

Neutral Higgs production on LHC in the two-Higgs-doublet model with spontaneous CP violation

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Spontaneous CP violation motivates the introduction of two Higgs doublets in the electroweak theory, such a simple extension of the standard model has five physical Higgs bosons and rich CP -violating sources. Exploration on more than one Higgs boson is a direct evidence for new physics beyond the standard model. The neutral Higgs production at LHC is investigated in such a general two-Higgs-doublet model with spontaneous CP violation, it is shown that the production cross section and decays of the neutral Higgs boson can significantly be different from the predictions from the standard model.

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I. INTRODUCTION

In the standard model (SM), the fermions and gauge bosons get masses through Higgs mechanism with a single weak-isospin doublet Higgs field. After the electroweak symmetry breaking, three Goldstone modes were absorbed to build up the longitudinal W and Z gauge bosons, only one physical scalar called the SM Higgs boson is left. Although the value of the Higgs mass can not be predicted in the SM, for the theoretical self-consistence, the unitarity [1] require $m_h < 1$ TeV. On the other hand, the analysis of all the LEP measurements [2] leads to the best fitting Higgs mass $m_h = 114^{+69}_{-45}$ GeV or the one-side 95% C.L. upper limit of $m_h < 260$ GeV. Once the Higgs mass is known in the SM, the properties of the Higgs boson, such as the decay width and production cross section, can be predicted. Nevertheless, if there exists new physics beyond the SM, the production cross section and decay width of the Higgs boson as well as the mass constraint to the Higgs boson can be different.

It has been shown that if the SM Higgs mass lies between 130 and 200 GeV [3], the SM can in general be valid at energy scales all the way up to the Planck scale. Nevertheless, the SM cannot be a fundamental theory, there are still some unknown puzzles in the SM, such as the origin of CP violation, the smallness of neutrino masses, the dark matter, and so on. They all suggest the existence of new physics beyond the SM. Thus, many extensions or modifications of the SM have been studied. In this paper, we are going to focus on the simplest extension of the SM with adding an extra Higgs doublet motivated from spontaneous CP violation [4–8], such a general two-Higgs-doublet model (2HDM) with spontaneous CP violation is also called Type III 2HDM. It has been shown that if one Higgs doublet is needed for the mass generation, then the additional extra Higgs doublet is necessary for the origin of CP violation, so that the CP violation is originated from a single relative phase of two vacuum expect-

tation values, which gives not only an explanation for the Kobayashi-Maskawa CP -violating mechanism [9] in the standard model, but also leads to a new type of CP -violating source [6,7], which has been studied broadly.

The complex Higgs doublets in the Type III 2HDM are generally expressed as [6,7,10]

$$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \phi_1^0 \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix}, \quad (1)$$

and the Higgs potential is

$$\begin{aligned} V = & -\mu_1^2 \Phi_1^\dagger \Phi_1 - \mu_2^2 \Phi_2^\dagger \Phi_2 - \frac{\mu_{12}^2}{2} \Phi_1^\dagger \Phi_2 - \frac{\mu_{12}^{*2}}{2} \Phi_2^\dagger \Phi_1 \\ & + \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \frac{\lambda_3}{4} (\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1)^2 \\ & - \frac{\lambda_4}{4} (\Phi_1^\dagger \Phi_2 - \Phi_2^\dagger \Phi_1)^2 + \lambda_5 \Phi_1^\dagger \Phi_1 \Phi_2^\dagger \Phi_2 \\ & + \frac{1}{2} (\lambda_6 \Phi_1^\dagger \Phi_1 + \lambda_7 \Phi_2^\dagger \Phi_2) (\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1). \end{aligned} \quad (2)$$

The Yukawa interaction terms have the following general form:

$$\mathcal{L}_Y = \eta_{ij}^{(k)} \bar{\psi}_{i,L} \tilde{\Phi}_k U_{j,R} + \xi_{ij}^{(k)} \bar{\psi}_{i,L} \Phi_k D_{j,R} + \text{H.c.}, \quad (3)$$

where $\eta_{ij}^{(k)}$ and $\xi_{ij}^{(k)}$ are real Yukawa coupling constants, so that the interactions are CP invariant. The major issue with respect to the model is that it allows flavor changing neutral current (FCNC) at the tree level through the neutral Higgs boson exchanges, which should strongly be suppressed based on the experimental observations. In order to prevent the FCNC at the tree level, an *ad hoc* discrete symmetry [11] is often imposed:

$$\begin{aligned} \Phi_1 & \rightarrow -\Phi_1 \quad \text{and} \quad \Phi_2 \rightarrow \Phi_2, \\ U_R & \rightarrow -U_R \quad \text{and} \quad D_R \rightarrow \mp D_R, \end{aligned} \quad (4)$$

which leads to the so-called Type I and Type II 2HDM relying on whether the up- and down-type quarks are coupled to the same or different Higgs doublet, respectively. Some interesting phenomena for various cases in

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such types of models without FCNC have been investigated in detail in Refs. [12,13]. When the discrete symmetry was introduced, there will be $\mu_{12} = 0$ and $\lambda_6 = \lambda_7 = 0$, which implies no spontaneous CP violation any more [14]. It should be noted that the supersymmetry also requires more than one Higgs doublet. The Higgs sector and the relevant Yukawa interactions in the minimal supersymmetric standard model (MSSM) is analogous to the Type II 2HDM. As the FCNC is the interesting phenomena observed in experiments in the weak interactions though it is strongly suppressed, we shall abandon the discrete symmetries and consider the small off-diagonal Yukawa couplings concerning the FCNC, the naturalness for such small Yukawa couplings may be understood from the approximate global $U(1)$ family symmetries [6,7,15–17]. This may be explained as follows: if all the up-type quarks and also the down-type quarks have the same masses and no mixing, the theory has an $U(3)$ family symmetry for three generations, while when all quarks have different masses but remain no mixing, the theory has the $U(1) \otimes U(1) \otimes U(1)$ family symmetries and the Cabibbo-Kobayashi-Maskawa quark mixing matrix is a unit matrix, in this case both the direct FCNC and induced FCNC are absent. In the real world, there are some FCNC processes observed, thus the $U(1)$ family symmetries should be broken down. As all the observed FCNC processes are strongly suppressed, the theory should possess approximate $U(1)$ family symmetries with small off-diagonal mixing among three generations. In this sense, the approximate $U(1)$ family symmetries are enough to ensure the naturalness of the observed smallness of FCNC.

After spontaneous symmetry breaking, the neutral Higgs bosons will get the vacuum expectation values

$$\langle \phi_1^0 \rangle = \frac{1}{\sqrt{2}} v_1 e^{i\delta_1}, \quad \langle \phi_2^0 \rangle = \frac{1}{\sqrt{2}} v_2 e^{i\delta_2}, \quad (5)$$

where one of the phases can be rotated away due to the global $U(1)$ symmetry. Without losing generality, we may take $\delta_1 = 0$ and $\delta_2 = \delta$. It is then convenient to make a unitary transformation

$$\begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = U \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix}, \quad (6)$$

with

$$U = \begin{pmatrix} \cos\beta & \sin\beta e^{-i\delta} \\ -\sin\beta & \cos\beta e^{-i\delta} \end{pmatrix} \quad (7)$$

and $\tan\beta = v_2/v_1$. After making the above transformation, we can re-express the Higgs doublets as follows:

$$H_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + \phi_1^0 \end{pmatrix} + \mathcal{G}, \quad H_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}H^+ \\ \phi_2^0 + i\phi_3^0 \end{pmatrix}, \quad (8)$$

with $v^2 = v_1^2 + v_2^2$ and $v \simeq 246$ GeV which is the same as

in the standard model. Thus, the Higgs doublet H_1 in the new basis plays the role of the standard model Higgs and gives masses to the gauge bosons ($m_W = gv/2$) and quarks and leptons. The Fermi constant is then given by the same value as in the standard model $G_F = g^2/(4\sqrt{2}m_W^2) = 1/(\sqrt{2}v^2)$. The Higgs field \mathcal{G} are the Goldstone particles absorbed by the gauge bosons, while H^\pm are mass eigenstates of the charged scalar Higgs, and $(\phi_1^0, \phi_2^0, \phi_3^0)$ are the neutral Higgs bosons in the electroweak eigenstates, they are in general not the same ones (h, H, A) in the mass eigenstates, but can be expressed as linear combinations of the mass eigenstates (h, H, A) via an orthogonal $SO(3)$ transformation which depends on the λ_i s and μ_i^2 s in the Higgs potential. In the new basis after the unitary transformation, the phase δ appears in the Yukawa coupling terms

$$\begin{aligned} \mathcal{L}_Y = & \eta_{ij}^U \bar{\psi}_{i,L} \tilde{H}_1 U_{j,R} + \eta_{ij}^D \bar{\psi}_{i,L} H_1 D_{j,R} + \xi_{ij}^U \bar{\psi}_{i,L} \tilde{H}_2 U_{j,R} \\ & + \xi_{ij}^D \bar{\psi}_{i,L} H_2 D_{j,R} + \text{H.c.}, \end{aligned} \quad (9)$$

where

$$\begin{aligned} \eta_{ij}^U &= \eta_{ij}^{(1)} \cos\beta + \eta_{ij}^{(2)} e^{-i\delta} \sin\beta \equiv \sqrt{2} M_{ij}^U / v, \\ \xi_{ij}^U &= -\eta_{ij}^{(1)} e^{-i\delta} \sin\beta + \eta_{ij}^{(2)} \cos\beta, \\ \eta_{ij}^D &= \xi_{ij}^{(1)} \cos\beta + \xi_{ij}^{(2)} e^{-i\delta} \sin\beta \equiv \sqrt{2} M_{ij}^D / v, \\ \xi_{ij}^D &= -\xi_{ij}^{(1)} e^{-i\delta} \sin\beta + \xi_{ij}^{(2)} \cos\beta. \end{aligned} \quad (10)$$

As the Yukawa coupling terms η^U and η^D become complex due to the vacuum phase δ , the resulting mass matrices are also complex. It then requires a unitary transformation to diagonalize the mass matrices for transforming the quark and lepton fields from the weak interaction states to the mass eigenstates. In the mass eigenstates, the Yukawa interaction terms are given by

$$\begin{aligned} \mathcal{L}_Y = & \bar{U}_L \frac{m^U}{v} U_R (v + \phi_1^0) + \bar{D}_L \frac{m^D}{v} D_R (v + \phi_1^0) \\ & + \frac{1}{\sqrt{2}} \bar{U}_L \xi^U U_R (\phi_2^0 - i\phi_3^0) + \bar{D}_L \hat{\xi}^U U_R H^- \\ & + \bar{U}_L \hat{\xi}^D D_R H^+ + \frac{1}{\sqrt{2}} \bar{D}_L \xi^D D_R (\phi_2^0 + i\phi_3^0) + \text{H.c.}, \end{aligned} \quad (11)$$

with

$$\hat{\xi}^U = \xi^U V_{\text{CKM}}, \quad \hat{\xi}^D = V_{\text{CKM}} \xi^D, \quad (12)$$

and

$$\begin{aligned} U_{L(R)} &= (u_{L(R)}, c_{L(R)}, t_{L(R)}), \\ D_{L(R)} &= (d_{L(R)}, s_{L(R)}, b_{L(R)}). \end{aligned} \quad (13)$$

It can be seen that the scalar ϕ_1^0 plays the role of the Higgs in the SM except considering the large mixing effects among ϕ_1^0 , ϕ_2^0 and ϕ_3^0 , which will be discussed later on.

Here, we shall use $\xi^{U(D)}$ and the masses of quarks as the independent input parameters instead of the original Yukawa couplings in Eq. (3) and the parameter β . It is noticed that the off-diagonal elements of the parameters $\xi_{ij}^{U(D)}$ ($i \neq j$) will cause the tree level FCNC, while their effects can be made to be small enough to fit the current experimental data by imposing approximate U(1) family symmetries.

Note that the Type II 2HDM can be regarded as a special case of the Type III 2HDM with spontaneous CP violation by setting $\eta_{ij}^{(1)} = 0$ and $\xi_{ij}^{(2)} = 0$, which are ensured by a discrete symmetry,

$$\begin{aligned}\eta_{ij}^U &= \eta_{ij}^{(2)} \sin\beta \equiv \sqrt{2}M_{ij}^U/v, \\ \xi_{ij}^U &= \eta_{ij}^{(2)} \cos\beta \equiv \sqrt{2}M_{ij}^U/v \cot\beta, \\ \eta_{ij}^D &= \xi_{ij}^{(1)} \cos\beta \equiv \sqrt{2}M_{ij}^D/v, \\ \xi_{ij}^D &= -\xi_{ij}^{(1)} \sin\beta \equiv -\sqrt{2}M_{ij}^D/v \tan\beta,\end{aligned}\quad (14)$$

with M_{ij}^U and M_{ij}^D being the mass matrices. Thus in the Type II 2HDM, the Yukawa couplings are almost fixed by the masses of quarks and Cabibbo-Kobayashi-Maskawa matrix elements, the angle β or the ratio of two vacuum expectation values is manifestly an important parameter as it uniquely characterizes the amplitude of Yukawa couplings for the up-type quarks ξ_{ij}^U and down-type quarks ξ_{ij}^D in the mass eigenstates. The similar situation occurs for the MSSM. When parameterizing the Yukawa couplings by using the quark mass scales,

$$\xi_{ij}^{U,D} \equiv \lambda_{ij} \sqrt{2m_i m_j}/v, \quad (15)$$

where the smallness of the off-diagonal matrix elements is characterized by the hierarchical mass scales of quarks and the parameters λ_{ij} . In terms of this parametrization, one has the following simple relations in the Type II 2HDM,

$$\begin{aligned}\xi^t &\equiv \xi_{tt}^U = \sqrt{2}m_t/v \cot\beta, & \lambda_t &\equiv \lambda_{tt}^U \sim \cot\beta, \\ \xi^b &\equiv \xi_{bb}^D = \sqrt{2}m_b/v \tan\beta, & \lambda_b &\equiv \lambda_{bb}^D \sim -\tan\beta.\end{aligned}\quad (16)$$

The situation is obviously different from the general Type III 2HDM with spontaneous CP violation, where each Higgs doublet couples to both up-type and down-type quarks, there are physically meaningful Yukawa coupling constants which are twice as the ones in the Type II 2HDM, thus the dependence on parameter β is not manifest as in the Type II 2HDM. There are more free parameters in the Yukawa interactions, they are in general determined only through various experiments like in the standard model. Whereas it provides, from the phenomenological points of view, an interesting window for exploring possible new physics effects inspired from the Type III 2HDM with spontaneous CP violation. It is seen that an

additionally imposed discrete symmetry is much stronger than the CP symmetry in the 2HDM. It would be interesting to have a detailed study for both the Type II (or Type I) 2HDM and Type III 2HDM, as the Type II or Type I model was motivated from the assumption of natural flavor conservation, while Type III model was initiated from the origin of CP violation with spontaneous symmetry breaking.

It was observed in the Type III 2HDM [6,7,15] that the charged Higgs interactions involving the Yukawa couplings $\hat{\xi}^{U(D)}$ in Eq. (11) lead to a new type of CP -violating FCNC even if the neutral current couplings $\xi^{U(D)}$ are diagonal. For the parameters concerning the third generation, we may express as

$$\begin{aligned}\xi^t &\equiv \xi_{tt}^U, & \xi^t &= |\xi^t| e^{i\delta_t}, \\ \xi^b &\equiv \xi_{tt}^D, & \xi^b &= |\xi^b| e^{i\delta_b}.\end{aligned}\quad (17)$$

The general constraints on the FCNC and the relevant parameter spaces have been investigated in [10,18–22]. Here, we may consider the following three typical parameter spaces for the neutral Yukawa couplings of b quark and t quark $\xi^q/\sqrt{2} = \lambda_q m_q/v$,

$$\begin{aligned}\text{Case A: } &|\xi^t/\sqrt{2}| = 0.2 (\lambda_t = 0.3); \\ &|\xi^b/\sqrt{2}| = 0.5 (\lambda_b = 30), \\ \text{Case B: } &|\xi^t/\sqrt{2}| = 0.1 (\lambda_t = 0.15); \\ &|\xi^b/\sqrt{2}| = 0.8 (\lambda_b = 50), \\ \text{Case C: } &|\xi^t/\sqrt{2}| = 0.01 (\lambda_t = 0.015); \\ &|\xi^b/\sqrt{2}| = 1.0 (\lambda_b = 60),\end{aligned}\quad (18)$$

which is consistent with the current experimental constraints in the flavor sector, including the B meson decays [23,24] even when the neutral Higgs masses are taken to be the typical low values

$$\begin{aligned}m_A &= 120 \text{ GeV}, & m_h &= 115 \text{ GeV}, \\ m_H &= 160 \text{ GeV}.\end{aligned}\quad (19)$$

For the charged Higgs mass, when $\frac{1}{\sqrt{2}}|\xi^t| \approx 0.2$ (or $|\lambda_t| \sim 0.3$), the lower bound for the charged Higgs mass can reach to be about $m_{H^+} \sim 160$ GeV from the $B^0 - \bar{B}^0$ mixing at the 1σ level (or about $m_{H^+} \sim 60$ GeV at 2σ level). In general, a smaller value of $|\xi^t|$ leads to a lower bound on the charged Higgs mass. The strong constraints may arise from the radiative bottom-quark decay $b \rightarrow s\gamma$. In fact, its mass was found to be severely constrained from the $b \rightarrow s\gamma$ decay in the Type II 2HDM, the lower bound on the charged Higgs mass can be as large as $m_{H^+} \approx 350$ GeV, which is corresponding to the special case in the Type III 2HDM with the parameter $|\xi^t||\xi^b| \sim 0.02$ (or $|\lambda_t \lambda_b| \sim 1$) and a relative phase $\delta_t - \delta_b = 180^\circ$. Nevertheless, the constraints can significantly be relaxed

in the Type III 2HDM due to the freedom of the parameters ξ^t and ξ^b as well as their relative phase ($\delta_t - \delta_b$). In general, when the combined parameter $|\xi^t||\xi^b|$ becomes smaller, the resulting bound to the charged Higgs mass goes to be lower. While an interesting feature arises in the Type III 2HDM, when the relative phase makes the charged Higgs amplitude to interfere destructively with the standard model amplitude, the allowed charged Higgs mass can remain small even for a large value of combined parameter $|\xi^t||\xi^b|$. For instance, when $|\xi^t||\xi^b| \lesssim 0.025$ (or $|\lambda_t \lambda_b| \lesssim 1.0$), the allowed charged Higgs mass can be in all range for a large range of the relative phase ($\delta_t - \delta_b \simeq \pi/4 \sim \pi/2$) [18], even when taking the combined parameter to be large $|\xi^t \xi^b| \simeq 0.07$ (or $|\lambda_t \lambda_b| \sim 3$), the resulting bound on the charge Higgs mass can still be as low as $m_{H^\pm} \sim 100$ GeV for a certain range of the relative phase. Some strong constraints to the charged Higgs mass may arise from the neutron electric dipole momentum, but we shall not consider such possible constraints as it involves large uncertainties caused by the hadronic matrix elements and also receives various contributions from several CP -violating sources in the Type III 2HDM [6]. Some upper limit on the charged Higgs mass may arise from the ρ parameter [18], which needs a more precise measurement.

In 2HDM, there are three neutral and one charged Higgs bosons. The charged Higgs is totally different to the particles in SM, and its effect to lower-energy phenomenology and direct search have been studied by many authors. As there are more neutral Higgs, in paper [25] the authors discussed the pair production of the neutral Higgs $gg \rightarrow hh$, which is sensitive to the triple couplings in the Higgs potential. In our present paper, as we only study the neutral Higgs production and decays, which does not involve the charged Higgs boson and the triple couplings in the potential at lowest order, thus we may consider the allowed parameter space of ξ^t and ξ^b to be as large as possible, and take the combined parameter $|\xi^t||\xi^b|$ to range from $|\xi^t||\xi^b| = 0.02$ to $|\xi^t||\xi^b| = 0.2$, which is covered from the above given three typical parameter spaces. Where the values of $|\xi^t|$ are taken to be small so as to fit the constraints from the $B^0 - \bar{B}^0$ mixing. For a large value of $|\xi^b|$, it may naturally be resulted for a large value of $\tan\beta \sim 30$, but it is not a necessary requirement in the Type III 2HDM. For any given values of $\tan\beta$, one can always find appropriate parameter space of $\xi_u^{(1)}$ and $\xi_u^{(2)}$, so as to fit the bottom-quark mass $m_b = \sqrt{2}|\eta_{bb}^D|/\nu$ and meanwhile allow a large Yukawa coupling $|\xi^b| \equiv |\xi_{bb}^D|$. For the same reason, one can find the appropriate parameter space of $\eta_u^{(1)}$ and $\eta_u^{(2)}$ to yield a small Yukawa coupling $|\xi^t| \equiv |\xi_{tt}^U|$ with simultaneously fitting the top-quark mass. Therefore, we shall take the independent parameters $|\xi^t|$ and $|\xi^b|$ as the free parameters in the Type III 2HDM instead of using the parameter $\tan\beta$, which is unique in the Type II 2HDM and MSSM.

In the following sections, we shall calculate the neutral Higgs productions and decays with the above three typical Yukawa couplings and free neutral Higgs Masses (< 1 TeV). We first consider the Higgs production in Sec. II, and then the Higgs decay in Sec. III. In Sec. IV, we shall discuss the effects of the mixing between the neutral Higgs bosons. Our conclusion is presented in the final section.

II. THE HIGGS PRODUCTIONS

According to QCD, the quarks and gluons are the fundamental degrees of freedom to participate in strong interactions at high energy, the QCD parton model plays a pivotal role in understanding hadron collisions [26,27]. Because of the gluon luminosity, the gluon fusion is the main production channel of Higgs bosons in proton-proton collisions throughout the entire Higgs mass range both in SM and 2HDM, and the first prediction for the production cross section of the SM Higgs was carried out in [28]. The gluon-gluon couple to the Higgs boson through the quark loop is shown in Fig. 1. Although the higher order corrections by QCD [29–35] and electroweak [36] to the process have been calculated and discussed, here we only take the lowest order for our present purpose as the parameters in 2HDM have not well been constrained and the uncertainty remains large. To lowest order, the parton cross section can be expressed as [30,31]

$$\begin{aligned} \hat{\sigma}(gg \rightarrow H) &= \frac{\alpha_s^2 G_f}{128\sqrt{2}\pi} \left| \sum_f \frac{y_f v}{m_f} A_f(\tau_f) \right|^2 \\ &= \frac{\alpha_s^2}{256\pi} \left| \sum_f \frac{y_f}{m_f} A_f(\tau_f) \right|^2, \end{aligned} \quad (20)$$

where y_f is the Yukawa coupling of f quark. The scaling variable is defined as

$$\tau_f = \frac{m_H^2}{4m_f^2}, \quad (21)$$

and the loop amplitude A_f has the form

$$A_f(\tau) = [\tau + (\tau - 1)F(\tau)]/\tau^2, \quad (22)$$

with

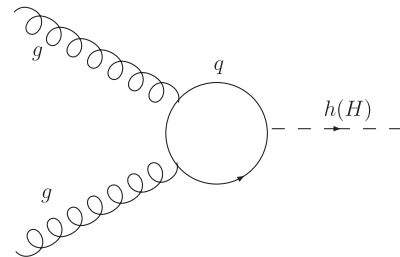


FIG. 1. The Feynman diagram of the Higgs production with gluon-gluon fusion.

$$F(\tau) = \begin{cases} \arcsin^2 \sqrt{\tau}, & \tau \leq 1, \\ -\frac{1}{4} [\log \frac{\sqrt{\tau} + \sqrt{\tau-1}}{\sqrt{\tau} - \sqrt{\tau-1}} - i\pi]^2, & \tau > 1. \end{cases} \quad (23)$$

Namely, if the Higgs mass is smaller than the threshold of the f -quark pair production, the amplitude is real, while above the threshold, the amplitude becomes complex according to the Cutkosky rule. As the Yukawa coupling of top quark is much larger than the Yukawa coupling of bottom quark in SM, the Higgs production is mainly through the triangular loop of top quark, and the contributions from other quarks can be omitted. While for the new neutral Higgs boson in 2HDM, the situation can be different due to the possible large Yukawa couplings of the b quarks such as shown in Eq. (18).

The running strong coupling $\alpha_s(\mu)$ is known to be as follows [37]:

$$\alpha_s(\mu) = \frac{\alpha_s(m_t)}{v(\mu)} \left(1 - \frac{\beta_1}{\beta_0} \frac{\alpha_s(m_t)}{4\pi v(\mu)} \ln v(\mu) \right), \quad (24)$$

where

$$\beta_0 = \frac{11N - 2f}{3}, \quad (25)$$

$$\beta_1 = \frac{34}{3}N^2 - \frac{10}{3}Nf - 2C_F f, \quad (26)$$

$$v(\mu) = 1 - \beta_0 \frac{\alpha_s(m_t)}{2\pi} \ln \frac{m_t}{\mu}, \quad (27)$$

with $\alpha_s(m_t = 174 \text{ GeV}) = 0.108$. This formula is valid at $\mu > m_b$ with $f = 5$ when $m_b < \mu < m_t$ and $f = 6$ when $\mu > m_t$.

The corresponding hadronic cross section σ can be obtained by convolution with the gluon-gluon luminosity $\mathcal{L}(\omega)$:

$$\begin{aligned} \sigma(PP \rightarrow H) &= \int dx_1 dx_2 g_1(x_1) g_2(x_2) \hat{\sigma}_{gg}(\hat{s} = x_1 \cdot x_2 S) \\ &= \int d\omega \hat{\sigma}_{gg}(\hat{s} = \omega \cdot S) \int \frac{dx_2}{x_2} g_1\left(\frac{\omega}{x_2}\right) g_2(x_2) \\ &\equiv \int d\omega \hat{\sigma}_{gg}(\hat{s} = \omega \cdot S) \mathcal{L}(\omega), \end{aligned} \quad (28)$$

where $S = 2P_1 \cdot P_2$ is the center-of-mass energy of the proton-proton collisions, and g_i is the parton distribution function [38] of the gluon from the i -th incoming proton.

We have shown in Fig. 2 the production cross section on LHC for the neutral Higgs bosons with the three typical Yukawa couplings mentioned in the previous section [Eq. (18)]. Where h denotes the SM-like Higgs in the 2HDM and H the new Higgs boson in the 2HDM. From the figure one can see that when the Higgs masses are light ($< 200 \text{ GeV}$), the production cross section for H is larger than the one for h . The reason is that at the lower mass the

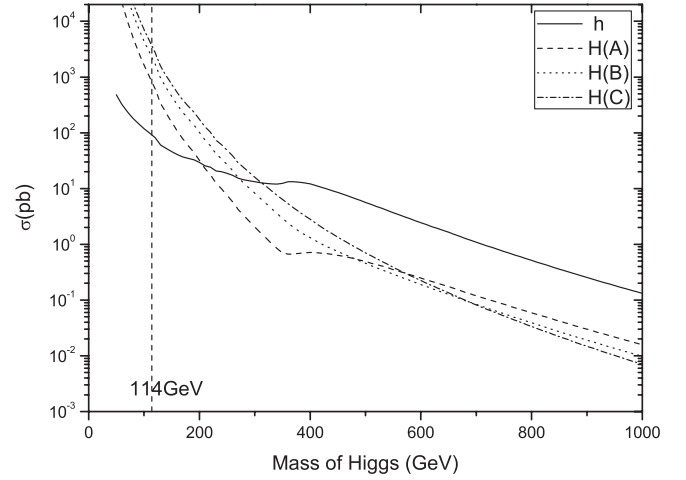


FIG. 2. The production cross section of the neutral Higgs. The solid line is the result of the SM-like Higgs h , and the A, B, C three lines are the results of H with different Yukawa couplings.

light quark contributions to the Higgs H production can not be omitted for the possible large Yukawa couplings. When the Higgs mass goes to be heavy, the loop contributions from top quark become dominant, as the top-quark Yukawa coupling of H is smaller than the one of h , thus the production cross section of H is smaller than the one of h when they are heavy.

Note that as the SM Higgs is similar to h in the 2HDM when neglecting the possible mixing among neutral Higgs bosons; therefore, it is hard to distinguish with the SM Higgs, unless the charged Higgs is very light, so that h in the 2HDM can decay to $H^+ H^-$, while the vertex comes from the Higgs potential which is strongly model dependent. In this paper, we will not consider the possible charged Higgs effect to the neutral Higgs decay modes. We are going to pay attention to the neutral Higgs mixing effect, which also make the h in the 2HDM differ to the SM Higgs. It is seen that when the masses of h and H are both at $200 \text{ GeV} \sim 300 \text{ GeV}$, the cross section of h and H are similar, but it will be shown below that they have very different decay modes.

In general, the Yukawa couplings of the new neutral Higgs bosons can be complex, namely, one can write $\xi^q = |\xi^q| e^{i\delta_q}$, so that the Yukawa interactions of the neutral Higgs bosons in Eq. (11) can be written into two parts:

$$\begin{aligned} \mathcal{L}_Y &= \bar{U} \frac{\xi^U}{\sqrt{2}} \frac{1 - \gamma^5}{2} U \phi_2^0 + \bar{U} \frac{\xi^{U\dagger}}{\sqrt{2}} \frac{1 + \gamma^5}{2} U \phi_2^0 \\ &\quad + (U \rightarrow D) + \dots \\ &= \frac{|\xi^q|}{\sqrt{2}} \cos \delta_q \bar{q} q \phi_2 - i \frac{|\xi^q|}{\sqrt{2}} \sin \delta_q \bar{q} \gamma^5 q \phi_2 + \dots, \end{aligned} \quad (29)$$

where the second part leads to additional contributions through a kind of anomaly loop diagrams, the resulting amplitude has the following form [39,40]:

$$\mathcal{M}^A = \sum_f \frac{ig_s^2 |\xi^f| \sin \delta_f \epsilon_{\mu\nu\rho\sigma} \epsilon_1^{a\mu} \epsilon_2^{b\nu} k_1^\rho k_2^\sigma \delta^{ab}}{4\sqrt{2}\pi^2 M_H} B_f(\tau_f), \quad (30)$$

where the a and b are the color index and the function

$B_f(\tau_f)$ is given by

$$B_f(\tau_f) = \frac{F(\tau_f)}{\sqrt{\tau_f}}, \quad (31)$$

where $F(\tau)$ is defined as Eq. (23).

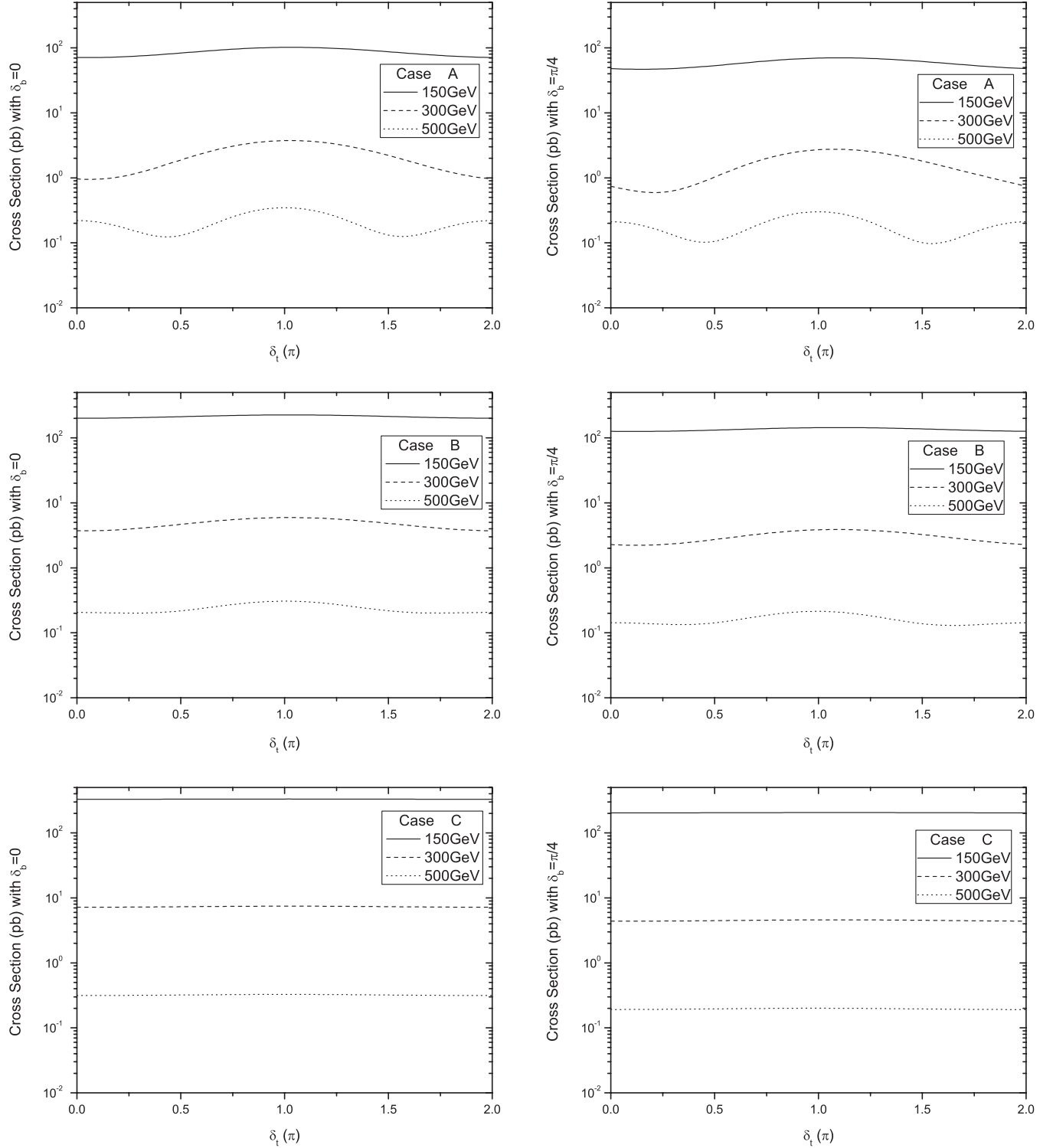


FIG. 3. The production cross section depends on the phase of the top-quark Yukawa coupling phase δ_t with the three typical absolute values given in Eq. (18) and different Higgs masses. The results for $\delta_b = 0$ are listed on the left side, and $\delta_b = \pi/4$ on the right side.

The cross section is calculated as the function of δ_t with $\delta_b = 0$ and $\delta_b = \pi/4$ plotted in Fig. 3, and as the function of δ_b with $\delta_t = 0$ and $\delta_t = \pi/4$ plotted in Fig. 4, where

we have also considered three typical cases given in Eq. (18) with the absolute values of the Yukawa couplings. It can be seen from the figures that: For the case A, the

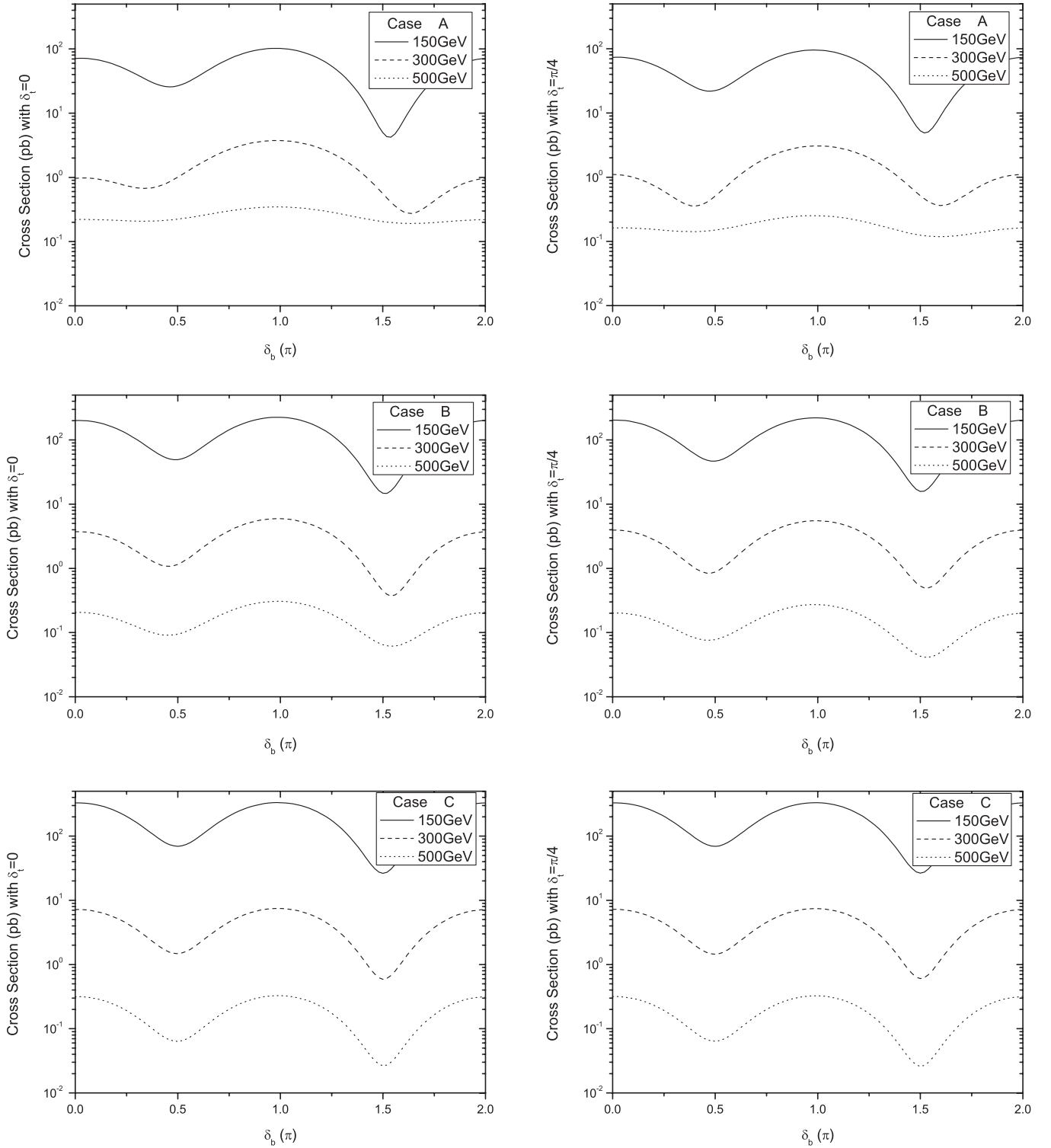


FIG. 4. The production cross section depends on the phase of the bottom-quark Yukawa coupling phase δ_b with the three typical absolute values given in Eq. (18) and different Higgs masses. The results for $\delta_t = 0$ are listed on the left side, and $\delta_t = \pi/4$ on the right side.

effect of the phase in top-quark Yukawa coupling becomes significant when the mass of the Higgs H increases, which is very different from the case with real Yukawa couplings and implies that the top-quark loop contribution to the cross section in gluon-gluon fusion is dominant for a heavy Higgs H , while for a light Higgs H , the bottom-quark contribution becomes important. However, the situation becomes different for the Case B and Case C when the top-quark Yukawa coupling goes to be small, especially for the Case C where the bottom-quark loop contribution becomes dominant, and the phase of the top-quark Yukawa coupling has less effect to the cross section.

III. THE HIGGS DECAYS

As the neutral Higgs bosons cannot be directly detected, they are probed only through the final states of their decays. For the SM-like Higgs h in the general 2HDM, without considering the mixings among the neutral Higgs, it has the same Yukawa coupling as the Higgs in SM at tree level (except the couplings with other Higgs). As shown in Fig. 5, which likes the SM Higgs decay [41], when its mass is lower than the WW threshold, the $b\bar{b}$ pair production is a dominant decay mode. However, the process $h \rightarrow \gamma\gamma$ is also sizable as shown in Fig. 5, which is known to be a golden channel for detecting the light neutral Higgs due to the clean background.

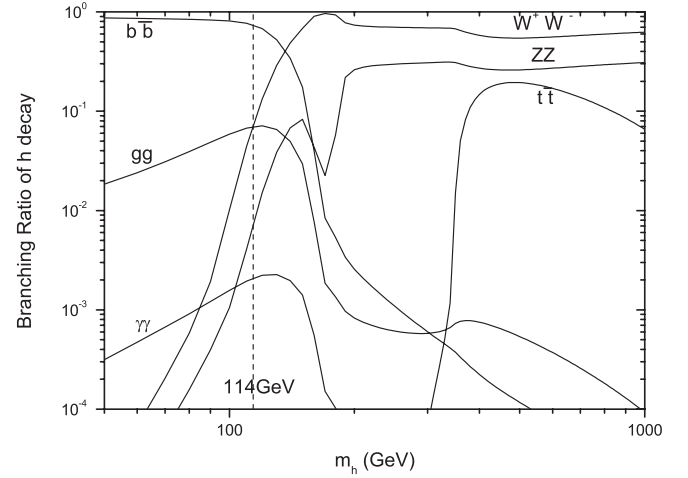


FIG. 5. The decay of the Higgs h in 2HDM without mixing, which likes the SM Higgs.

It can also be seen from Fig. 5 that if $M_h > 160$ GeV there are two dominant processes concerning the W and Z bosons, where the W and Z bosons can decay to quarks and leptons. They are the golden channels for searching the heavy neutral Higgs h . The situation is very different for the new neutral Higgs H . After the transformation of the Higgs in Eq. (6), the gauge part of the Higgs in the new basis can be written as [6]

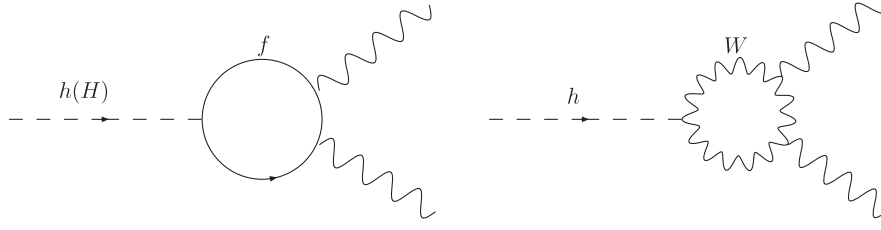
$$\begin{aligned}
 \mathcal{L}_G &= (D_\mu H_1)^\dagger (D^\mu H_1) + (D_\mu H_2)^\dagger (D^\mu H_2) \\
 &= \frac{1}{2} \partial_\mu \phi_1^0 \partial^\mu \phi_1^0 + \frac{(v + \phi_1^0)^2}{8} [(g'^2 + g^2) Z^2 + 2g^2 W^+ W^-] + \frac{1}{2} (\partial \phi_2^0 \partial^\mu \phi_2^0 + \partial \phi_3^0 \partial^\mu \phi_3^0) + \partial_\mu H^- \partial^\mu H^+ \\
 &\quad + e^2 H^+ H^- A^2 + \frac{(g^2 - g'^2)^2}{4(g^2 + g'^2)} H^+ H^- Z^2 + \frac{g^2}{2} H^+ H^- W^+ W^- + \frac{e(g^2 - g'^2)}{\sqrt{g^2 + g'^2}} H^+ H^- Z \cdot A + \frac{g^2}{4} W^+ W^- (\phi_2^{02} + \phi_3^{02}) \\
 &\quad + \frac{g^2 + g'^2}{8} (\phi_2^{02} + \phi_3^{02}) Z^2 + \left\{ i e A^\mu H^- \partial_\mu H^+ + i \frac{g^2 - g'^2}{2\sqrt{g^2 + g'^2}} Z^\mu H^- \partial_\mu H^+ + \frac{i g}{2} W_\mu^- (\phi_2^0 - i \phi_3^0) \partial^\mu H^+ \right. \\
 &\quad + \frac{e g}{2} A^\mu W_\mu^- H^+ (\phi_2^0 - i \phi_3^0) + \frac{g}{4} \frac{g^2 - g'^2}{\sqrt{g^2 + g'^2}} H^+ W_\mu^- Z^\mu (\phi_2^0 - i \phi_3^0) + \frac{i g}{2} H^- W_\mu^+ (\partial^\mu \phi_2^0 + i \partial^\mu \phi_3^0) \\
 &\quad \left. + \frac{i \sqrt{g^2 + g'^2}}{4} (\phi_2^0 - i \phi_3^0) Z^\mu (\partial_\mu \phi_2^0 + i \partial_\mu \phi_3^0) - \frac{g \sqrt{g^2 + g'^2}}{4} H^- W_\mu^+ Z^\mu (\phi_2^0 + i \phi_3^0) + \text{H.c.} \right\}. \quad (32)
 \end{aligned}$$

It is seen that without considering the mixing among the neutral Higgs bosons, namely, the neutral Higgs gauge interaction eigenstates $(\phi_1^0, \phi_2^0, \phi_3^0)$ are the same as the mass eigenstates (h, H, A) , there are no direct WWH and ZZH interactions, this is because the vacuum expectation value of the Higgs doublet H_2 in a rotating basis vanishes, $\langle H_2 \rangle = 0$. In this special case, the Higgs H cannot decay to WW and ZZ at the tree level, thus the $f\bar{f}$ channels are always the dominant decay modes of H for

its whole mass range. As there is no symmetry to forbid the mixing among the neutral Higgs bosons, in general we shall consider the mixing among the Higgs bosons which will be discussed later on.

A. The $\gamma\gamma$, WW and ZZ modes

The SM-like neutral Higgs h with mass M decays to $\gamma\gamma$ through the fermion loop and W loop as shown in Fig. 6, the decay width is given by

FIG. 6. The Feynman diagrams for the decay of the Higgs to $\gamma\gamma$. For $H \rightarrow \gamma\gamma$ only the fermion loop contributes.

$$\Gamma(\gamma\gamma) = \frac{G_F \alpha^2 M^3}{128\sqrt{2}\pi^3} \left| \sum_f N_c Q_f^2 \frac{y_f v}{m_f} A_f(\tau_f) + A_W(\tau_W) \right|^2, \quad (33)$$

where the color factor N_c is 3 for quarks and 1 for leptons. The amplitude A_W is contributed from the W loop [31]:

$$A_W(\tau) = -[2\tau^2 + 3\tau + 3(2\tau - 1)F(\tau)]/\tau^2. \quad (34)$$

As discussed in [31], the W -loop contribution is dominated when the Higgs mass is below 600 GeV. For the new neutral Higgs decay $H \rightarrow \gamma\gamma$, its decay width is smaller than the one of h when the Higgs mass is below 600 GeV, this is because there is no W -loop contribution to $H \rightarrow \gamma\gamma$. While the total decay width of H can be larger than the one of h as the $H \rightarrow b\bar{b}$ can be dominated and much larger than $h \rightarrow b\bar{b}$ due to a possible larger Yukawa coupling. Thus, the branching ratio of $H \rightarrow \gamma\gamma$ is very small in comparison with $\text{Br}(h \rightarrow \gamma\gamma)$ as shown in Fig. 7.

If the Higgs boson is very heavy, the WW , ZZ , $t\bar{t}$ channels open and become dominant. The fractional widths of the SM-like Higgs h decaying to WW and ZZ at tree level are given by [37]

$$\Gamma(WW) = \frac{G_F M^3 \beta_W}{32\pi\sqrt{2}} (4 - 4a_W + 3a_W^2), \quad (35)$$

$$\Gamma(ZZ) = \frac{G_F M^3 \beta_Z}{64\pi\sqrt{2}} (4 - 4a_Z + 3a_Z^2), \quad (36)$$

where $a_W = 1 - \beta_W^2 = 4m_W^2/M^2$ and $a_Z = 1 - \beta_Z^2 = 4m_Z^2/M^2$ with M is mass of the SM-like Higgs.

Note that the new neutral Higgs H cannot decay to WW and ZZ at tree level without considering the mixing with the Higgs h , which is going to be discussed in Sec. IV.

B. $H/h \rightarrow \bar{f}f$ decay

The fermion decay modes are dominated for both h and H when their masses are below the threshold of WW pair production. At tree level the fractional width of the Higgs (both h and H) decays to fermion-antifermion pair is given by

$$\Gamma(\bar{f}f) = N_c \frac{y_f^2}{4\pi M^2} (M^2 - 4m_f^2)^{3/2}, \quad (37)$$

which shows that when the Higgs mass M is much larger than the fermion mass m_f , the decay width is proportional to the Higgs mass. For the new neutral Higgs H , all other channels can in general be omitted in comparing with the $\bar{f}f$ channels when neglecting the large mixing effects between H and h . As the H cannot decay to WW and ZZ at tree level, the branching ratio of $\bar{f}f$ is approximately given by the ratio $N_c \xi_f^2 / \sum N_c \xi_f^2$, which is independent on the Higgs mass as shown in Fig. 8. For the neutral Higgs H , the Yukawa coupling of the bottom quark is taken to be larger than the one of top quark in our present consideration, thus the $H \rightarrow b\bar{b}$ decay width is always larger than the $H \rightarrow t\bar{t}$ decay width as shown in Fig. 8. However, $H \rightarrow b\bar{b}$ is overwhelmed by the combinatorial background from QCD b -jets production with $\sigma(gg \rightarrow b\bar{b}) \approx 500\mu b$. As a matter of fact, the $H \rightarrow b\bar{b}$ channel is now considered inaccessible at the LHC [42] and it seems to be left open to study only at a next linear collider [43,44]. Nevertheless, at LHC there are some other processes to be hoped, such as the associated production modes $W^\pm H(b\bar{b})$ and $ZH(b\bar{b})$ [45], and $\gamma\gamma \rightarrow H \rightarrow b\bar{b}$ [46] in proton-lead (p Pb) interactions. For the SM-like Higgs h , the situation is different as the Yukawa couplings are proportional to the fermion

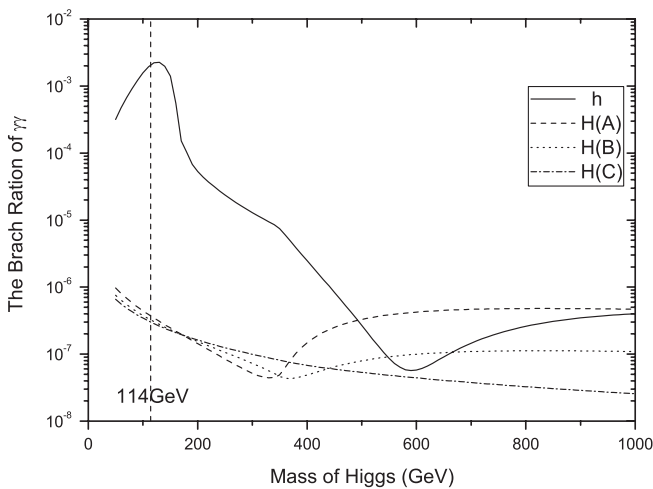


FIG. 7. The Higgs decay to $\gamma\gamma$ as the function of Higgs mass. The solid line is for the SM-like Higgs h , other three lines for the new Higgs H with three different Yukawa couplings given in Eq. (18).

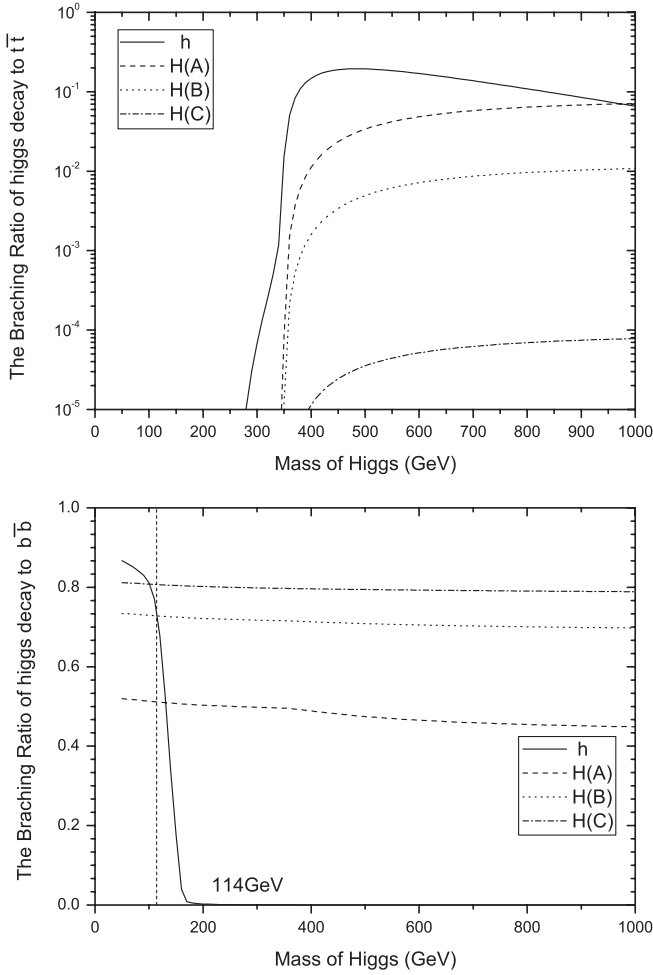


FIG. 8. The Higgs decay to $t\bar{t}$ and $b\bar{b}$ as the function of the Higgs mass. The solid line is for the SM-like Higgs h , other three lines for the new Higgs H with three different Yukawa couplings given in Eq. (18).

masses, $\xi_f \sim m_f$, its decay width to heavy quarks is larger than the one to light quarks. And when the SM-like Higgs is lighter than the threshold of WW , the $h \rightarrow b\bar{b}$ is the main decay mode but also overwhelmed by the QCD background, so that the $h \rightarrow \gamma\gamma$ studied as the golden channel to detect a light Higgs.

IV. THE MIXING EFFECTS OF THE NEUTRAL HIGGS

In general, there is no symmetry to forbid the mixing between the neutral Higgs bosons. Let us now consider the mixing between the scalar neutral Higgs bosons H and h , but without considering their mixing with the pseudoscalar A for simplicity. The Higgs bosons h and H in the mass eigenstate are the linear combinations of the Higgs bosons in the electroweak eigenstate denoted in Eq. (8):

$$h = \cos\theta\phi_1 + \sin\theta\phi_2, \quad (38)$$

$$H = -\sin\theta\phi_1 + \cos\theta\phi_2. \quad (39)$$

In the mass eigenstate, the Yukawa terms in Eq. (12) becomes

$$\begin{aligned} \mathcal{L}_Y &= \bar{f} \frac{m_f}{v} f \phi_1 + \frac{\xi^f}{\sqrt{2}} \bar{f} f \phi_2 + \dots \\ &= m_f \left(\cos\theta \frac{1}{v} + \sin\theta \frac{\xi^f}{\sqrt{2}m_f} \right) \bar{f} f h \\ &\quad + m_f \left(-\sin\theta \frac{1}{v} + \cos\theta \frac{\xi^f}{\sqrt{2}m_f} \right) \bar{f} f H + \dots, \end{aligned} \quad (40)$$

with $\theta \in (0, \pi)$. Note that if one renames h as H and H as $-h$, it is the same as the replacement: $\theta + \frac{\pi}{2}$ to θ with $\theta \in (0, \frac{\pi}{2})$. Thus, in the later formulas and calculations, we only need to consider $\theta \in (0, \frac{\pi}{2})$.

The Higgs production cross section is given by

$$\begin{aligned} \hat{\sigma}(gg \rightarrow \phi) &= \frac{\alpha_s^2}{256\pi} \left| \sum_f \left[\left(\cos\theta_\phi \frac{1}{v} + \sin\theta_\phi \frac{\xi^f}{\sqrt{2}m_f} \right) \right. \right. \\ &\quad \left. \left. \times A_f(\tau_f) \right] \right|^2, \end{aligned} \quad (41)$$

where ϕ denotes the Higgs boson H or h . The decay widths of Higgs boson to $\gamma\gamma$ and ZZ at tree level are given by

$$\begin{aligned} \Gamma(\phi \rightarrow \gamma\gamma) &= \frac{G_F \alpha^2 M^3}{128\sqrt{2}\pi^3} \left| \sum_f N_c Q_f^2 \right. \\ &\quad \times \left(\cos\theta_\phi + \sin\theta_\phi \frac{\xi^f v}{\sqrt{2}m_f} \right) A_f(\tau_f) \\ &\quad \left. + \cos\theta_\phi A_W(\tau_W) \right|^2, \end{aligned} \quad (42)$$

$$\Gamma(\phi \rightarrow ZZ) = \frac{G_F M^3 \beta_W}{64\pi\sqrt{2}} (4 - 4a_W + 3a_W^2) \cos^2(\theta_\phi), \quad (43)$$

where $\theta_h = \theta$ for $\phi = h$ and $\theta_H = \theta + \frac{\pi}{2}$ for $\phi = H$.

Our results are shown in Figs. 9–12. From Figs. 9 and 11, it can be seen that the mixing effects could become significant for the SM-like Higgs h , its cross section can be much suppressed when the mixing angle becomes large.

V. CONCLUSION

We have investigated, in the general 2HDM with spontaneous CP violation, the production and decays of the SM-like Higgs h and the new neutral Higgs H . Numerically, we have considered three typical sets of Yukawa couplings for the Higgs boson H , which is consistent with the current experimental bounds from the flavor sector even when the Higgs boson mass is as low as $M \simeq 160$ GeV. It has been seen that when h and H are both light, i.e., $M < 200$ GeV, the production cross section

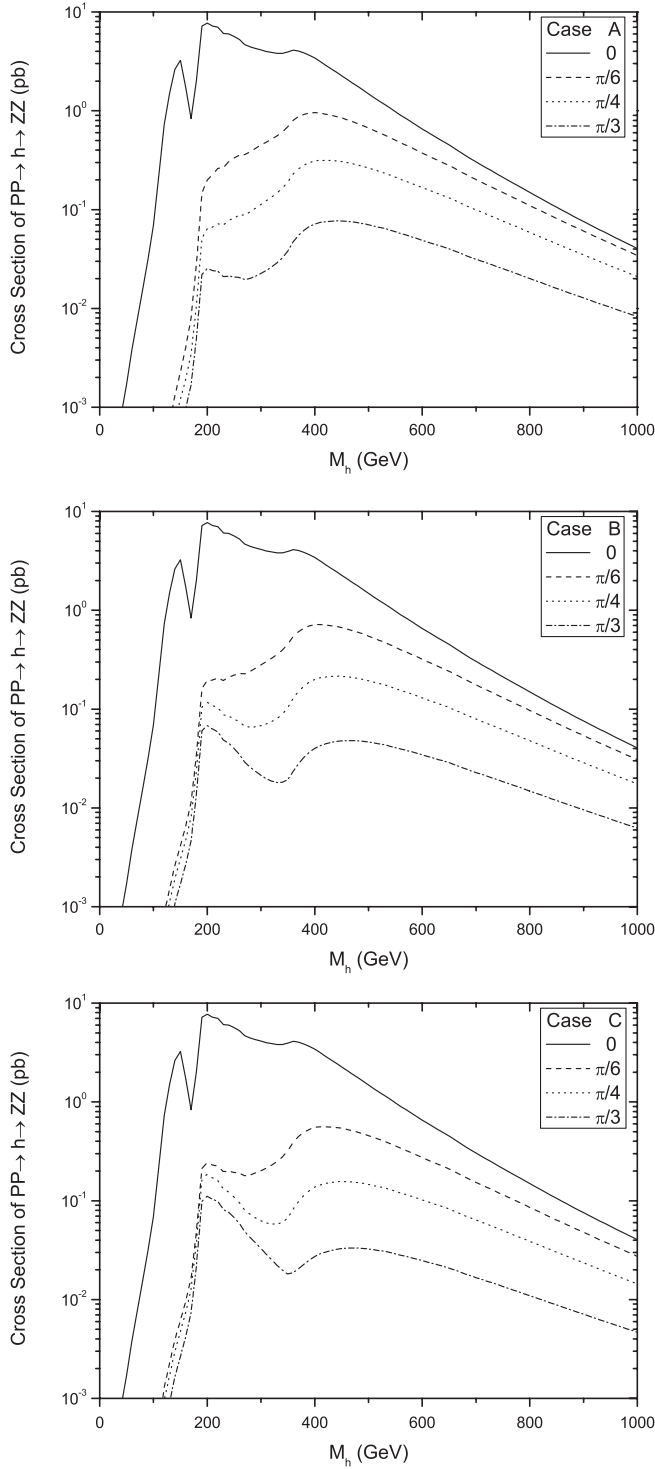


FIG. 9. The Higgs mixing effect to the cross section of $PP \rightarrow h \rightarrow ZZ$, which is regarded as the golden channel to search for a heavy SM-like Higgs h . The solid line is for the case without mixing which is similar to SM, while other lines are for the cases with mixing angles $\theta = \pi/6, \pi/4, \pi/3$.

of H is in general larger than the one of h , while for $M > 200$ GeV, the production cross section of h becomes larger than the one of H . As the Yukawa couplings of H can be

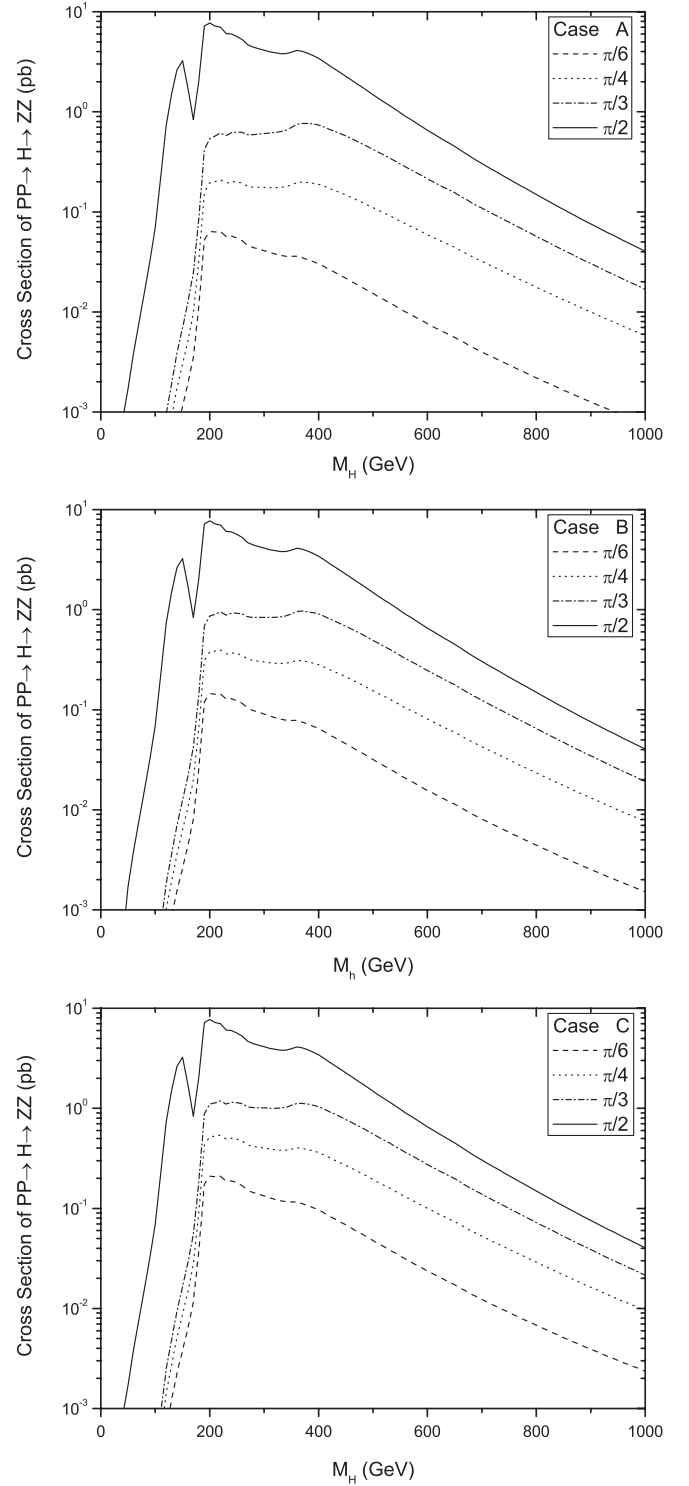


FIG. 10. The Higgs mixing effect to the cross section of $PP \rightarrow H \rightarrow ZZ$. For $\theta = 0$, the H cannot decay to ZZ at tree level, but for $\theta = \pi/2$, the H plays the role of the Higgs in SM as shown from the solid lines.

complex, its production cross section can strongly rely on the CP -violating phase and be affected significantly. The $b\bar{b}$ decay mode is the dominant channel for both h and H ,

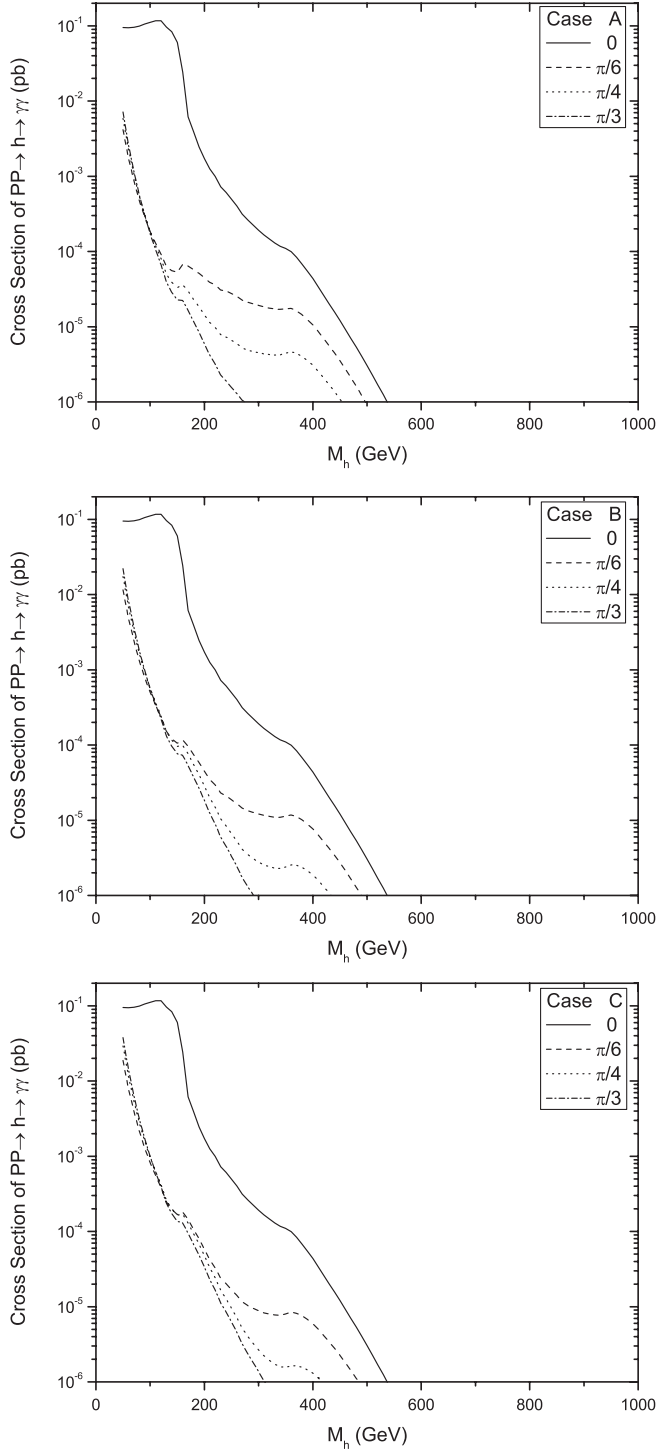


FIG. 11. The Higgs mixing effect to the cross section of $PP \rightarrow h \rightarrow \gamma\gamma$, which is regarded as the golden channel to search for a heavy SM-like Higgs h . The solid line is for the case without mixing which is similar to SM, while other lines are for the cases with mixing angles $\theta = \pi/6, \pi/4, \pi/3$.

while the $H \rightarrow \gamma\gamma$ and $H \rightarrow gg$ are both smaller than $h \rightarrow \gamma\gamma$ and $h \rightarrow gg$, thus it is not difficult to distinguish them. The SM-like Higgs h can be detected via the golden channel $h \rightarrow ZZ \rightarrow 4l$, while the new neutral Higgs H

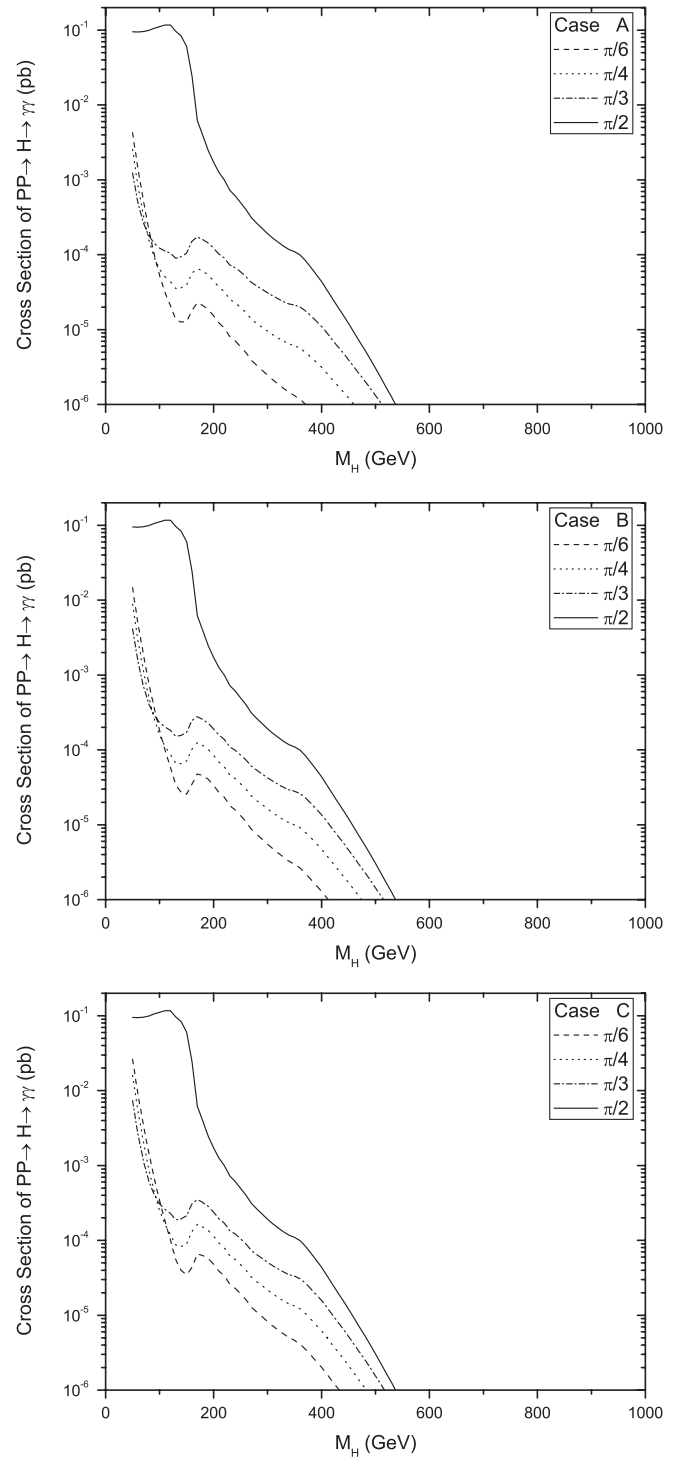


FIG. 12. The Higgs mixing effect to the cross section of $PP \rightarrow H \rightarrow \gamma\gamma$. For $\theta = \pi/2$, the H plays the role of the Higgs in SM as shown from the solid lines.

has no such a channel at tree level if without considering the neutral Higgs mixing, it is mainly detected via $H \rightarrow \bar{b}b$ as it can be very different from $h \rightarrow \bar{b}b$ due to different Yukawa coupling. When the mixing between h and H becomes very large and their mass difference is very small,

it is then not very easy to distinguish them from the production signals. It is noted that LHC does not favor 2HDM with all parameter spaces, especially in the decoupling limit with a small Yukawa coupling $\xi^f \ll m_f/v$ and a small mixing $\theta \simeq 0$ between h and H , in this case h looks the same as the SM Higgs and it then becomes hard to detect the new Higgs bosons, thus one is not able to distinguish the Type III 2HDM and SM from a direct detection. In general, the mixing between the neutral Higgs bosons h and H is characterized by a free parameter θ which can be large, so that the production cross section and decays of the neutral Higgs boson can significantly be

different from the predictions from the standard model. It would be very interesting to search for the possible new Higgs boson effects at LHC or at ILC.

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- [1] B. W. Lee, C. Quigg, and H. B. Thacker, *Phys. Rev. Lett.* **38**, 883 (1977); *Phys. Rev. D* **16**, 1519 (1977).
 - [2] D. Abbaneo *et al.* (LEP Electroweak Working Group) and (SLD Electroweak and Heavy Flavor Groups), arXiv:hep-ex/0312023; R. Barate *et al.*, *Phys. Lett. B* **565**, 61 (2003).
 - [3] T. Hambye and K. Riesselmann, *Phys. Rev. D* **55**, 7255 (1997); M. Lindner, *Z. Phys. C* **31**, 295 (1986).
 - [4] T. D. Lee, *Phys. Rev. D* **8**, 1226 (1973); *Phys. Rep.* **9**, 143 (1974).
 - [5] S. Weinberg, *Phys. Rev. Lett.* **37**, 657 (1976).
 - [6] Y. L. Wu, Carnegie-Mellon Univ. Report No. CMU-HEP94-01, 1994; in *Proceedings of the 5th Conference on the Intersections of Particle and Nuclear Physics, St. Petersburg, FL, 1994*, edited by S. J. Seestrom (AIP, New York, 1995), p. 338.
 - [7] Y. L. Wu and L. Wolfenstein, *Phys. Rev. Lett.* **73**, 1762 (1994).
 - [8] G. C. Branco, L. Lavoura, and J. P. Silva, *CP Violation* (Oxford University Press, Oxford, 1999).
 - [9] M. Kobayashi and T. Maskawa, *Prog. Theor. Phys.* **49**, 652 (1973).
 - [10] D. Atwood, L. Reina, and A. Soni, *Phys. Rev. D* **55**, 3156 (1997); D. Atwood, S. Bar-Shalom, G. Eilam, and A. Soni, *Phys. Rep.* **347**, 1 (2001), and reference therein.
 - [11] S. Glashow and S. Weinberg, *Phys. Rev. D* **15**, 1958 (1977).
 - [12] V. D. Barger, J. L. Hewett, and R. J. N. Phillips, *Phys. Rev. D* **41**, 3421 (1990).
 - [13] V. D. Barger, H. E. Logan, and G. Shaughnessy, *Phys. Rev. D* **79**, 115018 (2009).
 - [14] G. C. Branco and M. N. Rebelo, *Phys. Lett.* **160B**, 117 (1985).
 - [15] L. Wolfenstein and Y. L. Wu, *Phys. Rev. Lett.* **73**, 2809 (1994).
 - [16] A. Antaramian, L. J. Hall, and A. Rasin, *Phys. Rev. Lett.* **69**, 1871 (1992); M. J. Savage, *Phys. Lett. B* **266**, 135 (1991); W. S. Hou, *Phys. Lett. B* **296**, 179 (1992); D. W. Chang, W. S. Hou, and W. Y. Keung, *Phys. Rev. D* **48**, 217 (1993).
 - [17] L. J. Hall and S. Weinberg, *Phys. Rev. D* **48**, R979 (1993).
 - [18] D. Bowser-Chao, K. Cheung, and W.-Y. Keung, *Phys. Rev. D* **59**, 115006 (1999).
 - [19] Y. L. Wu and Y. F. Zhou, *Phys. Rev. D* **61**, 096001 (2000).
 - [20] C. S. Huang and S. H. Zhu, *Phys. Rev. D* **68**, 114020 (2003); Y. B. Dai, C. S. Huang, J. T. Li, and W. J. Li, *Phys. Rev. D* **67**, 096007 (2003).
 - [21] Y. L. Wu, *Chin. Phys. Lett.* **16**, 339 (1999).
 - [22] C. S. Huang and J. T. Li, *Int. J. Mod. Phys. A* **20**, 161 (2005).
 - [23] Y. L. Wu and C. Zhuang, *Phys. Rev. D* **75**, 115006 (2007).
 - [24] S. S. Bao, F. Su, Y. L. Wu, and C. Zhuang, *Phys. Rev. D* **77**, 095004 (2008).
 - [25] A. Arhrib, R. Benbrik, C. H. Chen, R. Guedes, and R. Santos, *J. High Energy Phys.* **08** (2009) 035.
 - [26] T. Han, in *Physics in D >= 4, Proceedings of the Theoretical Advance Study Institute in Elementary Particle Physics, Boulder, Colorado, 6 Jun 2 Jul 2004*, edited by J. Terning, C. E. M. Wagner, and D. Zeppenfeld (World Scientific, Singapore, 2006), p. 407.
 - [27] V. D. Barger and R. J. N. Phillips, *Collider Physics* (Addison-Wesley, Redwood City, CA, 1988).
 - [28] H. M. Georgi, S. L. Glashow, M. E. Machacek, and D. V. Nanopoulos, *Phys. Rev. Lett.* **40**, 692 (1978).
 - [29] D. Graudenz, M. Spira, and P. M. Zerwas, *Phys. Lett.* **70**, 1372 (1993).
 - [30] S. Marzani, R. D. Ball, V. Del Duca, S. Forte, and A. Vicini, *Nucl. Phys. B* **800**, 127 (2008).
 - [31] M. Spira, A. Djouadi, D. Graudenz, and P. M. Zerwas, *Nucl. Phys. B* **453**, 17 (1995).
 - [32] R. V. Harlander and W. B. Kilgore, *Phys. Rev. Lett.* **88**, 201801 (2002).
 - [33] C. Anastasiou and K. Melnikov, *Nucl. Phys. B* **646**, 220 (2002); V. Ravindran, J. Smith, and W. L. van Neerven, *Nucl. Phys. B* **665**, 325 (2003).
 - [34] S. Moch and A. Vogt, *Phys. Lett. B* **631**, 48 (2005); E. Laenen and L. Magnea, *Phys. Lett. B* **631**, 48 (2005).
 - [35] A. Idilbi *et al.*, *Phys. Rev. D* **73**, 077501 (2006); V. Ravindran, *Nucl. Phys. B* **752**, 173 (2006).
 - [36] R. Bonciani, *J. Phys. Conf. Ser.* **110**, 042004 (2008); A. Djouadi and P. Gambino, *Phys. Rev. Lett.* **73**, 2528 (1994); U. Aglietti, R. Bonciani, G. Degrossi, and A. Vicini, *Phys. Lett. B* **595**, 432 (2004).
 - [37] C. Amsler *et al.* (Particle Data Group), *Phys. Lett. B* **667**, 1 (2008).

- [38] H. L. Lai *et al.* (CTEQ Collaboration), *Eur. Phys. J. C* **12**, 375 (2000).
- [39] S. L. Adler and W. A. Bardeen, *Phys. Rev.* **182**, 1517 (1969); W. Bardeen, *Phys. Rev.* **184**, 1848 (1969).
- [40] Y. L. Ma and Y. L. Wu, *Int. J. Mod. Phys. A* **21**, 6383 (2006).
- [41] A. Djouadi, J. Kalinowski, and M. Spira, *Comput. Phys. Commun.* **108**, 56 (1998).
- [42] G. Aad *et al.* (ATLAS Collaboration), [arXiv:0901.0512](#); G. L. Bayatian *et al.* (CMS Collaboration), *J. Phys. G* **34**, 995 (2007).
- [43] J. A. Aguilar-Saavedra *et al.* (ECFA/DESY LC Physics Working Group), [arXiv:hep-ph/0106315](#); K. Abe *et al.* (ACFA Linear Collider Working Group), [arXiv:hep-ph/0109166](#); E. Accomando *et al.* (CLIC Physics Working Group), [arXiv:hep-ph/0412251](#); G. Weiglein *et al.* (LHC/LC Study Group), *Phys. Rep.* **426**, 47 (2006).
- [44] D. Lopez-Val and J. Sola, *Phys. Rev. D* **81**, 033003 (2010); R. N. Hodgkinson, D. Lopez-Val, and J. Sola, *Phys. Lett. B* **673**, 47 (2009); E. Asakawa, D. Harada, S. Kanemura, Y. Okada, and K. Tsumura, [arXiv:0902.2458](#); *Phys. Lett. B* **672**, 354 (2009); A. Arhrib, R. Benbrik, C. H. Chen, and R. Santos, *Phys. Rev. D* **80**, 015010 (2009).
- [45] A. Del Fabbro and D. Treleani, *Phys. Rev. D* **61**, 077502 (2000); A. De Roeck and G. Polesello, *C.R. Physique* **8**, 1078 (2007); J. M. Butterworth, A. R. Davison, M. Rubin, and G. P. Salam, *Phys. Rev. Lett.* **100**, 242001 (2008).
- [46] D. d'Enterria and J. P. Lansberg, *Phys. Rev. D* **81**, 014004 (2010).