2 - \mathrm{Im}(\delta)\mathrm{Im}(\xi_f))\cosh\bigg(\frac{\Delta\Gamma_q t}{2}\bigg) + \mathrm{Im}(\delta)\mathrm{Im}(\xi_f)\cos(\Delta m_q t) + 2\,\mathrm{Re}(\xi_f)\sinh\bigg(\frac{\Delta\Gamma_q t}{2}\bigg)\bigg],\\ \Gamma_T[f,t] &= |A_f|^2 e^{-\Gamma_q t}\bigg[-\mathrm{Re}(\delta)\mathrm{Re}(\xi_f)\cosh\bigg(\frac{\Delta\Gamma_q t}{2}\bigg) + \mathrm{Re}(\delta)\mathrm{Re}(\xi_f)\cos(\Delta m_q t) - \mathrm{Re}(\delta)\sinh\bigg(\frac{\Delta\Gamma_q t}{2}\bigg)\\ &\quad + \{2\,\mathrm{Im}(\xi_f) - \mathrm{Im}(\delta)\}\sin(\Delta m_q t)\bigg]. \qquad (22)\end{aligned}$$

[3] ξ_f is a SM quantity, so it is theoretically calculable, but it may also contain other new physics which is *CPT* conserving, so it is better to obtain both real and imaginary parts of ξ_f and check whether $|\xi_f| = 1$.

It is clear from Eq. (22) how one can extract $\mathrm{Re}(\delta)$ and $\mathrm{Im}(\delta)$ by comparing the untagged and tagged analyses. Suppose we consider the B_s system where $\Delta\Gamma_s$ is non-negligible. The coefficient of the sinh term in Γ_T gives $\mathrm{Re}(\delta)$. However, there is an overall normalization uncertainty given by $|A_f|^2$. To remove this, one can consider a combined study of the coefficients of $\sinh(\frac{\Delta\Gamma_s t}{2})$ and $\cos(\Delta m_s t)$ from the untagged and tagged decay rates, respectively; their ratio allows for a clean extraction of $\mathrm{Re}(\delta)$. On the other hand, the ratio of the coefficients of $\cos(\Delta m_s t)$ in Γ_U and $\sin(\Delta m_s t)$ in Γ_T gives a clean measurement of $\mathrm{Im}(\delta)$, as $\mathrm{Im}(\xi_f)$ is known from the SM dynamics. A further check about the one-amplitude nature is provided from $|\mathrm{Re}(\xi_f)|^2 + |\mathrm{Im}(\xi_f)|^2 = 1$. In fact, as long as δ is small, one can extract both $\mathrm{Re}(\delta)$ and $\mathrm{Im}(\delta)$ even if $|\xi_f| \neq 1$, from the coefficients of the sine, cosine, and hyperbolic sine terms in Γ_U and Γ_T.

One may also define the time-dependent *CPT* asymmetry as

$$A_{CPT}(f,t) = \frac{\Gamma_T[f,t]}{\Gamma_U[f,t]} = \frac{\Gamma(B_q(t)\to f) - \Gamma(\bar{B}_q(t)\to f)}{\Gamma(B_q(t)\to f) + \Gamma(\bar{B}_q(t)\to f)}, \tag{23}$$

and the time-independent *CPT* asymmetry as

$$\begin{aligned} A_{CPT}(f) &= \frac{\int_0^\infty dt \Gamma_T[f,t]}{\int_0^\infty dt \Gamma_U[f,t]} \\ &= \frac{\int_0^\infty dt[\Gamma(B_q(t)\to f) - \Gamma(\bar{B}_q(t)\to f)]}{\int_0^\infty dt[\Gamma(B_q(t)\to f) + \Gamma(\bar{B}_q(t)\to f)]}. \end{aligned} \tag{24}$$

This goes to the usual *CP* asymmetry A_{CP} if $\delta = 0$; thus, any deviation from the expected *CP* asymmetry calculated from the SM would signal new physics, but one must check all the boxes to pinpoint the nature of the new physics. For example, there would not be any change in the semileptonic *CP* asymmetry if the new physics is only *CPT* violating in nature.

IV. ANALYSIS

There are five *a priori* unknowns: $\mathrm{Re}(\delta)$, $\mathrm{Im}(\delta)$, $\mathrm{Re}(\xi_f)$, $\mathrm{Im}(\xi_f)$, and $|A_f|^2$. For a one-amplitude process $|\xi_f|^2 = 1$ and the number of unknowns reduces to four. The tagged and untagged decay rates, the branching fraction, and the time-independent *CPT* asymmetry would provide information on all of these unknowns. Assuming the *CPT*-conserving physics to be purely that of the SM, one may calculate ξ_f following the CKM picture. In the analysis that follows, we take ξ_f to be known from the SM. We would like to point out the following features:

(i) The overall amplitude $|A_f|^2$ cancels in the *CPT* asymmetry. This, therefore, is going to be the observable one needs to measure most precisely.

(ii) It is enough to measure the coefficients of the trigonometric terms only. For the B_d system, $\Delta\Gamma_d$ is small anyway, and for the B_s system, $\Delta\Gamma_s$ has a large theoretical uncertainty.

(iii) The analysis holds even if the process under consideration is not a one-amplitude process. In fact, one may check whether there is a second *CPT* conserving new physics amplitude just by looking at the extracted values of $\mathrm{Re}(\xi_f)$ and $\mathrm{Im}(\xi_f)$.

(iv) The coefficient of $\sin(\Delta m_q t)$ in the expression for the tagged decay rate Γ_T gives the mixing phase in the box diagram. Thus, $\mathrm{Im}(\delta)$ may be constrained by the *CP* asymmetry measurements in the B_d system. On the other hand, even those constrained values generate a large mixing phase for the B_s system compatible with the CDF data.

A. The B_s system

For the B_s system, we take

$$\begin{aligned} &\Delta m_s = 17.77 \pm 0.12\ \mathrm{ps}^{-1}, \\ &\Delta\Gamma_s = 0.096 \pm 0.039\ \mathrm{ps}^{-1}, \qquad \frac{\Delta\Gamma_s}{\Gamma_s} = 0.147 \pm 0.060, \\ &\frac{1}{\Gamma_s} = 1.530 \pm 0.009\ \mathrm{ps}, \qquad \mathrm{Re}(\xi_f) = 0.99, \\ &\mathrm{Im}(\xi_f) = -0.04. \end{aligned} \tag{25}$$

In Fig. 1, we show the variation of A_{CPT} with $\mathrm{Re}(\delta)$. For our analysis, we take both $|\mathrm{Re}(\delta)|, |\mathrm{Im}(\delta)| < 0.1$, which is consistent with [10]. The variation of A_{CPT} with Δm_s and $\Delta\Gamma_s$ is negligible, of the order of 0.2%, so we fix them to their respective central values. Effects of δ in both Δm_s and $\Delta\Gamma_s$ are quadratic in δ, and hence we can use the SM values for them. In fact, A_{CPT} does not depend significantly on the choice of $\mathrm{Im}(\delta)$ either; the variation is less than 1%. This is due to the fact that here, $|\mathrm{Im}(\xi_f)| \ll |\mathrm{Re}(\xi_f)|$ and hence the coefficient of $\mathrm{Re}(\delta)$ is much greater than the coefficient of $\mathrm{Im}(\delta)$ in the expression of A_{CPT}. This feature does not hold for the B_d system. Note that A_{CPT} clearly

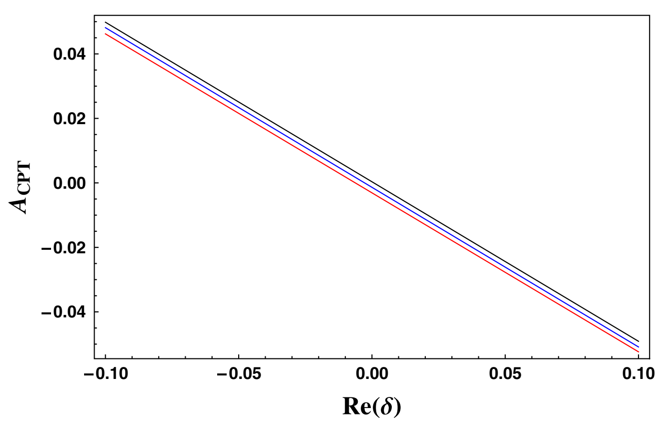

FIG. 1 (color online). Variation of A_{CPT} with $\mathrm{Re}(\delta)$ for the B_s system. The three lines, from top to bottom, are for $\mathrm{Im}(\delta) = -0.1$, 0, and 0.1, respectively.

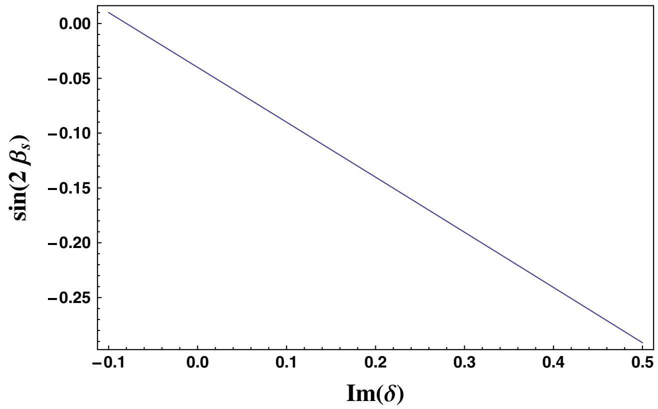

FIG. 2 (color online). Variation of $\sin(2\beta_s)$ with $\mathrm{Im}(\delta)$.

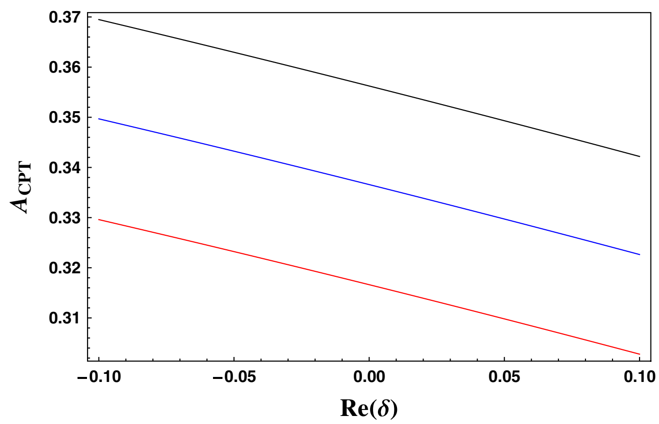

FIG. 3 (color online). Variation of A_{CPT} with $\mathrm{Re}(\delta)$ for the B_d system. The three lines, from top to bottom, are for $\mathrm{Im}(\delta) = -0.1$, 0, and 0.1, respectively.

gives the sign of $\mathrm{Re}(\delta)$. The small nonzero value of A_{CPT} for $\delta = 0$ indicates the small mixing phase in the $B_s^0 - \bar{B}_s^0$ box diagram. However, the apparent phase, i.e., the coefficient of $\sin(\Delta m_s t)$, can increase with $\mathrm{Im}(\delta)$, as can be seen from Fig. 2.

B. The B_d system

The inputs that we use for the B_d system are

$$\begin{aligned} &\Delta m_d = 0.507\ \mathrm{ps}^{-1}, \qquad \Delta\Gamma_d = 0, \\ &\mathrm{Re}(\xi_f) = 0.72, \qquad \mathrm{Im}(\xi_f) = 0.695. \end{aligned} \tag{26}$$

This follows from the CKM expectation of $\sin(2\beta_d) = 0.695 \pm 0.020$. The constraint on δ comes from the measurement of $\sin(2\beta_d)$ in the $b \to c\bar{c}s$ channel: 0.668 ± 0.028 [15].[4] Again, we can fix Δm_d at its central value. This time, due to the comparable values of $\mathrm{Re}(\xi_f)$ and $\mathrm{Im}(\xi_f)$, A_{CPT} is sensitive to both $\mathrm{Re}(\delta)$ and $\mathrm{Im}(\delta)$. The variations are shown in Fig. 3 for three values of $\mathrm{Im}(\delta)$ and Fig. 4 for three values of $\mathrm{Re}(\delta)$. It turns out that A_{CPT} is always positive for $\mathrm{Re}(\delta)$, $\mathrm{Im}(\delta) < 1$; this is a consistency check for the CPT violation. Note that the measured value of $\sin(2\beta_d)$ can go down from its CKM expectation for $\mathrm{Im}(\delta) > 0$, in fact, for $\mathrm{Im}(\delta) \approx 0.07$, $\sin(2\beta_d) \approx 0.66$, as can be seen from Fig. 5. While this value of $\mathrm{Im}(\delta)$ generates a mixing phase for the B_s system that is consistent with the CDF and D0 measurements at 1σ, one must remember that δ need not be a flavor-blind parameter.

[4]We do not take the measurements coming from $b \to s$ penguin channels because of their inherent uncertainties.

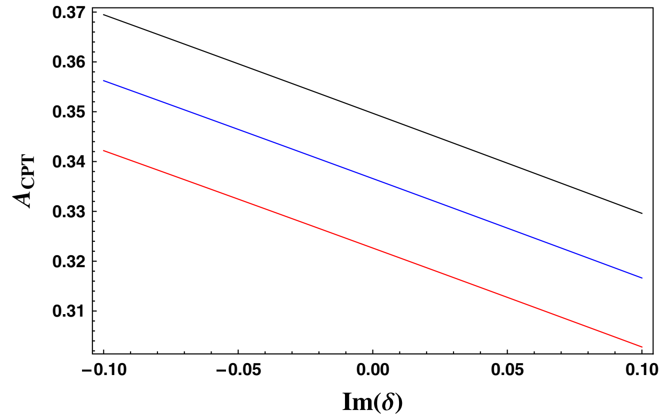

FIG. 4 (color online). Variation of A_{CPT} with $\mathrm{Im}(\delta)$ for the B_d system. The three lines, from top to bottom, are for $\mathrm{Re}(\delta) = -0.1$, 0, and 0.1, respectively.

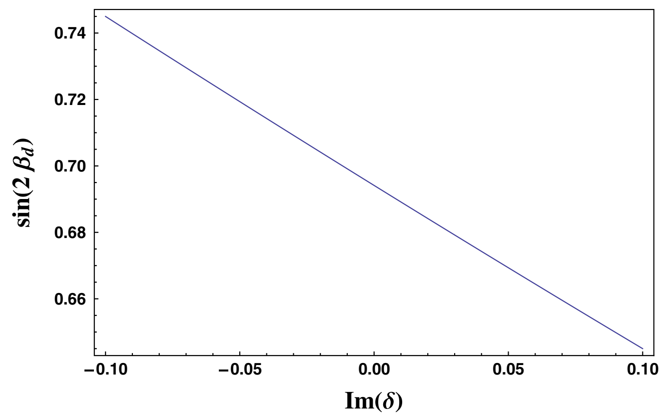

FIG. 5 (color online). Variation of $\sin(2\beta_d)$ with $\mathrm{Im}(\delta)$.

V. SUMMARY AND CONCLUSIONS

We have investigated the possibility of CPT violation in neutral B systems. CPT is a symmetry that is expected to be exact and the violation, even if it exists, should be quite small. However, it is possible to measure even a small CPT violation from the tagged and untagged decay rates of the neutral B mesons. In particular, for single-amplitude decay channels, the coefficients of the trigonometric terms $\sin(\Delta m t)$ and $\cos(\Delta m t)$ can effectively pinpoint the nature of the CPT violating parameter δ. This is an interesting possibility for the decays $B_s \to D_s^+ D_s^-$ and $B_S \to J/\psi\phi$ (with an angular analysis). Even a small *CPT* violation, allowed by the mixing constraints for the B_d system, can make the B_s mixing phase more compatible with the Tevatron measurements, at the level of about 1σ. On the other hand *CPT* violation should not affect the semileptonic *CP* asymmetries, as the corrections are quadratic in nature, and expected to be negligible for small δ. Thus, a correlated study of the *CP* asymmetries in $B_s \to J\psi\phi$ and $B_s \to D_s^+ D_s^-$ *vis-à-vis* $B_s \to D_s \ell \nu$ might be useful to pinpoint the *CPT* violating effects. This, we feel, is something that the experimentalists should look for in the coming years at the LHC.

ACKNOWLEDGMENTS

S. N. would like to thank Ulrich Nierste for useful discussions. His work is supported by a European Community's Marie-Curie Research Training Network under Contract No. MRTN-CT-2006-035505 "Tools and Precision Calculations for Physics Discoveries at Colliders." The work of A. K. was supported by BRNS, Government of India; CSIR, Government of India; and the DRS program of the University Grants Commission.

[1] V. Alan Kostelecky and N. Russell, arXiv:0801.0287.
[2] M. Kobayashi and A. I. Sanda, Phys. Rev. Lett. **69**, 3139 (1992).
[3] V. A. Kostelecky and A. Potting, Phys. Lett. B **381**, 89 (1996).
[4] S. Nussinov, arXiv:0907.3088.
[5] L. Lavoura, Ann. Phys. (N.Y.) **207**, 428 (1991).
[6] A. Datta, E. A. Paschos, and L. P. Singh, Phys. Lett. B **548**, 146 (2002).
[7] K. R. S. Balaji, W. Horn, and E. A. Paschos, Phys. Rev. D **68**, 076004 (2003).
[8] Z.-z. Xing, Phys. Rev. D **50**, R2957 (1994).
[9] Z.-z. Xing, Phys. Lett. B **450**, 202 (1999); P. Ren and Z.-z. Xing, Phys. Rev. D **76**, 116001 (2007).
[10] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. Lett. **100**, 131802 (2008).
[11] A. Lenz and U. Nierste, J. High Energy Phys. 06 (2007) 072.
[12] Y. Grossman, Phys. Lett. B **380**, 99 (1996).
[13] A. Dighe, A. Kundu, and S. Nandi, Phys. Rev. D **76**, 054005 (2007).
[14] T. Aaltonen *et al.* (CDF Collaboration), Phys. Rev. Lett. **100**, 161802 (2008); V. M. Abazov *et al.* (D0 Collaboration), Phys. Rev. Lett. **101**, 241801 (2008).
[15] See the website of UTFIT at http://www.utfit.org/.