Mass spectra of doubly heavy mesons in Regge phenomenology

Ke-Wei Wei (魏科伟)[1,2,*] and Xin-Heng Guo (郭新恒)[2,†]

[1]*Institute of High Energy Physics, CAS, P.O. Box 918(4), Beijing 100049, China*

[2]*College of Nuclear Science and Technology, Beijing Normal University, Beijing 100875, China*

(Received 27 January 2010; published 15 April 2010)

In Regge phenomenology, we first study the spectra of charmonium and bottomonium in the (n, M^2) plane and predict masses of some charmonium and bottomonium states which have not been observed now. Then, with the aid of the additivity of intercepts and inverse slopes, we calculate the masses of ground S-wave and P-wave $c\bar{b}$ meson states and their first radial excited states. The predictions are in reasonable agreement with the existing experimental data and those suggested in many other approaches. The predictions may be useful for the discovery of the unobserved $c\bar{c}$, $b\bar{b}$, and $c\bar{b}$ mesons and the J^P assignment of these states.

DOI: 10.1103/PhysRevD.81.076005 PACS numbers: 11.55.Jy, 12.40.Nn, 12.40.Yx, 14.40.−n

I. INTRODUCTION

According to the Particle Data Group's recent ''Review of Particle Physics'' [1], there are about 20 well-established charmonium and bottomonium states. However, many doubly heavy meson states ($c\bar{c}$, $b\bar{b}$, and $c\bar{b}$) are still absent from the ''Meson Summary Table,'' especially the $c\bar{b}$ (or $b\bar{c}$) meson states. Until recently, among $c\bar{b}$ meson states, only the pseudoscalar mesons $B_c^{\pm}$ have been observed experimentally [1,2]. Both $c\bar{c}$ and $b\bar{b}$ are heavy quarkonia. The $c\bar{b}$ (or $b\bar{c}$) meson states are special systems with two heavy quarks of different flavors. The presence of both such quarks impacts on the production, decay, and mass properties of the $c\bar{b}$ mesons. Recently, many charmoniumlike and bottomoniumlike states have been observed or measured more precisely, e.g. the recent observed charmoniumlike states, $X(3872)$ [3], $X(3940)$ [4,5], $Y(3940)$ [6,7], $X(4008)$ [8], $Y(4260)$ [9], $Y(4325)$ [10], $X(4350)$ [11], $Y(4360, 4660)$ [12], the newly observed bottomoniumlike states $\Upsilon(10890)$ [13], $\Upsilon(10876)$ [14], $\Upsilon(10992)$ [14], and the more precise measurement about $\eta_b(1S)$ by *BABAR* [15,16] and CLEO [17].

However, the properties of some recently observed states are not very clear, e.g. the hotly debated $X(3872)$ and $Y(4260)$. Some states in the conventional charmonium and bottomonium spectra are still missing while many unexpected states are observed. It seems that more excessive states exist experimentally than forepassed theoretical expectations in the $q\bar{q}$ picture. Thereafter, glueballs, hybrid states, tetraquark states, and molecular states are suggested to interpret them. Obviously, there is still a lot of work to be done both experimentally and theoretically. The copious productions of doubly heavy mesons and their radial and orbital excitations are expected at the experimental facilities such as the Large Hadron Collider (LHC) at CERN. Therefore, it is urgent to study the $c\bar{c}$, $b\bar{b}$, and $c\bar{b}$ mass spectra theoretically.

Quantum chromodynamics (QCD) has been verified as an appropriate theory to describe strong interaction at short distances. However, the application of QCD to the processes of hadronic interactions at large distances is still limited by the unsolved confinement problem. The simulations of lattice QCD still have large uncertainties and difficulties in dealing with higher excited states. Furthermore, calculating the heavy quarkonium spectrum from lattice QCD requires a more tremendous computational effort than calculating the light quarkonium spectrum [18]. Nowadays calculations of hadronic properties, which are related to the nonperturbative effects, are frequently carried out with the help of phenomenological models. Regge phenomenology (which was derived from the analysis of the properties of the scattering amplitude in the complex angular momentum plane [19]) is one of the simplest ones among these phenomenological models.

Regge theory is concerned with almost all aspects of strong interactions including the particle spectra, the forces between particles, and the high energy behavior of scattering amplitudes [20]. One of the most distinctive features of Regge theory is the Regge trajectory by which the mass M and the total spin J of a hadron are related. Until now, the Regge trajectory ansatz is still one of the popular and effective approaches for studying hadron spectra [21–29].

By analyzing the Regge trajectory in the (J, M^2) plane, the numerical values of the parameters of the Regge trajectories were extracted for mesons of different flavors [24–28] and for baryons of different flavors [29]. The feature of the radial Regge trajectories in the (n, M^2) plane (n denotes the radial quantum number) were studied in Refs. [20,26,30–41]. Trajectories for light radial excited mesons in the (n, M^2) plane are suggested to be linear for light meson spectra, though the values of parameters are different in different Refs. [26,35–39]. However, according to the experimental data [1], trajectories for vector (n^3S_1) bottomonium in the (n, M^2) plane deviate significantly

*weikw@ihep.ac.cn
†Corresponding author.
xhguo@bnu.edu.cn

from the linearity. The possible reason for such deviation is that the b quark is a heavy quark [41].

Mass spectroscopy is always one of the foundational subjects in particle physics. Besides Regge theory, there are many theoretical approaches to study mass spectroscopy such as lattice QCD [42,43], quark model [44–46], potential model [47–54], and the Bethe-Salpeter equation [55,56].

In the present work, we will focus on studying doubly heavy meson masses in Regge phenomenology. We will first study the trajectories of charmonium and bottomonium in the (n, M^2) plane and predict masses of some states which have not been observed up to now. Then, with the aid of the additivity of intercepts and inverse slopes, together with the quasilinear Regge trajectories in the (J, M^2) plane, we will extract a relation between the slope ratio and masses of three mesons involving two flavors in one spin-parity multiplet and a mass relation for six mesons involving three flavors in one spin-parity multiplet. The masses of $c\bar{b}$ meson states belonging to the 1^1S_0, 1^3S_1, 1^1P_1, 1^3P_0, 1^3P_1, 1^3P_2, 2^1S_0, 2^3S_1, 2^1P_1, 2^3P_0, 2^3P_1, and 2^3P_2 meson multiplets will be extracted. We will compare the results with those given in many other approaches [42–56].

The remainder of this paper is organized as follows. In Sec. II we briefly introduce Regge phenomenology and a relation for hyperfine splittings. In Sec. III we study the feature of trajectories for charmonium and bottomonium in the (n, M^2) plane and predict masses of some states which have not been observed up to now. The masses of $c\bar{b}$ $(1S, 1P, 2S, 2P)$ states are calculated in Sec IV. Finally, we give a short discussion and summary in Sec. V.

II. REGGE TRAJECTORY AND HYPERFINE SPLITTING

It is known from Regge theory that all mesons and baryons are associated with Regge poles which move in the complex angular momentum plane as a function of energy. The trajectory of a particular pole (Regge trajectory) is characterized by a set of internal quantum numbers (baryon number $\mathcal{B}$, intrinsic parity P, strangeness $\mathcal{S}$, charmness C, bottomness B, etc.) and by the evenness or oddness of the total spin J for mesons [21]. The plots of Regge trajectories of hadrons in the (J, M^2) plane are usually called Chew-Frautschi plots.

Assuming the existence of the quasilinear Regge trajectories for both light and heavy mesons, one can have

$$J = a_{i\bar{j}}(0) + \alpha'_{i\bar{j}} M^2_{i\bar{j}}, \tag{1}$$

where i and j denote the quark constituents, $a_{i\bar{j}}(0)$ and $\alpha'_{i\bar{j}}$ are, respectively, the intercept and slope of the trajectory on which the $i\bar{j}$ meson lies.

For a meson multiplet with spin-parity J^P (more exactly speaking, with quantum numbers $n^{2s+1}L_J$, where L and s denote the orbital quantum number and the intrinsic spin, respectively), the parameters $a(0)$ and α' with different quark constituents can be related by the following relations:

the additivity of intercepts [24,25,28,57–62],

$$a_{i\bar{i}}(0) + a_{j\bar{j}}(0) = 2a_{i\bar{j}}(0), \tag{2}$$

the additivity of inverse slopes [24,25,28,57],

$$\frac{1}{\alpha'_{i\bar{i}}} + \frac{1}{\alpha'_{j\bar{j}}} = \frac{2}{\alpha'_{i\bar{j}}}, \tag{3}$$

where i and j represent quark flavors. Equations (2) and (3) were derived in a model based on the topological expansion and the $q\bar{q}$-string picture of hadrons [28]. This model provides a microscopic approach to describe Regge phenomenology in terms of quark degrees of freedom [63]. In fact, Eq. (2) was first derived for light quarks in the dual-resonance model [58], and was found to be satisfied in two-dimensional QCD [59], the dual-analytic model [60], and the quark bremsstrahlung model [61]. Also, it saturates the inequality for Regge intercepts [62] which follows from the Schwarz inequality and the unitarity relation. Furthermore, it was shown that Eqs. (2) and (3) still hold even in the case of nonlinear Regge trajectories in the (J, M^2) plane [22,28]. In the present work, we will use Eqs. (2) and (3) together with Eq. (1) [the quasilinear Regge trajectories in the (J, M^2) plane] to calculate the masses of $c\bar{b}$ mesons (see Sec. IV).

Theoretical work on the Regge-pole model indicates that the expansion of the amplitude in the Khuri representation has singularity at $t = 0$ (t is the square of the momentum transferred) [32,33,64]. In order to ensure the analyticity of the Khuri amplitude, a popular suggestion is that there are further trajectories known as "daughters" which have singular residues that precisely cancel the singularities of the original "parent" trajectory [20,32–34]. It is also suggested that the radial excited resonances lie on the corresponding "daughter trajectories" and an equality is proposed to describe the mass relation between the ground state and its radial excited states [26,36–39,41],

$$M_n^2 = M_1^2 + \mu^2(n-1), \tag{4}$$

where M_1 is the mass of the ground state and μ^2 is a parameter. For the light meson spectra, M_n^2 is regarded as linear with respect to n [26,35–39].

In the string model, μ^2 and Regge slope α' are related [36,37],

$$\mu^2 = 2\pi\sigma = \frac{1}{\alpha'}, \tag{5}$$

where σ is the string tension. In Ref. [26], after analyzing the temporal experimental data for light mesons, the author pointed out that numerical values satisfy $\mu^2 \simeq \frac{1}{\alpha'}$, which agrees with Eq. (5) (the result of the string model). However, Bicudo studied light mesons in the quasiclassical picture described by the massless Salpeter equation with a linear confining potential and gave [38]

$$\mu^2 = \frac{\pi/2}{\alpha'}. \tag{6}$$

In a recent paper [39], mass spectra of light quark-antiquark mesons were calculated in the framework of the QCD-motivated relativistic quark model based on the quasipotential approach. The Regge trajectories of the calculated masses led to the following result:

$$\mu^2 \simeq \frac{1.3}{\alpha'}. \tag{7}$$

Therefore, the values of the parameter μ^2 for light mesons are significantly different in different references. The main reason is that the nature of most higher excited mesons is not undoubtedly uncovered experimentally. The interpretation of these higher excited mesons are different in different references.

Although the value of μ^2 for light mesons is still uncertain now, the linear radial Regge trajectory is widely accepted for light mesons. However, for heavy quarkonia such as vector bottomonium, the experimental data [1] do not lie on linear radial Regge trajectory. A detailed study for bottomonium and charmonium will be presented in Sec. III.

Hyperfine splittings (the mass difference between spin-singlet and spin-triplet states) have been discussed for a long time. In Refs. [47,65,66], there is an equality for P-wave hyperfine splittings,

$$9M_{n^1P_1} = M_{n^3P_0} + 3M_{n^3P_1} + 5M_{n^3P_2}. \tag{8}$$

In Refs. [44,49,67–71] the left-hand side and the right-hand side of Eq. (8) differ by a few MeV. The experimental data for $1P$ charmonium satisfy Eq. (8) very well. The masses of doubly heavy mesons are several GeV. The uncertainty of a few MeV is only about 0.1% of the doubly heavy meson masses. Therefore, we neglect such possible uncertainties and use Eq. (8) directly in this work.

III. BOTTOMONIUM AND CHARMONIUM IN THE (n, M^2) PLANE

In this section, we will first study the feature of the trajectories for n^3S_1 $b\bar{b}$ and $c\bar{c}$ in the (n, M^2) plane. Then, we will predict the masses of the unobserved $(1S, 1P, 2S, 2P)$ $b\bar{b}$ and $c\bar{c}$ states.

Using the present experimental data, we show the n^3S_1 bottomonium trajectory in the (n, M^2) plane in Fig. 1. There is an obvious curve in the (n, M^2) plane in Fig. 1. This is very different from the cases for light mesons shown in Refs. [20,26,32–35,40,41]. This means that the dependence of M^2 on n is not linear. We calculate the values of μ^2 for the well-established bottomonium. Results are given below:

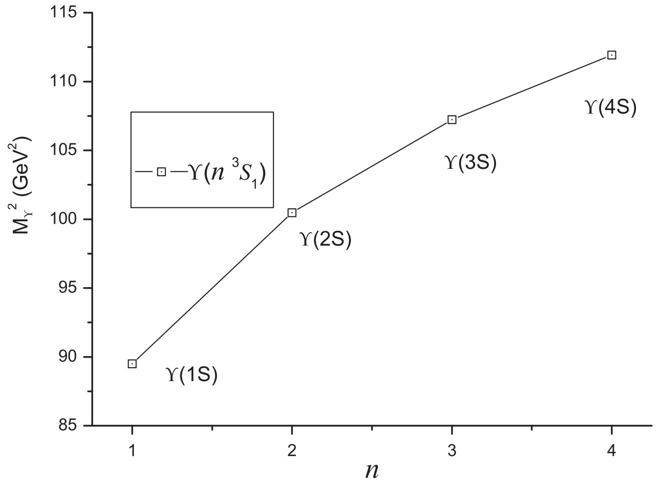

FIG. 1. The (n, M^2) plane for some $b\bar{b}$ (n^3S_1) mesons.

$$\mu^2|_{M^2_{\Upsilon(2S)}-M^2_{\Upsilon(1S)}} = 10.968\ \mathrm{GeV}^2,$$
$$\mu^2|_{M^2_{\Upsilon(3S)}-M^2_{\Upsilon(2S)}} = 6.764\ \mathrm{GeV}^2,$$
$$\mu^2|_{M^2_{\Upsilon(4S)}-M^2_{\Upsilon(3S)}} = 4.694\ \mathrm{GeV}^2.$$

These values obtained from experimental data manifest that for $b\bar{b}(n^3S_1)$, μ^2 is not a constant and sensitively depends upon the radial excitation quantum number n. The authors of Ref. [41] suggested that the possible reason for such deviation from linearity is that the massless quark approximation fails in the bottomonium case.

Let us see the case of vector charmonium states $c\bar{c}(n^3S_1)$. $J/\psi(1S)$ and $\psi(2S)$ are well established. $\psi(4040)$ is believed to be $\psi(3S)$ [40,41,72,73]. It is worthwhile to discuss the recently observed state $Y(4260)$ which is a candidate for $\psi(4S)$.

The state $Y(4260)$ [or $X(4260)$] was first reported by *BABAR* in the subsystem $(\pi^+\pi^-J/\psi)$ in the initial state radiation $e^+e^- \to \gamma_{\mathrm{ISR}}\pi^+\pi^-J/\psi$ [9]. The CLEO collaboration [74] and the Belle collaboration [8] confirmed this state soon in independent radiative return. In a direct scan, the CLEO collaboration confirmed the decay $X(4260) \to \gamma_{\mathrm{ISR}}\pi^+\pi^-J/\psi(11\sigma)$, made the first observation $X(4260) \to \pi^0\pi^0J/\psi(5.1\sigma)$, and found the first evidence for $X(4260) \to K^+K^-J/\psi(3.7\sigma)$ [75]. In the latest "Review of Particle Physics," the average mass and width are given as $M_{X(4260)} = 4263^{+8}_{-9}$ and $\Gamma_{X(4260)} = 95 \pm 14$ MeV. A new search reported by the *BABAR* collaboration found no evidence for $Y(4260)$ decaying into $D^*\bar{D}$ or $D^*\bar{D}^*$ [76].

After analyzing the properties of the state $Y(4260)$, Llanes-Estrada argued that the state $Y(4260)$ displaces $\psi(4415)$ as $\psi(4S)$ [77]. Llanes-Estrada also explained that $Y(4260)$ has not been observed to decay into a pseudoscalar pair because of the couplings of the S- and

TABLE I. The masses of the well-established charmonium and bottomonium states from PDG08 [1] and the values of μ^2 for these states calculated by us.

$n^{2s+1}L_J$	$^1S_0(c\bar{c})$	$^3S_1(c\bar{c})$	$^3S_1(b\bar{b})$	$^3P_0(b\bar{b})$	$^3P_1(b\bar{b})$	$^3P_2(b\bar{b})$
$M\|_{n=1}$ (GeV)	2.9803	3.096 916	9.4603	9.859 44	9.892 78	9.912 21
$M\|_{n=2}$ (GeV)	3.637	3.686 09	10.023 26	10.2325	10.255 46	10.268 65
$\mu^2 = M^2_{n=2} - M^2_{n=1}$ (GeV^2)	4.346	3.996	10.968	7.496	7.307	7.193

D-wave 1^{--} mesons. Furthermore, the mass of the state $Y(4260)$ is close to the mass of $\psi(4S)$ predicted by Quigg and Rosner using the logarithmic potential in 1977 (4233 MeV) [78] and by Li and Chao using the screened potential in 2009 (4273 MeV) [79]. Therefore, it is reasonable to take the mass of $Y(4260)$ as that of $\psi(4S)$.

Using the present experimental data, we show the n^3S_1 charmonium trajectory in the (n, M^2) plane in Fig. 2. We can see that there is an obvious curve in Fig. 2, which means that the parameter μ^2 for $c\bar{c}(n^3S_1)$ is not a constant either and depends upon the radial excitation quantum number n.

We list the values of μ^2 for the well-established charmonium and bottomonium states in Table I, from which we can see that the values of the parameter μ^2 are sensitive to quark flavor and orbital quantum number L. The values of the parameter μ^2 are also slightly related to the intrinsic spin s and the total spin J which is related to the spin-orbital coupling. It seems perplexing about the values of μ^2.

However, we may get an approximate law by analysis. First of all, both charm-quark and bottom-quark are heavy quarks. The charmonium states are composed of a charm-quark and an anti-charm-quark while the bottomonium states are composed of a bottom-quark and an anti-bottom-quark. The experimental data show that at the qualitative level, charmonium and bottomonium are remarkably similar to each other [1,78,80–82]. Based on the experimental data, we have the following values:

$$\frac{M^2_{c\bar{c}(2^3S_1)} - M^2_{c\bar{c}(1^3S_1)}}{M^2_{b\bar{b}(2^3S_1)} - M^2_{b\bar{b}(1^3S_1)}} = \frac{M^2_{\psi(2S)} - M^2_{J/\psi(1S)}}{M^2_{\Upsilon(2S)} - M^2_{\Upsilon(1S)}} = 0.364,$$

$$\frac{M^2_{c\bar{c}(3^3S_1)} - M^2_{c\bar{c}(2^3S_1)}}{M^2_{b\bar{b}(3^3S_1)} - M^2_{b\bar{b}(2^3S_1)}} = \frac{M^2_{\psi(3S)} - M^2_{\psi(2S)}}{M^2_{\Upsilon(3S)} - M^2_{\Upsilon(2S)}} = 0.403,$$

$$\frac{M^2_{c\bar{c}(4^3S_1)} - M^2_{c\bar{c}(3^3S_1)}}{M^2_{b\bar{b}(4^3S_1)} - M^2_{b\bar{b}(3^3S_1)}} = \frac{M^2_{\psi(4S)} - M^2_{\psi(3S)}}{M^2_{\Upsilon(4S)} - M^2_{\Upsilon(3S)}} = 0.396.$$

We plot Figs. 1 and 2 together to make a new figure (Fig. 3). It should be noted that the scales of the ordinate (M^2 coordinate) for charmonium and bottomonium are different (the axis scale for charmonium is 0.4 times of the axis scale for bottomonium). From Fig. 3, we can see that the trajectories for charmonium and bottomonium (with different axis scales) are approximately parallel.

This motivates us to introduce a parameter β to denote the ratio of $\mu^2_{c\bar{c}}$ and $\mu^2_{b\bar{b}}$ which refer to the same quantum numbers n, L, s, and J:

$$\beta = \frac{\mu^2_{c\bar{c}}(n, L, s, J)}{\mu^2_{b\bar{b}}(n, L, s, J)} = \frac{M^2_{c\bar{c}}|_{(n+1)^{2s+1}L_J} - M^2_{c\bar{c}}|_{n^{2s+1}L_J}}{M^2_{b\bar{b}}|_{(n+1)^{2s+1}L_J} - M^2_{b\bar{b}}|_{n^{2s+1}L_J}}. \quad (9)$$

Now we try to understand Eq. (9) in the heavy quark limit. The mass of the heavy quarkonium $Q\bar{Q}$ ($Q = c, b,$) with the radial quantum number n can be expressed as

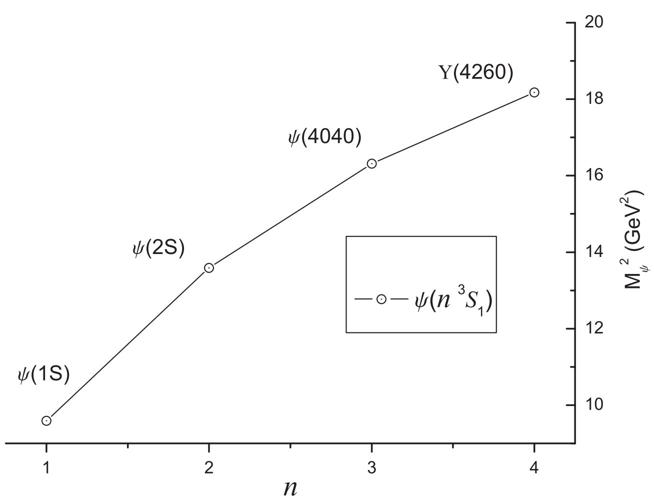

FIG. 2. The (n, M^2) plane for some $c\bar{c}$ (n^3S_1) mesons.

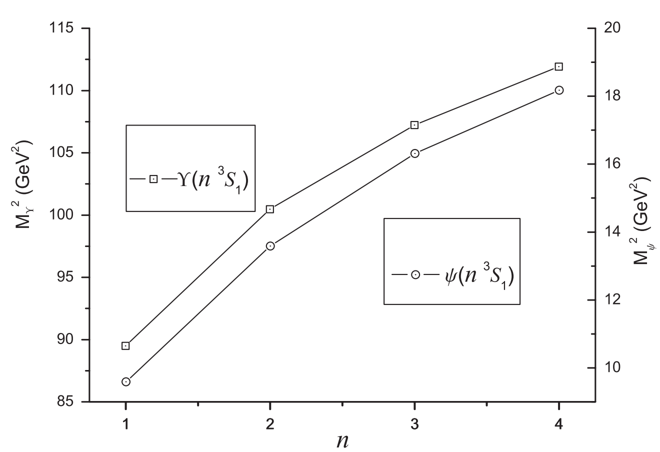

FIG. 3. The (n, M^2) plane for some $b\bar{b}$ (n^3S_1) and $c\bar{c}$ (n^3S_1) mesons.

$$M_{Q\bar{Q},n} = m_Q + m_Q + (E_n)_Q, \tag{10}$$

where $(E_n)_Q$ is the binding energy. Then we have

$$\beta = \frac{M^2_{c\bar{c},n+1} - M^2_{c\bar{c},n}}{M^2_{b\bar{b},n+1} - M^2_{b\bar{b},n}} = \frac{4m_c + (E_{n+1})_c + (E_n)_c}{4m_b + (E_{n+1})_b + (E_n)_b} \times \frac{(E_{n+1})_c - (E_n)_c}{(E_{n+1})_b - (E_n)_b}. \tag{11}$$

When $m_Q \to \infty$ (i.e. m_Q is much larger than the QCD scale $\Lambda_{\rm QCD}$), the binding energy is much less than the heavy quark mass [83], therefore,

$$\frac{4m_c + (E_{n+1})_c + (E_n)_c}{4m_b + (E_{n+1})_b + (E_n)_b} = \frac{m_c}{m_b}. \tag{12}$$

When $m_Q \to \infty$, the Coulomb potential is dominant, the binding energy has the following expression [83]:

$$(E_n)_Q = -\frac{\mu\kappa^2}{2n^2}, \tag{13}$$

where μ is the reduced mass, $\mu = (m_Q m_Q)/(m_Q + m_Q) = \frac{m_Q}{2}$, $\kappa = \frac{4}{3}\alpha_s(m_Q)$. From Eq. (13), we have

$$\frac{(E_{n+1})_c - (E_n)_c}{(E_{n+1})_b - (E_n)_b} = \frac{m_c\alpha_s^2(m_c)}{m_b\alpha_s^2(m_b)}. \tag{14}$$

Therefore, in the heavy quark limit, $\beta = [m_c^2\alpha_s^2(m_c)]/[m_b^2\alpha_s^2(m_b)]$, which is independent of the radial quantum number n. That we take β as a constant is reasonable.

For S wave, we have

$$\beta = \frac{M^2_{\psi(2S)} - M^2_{J/\psi(1S)}}{M^2_{\Upsilon(2S)} - M^2_{\Upsilon(1S)}} = \frac{M^2_{\eta_c(2S)} - M^2_{\eta_c(1S)}}{M^2_{\eta_b(2S)} - M^2_{\eta_b(1S)}}, \tag{15}$$

and for P wave, we have

$$\beta = \frac{M^2_{\chi_{c2}(2P)} - M^2_{\chi_{c2}(1P)}}{M^2_{\chi_{b2}(2P)} - M^2_{\chi_{b2}(1P)}} = \frac{M^2_{\chi_{c1}(2P)} - M^2_{\chi_{c1}(1P)}}{M^2_{\chi_{b1}(2P)} - M^2_{\chi_{b1}(1P)}} = \frac{M^2_{\chi_{c0}(2P)} - M^2_{\chi_{c0}(1P)}}{M^2_{\chi_{b0}(2P)} - M^2_{\chi_{b0}(1P)}}. \tag{16}$$

Assuming that β is approximately constant (for S wave and P wave at small radial quantum number n), based on Eqs. (15) and (16) and the corresponding meson masses in PDG [1], we have the masses of $\eta_b(2S)$, $\chi_{c0}(2P)$, $\chi_{c1}(2P)$, and $\chi_{c2}(2P)$. The results are also shown in Table II. Inserting the masses of $\chi_{b0,1,2}(1P)$, $\chi_{b0,1,2}(2P)$, and $\chi_{c0,1,2}(2P)$ into Eq. (8), we have the masses of $h_b(1P)$, $h_b(2P)$, and $h_c(2P)$, respectively. The results are shown in Table II. In Table II, we also list the results of Refs. [79,84] for comparison. The results of Ref. [79] were given in the screened potential model while the results of Ref. [84] were given in lattice QCD.

TABLE II. The masses of some unobserved $c\bar{c}$ and $b\bar{b}$ mesons given by Eqs. (8), (15), and (16) (in units of MeV). The results of Ref. [79] were given in the screened potential model while the results of Ref. [84] were given in lattice QCD.

States ($n^{2s+1}L_J$)	Prediction	[79]	[84]
$h_b(1P)$ (1^1P_1)	9900	9903	
$h_b(2P)$ (2^1P_1)	10 260	10 256	
$\eta_b(2S)$ (2^1S_0)	10 005	9987	
$h_c(2P)$ (2^1P_1)	3883	3908	3858 ± 70
$\chi_{c0}(2P)$ (2^3P_0)	3794	3842	3825 ± 88
$\chi_{c1}(2P)$ (2^3P_1)	3871	3901	3853 ± 57
$\chi_{c2}(2P)$ (2^3P_2)	3907	3937	

IV. MASS RELATIONS AND MASSES OF SOME $c\bar{b}$ MESONS

The mass of the ground pseudoscalar meson B_c^+ was first measured by the CDF collaboration in 1998 [85], with the mass to be $6.40 \pm 0.39 \pm 0.13$ GeV. After that, the OPAL [86] and the CDF [2,87] collaborations studied $B_c^\pm$ further. The CDF collaboration confirmed their earlier report [85,87] with higher statistical samples (with a significance greater than 8σ) and the more accurate mass was reported to be $6275.6 \pm 2.9 \pm 2.5$ GeV [2].

A. Relation about six mesons involving three flavors among one multiplet

Since Eqs. (2) and (3) are the relations of the parameters in Eq. (1) [the quasilinear Regge trajectories in the (J, M^2) plane], we can use Eqs. (1)–(3) together. Using Eqs. (1) and (2), one obtains

$$\alpha'_{i\bar{i}}M^2_{i\bar{i}} + \alpha'_{j\bar{j}}M^2_{j\bar{j}} = 2\alpha'_{i\bar{j}}M^2_{i\bar{j}}, \tag{17}$$

where the meson states $i\bar{i}$, $j\bar{j}$, and $i\bar{j}$ belong to the same $n^{2s+1}L_J$ multiplet. This relation can be reduced to the quadratic Gell-Mann–Okubo-type formula by assuming that all the slopes are independent of flavors ($\alpha'_{i\bar{i}} = \alpha'_{i\bar{j}} = \alpha'_{j\bar{j}}$). As demonstrated in our previous paper [29], when quark masses $m_i < m_j$, combining the relations (3) and (17), one can get the following relation between the slope ratio and meson masses:

$$\frac{\alpha'_{j\bar{j}}}{\alpha'_{i\bar{i}}} = \frac{1}{2M^2_{j\bar{j}}}\left[(4M^2_{i\bar{j}} - M^2_{i\bar{i}} - M^2_{j\bar{j}}) + \sqrt{(4M^2_{i\bar{j}} - M^2_{i\bar{i}} - M^2_{j\bar{j}})^2 - 4M^2_{i\bar{i}}M^2_{j\bar{j}}}\right]. \tag{18}$$

Equation (18) is a relation about three mesons involving two flavors in a spin-parity multiplet. Based on Eq. (18), with the help of arguments that the slope ratio is a real number and the Regge slope decreases with the increase of the quark mass [22,24,25,27,28], one can get two inequalities:

$$2M_{i\bar{j}} > M_{i\bar{i}} + M_{j\bar{j}}, \quad (19)$$

$$2M_{i\bar{j}}^2 < M_{i\bar{i}}^2 + M_{j\bar{j}}^2. \quad (20)$$

Both of the linear inequality (19) and the quadratic inequality (20) satisfy the experimental data. We will show later that the masses of $c\bar{c}$, $b\bar{b}$, and $c\bar{b}$ given in the present work also satisfy these two inequalities.

From Eq. (18), high-power meson mass equalities can be derived. Using Eq. (18) and the identical equation,

$$\frac{\alpha'_{c\bar{c}}}{\alpha'_{n\bar{n}}} \equiv \frac{\alpha'_{b\bar{b}}}{\alpha'_{n\bar{n}}} \times \frac{\alpha'_{c\bar{c}}}{\alpha'_{b\bar{b}}}, \quad (21)$$

and noticing $m_n < m_c < m_b$, we have

$$\frac{(4M_{n\bar{c}}^2 - M_{n\bar{n}}^2 - M_{c\bar{c}}^2) + \sqrt{(4M_{n\bar{c}}^2 - M_{n\bar{n}}^2 - M_{c\bar{c}}^2)^2 - 4M_{n\bar{n}}^2 M_{c\bar{c}}^2}}{2M_{c\bar{c}}^2} = \frac{[(4M_{n\bar{b}}^2 - M_{n\bar{n}}^2 - M_{b\bar{b}}^2) + \sqrt{(4M_{n\bar{b}}^2 - M_{n\bar{n}}^2 - M_{b\bar{b}}^2)^2 - 4M_{n\bar{n}}^2 M_{b\bar{b}}^2}]/2M_{b\bar{b}}^2}{[(4M_{c\bar{b}}^2 - M_{c\bar{c}}^2 - M_{b\bar{b}}^2) + \sqrt{(4M_{c\bar{b}}^2 - M_{c\bar{c}}^2 - M_{b\bar{b}}^2)^2 - 4M_{c\bar{c}}^2 M_{b\bar{b}}^2}]/2M_{b\bar{b}}^2}. \quad (22)$$

Relation (22) gives the high-power mass equalities about six mesons involving three flavors among one J^P multiplet. They can be used to predict the masses of unobserved states. In the following, we will apply Eq. (22) to predict the masses of B_c and B_c^*.

B. Calculation the masses of $M_{c\bar{b}}$

In the following, we will first use Eq. (22) to calculate the masses of B_c and B_c^*. Then, the slope ratios $(\alpha'_{b\bar{b}})/(\alpha'_{c\bar{c}})$ for the 1^1S_0 and 1^3S_1 multiplets can be calculated based on Eq. (18). After that, we will calculate the masses of $c\bar{b}$ ($1P$, $2S$, $2P$) mesons. The results will be compared with those given in other works.

Noticing $m_c < m_b$ and using Eq. (18), one can have

$$\frac{\alpha'_{b\bar{b}}}{\alpha'_{c\bar{c}}} = \frac{[(4M_{c\bar{b}}^2 - M_{c\bar{c}}^2 - M_{b\bar{b}}^2) + \sqrt{(4M_{c\bar{b}}^2 - M_{c\bar{c}}^2 - M_{b\bar{b}}^2)^2 - 4M_{c\bar{c}}^2 M_{b\bar{b}}^2}]}{2M_{b\bar{b}}^2}. \quad (23)$$

TABLE III. The predicted masses of $c\bar{b}$ mesons (in units of MeV) given by us and those in other approaches: nonlinear Regge trajectory [22], quasilinear Regge trajectory [24], lattice QCD [42,43], quark model [44–46], nonrelativistic quark model, potential model [47–54], Bethe-Salpeter equation [55,56].

$n^{2s+1}L_J$	$1^1S_0(B_c)$	$1^3S_1(B_c^*)$	1^1P_1	1^3P_0	1^3P_1	1^3P_2	2^1S_0	2^3S_1	2^1P_1	2^3P_0	2^3P_1	2^3P_2
Predicted	6270	6355	6761	6697	6750	6782	6863	6895	7103	7052	7095	7120
Experiment	6276											
[22]	6283	6356			6740	6780						
[24]	6263	6354			6788	6781		6894				
[42]	6278	6315	6796	6732	6778							
[43]	6280	6321	6765	6727	6743	6783	6960	6990				
[44]	6270	6332	6749	6699	6734	6762	6835	6881	7145	7091	7126	7156
[45]	6260	6340	6740	6680	6730	6760	6850	6900	7150	7100	7140	7160
[46]	6271	6338	6750	6704	6741	6768	6855	6887	7150	7122	7145	7164
[47]	6264	6337	6736	6700	6730	6747	6856	6899	7142	7108	7135	7153
[48]	6253	6317	6729	6683	6717	6743	6867	6902	7124	7088	7113	7134
[49]	6314	6355	6763	6728	6760	6773	6890	6917	7160	7134	7159	7166
[50]	6286	6341	6760	6701	6737	6772	6882	6914				
[52]	6349	6373	6738	6715	6726	6749	6821	6855	7136	7102	7119	7153
[53]	6247	6308	6757	6689	6738	6773	6853	6886				
[54]	6255	6320		6660	6740	6780		6900				
[55]	6380	6416	6775	6693	6772	6837	6875	6896	7139	7081	7136	7186
[56]	6258	6334						6883				

For the 1^1S_0 multiplet, when $i = n$, $j = c$, and $k = b$, inserting the masses of π, $\eta_c(1S)$, $\eta_b(1S)$ [15], D, and B into Eq. (22), we have $M_{B_c} = 6270$ MeV. Inserting the masses of $\eta_c(1S)$, $\eta_b(1S)$ [15], and $M_{B_c} = 6270$ MeV into Eq. (23), we have $(\alpha'_{b\bar{b}})/(\alpha'_{c\bar{c}})|_{1^1S_0} = 0.4667$. For the 1^3S_1 multiplet, when $i = n$, $j = c$, and $k = b$, inserting the masses of ρ, $J/\psi(1S)$, $\Upsilon(1S)$, D^*, and B^* into Eq. (22), we have $M_{B_c^*} = 6355$ MeV. Inserting the masses of $J/\psi(1S)$, $\Upsilon(1S)$, and $M_{B_c^*} = 6355$ MeV into Eq. (23), we have $(\alpha'_{b\bar{b}})/(\alpha'_{c\bar{c}})|_{1^3S_1} = 0.4702$. The comparison of the masses of B_c and B_c^* given in the present work and other references is shown in Table III. The mass splitting between B_c and B_c^* will be discussed in Sec. V.

Regge slopes are independent of charge conjugation in accordance with the C-invariance of QCD [22]. The Regge slope of the radial excited state is the same as that of the corresponding ground state [26] and the slopes of the parity partners' trajectories coincide [88]. Therefore, the authors of Ref. [24] divided all the multiplets into the 1^1S_0-like multiplets and the 1^3S_1-like multiplets. Mesons with the same flavors and belonging to the 1^3S_0-like multiplets (the 1^3S_1-like multiplets) have the same slopes.

The 1^1S_0-like multiplets include 1^1S_0, 1^1P_1, 1^3P_1, 2^1S_0, 2^1P_1, 2^3P_1, ... $\frac{\alpha'_{b\bar{b}}}{\alpha'_{c\bar{c}}}|_{1^1S_0} = 0.4667$.

The 1^3S_1-like multiplets include 1^3S_1, 1^3P_0, 1^3P_2, 2^3S_1, 2^3P_0, 2^3P_2, ... $(\alpha'_{b\bar{b}})/(\alpha'_{c\bar{c}})|_{1^1S_0} = 0.4702$.

Using the values of slope rations $((\alpha'_{b\bar{b}})/(\alpha'_{c\bar{c}})|_{1^1S_0} = 0.4667$, $(\alpha'_{b\bar{b}})/(\alpha'_{c\bar{c}})|_{1^3S_1} = 0.4702)$ and the masses of $c\bar{c}$ and $b\bar{b}$ obtained in Sec. III and given by experiment [1], we can have the masses of $c\bar{b}(1P, 2S, 2P)$ from Eq. (23). The comparison of the masses of excited $c\bar{b}(1P, 2S, 2P)$ mesons extracted in the present work and those given by other works is also shown in Table III.

From Table III, one can see that the mass of the B_c meson given in the present work is very close to the experimental data. The masses of the 12 $c\bar{b}$ mesons given in the present work are in reasonable agreement with those given in many other approaches. The masses of $c\bar{b}(1P, 2P)$ mesons satisfy Eq. (8) (equality for P-wave hyperfine splittings). A detailed discussion will be given in Sec. V.

V. DISCUSSION AND SUMMARY

In this work, we have studied the feature of radial Regge trajectories for charmonium and bottomonium in the (n, M^2) plane and predicted masses of some states which have not been observed up to now. Using the additivity of intercepts and inverse slopes for the quasilinear Regge trajectories in the (J, M^2) plane, we have extracted all the masses of the $c\bar{b}$ mesons belonging to the 1^1S_0, 1^3S_1, 1^1P_1, 1^3P_0, 1^3P_1, 1^3P_2, 2^1S_0, 2^3S_1, 2^1P_1, 2^3P_0, 2^3P_1, and 2^3P_2 multiplets. Therefore, all the masses of the unobserved doubly heavy mesons $(1S, 1P, 2S, 2P)$ have been extracted. The present results of the $(2P)$ $c\bar{c}$ mesons (Table II) and $(1P, 2P)$ $c\bar{b}$ mesons (Table III) satisfy Eq. (8) (equality for P-wave hyperfine splittings) very well. All the present results satisfy the linear mass inequality (19) and the quadratic mass inequality (20). The predictions are in reasonable agreement with the existing experimental data and those suggested in many other different approaches.

In the calculations, all the input masses are the masses of the well-established mesons whose mass uncertainties are very small. We have analyzed the relation between the value of $M_{c\bar{b}}$ and the value of $\alpha'_{b\bar{b}}/\alpha'_{c\bar{c}}$ from Eq. (23), and have found that the 10% uncertainty of $\alpha'_{b\bar{b}}/\alpha'_{c\bar{c}}$ only leads to less than 1% uncertainty of $M_{c\bar{b}}$.

Based on the experimental data [1], Tables II and III, one can have the following hyperfine splittings:

Charmonium hyperfine splittings, $M_{J/\psi(1S)} - M_{\eta_c(1S)} = 116$ MeV, $M_{J/\psi(2S)} - M_{\eta_c(2S)} = 49$ MeV;

Bottomonium hyperfine splittings, $M_{\Upsilon(1S)} - M_{\eta_b(1S)} = 71$ MeV, $M_{\Upsilon(2S)} - M_{\eta_b(2S)} = 18$ MeV.

The $c\bar{b}$ hyperfine splittings, $M_{B_c^*} - M_{B_c} = 85$ MeV, $M_{B_c^*(2S)} - M_{B_c(2S)} = 32$ MeV are between the splittings of charmonium and bottomonium as expected.

In Ref. [89], the authors gave an approximate relation for mass splittings,

$$M_{B_c^*} - M_{B_c} \simeq (0.7)(M_{J/\psi} - M_{\eta_c})^{0.65}(M_\Upsilon - M_{\eta_b})^{0.35}, \tag{24}$$

and pointed out that the uncertainty of Eq. (24) is at the order of 10%.

We list these splittings given in the present work and those given in many other works in Table IV. From Tables III and IV, we can see that our predicted splittings agree with some works and are different from others.

From Table II, one can see that our predicted mass of $\chi_{c1}(2P)$ (2^3P_1) is 3871 MeV, which is in excellent agreement with the mass of the hotly debated charmoniumlike state $X(3872)$.

$X(3872)$ is one of the most complex charmoniumlike states discovered recently. It was discovered by the Belle collaboration in the exclusive decay process $B^\pm \to K^\pm\pi^+\pi^- J/\psi$ in 2003 [3]. After that, there are many theoretical and experimental works focused on the nature

TABLE IV. The mass splittings (in units of MeV).

Mass splittings	Predicted	Experiment	[44]	[45]	[46]	[47]	[48]	[50]	[53]	[56]	[67]	[90]	[91]	[92]	[93]
$B_c^*(1S)$-$B_c(1S)$	85		62	80	67	73	64	55	61	77	70	68–83	65–90	75 ± 28	$65 \pm 24^{+19}_{-16}$
$B_c^*(2S)$-$B_c(2S)$	32		46	50	32	43	35	32	33	42	40	31–42			

of $X(3872)$ (see Ref. [81] and references therein). In PDG2008 [1], the average mass and width of $X(3872)$ are 3872.2 ± 0.8 MeV and 775 ± 4 MeV, respectively. In the recent report of the CDF collaboration, it was pointed that the data of $X(3872)$ are consistent with a single state, with no evidence for two states [94]. With the single-state model the $X(3872)$ mass is measured to be $3871.61 \pm 0.16 \pm 0.19$ MeV, which is the most precise determination to date. Angular distribution analysis by the Belle collaboration [95] and the CDF collaboration [96] both favor the quantum number $J^{PC} = 1^{++}$. Therefore, interpreting the $X(3872)$ as the $\chi_{c1}(2P)$ seems favorable. This assignment can explain most of the experimental features of $X(3872)$, but fails in the isospin-violating decay $X(3872) \to J/\psi\rho$. In Refs. [79,97], the authors use the final state rescattering mechanism to explain this isospin-violating decay. After a comprehensive analysis, the authors of Refs. [79,97] assign $X(3872)$ as a $\chi_{c1}(2P)$-dominated charmonium state.

In Table II, we give the mass of $\chi_{c2}(2P)$ (2^3P_2) as 3907 MeV. The Belle collaboration reported a $\chi_{c2}(2P)$ (2^3P_2) candidate in $\gamma\gamma \to D\bar{D}$ production [98]. The mass and width are $3929 \pm 5 \pm 2$ MeV and $29 \pm 10 \pm 2$ MeV, respectively. The observed transverse momentum of the $D\bar{D}$ conflicts with the possible molecules $D\bar{D}^*$ or $D^*\bar{D}^*$. In many papers it is referred to as $Z(3930)$ or $X(3930)$. The observation of $Z(3930) \to D\bar{D}$ decay can rule out the interpretation as $\eta_c(3S)$. The fact that $Z(3930)$ was produced from $\gamma\gamma$ can exclude the interpretation as $\psi_3(1^3D_3)$. By assuming that $M_{\chi_{c2}(2P)} = 3931$ MeV, the authors of Ref. [99] got $\Gamma_{\chi_{c2}(2P)} = 28.6$ MeV which is in good agree with experimental data. The mass of $\chi_{c2}(2P)$ calculated in our approach deviates from the experimental mass of $Z(3930)$ by only about 20 MeV. Therefore, we support the assignment of the state $Z(3930)$ as the charmonium state $\chi_{c2}(2P)$.

However, the nature of many of the recently discovered charmoniumlike and bottomoniumlike states are still unclear. Some states in the conventional charmonium and bottomonium spectra are still missing while some unexpected states are observed. All of these indicate that our understanding of charmonium (bottomonium) states above the open charm (bottom) threshold is still poor.

In Sec. III, we have shown that the trajectories in the (n, M^2) plane are indeed nonlinear for bottomonium (bend down). There is a curve (bend down) for charmonium if the mass of $\psi(4S)$ is close to the mass of $Y(4260)$. The convex radial Regge trajectory (bend down) in the (n, M^2) plane implies that the spectrum condenses at high energies. This view is not isolated. In a recent paper [100], for light mesons in AdS/QCD models, Afonin pointed out that the dependence of m^2 on n is nonlinear at larger n. The holographic models reflect thereby the merging of resonances into continuum and the breaking of gluon string at sufficiently large quark-antiquark separation that causes the linear Regge trajectories to bend down. The authors of Ref. [79] found the masses of higher charmonium and bottomonium states with screened potential are considerably lower than that with unscreened potential.

From the above analysis, we can give a summary as the following:

(1) The trajectories in the (n, M^2) plane are indeed nonlinear for bottomonium (bend down). There is a similar curve (bend down) for charmonium if the mass of $\psi(4S)$ is close to the mass of $Y(4260)$. The fact that trajectories in the (n, M^2) plane are convex (bend down) implies denser spectra than the linear case. This feature agrees with experiment and some recent theoretical work.

(2) We support that $X(3872)$ is a $\chi_{c1}(2P)$-dominated charmonium state and $Z(3930)$ is the charmonium state $\chi_{c2}(2P)$.

(3) Our predictions (masses of three bottomonium, four charmonium, and 12 $c\bar{b}$ states) are in reasonable agreement with the existing experimental data and those suggested in many other different approaches. The predictions may be useful for the discovery of the unobserved $c\bar{c}$, $b\bar{b}$, and $c\bar{b}$ mesons and the J^P assignment of these states.

ACKNOWLEDGMENTS

One of us (Ke-Wei Wei) would like to thank Professor Bing-Song Zou and Professor De-Min Li for support and discussions. This work was supported in part by the National Natural Science Foundation of China (Projects No. 10675022, No. 10875133, No. 10821063, and No. 10975018), the Special Grants from Beijing Normal University, and the Chinese Academy of Sciences Knowledge Innovation Project (Project No. KJCX3-SYW-N2).

[1] C. Amsler *et al.* (Particle Data Group), Phys. Lett. B **667**, 1 (2008), and 2009 partial update for the 2010 edition.
[2] T. Aaltonen *et al.* (CDF Collaboration), Phys. Rev. Lett. **100**, 182002 (2008).
[3] S. K. Choi *et al.* (Belle Collaboration), Phys. Rev. Lett. **91**, 262001 (2003).
[4] K. Abe *et al.* (Belle Collaboration), Phys. Rev. Lett. **98**, 082001 (2007).

[5] P. Pakhlov *et al.* (Belle Collaboration), Phys. Rev. Lett. **100**, 202001 (2008).
[6] K. Abe *et al.* (Belle Collaboration), Phys. Rev. Lett. **94**, 182002 (2005).
[7] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. Lett. **101**, 082001 (2008).
[8] C. Z. Yuan *et al.* (Belle Collaboration), Phys. Rev. Lett. **99**, 182004 (2007).
[9] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. Lett. **95**, 142001 (2005).
[10] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. Lett. **98**, 212001 (2007).
[11] C. P. Shen *et al.* (Belle Collaboration), Phys. Rev. Lett. **104**, 112004 (2010).
[12] X. L. Wang *et al.* (Belle Collaboration), Phys. Rev. Lett. **99**, 142002 (2007).
[13] I. Adachi *et al.* (Belle Collaboration), arXiv:0808.2445.
[14] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. Lett. **102**, 012001 (2009).
[15] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. Lett. **101**, 071801 (2008).
[16] B. Aubert (*BABAR* Collaboration), Phys. Rev. Lett. **103**, 161801 (2009).
[17] G. Bonvicini *et al.* (CLEO Collaboration), Phys. Rev. D **81**, 031104 (2010).
[18] N. Brambilla *et al.* (Quarkonium Working Group), arXiv: hep-ph/0412158.
[19] T. Regge, Nuovo Cimento **14**, 951 (1959); **18**, 947 (1960).
[20] P. D. B. Collins, *An Introduction to Regge theory and High Energy Physics* (Cambridge University Press, Cambridge, England, 1977); V. N. Gribov, *The Theory of Complex Angular Momenta: Gribov Lectures on Theoretical Physics* (Cambridge University Press, Cambridge, England, 2003).
[21] G. F. Chew and S. C. Frautschi, Phys. Rev. Lett. **7**, 394 (1961); **8**, 41 (1962).
[22] M. M. Brisudova, L. Burakovsky, and T. Goldman, Phys. Rev. D **61**, 054013 (2000).
[23] L. Burakovsky, T. Goldman, and L. P. Horwitz, Phys. Rev. D **56**, 7124 (1997).
[24] D.-M. Li, B. Ma, Y.-X. Li, Q.-K. Yao, and H. Yu, Eur. Phys. J. C **37**, 323 (2004).
[25] L. Burakovsky, T. Goldman, and L. P. Horwitz, Phys. Rev. D **56**, 7119 (1997); J. Phys. G **24**, 771 (1998).
[26] A. V. Anisovich, V. V. Anisovich, and A. V. Sarantsev, Phys. Rev. D **62**, 051502(R) (2000); V. V. Anisovich, AIP Conf. Proc. **619**, 197 (2002); Usp. Fiz. Nauk **174**, 49 (2004); [Phys. Usp. **47**, 45 (2004)]; AIP Conf. Proc. **717**, 441 (2004).
[27] A. Zhang, Phys. Rev. D **72**, 017902 (2005); Phys. Lett. B **647**, 140 (2007).
[28] A. B. Kaidalov, Z. Phys. C **12**, 63 (1982).
[29] X.-H. Guo, K.-W. Wei, and X.-H. Wu, Phys. Rev. D **78**, 056005 (2008).
[30] A. M. Badalian, B. L. G. Bakker, and Yu. A. Simonov, Phys. Rev. D **66**, 034026 (2002).
[31] S. S. Afonin, Int. J. Mod. Phys. A **22**, 4537 (2007).
[32] D. Z. Freedman and J.-M. Wang, Phys. Rev. Lett. **17**, 569 (1966); **18**, 863 (1967); Phys. Rev. **153**, 1596 (1967); **160**, 1560 (1967).
[33] M. L. Goldberger and C. E. Jones, Phys. Rev. **150**, 1269 (1966); D. Z. Freedman, C. E. Jones, and J.-M. Wang, Phys. Rev. **155**, 1645 (1967).
[34] G. Veneziano, Nuovo Cimento A **57**, 190 (1968).
[35] D. V. Bugg, Phys. Rep. **397**, 257 (2004); A. V. Anisovich *et al.*, Phys. Lett. B **491**, 47 (2000).
[36] Yu. S. Kalashnikova, A. V. Nefediev, and Yu. A. Simonov, Phys. Rev. D **64**, 014037 (2001).
[37] L. D. Solovev, Phys. Rev. D **61**, 015009 (1999).
[38] S. S. Afonin, Phys. Rev. C **76**, 015202 (2007).
[39] D. Ebert, R. N. Faustov, and V. O. Galkin, Phys. Rev. D **79**, 114029 (2009).
[40] D.-M. Li, B. Ma, and Y.-H. Liu, Eur. Phys. J. C **51**, 359 (2007).
[41] S. S. Gershtein, A. K. Likhoded, and A. V. Luchinsky, Phys. Rev. D **74**, 016002 (2006).
[42] T.-W. Chiu, T.-H. Hsieh, C.-H. Huang, and K. Ogawa (TWQCD Collaboration), Phys. Lett. B **651**, 171 (2007).
[43] C. T. H. Davies, K. Hornbostel, G. P. Lepage, A. J. Lidsey, J. Shigemitsu, and J. H. Sloan, Phys. Lett. B **382**, 131 (1996).
[44] D. Ebert, R. N. Faustov, and V. O. Galkin, Phys. Rev. D **67**, 014027 (2003).
[45] J. Zeng, J. W. Van Orden, and W. Roberts, Phys. Rev. D **52**, 5229 (1995).
[46] S. Godfrey, Phys. Rev. D **70**, 054017 (2004).
[47] E. J. Eichten and C. Quigg, Phys. Rev. D **49**, 5845 (1994).
[48] S. S. Gershtein, V. V. Kiselev, A. K. Likhoded, and A. V. Tkabladze, Phys. Rev. D **51**, 3613 (1995).
[49] Y.-Q. Chen and Y.-P. Kuang, Phys. Rev. D **46**, 1165 (1992). See also Y.-Q. Chen, Ph.D. thesis, Institute of Theoretical Physics, Chinese Academic of Science, 1992.
[50] L. P. Fulcher, Phys. Rev. D **60**, 074006 (1999).
[51] B. Patel and P. C. Vinodkumar, J. Phys. G **36**, 035003 (2009).
[52] A. K. Rai and P. C. Vinodkumar, Pramana **66**, 953 (2006).
[53] S. N. Gupta and J. M. Johnson, Phys. Rev. D **53**, 312 (1996).
[54] R. Roncaglia, A. Dzierba, D. B. Lichtenberg, and E. Predazzi, Phys. Rev. D **51**, 1248 (1995).
[55] A. Abd El-Hady, J. R. Spence, and J. P. Vary, Phys. Rev. D **71**, 034006 (2005).
[56] M. Baldicchi and G. M. Prosperi, Phys. Rev. D **62**, 114024 (2000).
[57] L. Burakovsky and T. Goldman, Phys. Lett. B **434**, 251 (1998).
[58] K. Kawarabayashi, S. Kitakado, and H. Yabuki, Phys. Lett. B **28**, 432 (1969).
[59] R. C. Brower, J. Ellis, M. G. Schmidt, and J. H. Weis, Nucl. Phys. B **128**, 175 (1977).
[60] N. A. Kobylinsky, E. S. Martynov, and A. B. Prognimak, Ukr. Fiz. Zh. **24**, 969 (1979).
[61] V. V. Dixit and L. A. P. Balazs, Phys. Rev. D **20**, 816 (1979).
[62] K. Igi and S. Yazaki, Phys. Lett. B **71**, 158 (1977).
[63] W. Cassing, L. A. Kondratyuk, G. I. Lykasov, and M. V. Rzjanin, Phys. Lett. B **513**, 1 (2001).
[64] N. N. Khuri, Phys. Rev. **132**, 914 (1963).
[65] A. Zhang, Phys. Lett. B **647**, 140 (2007); Y.-M. Kong and A. Zhang, Phys. Lett. B **657**, 192 (2007).
[66] L. Motyka and K. Zalewski, Eur. Phys. J. C **4**, 107 (1998).
[67] S. Godfrey and N. Isgur, Phys. Rev. D **32**, 189 (1985).

[68] M. Okamoto *et al.* (CP-PACS Collaboration), Phys. Rev. D **65**, 094508 (2002).
[69] I. Haysak, Yu. Fekete, V. Morokhovych, S. Chalupka, and M. Salak, Czech. J. Phys. **55**, 541 (2005).
[70] J. T. Pantaleone and S. H. H. Tye, Phys. Rev. D **37**, 3337 (1988); J. T. Pantaleone, S. H. H. Tye, and Y. J. Ng, Phys. Rev. D **33**, 777 (1986).
[71] S. Recksiegel and Y. Sumino, Phys. Lett. B **578**, 369 (2004).
[72] V. V. Anisovich, L. G. Dakhno, M. A. Matveev, V. A. Nikonov, and A. V. Sarantsev, Phys. At. Nucl. **70**, 364 (2007).
[73] T. Barnes, S. Godfrey, and E. S. Swanson, Phys. Rev. D **72**, 054026 (2005); E. Swanson, AIP Conf. Proc. **814**, 203 (2006); Int. J. Mod. Phys. A **21**, 733 (2006).
[74] Q. He *et al.* (CLEO Collaboration), Phys. Rev. D **74**, 091104 (2006).
[75] T. E. Coan *et al.* (CLEO Collaboration), Phys. Rev. Lett. **96**, 162003 (2006).
[76] B. Aubert (*BABAR* Collaboration), Phys. Rev. D **79**, 092001 (2009).
[77] F. J. Llanes-Estrada, Phys. Rev. D **72**, 031503 (2005).
[78] C. Quigg and J. L. Rosner, Phys. Lett. B **71**, 153 (1977).
[79] B.-Q. Li and K.-T. Chao, Phys. Rev. D **79**, 094004 (2009); Commun. Theor. Phys. **52**, 653 (2009).
[80] J. L. Rosner, J. Phys. G **34**, S127 (2007); AIP Conf. Proc. **870**, 63 (2006).
[81] E. Eichten, S. Godfrey, H. Mahlke, and J. L. Rosner, Rev. Mod. Phys. **80**, 1161 (2008).
[82] W. R. Innes *et al.*, Phys. Rev. Lett. **39**, 1240 (1977).
[83] W. Lucha, F. F. Schoberl, and D. Gromes, Phys. Rep. **200**, 127 (1991).
[84] Y. Chen, C. Liu, Y. Liu, J. Ma, and J. Zhang (CLQCD Collaboration), arXiv:hep-lat/0701021.
[85] F. Abe *et al.* (CDF Collaboration), Phys. Rev. Lett. **81**, 2432 (1998).
[86] K. Ackerstaff *et al.* (OPAL Collaboration), Phys. Lett. B **420**, 157 (1998).
[87] A. Abulencia *et al.* (CDF Collaboration), Phys. Rev. Lett. **96**, 082002 (2006).
[88] H.-M. Chan, C. S. Hsue, C. Quigg, and J.-M. Wang, Phys. Rev. Lett. **26**, 672 (1971).
[89] S. J. Collins, T. D. Imbo, B. A. King, and E. C. Martell, Phys. Lett. B **393**, 155 (1997).
[90] S. M. Ikhdair, arXiv:hep-ph/0504176.
[91] W. Kwong and J. L. Rosner, Phys. Rev. D **44**, 212 (1991).
[92] E. Bagan, H. G. Dosch, P. Gosdzinsky, S. Narison, and J.-M. Richard, Z. Phys. C **64**, 57 (1994).
[93] A. A. Penin, A. Pineda, V. A. Smirnov, and M. Steinhauser, Phys. Lett. B **593**, 124 (2004).
[94] T. Aaltonen *et al.* (CDF Collaboration), Phys. Rev. Lett. **103**, 152001 (2009).
[95] K. Abe *et al.* (Belle Collaboration), arXiv:hep-ex/0505038; G. Gokhroo *et al.* (Belle Collaboration), Phys. Rev. Lett. **97**, 162002 (2006).
[96] A. Abulencia *et al.* (CDF Collaboration), Phys. Rev. Lett. **98**, 132002 (2007).
[97] B.-Q. Li, C. Meng, and K.-T. Chao, Phys. Rev. D **80**, 014012 (2009); O. Zhang, C. Meng, and H.-Q. Zheng, Phys. Lett. B **680**, 453 (2009).
[98] S. Uehara *et al.* (Belle Collaboration), Phys. Rev. Lett. **96**, 082003 (2006).
[99] E. J. Eichten, K. Lane, and C. Quigg, Phys. Rev. D **73**, 014014 (2006).
[100] S. S. Afonin, Phys. Lett. B **675**, 54 (2009).