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Branching ratios of $B_c \to AP$ decays in the perturbative QCD approach

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In this paper we calculate the branching ratios (BRs) of the 32 charmless hadronic $B_c \to AP$ decays ($A = a_1(1260), b_1(1235), K_1(1270), K_1(1400), f_1(1285), f_1(1420), h_1(1170), h_1(1380)$) by employing the perturbative QCD factorization approach. These considered decay channels can only occur via annihilation type diagrams in the standard model. From the numerical calculations and phenomenological analysis, we found the following results: (a) the perturbative QCD predictions for the BRs of the considered B_c decays are in the range of 10^{-6} to 10^{-8}, while the CP-violating asymmetries are absent because only one type tree operator is involved here; (b) the BRs of $\Delta S = 0$ processes are generally much larger than those of $\Delta S = 1$ ones due to the large Cabibbo-Kobayashi-Maskawa factor of $|V_{ud}/V_{us}|^2 \sim 19$; (c) since the behavior for the $^1P_1$ meson is much different from that of the $^3P_1$ meson, the BRs of $B_c \to A(^1P_1)P$ decays are generally larger than those of $B_c \to A(^3P_1)P$ decays; (d) the perturbative QCD predictions for the BRs of $B_c \to (K_1(1270), K_1(1400))\eta^{(\prime)}$ and $(K_1(1270), K_1(1400))K$ decays are rather sensitive to the value of the mixing angle θ_K.

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I. INTRODUCTION

Unlike the ordinary light B_q ($q = u, d, s$) mesons, the B_c meson is the only heavy meson consisting of two heavy quarks b and c and plays a special role in the precision test of the standard model (SM) [1]. Moreover, a large number of B_c meson events will be collected with the running of Large Hadron Collider (LHC) experiments and this will provide great opportunities for both theorists and experimentalists to study the perturbative and nonperturbative QCD dynamics, final state interactions, etc.

In two recent works [2,3], the pure annihilation $B_c \to PP$, PV/VP, VV decays (here P and V stand for the light pseudoscalar and vector mesons) have been studied by employing the SU(3) flavor symmetry and the perturbative QCD (pQCD) factorization approach [4–6], respectively.

In the present work, we will study the two body charmless hadronic $B_c \to AP$ decays (here A denotes the light axial-vector mesons), which can only occur via annihilation type diagrams in the SM. First of all, the size of annihilation contributions is an important issue in the B meson physics, and has been studied extensively, for example, in Refs. [4,5,7–10]. Second, the internal structure of the axial-vector mesons has been one of the hot topics in recent years [11–13]. Although many efforts on both theoretical and experimental sides have been made [14–20] to explore it through the studies for the relevant decay rates, the CP-violating asymmetries, polarization fractions, and the form factors, etc., we currently still know little about the nature of the axial-vector mesons.

In the quark model, there are two different types of light axial-vector mesons: $^3P_1$ and $^1P_1$, which carry the quantum numbers $J^{PC} = 1^{++}$ and 1^{+-}, respectively. The 1^{++} nonet consists of $a_1(1260)$, $f_1(1285)$, $f_1(1420)$, and K_{1A}, while the 1^{+-} nonet has $b_1(1235)$, $h_1(1170)$, $h_1(1380)$, and K_{1B}.[1] In the SU(3) limit, these mesons cannot mix with each other. Because the s quark is heavier than u, d quarks, the meson $K_1(1270)$ and $K_1(1400)$ are not purely a 1^3P_1 or 1^1P_1 state, but a mixture of K_{1A} ($^3P_1$ state) and K_{1B} ($^1P_1$ state). Analogous to the η-η' system, the flavor-singlet and flavor-octet axial-vector mesons can also mix with each other. It is worth mentioning that the mixing angles can be determined by the relevant data, but unfortunately, there is not enough data now for these mesons, which leaves the mixing angles basically free parameters.

In this paper, we will calculate the branching ratios of the 32 nonleptonic charmless $B_c \to AP$ decays by employing the low energy effective Hamiltonian [21] and the pQCD factorization approach based on the framework of the k_T factorization theorem. By keeping the transverse momentum k_T of the quarks, the pQCD approach is free of end-point singularity and the Sudakov formalism makes it more self-consistent. In the pQCD approach one can do the quantitative calculations of the annihilation type diagrams directly, which can be seen, for instance, in Refs. [3–5,7,9].

The pure annihilation $B_c \to PP$, PV/VP, VV decays and $B_c \to AP$ decays considered in Refs. [2,3] and in this paper generally have very small branching ratios: at the

[1]For the sake of simplicity, we will adopt the forms a_1 and b_1 to denote the nonstrange axial-vector mesons $a_1(1260)$ and $b_1(1235)$, respectively, in the following section. We will also use K_1 to denote $K_1(1270)$ and $K_1(1400)$ for convenience unless otherwise stated.

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order of 10^{-6} to 10^{-9}. According to the discussions as given in Ref. [2], the charmless hadronic B_c decays with decay rates at the level of 10^{-6} could be measured at LHC experiments with the accuracy required for the phenomenological analysis, while it may be difficult to measure those B_c decays if their branching ratios are much less than 10^{-6}.

The paper is organized as follows. In Sec. II, we present the formalism of the considered B_c meson decays. Then we perform the analytic calculations for considered decay channels by using the pQCD approach in Sec. III. The numerical results and phenomenological analysis are given in Sec. IV. Finally, Sec. V contains a short summary and some discussions.

II. FORMALISM

In the pQCD approach, the decay amplitude of the two body decay $B_c \to M_1 M_2$ (M_1, M_2 stand for the two final state mesons) can be written conceptually as the convolution,

$$\mathcal{A}(B_c \to M_1 M_2) \sim \int d^4k_1 d^4k_2 d^4k_3 \,\mathrm{Tr}[C(t)\Phi_{B_c}(k_1) \times \Phi_{M_1}(k_2)\Phi_{M_2}(k_3)H(k_1,k_2,k_3,t)], \quad (1)$$

where k_i's are momenta of light quarks included in each meson, and Tr denotes the trace over Dirac and color indices. $C(t)$ is the Wilson coefficient which results from the radiative corrections at short distance. In the above convolution, $C(t)$ includes the harder dynamics at a larger scale than the m_{B_c} scale and describes the evolution of local 4-Fermi operators from m_W (the W boson mass) down to the $t \sim \mathcal{O}(\sqrt{\bar{\Lambda} m_{B_c}})$ scale, where $\bar{\Lambda} \equiv m_{B_c} - m_b$. The function $H(k_1, k_2, k_3, t)$ describes the four quark operator and the spectator quark connected by a hard gluon whose q^2 is in the order of $\bar{\Lambda} m_{B_c}$, and includes the $\mathcal{O}(\sqrt{\bar{\Lambda} m_{B_c}})$ hard dynamics. Therefore, this hard part H can be perturbatively calculated. The function Φ_M is the wave function which describes hadronization of the quark and antiquark to the meson M. In the present work, since the B_c meson is composed of two heavy quarks b and c, we will take the nonrelativistic approximation form $\delta(x - m_c/m_{B_c})$ [22] for the distribution amplitude $\phi_{B_c}(x)$. For light meson A and P, we adopt the light-cone distribution amplitudes directly, which will be displayed in the Appendix. While the function H depends on the processes considered, the wave function Φ_M is independent of the specific processes. Using the wave functions determined from other well measured processes, one can make quantitative predictions here.

Since the b quark is rather heavy, we work in the frame with the B_c meson at rest, i.e., with the B_c meson momentum $P_1 = (m_{B_c}/\sqrt{2})(1, 1, \mathbf{0}_T)$ in the light-cone coordinates. For the charmless hadronic $B_c \to AP$ decays, we assume that the A (P) meson moves in the plus (minus) z direction carrying the momentum P_2 (P_3), and with the polarization vector ϵ_2 for the A meson. Then the two final state meson momenta can be written as

$$P_2 = \frac{m_{B_c}}{\sqrt{2}}(1, r_A^2, \mathbf{0}_T), \qquad P_3 = \frac{m_{B_c}}{\sqrt{2}}(0, 1 - r_A^2, \mathbf{0}_T), \quad (2)$$

respectively, where $r_A = m_A/m_{B_c}$ and the mass of light pseudoscalar mesons (K, π, and $\eta^{(\prime)}$) has been neglected. For the axial-vector meson A, its longitudinal polarization vector, ϵ_2^L, can be defined as

$$\epsilon_2^L = \frac{m_{B_c}}{\sqrt{2} m_A}(1, -r_A^2, \mathbf{0}_T). \quad (3)$$

Putting the (light) quark momenta in B_c, A, and P mesons as k_1, k_2, and k_3, respectively, we can choose

$$k_1 = (x_1 P_1^+, 0, \mathbf{k}_{1T}), \qquad k_2 = (x_2 P_2^+, 0, \mathbf{k}_{2T}), \qquad k_3 = (0, x_3 P_3^-, \mathbf{k}_{3T}). \quad (4)$$

Then, for $B_c \to AP$ decays, the integration over k_1^-, k_2^-, and k_3^+ will lead to the decay amplitudes in the pQCD approach,

$$\mathcal{A}(B_c \to AP) \sim \int dx_1 dx_2 dx_3 b_1 db_1 b_2 db_2 b_3 db_3 \cdot \mathrm{Tr}[C(t)\Phi_{B_c}(x_1, b_1)\Phi_A(x_2, b_2) \times \Phi_P(x_3, b_3) H(x_i, b_i, t) S_t(x_i) e^{-S(t)}], \quad (5)$$

where b_i is the conjugate space coordinate of k_{iT}, and t is the largest energy scale in function $H(x_i, b_i, t)$. The large logarithms $\ln(m_W/t)$ are included in the Wilson coefficients $C(t)$. The large double logarithms ($\ln^2 x_i$) are summed by the threshold resummation [23], and they lead to $S_t(x_i)$ which smears the end-point singularities on x_i. The last term, $e^{-S(t)}$, is the Sudakov form factor which suppresses the soft dynamics effectively [24]. Thus it makes the perturbative calculation of the hard part H applicable at the intermediate scale, i.e., m_{B_c} scale. We will calculate analytically the function $H(x_i, b_i, t)$ for the considered decays at leading order in α_s expansion and give the convoluted amplitudes in next section.

For these considered decays, the related weak effective Hamiltonian H_{eff} [21] is given by

$$H_{\mathrm{eff}} = \frac{G_F}{\sqrt{2}}[V_{cb}^* V_{uD}(C_1(\mu)O_1(\mu) + C_2(\mu)O_2(\mu))], \quad (6)$$

with the current-current operators $O_{1,2}$,

$$O_1 = \bar{u}_\beta \gamma^\mu (1 - \gamma_5) D_\alpha \bar{c}_\beta \gamma^\mu (1 - \gamma_5) b_\alpha, \qquad O_2 = \bar{u}_\beta \gamma^\mu (1 - \gamma_5) D_\beta \bar{c}_\alpha \gamma^\mu (1 - \gamma_5) b_\alpha, \quad (7)$$

where V_{cb}, V_{uD} are the Cabibbo-Kobayashi-Maskawa

(CKM) matrix elements, D denotes the light down quark d or s, and $C_i(\mu)$ are Wilson coefficients at the renormalization scale μ. For the Wilson coefficients $C_{1,2}(\mu)$, we will also use the leading order expressions, although the next-to-leading order calculations already exist in the literature [21]. This is the consistent way to cancel the explicit μ dependence in the theoretical formulas. For the renormalization group evolution of the Wilson coefficients from higher scale to lower scale, we use the formulas as given in Ref. [5] directly.

III. ANALYTIC CALCULATIONS IN THE PQCD APPROACH

In this section, we will calculate the decay amplitudes for 32 charmless hadronic $B_c \to AP/PA$ decays. Analogous to $B_c \to PV/VP$ decays in Ref. [3], there are four kinds of annihilation Feynman diagrams contributing to these considered decays, as illustrated in Fig. 1. By analytical evaluation of the two factorizable annihilation (fa) diagrams Figs. 1(a) and 1(b), we find the corresponding decay amplitude

$$\begin{aligned} F_{fa}^{AP} = {} & -8\pi C_F m_{B_c}^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 \{h_{fa}(1-x_3, x_2, b_3, b_2) E_{fa}(t_a)[x_2 \phi_A(x_2)\phi_P^A(x_3) \\ & + 2r_A r_0^P \phi_P^P(x_3)((x_2+1)\phi_A^s(x_2) + (x_2-1)\phi_A^t(x_2))] + h_{fa}(x_2, 1-x_3, b_2, b_3)E_{fa}(t_b)[(x_3-1)\phi_A(x_2)\phi_P^A(x_3) \\ & + 2r_A r_0^P \phi_A^s(x_2)((x_3-2)\phi_P^P(x_3) - x_3\phi_P^T(x_3))]\}, \end{aligned} \tag{8}$$

where ϕ_A, $\phi_A^{s,t}$, and $\phi_P^{A,P,T}$ denote the distribution amplitudes of the axial-vector and pseudoscalar mesons, $r_0^P = m_0^P/m_{B_c}$ with m_0^P standing for the chiral scale of the pseudoscalar meson (P), and $C_F = 4/3$ is a color factor. In Eq. (8), the terms proportional to $(r_A(r_0^P))^2$ have been neglected because they are small: less than 7% numerically. The function h_{fa}, the scales t_i, and $E_{fa}(t)$ can be found in Appendix B of Ref. [3].

For the two nonfactorizable annihilation (na) diagrams Figs. 1(c) and 1(d), all three meson wave functions are involved. The integration of b_3 can be performed using δ function $\delta(b_3 - b_2)$, leaving only integration of b_1 and b_2. The corresponding decay amplitude is

$$\begin{aligned} M_{na}^{AP} = {} & -\frac{16\sqrt{6}}{3}\pi C_F m_{B_c}^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \{h_{na}^c(x_2, x_3, b_1, b_2)E_{na}(t_c)[(r_c - x_3 + 1)\phi_A(x_2)\phi_P^A(x_3) \\ & + r_A r_0^P(\phi_A^s(x_2)((3r_c + x_2 - x_3 + 1)\phi_P^P(x_3) - (r_c - x_2 - x_3 + 1)\phi_P^T(x_3)) + \phi_A^t(x_2)((r_c - x_2 - x_3 + 1)\phi_P^P(x_3) \\ & + (r_c - x_2 + x_3 - 1)\phi_P^T(x_3)))] - E_{na}(t_d)[(r_b + r_c + x_2 - 1)\phi_A(x_2)\phi_P^A(x_3) + r_A r_0^P(\phi_A^s(x_2) \\ & \times ((4r_b + r_c + x_2 - x_3 - 1)\phi_P^P(x_3) - (r_c + x_2 + x_3 - 1)\phi_P^T(x_3)) + \phi_A^t(x_2)((r_c + x_2 + x_3 - 1)\phi_P^P(x_3) \\ & - (r_c + x_2 - x_3 - 1)\phi_P^T(x_3)))]h_{na}^d(x_2, x_3, b_1, b_2)\}, \end{aligned} \tag{9}$$

where $r_b = m_b/m_{B_c}$, $r_c = m_c/m_{B_c}$, and $r_b + r_c \approx 1$ for B_c meson.

By exchanging the position of the final state mesons A and P, we can obtain the phenomenological topology for $B_c \to PA$ decays easily. The corresponding decay amplitudes for this type of decay channels can be obtained directly by the following replacements in Eqs. (8) and (9),

$$\phi_A \leftrightarrow \phi_P^A, \qquad \phi_A^s \leftrightarrow \phi_P^P, \qquad \phi_A^t \leftrightarrow \phi_P^T, \qquad r_A \leftrightarrow r_0^P. \tag{10}$$

Before we put the things together to write down the decay amplitudes for the studied decay modes, we give a

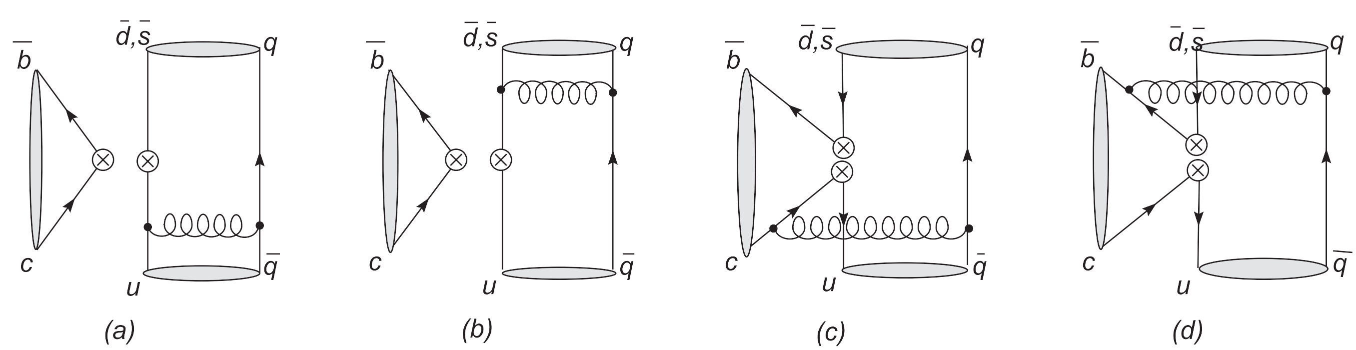

FIG. 1. Typical Feynman diagrams for the charmless hadronic $B_c \to AP$ decays.

brief discussion about the K_{1A}-K_{1B}, f_1-f_8, and h_1-h_8 mixing.

The physical states $K_1(1270)$ and $K_1(1400)$ are the mixtures of the K_{1A} and K_{1B}. K_{1A} and K_{1B} are not mass eigenstates, and can be mixed together due to the strange and nonstrange light quark mass difference. The mixing of K_{1A} and K_{1B} can be written as

$$|K_1(1270)\rangle = |K_{1A}\rangle \sin\theta_K + |K_{1B}\rangle \cos\theta_K, \tag{11}$$

$$|K_1(1400)\rangle = |K_{1A}\rangle \cos\theta_K - |K_{1B}\rangle \sin\theta_K. \tag{12}$$

If the SU(3) flavor symmetry between (u, d, s) quarks was an exact symmetry, K_{1A} and K_{1B} would not be mixed with each other. As mentioned in the introduction, the mixing angle θ_K is still not well determined because of the poor experimental data. In this paper, for simplicity, we will adopt two reference values as used in Ref. [13]: $\theta_K = \pm 45°$.

Analogous to the η-η' mixing in the pseudoscalar sector, $f_1(1285)$ and $f_1(1420)$ (the 1^3P_1 states) will mix in the form of

$$\begin{pmatrix} f_1(1285) \\ f_1(1420) \end{pmatrix} = \begin{pmatrix} \cos\theta_3 & \sin\theta_3 \\ -\sin\theta_3 & \cos\theta_3 \end{pmatrix} \begin{pmatrix} f_1 \\ f_8 \end{pmatrix}. \tag{13}$$

Likewise, the $h_1(1170)$ and $h_1(1380)$ (1^1P_1 states) system can be mixed in terms of the pure singlet $|h_1\rangle$ and octet $|h_8\rangle$,

$$\begin{pmatrix} h_1(1170) \\ h_1(1380) \end{pmatrix} = \begin{pmatrix} \cos\theta_1 & \sin\theta_1 \\ -\sin\theta_1 & \cos\theta_1 \end{pmatrix} \begin{pmatrix} h_1 \\ h_8 \end{pmatrix}, \tag{14}$$

where the component of $|f_1\rangle, |h_1\rangle$ and $|f_8\rangle, |h_8\rangle$ can be written as

$$\begin{aligned} |f_1\rangle, |h_1\rangle &= \frac{1}{\sqrt{3}}(|\bar{q}q\rangle + |\bar{s}s\rangle), \\ |f_8\rangle, |h_8\rangle &= \frac{1}{\sqrt{6}}(|\bar{q}q\rangle - 2|\bar{s}s\rangle), \end{aligned} \tag{15}$$

where $q = (u, d)$. The values of the mixing angles for 1^3P_1 and 1^1P_1 states are chosen as [13]

$$\theta_3 = 38° \quad \text{or} \quad 50°; \qquad \theta_1 = 10° \quad \text{or} \quad 45°. \tag{16}$$

By putting all things together, we can write down the general expression of the total decay amplitude for the considered decays:

$$\mathcal{A}(B_c \to AP) = V_{cb}^* V_{uD}\{f_{B_c} F_{fa}^{AP/(PA)} a_1 + M_{na}^{AP/(PA)} C_1\}, \tag{17}$$

where $a_1 = C_1/3 + C_2$. Now it is straightforward to present the explicit expressions of the decay amplitudes for all 32 considered $B_c \to AP$ decays.

(1) For $\Delta S = 0$ processes,

$$\mathcal{A}(B_c \to \pi^+ a_1^0) = V_{cb}^* V_{ud}\{[f_{B_c} F_{fa}^{\pi a_{1u}^0} a_1 + M_{na}^{\pi a_{1u}^0} C_1] - [f_{B_c} F_{fa}^{a_{1d}^0 \pi} a_1 + M_{na}^{a_{1d}^0 \pi} C_1]\}/\sqrt{2}, \tag{18}$$

$$\begin{aligned} \mathcal{A}(B_c \to a_1^+ \pi^0) &= -\mathcal{A}(B_c \to \pi^+ a_1^0) \\ &= V_{cb}^* V_{ud}\{[f_{B_c} F_{fa}^{a_1 \pi_u^0} a_1 + M_{na}^{a_1 \pi_u^0} C_1] - [f_{B_c} F_{fa}^{\pi_d^0 a_1} a_1 + M_{na}^{\pi_d^0 a_1} C_1]\}/\sqrt{2}, \end{aligned} \tag{19}$$

$$\mathcal{A}(B_c \to a_1^+ \eta) = V_{cb}^* V_{ud} \cos\phi\{[f_{B_c} F_{fa}^{a_1 \eta_u} a_1 + M_{na}^{a_1 \eta_u} C_1] + [f_{B_c} F_{fa}^{\eta_d a_1} a_1 + M_{na}^{\eta_d a_1} C_1]\}/\sqrt{2}, \tag{20}$$

$$\mathcal{A}(B_c \to a_1^+ \eta') = V_{cb}^* V_{ud} \sin\phi\{[f_{B_c} F_{fa}^{a_1 \eta_u} a_1 + M_{na}^{a_1 \eta_u} C_1] + [f_{B_c} F_{fa}^{\eta_d a_1} a_1 + M_{na}^{\eta_d a_1} C_1]\}/\sqrt{2}, \tag{21}$$

$$\mathcal{A}(B_c \to \pi^+ b_1^0) = V_{cb}^* V_{ud}\{[f_{B_c} F_{fa}^{\pi b_{1u}^0} a_1 + M_{na}^{\pi b_{1u}^0} C_1] - [f_{B_c} F_{fa}^{b_{1d}^0 \pi} a_1 + M_{na}^{b_{1d}^0 \pi} C_1]\}/\sqrt{2}, \tag{22}$$

$$\begin{aligned} \mathcal{A}(B_c \to b_1^+ \pi^0) &= -\mathcal{A}(B_c \to \pi^+ b_1^0) \\ &= V_{cb}^* V_{ud}\{[f_{B_c} F_{fa}^{b_1 \pi_u^0} a_1 + M_{na}^{b_1 \pi_u^0} C_1] - [f_{B_c} F_{fa}^{\pi_d^0 b_1} a_1 + M_{na}^{\pi_d^0 b_1} C_1]\}/\sqrt{2}, \end{aligned} \tag{23}$$

$$\mathcal{A}(B_c \to b_1^+ \eta) = V_{cb}^* V_{ud} \cos\phi\{[f_{B_c} F_{fa}^{b_1 \eta_u} a_1 + M_{na}^{b_1 \eta_u} C_1] + [f_{B_c} F_{fa}^{\eta_d b_1} a_1 + M_{na}^{\eta_d b_1} C_1]\}/\sqrt{2}, \tag{24}$$

$$\mathcal{A}(B_c \to b_1^+ \eta') = V_{cb}^* V_{ud} \sin\phi\{[f_{B_c} F_{fa}^{b_1 \eta_u} a_1 + M_{na}^{b_1 \eta_u} C_1] + [f_{B_c} F_{fa}^{\eta_d b_1} a_1 + M_{na}^{\eta_d b_1} C_1]\}/\sqrt{2}, \tag{25}$$

$$\mathcal{A}(B_c \to \pi^+ f_1(1285)) = V_{cb}^* V_{ud}\bigg\{\frac{\cos\theta_3}{\sqrt{3}}[f_{B_c}(F_{fa}^{\pi f_1^u} + F_{fa}^{f_1^d \pi})a_1 + (M_{na}^{\pi f_1^u} + M_{na}^{f_1^d \pi})C_1] + \frac{\sin\theta_3}{\sqrt{6}}[f_{B_c}(F_{fa}^{\pi f_8^u} + F_{fa}^{f_8^d \pi})a_1 + (M_{na}^{\pi f_8^u} + M_{na}^{f_8^d \pi})C_1]\bigg\}, \tag{26}$$

$$\mathcal{A}(B_c \to \pi^+ f_1(1420)) = V_{cb}^* V_{ud}\bigg\{\frac{-\sin\theta_3}{\sqrt{3}}[f_{B_c}(F_{fa}^{\pi f_1^u} + F_{fa}^{f_1^d \pi})a_1 + (M_{na}^{\pi f_1^u} + M_{na}^{f_1^d \pi})C_1] + \frac{\cos\theta_3}{\sqrt{6}}[f_{B_c}(F_{fa}^{\pi f_8^u} + F_{fa}^{f_8^d \pi})a_1 + (M_{na}^{\pi f_8^u} + M_{na}^{f_8^d \pi})C_1]\bigg\}, \tag{27}$$

$$\mathcal{A}(B_c \to \pi^+ h_1(1170)) = V_{cb}^* V_{ud}\bigg\{\frac{\cos\theta_1}{\sqrt{3}}[f_{B_c}(F_{fa}^{\pi h_1^u} + F_{fa}^{h_1^d \pi})a_1 + (M_{na}^{\pi h_1^u} + M_{na}^{h_1^d \pi})C_1] + \frac{\sin\theta_1}{\sqrt{6}}[f_{B_c}(F_{fa}^{\pi h_8^u} + F_{fa}^{h_8^d \pi})a_1 + (M_{na}^{\pi h_8^u} + M_{na}^{h_8^d \pi})C_1]\bigg\}, \tag{28}$$

$$\mathcal{A}(B_c \to \pi^+ h_1(1380)) = V_{cb}^* V_{ud}\bigg\{\frac{-\sin\theta_1}{\sqrt{3}}[f_{B_c}(F_{fa}^{\pi h_1^u} + F_{fa}^{h_1^d \pi})a_1 + (M_{na}^{\pi h_1^u} + M_{na}^{h_1^d \pi})C_1] + \frac{\cos\theta_1}{\sqrt{6}}[f_{B_c}(F_{fa}^{\pi h_8^u} + F_{fa}^{h_8^d \pi})a_1 + (M_{na}^{\pi h_8^u} + M_{na}^{h_8^d \pi})C_1]\bigg\}, \tag{29}$$

$$\mathcal{A}(B_c \to \bar{K}^0 K_1(1270)^+) = V_{cb}^* V_{ud}\{\sin\theta_K[f_{B_c}F_{fa}^{\bar{K}^0 K_{1A}}a_1 + M_{na}^{\bar{K}^0 K_{1A}}C_1] + \cos\theta_K[f_{B_c}F_{fa}^{\bar{K}^0 K_{1B}}a_1 + M_{na}^{\bar{K}^0 K_{1B}}C_1]\}, \tag{30}$$

$$\mathcal{A}(B_c \to \bar{K}^0 K_1(1400)^+) = V_{cb}^* V_{ud}\{\cos\theta_K[f_{B_c}F_{fa}^{\bar{K}^0 K_{1A}}a_1 + M_{na}^{\bar{K}^0 K_{1A}}C_1] - \sin\theta_K[f_{B_c}F_{fa}^{\bar{K}^0 K_{1B}}a_1 + M_{na}^{\bar{K}^0 K_{1B}}C_1]\}, \tag{31}$$

$$\mathcal{A}(B_c \to \bar{K}_1(1270)^0 K^+) = V_{cb}^* V_{ud}\{\sin\theta_K[f_{B_c}F_{fa}^{\bar{K}_{1A}^0 K}a_1 + M_{na}^{\bar{K}_{1A}^0 K}C_1] + \cos\theta_K[f_{B_c}F_{fa}^{\bar{K}_{1B}^0 K}a_1 + M_{na}^{\bar{K}_{1B}^0 K}C_1]\}, \tag{32}$$

$$\mathcal{A}(B_c \to \bar{K}_1(1400)^0 K^+) = V_{cb}^* V_{ud}\{\cos\theta_K[f_{B_c}F_{fa}^{\bar{K}_{1A}^0 K}a_1 + M_{na}^{\bar{K}_{1A}^0 K}C_1] - \sin\theta_K[f_{B_c}F_{fa}^{\bar{K}_{1B}^0 K}a_1 + M_{na}^{\bar{K}_{1B}^0 K}C_1]\}. \tag{33}$$

(2) For $\Delta S = 1$ processes,

$$\mathcal{A}(B_c \to K^0 a_1^+) = \sqrt{2}\mathcal{A}(B_c \to K^+ a_1^0) = V_{cb}^* V_{us}\{f_{B_c}F_{fa}^{K^0 a_1}a_1 + M_{na}^{K^0 a_1}C_1\}, \tag{34}$$

$$\mathcal{A}(B_c \to K^0 b_1^+) = \sqrt{2}\mathcal{A}(B_c \to K^+ b_1^0) = V_{cb}^* V_{us}\{f_{B_c}F_{fa}^{K^0 b_1}a_1 + M_{na}^{K^0 b_1}C_1\}, \tag{35}$$

$$\mathcal{A}(B_c \to K_1(1270)^0 \pi^+) = \sqrt{2}\mathcal{A}(B_c \to K_1(1270)^+ \pi^0) = V_{cb}^* V_{us}\{\sin\theta_K[f_{B_c}F_{fa}^{K_{1A}^0 \pi}a_1 + M_{na}^{K_{1A}^0 \pi}C_1] + \cos\theta_K[f_{B_c}F_{fa}^{K_{1B}^0 \pi}a_1 + M_{na}^{K_{1B}^0 \pi}C_1]\}, \tag{36}$$

$$\mathcal{A}(B_c \to K_1(1400)^0 \pi^+) = \sqrt{2}\mathcal{A}(B_c \to K_1(1400)^+ \pi^0) = V_{cb}^* V_{us}\{\cos\theta_K[f_{B_c}F_{fa}^{K_{1A}^0 \pi}a_1 + M_{na}^{K_{1A}^0 \pi}C_1] - \sin\theta_K[f_{B_c}F_{fa}^{K_{1B}^0 \pi}a_1 + M_{na}^{K_{1B}^0 \pi}C_1]\}, \tag{37}$$

$$\mathcal{A}(B_c \to K^+ f_1(1285)) = V_{cb}^* V_{us}\left\{\frac{\cos\theta_3}{\sqrt{3}}[f_{B_c}(F_{fa}^{Kf_1^u} + F_{fa}^{f_1^s K})a_1 + (M_{na}^{Kf_1^u} + M_{na}^{f_1^s K})C_1] + \frac{\sin\theta_3}{\sqrt{6}}[f_{B_c}(F_{fa}^{Kf_8^u} - 2F_{fa}^{f_8^s K})a_1 + (M_{na}^{Kf_8^u} - 2M_{na}^{f_8^s K})C_1]\right\}, \tag{38}$$

$$\mathcal{A}(B_c \to K^+ f_1(1420)) = V_{cb}^* V_{us}\left\{\frac{-\sin\theta_3}{\sqrt{3}}[f_{B_c}(F_{fa}^{Kf_1^u} + F_{fa}^{f_1^s K})a_1 + (M_{na}^{Kf_1^u} + M_{na}^{f_1^s K})C_1] + \frac{\cos\theta_3}{\sqrt{6}}[f_{B_c}(F_{fa}^{Kf_8^u} - 2F_{fa}^{f_8^s K})a_1 + (M_{na}^{Kf_8^u} - 2M_{na}^{f_8^s K})C_1]\right\}, \tag{39}$$

$$\mathcal{A}(B_c \to K^+ h_1(1170)) = V_{cb}^* V_{us}\left\{\frac{\cos\theta_1}{\sqrt{3}}[f_{B_c}(F_{fa}^{Kh_1^u} + F_{fa}^{h_1^s K})a_1 + (M_{na}^{Kh_1^u} + M_{na}^{h_1^s K})C_1] + \frac{\sin\theta_1}{\sqrt{6}}[f_{B_c}(F_{fa}^{Kh_8^u} - 2F_{fa}^{h_8^s K})a_1 + (M_{na}^{Kh_8^u} - 2M_{na}^{h_8^s K})C_1]\right\}, \tag{40}$$

$$\mathcal{A}(B_c \to K^+ h_1(1380)) = V_{cb}^* V_{us}\left\{\frac{-\sin\theta_1}{\sqrt{3}}[f_{B_c}(F_{fa}^{Kh_1^u} + F_{fa}^{h_1^s K})a_1 + (M_{na}^{Kh_1^u} + M_{na}^{h_1^s K})C_1] + \frac{\cos\theta_1}{\sqrt{6}}[f_{B_c}(F_{fa}^{Kh_8^u} - 2F_{fa}^{h_8^s K})a_1 + (M_{na}^{Kh_8^u} - 2M_{na}^{h_8^s K})C_1]\right\}, \tag{41}$$

$$\mathcal{A}(B_c \to K_1(1270)^+ \eta) = V_{cb}^* V_{us}\{\sin\theta_K[f_{B_c}(\cos\phi F_{fa}^{K_{1A}\eta_q} - \sin\phi F_{fa}^{\eta_s K_{1A}})a_1 + (\cos\phi M_{na}^{K_{1A}\eta_q} - \sin\phi M_{na}^{\eta_s K_{1A}})C_1] + \cos\theta_K[f_{B_c}(\cos\phi F_{fa}^{K_{1B}\eta_q} - \sin\phi F_{fa}^{\eta_s K_{1B}})a_1 + (\cos\phi M_{na}^{K_{1B}\eta_q} - \sin\phi M_{na}^{\eta_s K_{1B}})C_1]\}, \tag{42}$$

$$\mathcal{A}(B_c \to K_1(1400)^+ \eta) = V_{cb}^* V_{us}\{\cos\theta_K[f_{B_c}(\cos\phi F_{fa}^{K_{1A}\eta_q} - \sin\phi F_{fa}^{\eta_s K_{1A}})a_1 + (\cos\phi M_{na}^{K_{1A}\eta_q} - \sin\phi M_{na}^{\eta_s K_{1A}})C_1] - \sin\theta_K[f_{B_c}(\cos\phi F_{fa}^{K_{1B}\eta_q} - \sin\phi F_{fa}^{\eta_s K_{1B}})a_1 + (\cos\phi M_{na}^{K_{1B}\eta_q} - \sin\phi M_{na}^{\eta_s K_{1B}})C_1]\}, \tag{43}$$

$$\mathcal{A}(B_c \to K_1(1270)^+ \eta') = V_{cb}^* V_{us}\{\sin\theta_K[f_{B_c}(\sin\phi F_{fa}^{K_{1A}\eta_q} + \cos\phi F_{fa}^{\eta_s K_{1A}})a_1 + (\sin\phi M_{na}^{K_{1A}\eta_q} + \cos\phi M_{na}^{\eta_s K_{1A}})C_1] + \cos\theta_K[f_{B_c}(\sin\phi F_{fa}^{K_{1B}\eta_q} + \cos\phi F_{fa}^{\eta_s K_{1B}})a_1 + (\sin\phi M_{na}^{K_{1B}\eta_q} + \cos\phi M_{na}^{\eta_s K_{1B}})C_1]\}, \tag{44}$$

$$\mathcal{A}(B_c \to K_1(1400)^+ \eta') = V_{cb}^* V_{us}\{\cos\theta_K[f_{B_c}(\sin\phi F_{fa}^{K_{1A}\eta_q} + \cos\phi F_{fa}^{\eta_s K_{1A}})a_1 + (\sin\phi M_{na}^{K_{1A}\eta_q} + \cos\phi M_{na}^{\eta_s K_{1A}})C_1] - \sin\theta_K[f_{B_c}(\sin\phi F_{fa}^{K_{1B}\eta_q} + \cos\phi F_{fa}^{\eta_s K_{1B}})a_1 + (\sin\phi M_{na}^{K_{1B}\eta_q} + \cos\phi M_{na}^{\eta_s K_{1B}})C_1]\}. \tag{45}$$

IV. NUMERICAL RESULTS AND DISCUSSIONS

In this section, we will calculate the branching ratios for those considered 32 charmless hadronic $B_c \to AP$ decay modes. The input parameters and the wave functions to be used are given in the Appendix. In numerical calculations, central values of input parameters will be used implicitly unless otherwise stated.

For $B_c \to AP$ decays, the decay rate can be written as

$$\Gamma = \frac{G_F^2 m_{B_c}^3}{32\pi}(1 - r_A^2)|\mathcal{A}(B_c \to AP)|^2, \tag{46}$$

where the corresponding decay amplitudes $\mathcal{A}$ have been given explicitly in Eqs. (18)–(45). With the complete decay amplitudes as given in the last section, by employing Eq. (46) and the input parameters and wave functions as given in the Appendix, we calculate and present the pQCD predictions for the CP-averaged branching ratios of the considered decays with errors, as shown in Tables I, II, III, and IV. The dominant errors come from the uncertainties of charm quark mass $m_c = 1.5 \pm 0.15$ GeV, the combined Gegenbauer moments a_i of the relevant meson distribution amplitudes, and the chiral enhancement factors $m_0^\pi = 1.4 \pm 0.3$ GeV and $m_0^K = 1.6 \pm 0.1$ GeV, respectively.

Based on the numerical results as given in Tables I, II, III, and IV, we have the following remarks:

(i) The pQCD predictions for the CP-averaged branching ratios of considered B_c decays vary in the range of 10^{-6} to 10^{-8}. There is no CP violation for all these decays within the standard model, since there

TABLE I. The pQCD predictions of branching ratios (BRs) for $B_c \to (a_1, b_1)P$ decays. The source of the dominant errors is explained in the text.

$\Delta S = 0$ Decay modes	BRs(10^{-7})	$\Delta S = 0$ Decay modes	BRs(10^{-6})
$B_c \to \pi^+ a_1^0$	$3.0^{+0.1}_{-0.3}(m_c)^{+2.3}_{-1.7}(a_i)^{+1.5}_{-1.2}(m_0)$	$B_c \to \pi^+ b_1^0$	$4.3^{+1.9}_{-1.4}(m_c)^{+1.8}_{-1.5}(a_i)^{+0.0}_{-0.1}(m_0)$
$B_c \to a_1^+ \pi^0$	$2.9^{+0.1}_{-0.3}(m_c)^{+2.2}_{-1.7}(a_i)^{+1.4}_{-1.2}(m_0)$	$B_c \to b_1^+ \pi^0$	$4.3^{+2.0}_{-1.4}(m_c)^{+2.0}_{-1.5}(a_i)^{+0.1}_{-0.2}(m_0)$
$B_c \to a_1^+ \eta$	$6.8^{+2.4}_{-1.2}(m_c)^{+2.7}_{-2.1}(a_i)^{+0.0}_{-0.0}(m_0)$	$B_c \to b_1^+ \eta$	$0.6^{+0.3}_{-0.1}(m_c)^{+0.2}_{-0.1}(a_i)^{+0.0}_{-0.0}(m_0)$
$B_c \to a_1^+ \eta'$	$4.6^{+1.6}_{-0.8}(m_c)^{+1.7}_{-1.4}(a_i)^{+0.0}_{-0.0}(m_0)$	$B_c \to b_1^+ \eta'$	$0.4^{+0.2}_{-0.1}(m_c)^{+0.1}_{-0.1}(a_i)^{+0.0}_{-0.0}(m_0)$
$\Delta S = 1$ Decay modes	BRs(10^{-8})	$\Delta S = 1$ Decay modes	BRs(10^{-7})
$B_c \to a_1^+ K^0$	$3.4^{+1.1}_{-1.2}(m_c)^{+3.2}_{-2.3}(a_i)^{+0.6}_{-0.2}(m_0)$	$B_c \to b_1^+ K^0$	$5.4^{+0.9}_{-0.9}(m_c)^{+3.2}_{-2.0}(a_i)^{+0.2}_{-0.0}(m_0)$
$B_c \to K^+ a_1^0$	$1.7^{+0.6}_{-0.6}(m_c)^{+1.6}_{-1.1}(a_i)^{+0.3}_{-0.1}(m_0)$	$B_c \to K^+ b_1^0$	$2.7^{+0.5}_{-0.5}(m_c)^{+1.5}_{-1.1}(a_i)^{+0.1}_{-0.0}(m_0)$

is only one kind of tree operator involved in the decay amplitude of all considered B_c decays, which can be seen from Eq. (17).

(ii) Among the considered $B_c \to AP$ decays, the pQCD predictions for the branching ratios of those $\Delta S = 0$ processes are generally much larger than those of $\Delta S = 1$ channels (one of the two final state mesons is a strange meson); the main reason is the enhancement of the large CKM factor $|V_{ud}/V_{us}|^2 \sim 19$ for those $\Delta S = 0$ decays as generally expected. For $B_c \to (a_1^+, b_1^+)(\pi^0, K^0)$ decays, however, the difference is not so large, because the enhancement due to the CKM factor is partially canceled by the differences between the magnitude of individual decay amplitude $|F_{fa}^{a(b)_1^+ \pi^0}|$ and $|F_{fa}^{a(b)_1^+ K^0}|$.

(iii) For $B_c \to (a_1, b_1)\pi$ decays, the same component of $\bar{u}u - \bar{d}d$ is involved in both the axial-vector (a_1^0, b_1^0) and the pseudoscalar π^0 meson at the quark level. We therefore find the same branching ratios for $B_c \to \pi^+ a_1^0$ and $B_c \to a_1^+ \pi^0$, and for $B_c \to \pi^+ b_1^0$ and $B_c \to b_1^+ \pi^0$, respectively.

(iv) From the numerical results as shown in Table I, one can see that

$$\mathrm{Br}(B_c \to b_1 \pi) \sim 14 \times \mathrm{Br}(B_c \to a_1 \pi), \quad \mathrm{Br}(B_c \to b_1 K) \sim 16 \times \mathrm{Br}(B_c \to a_1 K). \tag{47}$$

This pattern agrees well with that as given in Refs. [14,16].

(v) Unlike $B_c \to (a_1, b_1)(\pi, K)$ decays, we find that

$$\mathrm{Br}(B_c \to a_1(\eta, \eta')) \sim \mathrm{Br}(B_c \to b_1(\eta, \eta')). \tag{48}$$

The main reason is that the suppressed factorizable annihilation amplitudes cancel each other for

TABLE II. Same as Table I but for $B_c \to (K_1(1270), K_1(1400))(\pi, K, \eta, \eta')$ decays.

$\Delta S = 0$ Decay modes	BRs(10^{-7}) $\theta_K = 45°$	BRs(10^{-7}) $\theta_K = -45°$
$B_c \to \bar{K}^0 K_1(1270)^+$	$8.2^{+1.1}_{-0.5}(m_c)^{+16.3}_{-8.1}(a_i)^{+0.0}_{-0.4}(m_0)$	$17.4^{+3.2}_{-4.1}(m_c)^{+25.2}_{-16.1}(a_i)^{+0.0}_{-1.5}(m_0)$
$B_c \to \bar{K}^0 K_1(1400)^+$	$17.3^{+3.1}_{-4.2}(m_c)^{+24.6}_{-16.1}(a_i)^{+0.0}_{-1.6}(m_0)$	$8.1^{+1.1}_{-0.5}(m_c)^{+16.1}_{-7.9}(a_i)^{+0.0}_{-0.4}(m_0)$
$B_c \to \bar{K}_1(1270)^0 K^+$	$15.8^{+7.1}_{-3.3}(m_c)^{+15.6}_{-8.1}(a_i)^{+1.6}_{-0.0}(m_0)$	$32.0^{+14.4}_{-7.3}(m_c)^{+20.2}_{-19.7}(a_i)^{+0.0}_{-2.0}(m_0)$
$B_c \to \bar{K}_1(1400)^0 K^+$	$31.7^{+14.3}_{-7.2}(m_c)^{+20.0}_{-19.5}(a_i)^{+0.0}_{-1.9}(m_0)$	$15.7^{+7.0}_{-3.4}(m_c)^{+15.2}_{-8.1}(a_i)^{+1.6}_{-0.0}(m_0)$
$\Delta S = 1$ Decay modes	BRs(10^{-8}) $\theta_K = 45°$	BRs(10^{-8}) $\theta_K = -45°$
$B_c \to K_1(1270)^0 \pi^+$	$6.8^{+5.1}_{-3.3}(m_c)^{+6.5}_{-4.5}(a_i)^{+0.8}_{-1.3}(m_0)$	$5.9^{+1.5}_{-0.7}(m_c)^{+3.5}_{-1.9}(a_i)^{+0.6}_{-0.1}(m_0)$
$B_c \to K_1(1400)^0 \pi^+$	$5.8^{+1.5}_{-0.6}(m_c)^{+3.6}_{-1.8}(a_i)^{+0.6}_{-0.0}(m_0)$	$6.8^{+5.0}_{-3.3}(m_c)^{+6.3}_{-4.5}(a_i)^{+0.7}_{-1.3}(m_0)$
$B_c \to K_1(1270)^+ \pi^0$	$3.4^{+2.5}_{-1.6}(m_c)^{+3.3}_{-2.2}(a_i)^{+0.4}_{-0.6}(m_0)$	$3.0^{+0.7}_{-0.4}(m_c)^{+1.7}_{-1.0}(a_i)^{+0.3}_{-0.1}(m_0)$
$B_c \to K_1(1400)^+ \pi^0$	$2.9^{+0.7}_{-0.3}(m_c)^{+1.8}_{-0.9}(a_i)^{+0.3}_{-0.0}(m_0)$	$3.4^{+2.5}_{-1.7}(m_c)^{+3.1}_{-2.2}(a_i)^{+0.4}_{-0.7}(m_0)$
$\Delta S = 1$ Decay modes	BRs(10^{-8}) $\theta_K = 45°$	BRs(10^{-8}) $\theta_K = -45°$
$B_c \to K_1(1270)^+ \eta$	$16.8^{+5.0}_{-3.6}(m_c)^{+12.1}_{-10.1}(a_i)^{+0.0}_{-0.0}(m_0)$	$27.2^{+9.0}_{-8.4}(m_c)^{+14.8}_{-12.9}(a_i)^{+0.0}_{-0.0}(m_0)$
$B_c \to K_1(1400)^+ \eta$	$26.9^{+8.9}_{-8.3}(m_c)^{+14.7}_{-12.8}(a_i)^{+0.0}_{-0.0}(m_0)$	$16.6^{+5.0}_{-3.5}(m_c)^{+12.2}_{-9.9}(a_i)^{+0.0}_{-0.0}(m_0)$
$B_c \to K_1(1270)^+ \eta'$	$2.7^{+0.4}_{-0.0}(m_c)^{+4.2}_{-2.7}(a_i)^{+0.0}_{-0.0}(m_0)$	$11.6^{+1.8}_{-2.1}(m_c)^{+5.0}_{-4.2}(a_i)^{+0.0}_{-0.0}(m_0)$
$B_c \to K_1(1400)^+ \eta'$	$11.5^{+1.7}_{-2.1}(m_c)^{+5.0}_{-4.2}(a_i)^{+0.0}_{-0.0}(m_0)$	$2.7^{+0.4}_{-0.0}(m_c)^{+4.1}_{-2.7}(a_i)^{+0.0}_{-0.0}(m_0)$

TABLE III. Same as Table I but for $B_c \to (f_1(1285), f_1(1420))(\pi, K)$ decays.

$\Delta S = 0$ Decay modes	BRs(10^{-8}) $\theta_3 = 38°$	BRs(10^{-8}) $\theta_3 = 50°$
$B_c \to \pi^+ f_1(1285)$	$52.4^{+9.3}_{-7.4}(m_c)^{+30.2}_{-23.2}(a_i)^{+21.7}_{-19.0}(m_0)$	$44.8^{+7.0}_{-6.7}(m_c)^{+23.6}_{-19.1}(a_i)^{+19.8}_{-16.8}(m_0)$
$B_c \to \pi^+ f_1(1420)$	$8.5^{+0.5}_{-1.4}(m_c)^{+6.0}_{-5.5}(a_i)^{+0.7}_{-2.6}(m_0)$	$16.0^{+2.8}_{-2.0}(m_c)^{+11.5}_{-9.2}(a_i)^{+0.2}_{-1.5}(m_0)$
$\Delta S = 1$ Decay modes	BRs(10^{-8}) $\theta_3 = 38°$	BRs(10^{-8}) $\theta_3 = 50°$
$B_c \to K^+ f_1(1285)$	$1.6^{+1.0}_{-0.7}(m_c)^{+3.4}_{-1.8}(a_i)^{+0.5}_{-0.4}(m_0)$	$1.5^{+1.0}_{-0.5}(m_c)^{+3.9}_{-1.4}(a_i)^{+0.5}_{-0.3}(m_0)$
$B_c \to K^+ f_1(1420)$	$7.4^{+0.3}_{-0.0}(m_c)^{+3.2}_{-2.8}(a_i)^{+0.4}_{-0.2}(m_0)$	$7.5^{+0.3}_{-0.0}(m_c)^{+3.2}_{-3.1}(a_i)^{+0.4}_{-0.4}(m_0)$

$B_c \to a_1 \eta^{(\prime)}$ decays, while the enhanced nonfactorizable ones cancel each other for $B_c \to b_1 \eta^{(\prime)}$ decays.

(vi) For $B_c \to \bar{K}^0(K_1(1270)^+, K_1(1400)^+)$ and $B_c \to (\bar{K}_1(1270)^0, \bar{K}_1(1400)^0)K^+$ decays, their BRs strongly depend on the value of the mixing angle θ_K of the K_{1A}-K_{1B} system. From Table II, one can see that

$$\frac{\mathrm{Br}(B_c \to \bar{K}^0 K_1(1400)^+)}{\mathrm{Br}(B_c \to \bar{K}^0 K_1(1270)^+)} \approx \frac{\mathrm{Br}(B_c \to K^+ \bar{K}_1(1400)^0)}{\mathrm{Br}(B_c \to K^+ \bar{K}_1(1270)^0)} \approx 2, \quad (49)$$

for $\theta_K = 45°$, while

$$\frac{\mathrm{Br}(B_c \to \bar{K}^0 K_1(1400)^+)}{\mathrm{Br}(B_c \to \bar{K}^0 K_1(1270)^+)} \approx \frac{\mathrm{Br}(B_c \to K^+ \bar{K}_1(1400)^0)}{\mathrm{Br}(B_c \to K^+ \bar{K}_1(1270)^0)} \approx \frac{1}{2}, \quad (50)$$

for $\theta_K = -45°$. This means that one can determine the sign and size of θ_K after enough B_c events become available at the LHC experiment.

(vii) For the $\Delta S = 1$ $B_c \to K_1 \pi$ decays, their decay rates have a very weak dependence on the value of mixing angle θ_K:

$$\mathrm{Br}(B_c \to K_1(1270)^0 \pi^+) \approx \mathrm{Br}(B_c \to K_1(1400)^0 \pi^+) \approx 6 \times 10^{-8}, \quad (51)$$

$$\mathrm{Br}(B_c \to K_1(1270)^+ \pi^0) \approx \mathrm{Br}(B_c \to K_1(1400)^+ \pi^0) \approx 3 \times 10^{-8}, \quad (52)$$

for both $\theta_K = 45°$ and $-45°$. This point will also be tested at LHC.

(viii) For $B_c \to K_1 \eta^{(\prime)}$ decays, the pQCD predictions have a strong θ_K dependence:

$$\frac{\mathrm{Br}(B_c \to K_1(1400)^+ \eta)}{\mathrm{Br}(B_c \to K_1(1270)^+ \eta)} \approx 1.6, \qquad \frac{\mathrm{Br}(B_c \to K_1(1400)^+ \eta')}{\mathrm{Br}(B_c \to K_1(1270)^+ \eta')} \approx 4.3, \quad (53)$$

for $\theta_K = 45°$, while

$$\frac{\mathrm{Br}(B_c \to K_1(1400)^+ \eta)}{\mathrm{Br}(B_c \to K_1(1270)^+ \eta)} \approx \frac{1}{1.6}, \qquad \frac{\mathrm{Br}(B_c \to K_1(1400)^+ \eta')}{\mathrm{Br}(B_c \to K_1(1270)^+ \eta')} \approx \frac{1}{4.3}, \quad (54)$$

for $\theta_K = -45°$. It is easy to see that these $B_c \to K_1 \eta^{(\prime)}$ decays are sensitive to the mixing angle θ_K. Analogous to the $B_c \to K^*(\eta, \eta')$ decays [3], the above four decays are dominated by the factorizable annihilation diagrams.

(ix) The theoretical predictions for the branching ratios of $B_c \to K_1(1270)P$ and $B_c \to K_1(1400)P$ for $\theta_K = 45°$, as listed in column two of Table II, are roughly exchanged with respect to those of the third column for the choice of $\theta_K = -45°$. Such simple relation comes from the fact that the two states $K_1(1270)$ and $K_1(1400)$ can go one into another as a mixture of K_{1A} and K_{1B} states when one sets the mixing angle $\theta_K = 45°$ or $-45°$, respectively, as can be seen from Eqs. (11) and (12),

TABLE IV. Same as Table I but for $B_c \to (h_1(1170), h_1(1380))(\pi, K)$ decays.

$\Delta S = 0$ Decay modes	BRs(10^{-8}) $\theta_1 = 10°$	BRs(10^{-8}) $\theta_1 = 45°$
$B_c \to \pi^+ h_1(1170)$	$60.5^{+28.6}_{-0.0}(m_c)^{+32.1}_{-16.9}(a_i)^{+25.8}_{-8.0}(m_0)$	$49.1^{+28.8}_{-4.9}(m_c)^{+33.7}_{-10.9}(a_i)^{+20.7}_{-5.5}(m_0)$
$B_c \to \pi^+ h_1(1380)$	$1.2^{+2.4}_{-1.0}(m_c)^{+3.7}_{-0.2}(a_i)^{+0.5}_{-0.0}(m_0)$	$12.4^{+3.9}_{-0.0}(m_c)^{+3.3}_{-3.7}(a_i)^{+5.5}_{-2.2}(m_0)$
$\Delta S = 1$ Decay modes	BRs(10^{-8}) $\theta_1 = 10°$	BRs(10^{-8}) $\theta_1 = 45°$
$B_c \to K^+ h_1(1170)$	$14.9^{+2.0}_{-1.8}(m_c)^{+12.6}_{-8.0}(a_i)^{+0.0}_{-0.3}(m_0)$	$16.8^{+5.7}_{-4.5}(m_c)^{+6.9}_{-5.7}(a_i)^{+0.3}_{-1.6}(m_0)$
$B_c \to K^+ h_1(1380)$	$22.2^{+14.5}_{-7.3}(m_c)^{+15.6}_{-13.1}(a_i)^{+0.0}_{-0.2}(m_0)$	$20.2^{+10.6}_{-4.5}(m_c)^{+11.7}_{-8.2}(a_i)^{+0.0}_{-0.8}(m_0)$

$$|K_1(1270)\rangle|_{\theta_K=45^\circ} = |K_1(1400)\rangle|_{\theta_K=-45^\circ}, \quad |K_1(1400)\rangle|_{\theta_K=45^\circ} = -|K_1(1270)\rangle|_{\theta_K=-45^\circ}. \tag{55}$$

This relation further leads to the following relations between the decay amplitudes of $B_c \to K_1 P$,

$$\mathcal{A}(B_c \to K_1(1270)P)|_{\theta_K=45^\circ} = \mathcal{A}(B_c \to K_1(1400)P)|_{\theta_K=-45^\circ}, \quad \mathcal{A}(B_c \to K_1(1400)P)|_{\theta_K=45^\circ} = -\mathcal{A}(B_c \to K_1(1270)P)|_{\theta_K=-45^\circ}, \tag{56}$$

and finally we obtain the special pattern of branching ratios as listed in Table II. The small differences in corresponding decay rates are due to the difference in the masses of $K_1(1270)$ and $K_1(1400)$ mesons. The numerical relations as shown in Eqs. (49), (50), (53), and (54) are also induced by the same mechanism.

(x) For the four $B_c \to f_1(K, \pi)$ decays, one can see from Table III that

$$\frac{\mathrm{Br}(B_c \to \pi^+ f_1(1285))}{\mathrm{Br}(B_c \to \pi^+ f_1(1420))} \approx \begin{cases} 6.2 & \text{for } \theta_3 = 38^\circ \\ 2.8 & \text{for } \theta_3 = 50^\circ \end{cases} \tag{57}$$

and

$$\frac{\mathrm{Br}(B_c \to K^+ f_1(1285))}{\mathrm{Br}(B_c \to K^+ f_1(1420))} \approx 0.2 \tag{58}$$

for $\theta_3 = 38^\circ$ and 50°. The relations in Eqs. (57) and (58) can be understood as follows: (a) Since $f_1(1285)$ and $f_1(1420)$ are the mixed states of f_1 and f_8 [see Eq. (13)] and both $\sin\theta_3$ and $\cos\theta_3$ are positive for $\theta_3 = 38^\circ$ and 50°, the contribution from the common component $(\bar{q}q)$ of f_1 and f_8 will interfere constructively (destructively) for $B_c \to \pi^+ f_1(1285)$ ($B_c \to \pi^+ f_1(1420)$) decay. This results in the large difference for the decay rate of the two decays. (b) For the two $\Delta S = 1$ decays, however, the new component $(\bar{s}s)$ will provide additional contributions to the considered decays. Furthermore, the contributions from $(\bar{s}s)$ and $\bar{q}q$ interfere constructively for $B_c \to K^+ f_1(1420)$, but destructively for $B_c \to K^+ f_1(1285)$ decay.

(xi) The pQCD predictions for $B_c \to (h_1(1170), h_1(1380))(K, \pi)$ decays, as given in Table IV, can be explained in a similar way as for $B_c \to (f_1(1285), f_1(1400))(K, \pi)$ decays.

(xii) Since the LHC experiment can measure the B_c decays with a branching ratio at the 10^{-6} level [2], our pQCD predictions for the branching ratios of $B_c \to K(K_1(1270), K_1(1400))$ and $b_1\pi$ decays could be tested in the forthcoming LHC experiments.

It is worth stressing that the theoretical predictions in the pQCD approach still have large theoretical errors induced by the still large uncertainties of many input parameters, e.g., Gegenbauer moments a_i. For most considered pure annihilation B_c decays, it is hard to observe them even in LHC due to their tiny decay rate. Their observation at LHC, however, would mean a large nonperturbative contribution or a signal for new physics beyond the SM.

We here calculated the branching ratios of the pure annihilation $B_c \to AP$ decays by employing the pQCD approach. We do not consider the possible long-distance contributions, such as the rescattering effects, although they may be large and affect the theoretical predictions. They are beyond the scope of this work.

V. SUMMARY

In short, we studied the charmless hadronic $B_c \to AP$ decays by employing the pQCD factorization approach based on the k_T factorization theorem. These considered decay channels can occur only via the annihilation diagram in the SM and they will provide an important platform for testing the magnitude of the annihilation contribution and understanding the content of the axial-vector mesons.

The pQCD predictions for CP-averaged branching ratios are displayed in Tables I, II, III, and IV. From our numerical evaluations and phenomenological analysis, we found the following results:

(i) The pQCD predictions for the branching ratios vary in the range of 10^{-6} to 10^{-8}. The $B_c \to \bar{K}^0(K_1(1270)^+, K_1(1400)^+)$ and other decays with a decay rate at 10^{-6} or larger could be measured at the LHC experiment.

(ii) For $B_c \to AP$ decays, the branching ratios of $\Delta S = 0$ processes are generally much larger than those of $\Delta S = 1$ ones. Such differences are mainly induced by the CKM factors involved: $V_{ud} \sim 1$ for the former decays, while $V_{us} \sim 0.22$ for the latter ones.

(iii) Since the behavior for the $^1P_1$ meson is much different from that for the $^3P_1$ meson, the branching ratios of pure annihilation $B_c \to A(^1P_1)P$ are basically larger than those of $B_c \to A(^3P_1)P$, which can be tested in the LHC and Super-B experiments.

(iv) The pQCD predictions about the branching ratios of $B_c \to K_1\eta^{(\prime)}$ and K_1K decays are rather sensitive to the value of the mixing angle θ_K. One can determine θ_K through the measurement of these decays if enough B_c events become available at the LHC experiment.

(v) The pQCD predictions still have large theoretical uncertainties, mainly induced by the uncertainties of the Gegenbauer moments a_i in the meson distribution amplitudes.

(vi) Because only tree operators are involved, the CP-violating asymmetries for these considered B_c decays are absent naturally.

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APPENDIX: INPUT PARAMETERS AND DISTRIBUTION AMPLITUDES

The masses (GeV), decay constants (GeV), QCD scale (GeV), and B_c meson lifetime (ps) are

$$\begin{aligned}
&\Lambda^{(f=4)}_{\overline{\rm MS}} = 0.250, \quad m_W = 80.41, \quad m_{B_c} = 6.286, \quad f_{B_c} = 0.489, \quad m_{a_1} = 1.23, \quad f_{a_1} = 0.238,\\
&m_{b_1} = 1.21, \quad f_{b_1} = 0.180, \quad m_{K_{1A}} = 1.32, \quad f_{K_{1A}} = 0.250 \quad m_{K_{1B}} = 1.34, \quad f_{K_{1B}} = 0.190,\\
&f_{f_1} = 0.245, \quad m_{f_1} = 1.28, \quad f_{f_8} = 0.239, \quad m_{f_8} = 1.28, \quad f_{h_1} = 0.180, \quad m_{h_1} = 1.23,\\
&f_{h_8} = 0.190, \quad m_{h_8} = 1.37, \quad m_0^{\pi} = 1.4, \quad m_0^K = 1.6, \quad m_0^{\eta_q} = 1.08, \quad m_0^{\eta_s} = 1.92,\\
&m_b = 4.8, \quad f_\pi = 0.131, \quad f_K = 0.16, \quad \tau_{B_c^+} = 0.46.
\end{aligned} \tag{A1}$$

For the CKM matrix elements, here we adopt the Wolfenstein parametrization for the CKM matrix, and take $A = 0.814$ and $\lambda = 0.2257$, $\bar{\rho} = 0.135$ and $\bar{\eta} = 0.349$ [19].

For the distribution amplitudes of pseudoscalar mesons, we adopt the same forms as used in the literature (see Ref. [3] and references therein).

The twist-2 distribution amplitudes for the longitudinally polarized axial-vector ${}^3P_1$ and ${}^1P_1$ mesons can be parametrized as [13,18]

$$\phi_A(x) = \frac{f}{2\sqrt{2N_c}}\left\{6x(1-x)\left[a_0^{\parallel} + 3a_1^{\parallel} t + a_2^{\parallel}\frac{3}{2}(5t^2-1)\right]\right\}. \tag{A2}$$

As for twist-3 light-cone distribution amplitudes, we use the following form:

$$\phi_A^t(x) = \frac{3f}{2\sqrt{2N_c}}\left\{a_0^{\perp} t^2 + \frac{1}{2}a_1^{\perp} t(3t^2-1)\right\}, \tag{A3}$$

$$\phi_A^s(x) = \frac{3f}{2\sqrt{2N_c}}\frac{d}{dx}\{x(1-x)(a_0^{\perp} + a_1^{\perp} t)\}, \tag{A4}$$

where f is the decay constant and $t = 2x - 1$. It should be noted that for the distribution amplitudes of strange axial-vector mesons K_{1A} and K_{1B}, x stands for the momentum fraction carrying by the s quark.

Here, the definition of these distribution amplitudes $\phi_A(x)$ satisfies the following relation:

$$\int_0^1 \phi_{3P_1}(x) = \frac{f}{2\sqrt{2N_c}}, \qquad \int_0^1 \phi_{1P_1}(x) = a_0^{\parallel, {}^1P_1}\frac{f}{2\sqrt{2N_c}}, \tag{A5}$$

where we have used $a_0^{\parallel, {}^3P_1} = 1$.

The Gegenbauer moments have been studied extensively in the literature (see Ref. [13] and references therein). Here we adopt the following values:

$$\begin{aligned}
&a_2^{\parallel, a_1} = -0.02 \pm 0.02; \quad a_1^{\perp, a_1} = -1.04 \pm 0.34; \quad a_1^{\parallel, b_1} = -1.95 \pm 0.35; \quad a_2^{\parallel, f_1} = -0.04 \pm 0.03;\\
&a_1^{\perp, f_1} = -1.06 \pm 0.36; \quad a_1^{\parallel, h_1} = -2.00 \pm 0.35; \quad a_2^{\parallel, f_8} = -0.07 \pm 0.04; \quad a_1^{\perp, f_8} = -1.11 \pm 0.31;\\
&a_1^{\parallel, h_8} = -1.95 \pm 0.35; \quad a_1^{\parallel, K_{1A}} = 0.00 \pm 0.26; \quad a_2^{\parallel, K_{1A}} = -0.05 \pm 0.03; \quad a_0^{\perp, K_{1A}} = 0.08 \pm 0.09;\\
&a_1^{\perp, K_{1A}} = -1.08 \pm 0.48; \quad a_0^{\parallel, K_{1B}} = 0.14 \pm 0.15; \quad a_1^{\parallel, K_{1B}} = -1.95 \pm 0.45; \quad a_2^{\parallel, K_{1B}} = 0.02 \pm 0.10;\\
&a_1^{\perp, K_{1B}} = 0.17 \pm 0.22.
\end{aligned} \tag{A6}$$

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