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Understanding the branching ratios of $\chi_{c1} \to \phi\phi$, $\omega\omega$, $\omega\phi$ observed at BES-III

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In this work, we discuss the contribution of the mesonic loops to the decay rates of $\chi_{c1} \to \phi\phi$, $\omega\omega$, which are suppressed by the helicity selection rules and $\chi_{c1} \to \phi\omega$, which is a double-Okubo-Zweig-Iizuka forbidden process. We find that the mesonic loop effects naturally explain the clear signals of $\chi_{c1} \to \phi\phi$, $\omega\omega$ decay modes observed by the BES Collaboration. Moreover, we investigate the effects of the $\omega - \phi$ mixing, which may result in the order of magnitude of the branching ratio $\mathrm{BR}(\chi_{c1} \to \omega\phi)$ being 10^{-7}. Thus, we are waiting for the accurate measurements of the $\mathrm{BR}(\chi_{c1} \to \omega\omega)$, $\mathrm{BR}(\chi_{c1} \to \phi\phi)$, and $\mathrm{BR}(\chi_{c1} \to \omega\phi)$, which may be very helpful for testing the long-distant contribution and the $\omega - \phi$ mixing in $\chi_{c1} \to \phi\phi$, $\omega\omega$, $\omega\phi$ decays.

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I. INTRODUCTION

Charm physics is an active field full of chances and challenges [1]. Decays of charmonia may provide an ideal laboratory to study perturbative as well as nonperturbative QCD. Until now, most of hadronic decays of P-wave charmonium states χ_{cJ} ($J = 0$, 1, 2) have not been well understood compared to J/ψ decays, so they have become of great interest to experimentalists and theorists to further explore the decay behavior of χ_{cJ}.

In the Hadron 2009 conference, the BES-III Collaboration announced its observations of χ_{cJ} ($J = 0$, 1, 2) decaying into light vector mesons, where the data of χ_{cJ} are taken from 110×10^6 radiative-decay events of $\psi(2S)$ collected by the BES-III. Among those decay modes of χ_{cJ} decaying into light vector mesons, $\chi_{c1} \to \phi\phi$, $\omega\omega$ processes were measured for the first time, and the doubly Okubo-Zweig-Iizuka (OZI) suppressed process $\chi_{cJ} \to \phi\omega$ had not been measured before the BES-III observation [2].

Generally, the decays of χ_{c1} into two light vector mesons are suppressed compared to the corresponding decays of χ_{c0} and χ_{c2} due to the helicity selection rule [3]. Besides, $\chi_{c1} \to \omega\phi$ suffers from the double-OZI suppression [4–7]. Thus, in this sense, the branching ratios of the channels $\chi_1 \to \phi\phi$, $\omega\omega$, $\phi\omega$ should be small, and it would be difficult to observe them in experiments, especially $\chi_{c1} \to \phi\omega$. However, the observation of $\chi_{c1} \to \phi\phi$, $\omega\omega$, $\phi\omega$ seems to be surprising and compel us to reconsider what mechanism plays the dominant role in those decays. It would be definitely different from that responsible for $\chi_{c1} \to \phi\phi$, $\omega\omega$, $\phi\omega$ decays. To understand the governing mechanism that results in sizable ratios for $\chi_{c1} \to \phi\phi$, $\omega\omega$, $\phi\omega$, two questions must be answered: (1) what is the source to alleviate the helicity selection rule for χ_{c1}; (2) why the double-OZI suppression is violated for $\chi_{c1} \to \phi\omega$ decay.

The conventional decay mechanism depicting χ_{cJ} into two light vector mesons is that c and $\bar{c}$ annihilate into a pair of gluons, which then transit into quark-antiquark pairs to form the light vector mesons in the final state. The helicity selection rule manifests in the χ_{cJ} decays, and results in the suppression of χ_{c1} decaying into two light mesons. In χ_{c1} decays, the nonperturbative QCD effect plays a crucial role. As an important nonperturbative effect, the hadronic loop contributions, which were introduced in Refs. [8,9] and applied to study charmonium decay [10–12] and open-charm and hidden charm decays of charmoniumlike states X, Y, Z [13–15] extensively, would change the whole scenario from the conventional decay mechanism of charmonium.

For χ_{c1} into two light vector mesons, a quark level description of the hadronic loop contribution is presented in Fig. 1. Here, the red fermion line denotes charm quark, and the blue and green lines represent the light quarks. χ_{c1} first dissolves into two virtual charmed mesons, then by exchanging an appropriate hadron (i.e. it possesses appropriate charge, flavor, spin, and isospin) they turn into two on-shell real light hadrons, which can be caught by a detector [9]. The matrix element of χ_{c1} into two light vector mesons via hadronic loop effect can be described as

$$\mathcal{M}(\chi_{c1} \to VV) = \sum_i \langle VV|\mathcal{H}^{(2)}|i\rangle\langle i|\mathcal{H}^{(1)}|\chi_{c1}\rangle. \quad (1)$$

The depiction at the hadron level corresponding to the quark level diagrams are presented in Fig. 1. Here, the suitable intermediated charmed mesons for the decay of χ_{c1} into two light vector mesons should be $D_{(s)}\bar{D}^*_{(s)}$ + H.c., which interact with χ_{c1} via the S wave. The exchanged mesons include pseudoscalar and vector charmed mesons.

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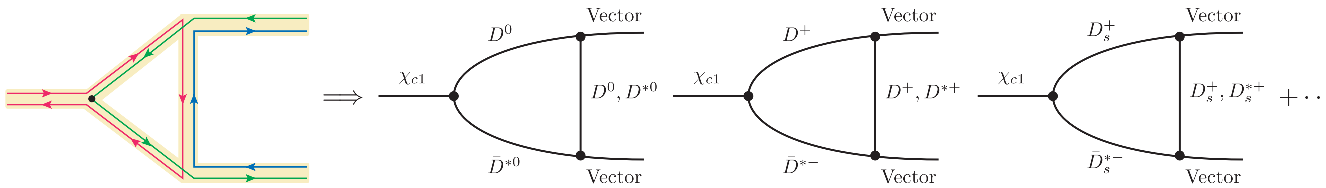

FIG. 1 (color online). The diagrams of hadronic loop contributions to $\chi_{c1} \to$ vector + vector mesons depicted at the quark level and hadron level. Here, ''Vector'' means the light vector meson and ''ellipsis'' denotes other diagrams at the hadron level, which can be obtained by performing a charge conjugation $D_{(s)}^{(*)} \rightleftharpoons \bar{D}_{(s)}^{(*)}$.

Thus, by the hadronic loop mechanism, the transition of χ_{c1} into two light vector mesons would not be suppressed by the helicity selection rule.

Another important motivation is how $\chi_{c1} \to \phi\omega$ evades the double-OZI suppression, assuming ω and ϕ mesons are ideal mixtures of the flavor $SU(3)$ octet $\omega_8 = (u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6}$ and the singlet $\phi_0 = (u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3}$. In terms of the hadronic loop mechanism, such $\chi_{c1} \to \phi\omega$ decay is fully forbidden. In reality ω and ϕ are not ideal mixtures of the flavor $SU(3)$ octet and singlet [16–21], which would provide a source that violates the double-OZI suppression rule for $\chi_{c1} \to \phi\omega$.

In this work, we will be combining the hadronic loop effect with the $\omega - \phi$ mixing to study $\chi_{c1} \to \phi\phi$, $\omega\omega$, $\phi\omega$. The paper is organized as follows: After the introduction, we present the formula of hadronic loop contribution to $\chi_{c1} \to \phi\phi$, $\omega\omega$, $\phi\omega$ with the mixing schemes for $\omega - \phi$. Then, the numerical results about $\chi_{c1} \to \omega\omega$, $\phi\phi$, $\phi\omega$ are given in Sec. 1. Finally, the paper ends with a discussion and a short summary.

II. HADRONIC LOOP EFFECT ON $\chi_{c1} \to VV$ DECAYS UNDER TWO DIFFERENT MIXING SCHEMES OF $\omega - \phi$

First, we present the mixing scheme for $\omega - \phi$ used in this work

$$\begin{pmatrix} |\phi^p\rangle \\ |\omega^p\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |\phi^I\rangle \\ |\omega^I\rangle \end{pmatrix}, \tag{2}$$

where $|\phi^p\rangle$ ($|\omega^p\rangle$), and $|\phi^I\rangle$ ($|\omega^I\rangle$) are the physical and ideally mixing states, respectively. The flavor wave functions for the ideally mixing states $|\omega^I\rangle$ and $|\phi^I\rangle$ are $\omega^I = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $\phi^I = -s\bar{s}$. Taking mixing angle $\theta = 0°$ corresponds to the ideal mixing. According to the analysis in Refs. [16,17,22], the mixing angle θ should be $(3.4 \pm 0.2)°$ in the mixing scheme in Eq. (2).

The effective Lagrangians, which are responsible for the decay amplitudes for the diagrams in Fig. 1, are listed below [23–28]

$$\mathcal{L}_{\chi_{c1}\mathcal{D}\mathcal{D}^*} = ig_{\chi_{c1}\mathcal{D}\mathcal{D}^*}\chi_{c1} \cdot \mathcal{D}_i^{*\dagger}\mathcal{D}^i + \text{H.c.}, \tag{3}$$

$$\begin{aligned} \mathcal{L}_{H\bar{H}} = {} & i\langle H_b v^\mu \mathcal{D}_{\mu ba}\bar{H}_a\rangle + ig\langle H_b \gamma_\mu \gamma_5 A^\mu_{ba}\bar{H}_a\rangle \\ & + i\beta\langle H_b v^\mu (V_\mu - \rho_\mu)_{ba}\bar{H}_a\rangle \\ & + i\lambda\langle H_b \sigma^{\mu\nu}F_{\mu\nu}(\rho)_{ba}\bar{H}_a\rangle, \end{aligned} \tag{4}$$

where Eq. (4) is constructed under the chiral and heavy quark limits. The superfield H is given by $H = \frac{1+\not{v}}{2} \times (\mathcal{D}^{*\mu}\gamma_\mu - i\gamma_5\mathcal{D})$ and $\bar{H} = \gamma^0 H^\dagger \gamma^0$. $(V_\mu)_{ba}$ and $(A_\mu)_{ba}$ denote the matrix elements for vector and axial currents, respectively. The expansion in Eq. (4), which is related to the hadronic loop calculation, includes

$$\begin{aligned} \mathcal{L}_{\mathcal{D}^{(*)}\mathcal{D}^{(*)}\mathcal{V}} = {} & -ig_{\mathcal{D}\mathcal{D}\mathcal{V}}\mathcal{D}_i^\dagger \overleftrightarrow{\partial}_\mu \mathcal{D}^j(\mathcal{V}^\mu)^i_j \\ & - 2f_{\mathcal{D}^*\mathcal{D}\mathcal{V}}\epsilon_{\mu\nu\alpha\beta}(\partial^\mu\mathcal{V}^\nu)^i_j(\mathcal{D}_i^\dagger \overleftrightarrow{\partial}^\alpha \mathcal{D}^{*\beta j} \\ & - \mathcal{D}_i^{*\beta\dagger}\overleftrightarrow{\partial}^\alpha\mathcal{D}^j) \\ & + ig_{\mathcal{D}^*\mathcal{D}^*\mathcal{V}}\mathcal{D}_i^{*\nu\dagger}\overleftrightarrow{\partial}_\mu\mathcal{D}_\nu^{*j}(\mathcal{V}^\mu)^i_j \\ & + 4if_{\mathcal{D}^*\mathcal{D}^*\mathcal{V}}\mathcal{D}_{i\mu}^{*\dagger}(\partial^\mu\mathcal{V}^\nu - \partial^\nu\mathcal{V}^\mu)^i_j\mathcal{D}_\nu^{*j}, \end{aligned} \tag{5}$$

where $\mathcal{D}^{(*)\dagger} = (\bar{D}^{(*)0}, D^{(*)-}, D_s^{(*)-})$. The coupling constants relevant to the calculation include $g_{\mathcal{D}\mathcal{D}\mathcal{V}} = g_{\mathcal{D}^*\mathcal{D}^*\mathcal{V}} = \beta g_V/\sqrt{2}$, $f_{\mathcal{D}^*\mathcal{D}\mathcal{V}} = f_{\mathcal{D}^*\mathcal{D}^*\mathcal{V}}/m_{\mathcal{D}^*} = \lambda m_\rho/(\sqrt{2}f_\pi)$ with $\beta = 0.9$, $\lambda = 0.56\ \text{GeV}^{-1}$ and $f_\pi = 132$ MeV [23–26]. $g_{\chi_{c1}\mathcal{D}\mathcal{D}^*} = 21.4$ GeV for the D meson and $g_{\chi_{c1}\mathcal{D}\mathcal{D}^*} = 22.6$ GeV for the D_s meson are determined in Ref. [28]. Introducing the mixing scheme of $\omega - \phi$ as shown in Eq. (2), one defines the 3×3 matrix of the nonet vector mesons $\mathcal{V}$ as

$$\mathcal{V} = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \kappa\omega^p + \zeta\phi^p & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \kappa\omega^p + \zeta\phi^p & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \delta\omega^p + \sigma\phi^p \end{pmatrix}.$$

If we pre-assume that the mixing parameters in the matrix are not independent, but related to each other by a single variable θ, the coefficients κ, ζ, δ, σ are written as

$$\begin{aligned} \kappa &= \frac{1}{\sqrt{2}}\cos\theta, & \zeta &= \frac{1}{\sqrt{2}}\sin\theta, \\ \delta &= -\sin\theta, & \sigma &= \cos\theta. \end{aligned} \tag{6}$$

Thus, the decay amplitudes of $\chi_{c1} \to \omega\omega$, $\phi\phi$, $\phi\omega$ due to the hadronic loop effect are written as

$$\mathcal{M}[\chi_{c1}\to\omega\omega] = 2\begin{pmatrix}\kappa^2 & \kappa^2 & \delta^2\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}\cdot\begin{pmatrix}\mathcal{M}^{(P)}[D^-, D^{*+}, D^-, \omega, \omega]\\ \mathcal{M}^{(P)}[\bar{D}^0, D^{*0}, \bar{D}^0, \omega, \omega]\\ \mathcal{M}^{(P)}[D_s^-, D_s^{*+}, D_s^-, \omega, \omega]\end{pmatrix} + 2\begin{pmatrix}\kappa^2 & \kappa^2 & \delta^2\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}\cdot\begin{pmatrix}\mathcal{M}^{(V)}[D^-, D^{*+}, D^{*-}, \omega, \omega]\\ \mathcal{M}^{(V)}[\bar{D}^0, D^{*0}, \bar{D}^{*0}, \omega, \omega]\\ \mathcal{M}^{(V)}[D_s^-, D_s^{*+}, D_s^{*-}, \omega, \omega]\end{pmatrix}, \tag{7}$$

$$\mathcal{M}[\chi_{c1}\to\phi\phi] = 2\begin{pmatrix}0 & 0 & 0\\ \zeta^2 & \zeta^2 & \sigma^2\\ 0 & 0 & 0\end{pmatrix}\cdot\begin{pmatrix}\mathcal{M}^{(P)}[D^-, D^{*+}, D^-, \phi, \phi]\\ \mathcal{M}^{(P)}[\bar{D}^0, D^{*0}, \bar{D}^0, \phi, \phi]\\ \mathcal{M}^{(P)}[D_s^-, D_s^{*+}, D_s^-, \phi, \phi]\end{pmatrix} + 2\begin{pmatrix}0 & 0 & 0\\ \zeta^2 & \zeta^2 & \sigma^2\\ 0 & 0 & 0\end{pmatrix}\cdot\begin{pmatrix}\mathcal{M}^{(V)}[D^-, D^{*+}, D^{*-}, \phi, \phi]\\ \mathcal{M}^{(V)}[\bar{D}^0, D^{*0}, \bar{D}^{*0}, \phi, \phi]\\ \mathcal{M}^{(V)}[D_s^-, D_s^{*+}, D_s^{*-}, \phi, \phi]\end{pmatrix}, \tag{8}$$

$$\begin{aligned}\mathcal{M}[\chi_{c1}\to\phi\omega] = {}& 2\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ \kappa\zeta & \kappa\zeta & \delta\sigma\end{pmatrix}\cdot\begin{pmatrix}\mathcal{M}^{(P)}[D^-, D^{*+}, D^-, \omega, \phi]\\ \mathcal{M}^{(P)}[\bar{D}^0, D^{*0}, \bar{D}^0, \omega, \phi]\\ \mathcal{M}^{(P)}[D_s^-, D_s^{*+}, D_s^-, \omega, \phi]\end{pmatrix} + 2\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ \kappa\zeta & \kappa\zeta & \delta\sigma\end{pmatrix}\\ & \cdot\begin{pmatrix}\mathcal{M}^{(P)}[D^-, D^{*+}, D^-, \phi, \omega]\\ \mathcal{M}^{(P)}[\bar{D}^0, D^{*0}, \bar{D}^0, \phi, \omega]\\ \mathcal{M}^{(P)}[D_s^-, D_s^{*+}, D_s^-, \phi, \omega]\end{pmatrix} + 2\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ \kappa\zeta & \kappa\zeta & \delta\sigma\end{pmatrix}\cdot\begin{pmatrix}\mathcal{M}^{(V)}[D^-, D^{*+}, D^{*-}, \omega, \phi]\\ \mathcal{M}^{(V)}[\bar{D}^0, D^{*0}, \bar{D}^{*0}, \omega, \phi]\\ \mathcal{M}^{(V)}[D_s^-, D_s^{*+}, D_s^{*-}, \omega, \phi]\end{pmatrix}\\ & + 2\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ \kappa\zeta & \kappa\zeta & \delta\sigma\end{pmatrix}\cdot\begin{pmatrix}\mathcal{M}^{(V)}[D^-, D^{*+}, D^{*-}, \phi, \omega]\\ \mathcal{M}^{(V)}[\bar{D}^0, D^{*0}, \bar{D}^{*0}, \phi, \omega]\\ \mathcal{M}^{(V)}[D_s^-, D_s^{*+}, D_s^{*-}, \phi, \omega]\end{pmatrix},\end{aligned} \tag{9}$$

where the factor 2 is from the charge conjugation transformation. In the above expressions, $\mathcal{M}^{(P)}[\star, \cdots, \star]$ and $\mathcal{M}^{(V)}[\star, \cdots, \star]$ denote the amplitudes corresponding to pseudoscalar and vector charmed meson exchanges. The general expressions of the amplitudes are

$$\begin{aligned}\mathcal{M}^{(P)}[A(p_1), B(p_2), C(q), D(p_3), E(p_4)] = {}& \int\frac{d^4q}{(2\pi)^4}[ig_{\chi_{c1}\mathcal{D}\mathcal{D}^*}\epsilon_\sigma][-ig_{\mathcal{D}\mathcal{D}\mathcal{V}}(p_1+q)\cdot\epsilon_3][2if_{\mathcal{D}^*\mathcal{D}\mathcal{V}}\varepsilon_{\mu\nu\alpha\beta}p_4^\mu\epsilon_4^\nu(q^\alpha - p_2^\alpha)]\\ & \times\frac{i}{p_1^2-m_{\mathcal{D}}^2}\frac{i}{p_2^2-m_{\mathcal{D}^*}^2}\left(-g^{\sigma\beta}+\frac{p_2^\sigma p_2^\beta}{m_{\mathcal{D}^*}^2}\right)\frac{i}{q^2-m_{\mathcal{D}}^2}\mathfrak{F}_N^2(q^2, m_{\mathcal{D}}^2),\end{aligned} \tag{10}$$

$$\begin{aligned}\mathcal{M}^{(V)}[A(p_1), B(p_2), C(q), D(p_3), E(p_4)] = {}& \int\frac{d^4q}{(2\pi)^4}[ig_{\chi_{c1}\mathcal{D}\mathcal{D}^*}\epsilon_\sigma][2if_{\mathcal{D}^*\mathcal{D}\mathcal{V}}\varepsilon_{\mu\nu\alpha\beta}p_3^\mu\epsilon_3^\nu(p_1^\alpha+q^\alpha)][ig_{\mathcal{D}^*\mathcal{D}^*\mathcal{V}}(q-p_2)\\ & \cdot\epsilon_4 g_{\lambda\kappa} - 4if_{\mathcal{D}^*\mathcal{D}^*\mathcal{V}}(p_{4\kappa}\epsilon_{4\lambda} - p_{4\lambda}\epsilon_{4\kappa})]\frac{i}{p_1^2-m_{\mathcal{D}}^2}\frac{i}{p_2^2-m_{\mathcal{D}^*}^2}\left(-g^{\sigma\kappa}+\frac{p_2^\sigma p_2^\kappa}{m_{\mathcal{D}^*}^2}\right)\\ & \times\frac{i}{q^2-m_{\mathcal{D}^*}^2}\left(-g^{\beta\lambda}+\frac{q^\beta q^\lambda}{m_{\mathcal{D}^*}^2}\right)\mathfrak{F}_N^2(q^2, m_{\mathcal{D}^*}^2),\end{aligned} \tag{11}$$

which correspond to pseudoscalar and vector charmed meson exchanges, respectively. Here, A and B denote the intermediated charmed mesons. C is the exchanged charmed meson. D and E mean the light vector mesons in the final states. We adopt the form factor with the pole form

$$\mathfrak{F}_N(q^2, m^2) = \left(\frac{\Lambda^2 - m^2}{\Lambda^2 - q^2}\right)^N, \tag{12}$$

which depicts the inner structure of the effective vertex of the exchanged charmed meson and intermediated states. Meanwhile, the form factor with pole form also plays the role to make the ultraviolet divergence disappear, in analog to the cutoffs in the Pauli-Villas renormalization scheme. Here, the cutoff Λ can be parameterized as $\Lambda = m + \alpha\Lambda_{\rm QCD}$ with $\Lambda_{\rm QCD} = 220$ MeV and m is the mass of the exchanged meson [29].

III. NUMERICAL RESULTS

As a free parameter, α is introduced by the cutoff Λ. The value is usually dependent on the particular process and taken to be of the order of unity. The BES-II Collaboration

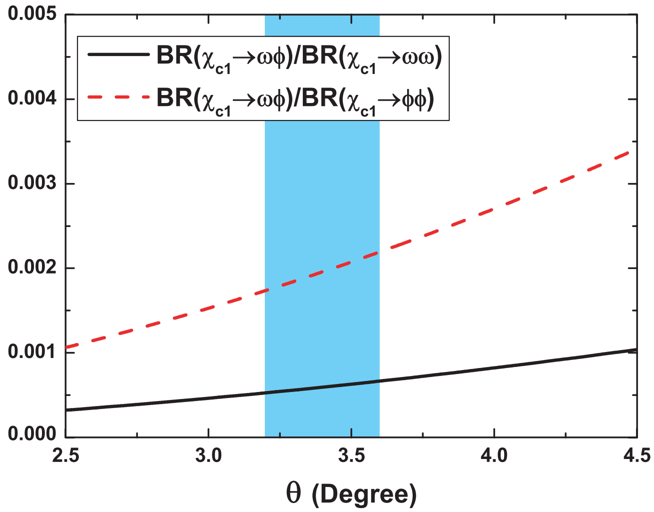

FIG. 2 (color online). The θ dependence of $\mathrm{BR}(\chi_{c1} \to \omega\phi)/\mathrm{BR}(\chi_{c1} \to \omega\omega)$ and $\mathrm{BR}(\chi_{c1} \to \omega\phi)/\mathrm{BR}(\chi_{c1} \to \phi\phi)$ without $SU(3)_f$ flavor symmetry breaking effect on $\omega - \phi$ mixing.

reported the branching ratio of $\chi_{c1} \to K^{*0}(892)\bar{K}^{*0}(892)$ as $\mathrm{BR}[\chi_{c1} \to K^{*0}\bar{K}^{*0}] = (1.67 \pm 0.32 \pm 0.31) \times 10^{-3}$ [30], which can be applied to determine α assuming that the hadronic loop effect is dominant in the process $\chi_{c1} \to K^{*0}\bar{K}^{*0}$. The formula of hadronic loop contribution to $\chi_{c1} \to K^{*0}\bar{K}^{*0}$ is similar to that for $\chi_{c1} \to \omega\omega$, $\phi\phi$, $\phi\omega$ decays, where the exchanged charmed meson matching to intermediated states $D^+\bar{D}^{*-}$ + H.c. and $D_s^+\bar{D}_s^{*-}$ + H.c. are $D_s^{(*)+}$ and $D^{(*)}$, respectively. Our study indicates that the dipole form factor, i.e. taking $N = 2$ in Eq. (12), can well reproduce the branching ratio of $\chi_{c1} \to K^{*0}\bar{K}^{*0}$ with $\alpha = 1.14 \sim 1.28$, which seems to be reasonable.[1]

In Fig. 2 we show the dependence of the ratios of $\mathrm{BR}(\chi_{c1} \to \omega\phi)/\mathrm{BR}(\chi_{c1} \to \phi\phi)$ and $\mathrm{BR}(\chi_{c1} \to \omega\phi)/\mathrm{BR}(\chi_{c1} \to \omega\omega)$ on θ considering the $\omega - \phi$ mixing. In our calculations, we find these two ratios weakly depend on the parameter α. In the figure a typical value of $\alpha = 1.20$ is employed. For the ideal mixing, i.e. $\theta = 0°$, the coefficients ζ and δ are zero, thus, the mesonic loop contribution to the decay $\chi_{c1} \to \omega\phi$ vanishes. Even though considering the deviation from the ideal mixing, the ratios of $\mathrm{BR}(\chi_{c1} \to \omega\phi)/\mathrm{BR}(\chi_{c1} \to \phi\phi)$ and $\mathrm{BR}(\chi_{c1} \to \omega\phi)/\mathrm{BR}(\chi_{c1} \to \omega\omega)$ are still rather small. In the region $\theta = (3.4 \pm 0.2)°$, the branching ratio of $\chi_{c1} \to \omega\phi$ is three orders smaller than those of $\chi_{c1} \to \omega\omega$ and $\chi_{c1} \to \phi\phi$. In Fig. 3, we present the θ and α dependence of the branching ratios of $\chi_{c1} \to \omega\omega$, $\chi_{c1} \to \phi\phi$, and $\chi_{c1} \to \omega\phi$. The corresponding ranges of $\mathrm{BR}(\chi_{c1} \to \omega\omega)$,

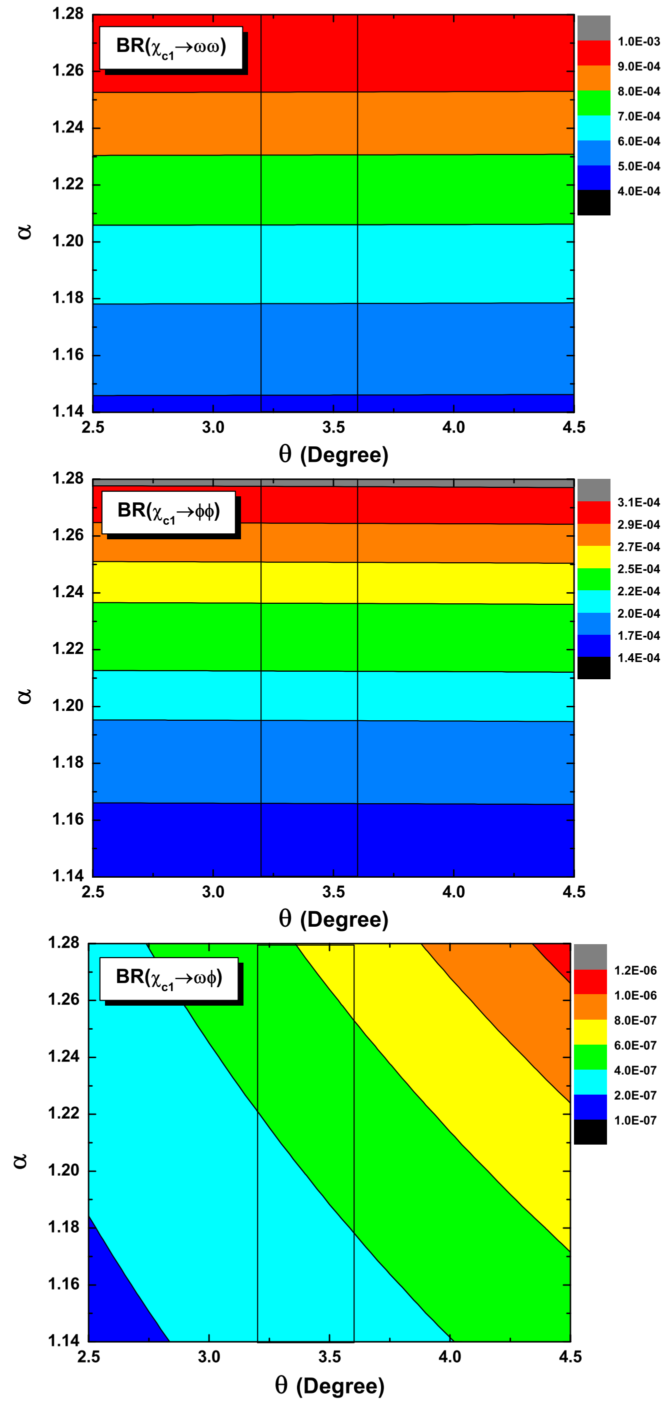

FIG. 3 (color online). The contour plot for the dependence of $\mathrm{BR}(\chi_{c1} \to \omega\omega)$, $\mathrm{BR}(\chi_{c1} \to \phi\phi)$ and $\mathrm{BR}(\chi_{c1} \to \omega\phi)$ on θ and α. Here, the range sandwiched between two vertical solid lines is allowed by the 1-sigma standard deviation of the mixing angle θ.

[1]In our calculation, we also tried to fit the measured $B(\chi_{c1} \to K^{*0}\bar{K}^{*0})$ by adopting the monopole form factor [setting $N = 1$ in Eq. (12)]. Although we can also describe the $\chi_{c1} \to K^{*0}\bar{K}^{*0}$ data, the obtained value of α is far away from order of unity. Thus it is more reasonable not to take the monopole form factor in our calculation. Then, we choose the dipole form factor instead. In Ref. [29], Cheng *et al.* preferred the $N = 1$ monopole form factor, whereas, in our work we adopt $N = 2$. The difference between Ref. [29] and this work is due to that the intermediate states of the hadronic loop for the processes discussed in Ref. [29] can be on shell, while the intermediate states in this work are off shell.

TABLE I. The ranges of the branching ratios of $\chi_{c1} \to \omega\omega$, $\phi\phi$, $\omega\phi$. Here, the experimental data of $\chi_{c1} \to K^{*0}\bar{K}^{*0}$ provide the central value with an error tolerance. The range in the $\chi_{c1} \to K^{*0}\bar{K}^{*0}$ is determined by the error existing in the experimental data. The experimental error also results in the range of α, i.e. $\alpha = 1.14 \sim 1.28$. By this determined range of α and considering the range of mixing angle [$\theta = (3.4 \pm 0.2)°$], we give the possible range of the branching ratios of $\chi_{c1} \to \omega\omega$, $\phi\phi$, $\omega\phi$.

Channel $\chi_{c1} \to$	Branching ratio	Experimental value
$K^{*0}\bar{K}^{*0}$	$(10.83 \sim 23.25) \times 10^{-4}$	$(16.7 \pm 3.2 \pm 3.1) \times 10^{-4}$
$\omega\omega$	$(4.822 \sim 10.366) \times 10^{-4}$	-
$\phi\phi$	$(1.465 \sim 3.238) \times 10^{-4}$	-
$\omega\phi$	$(2.542 \sim 6.893) \times 10^{-7}$	-

BR($\chi_{c1} \to \phi\phi$), BR($\chi_{c1} \to \omega\omega$) with $\alpha = 1.14 \sim 1.28$ and $\theta = (3.4 \pm 0.2)°$ are listed in the second column of Table I. In the table, one can notice, the branching ratio of $\chi_{c1} \to \omega\phi$ is at most of the order 10^{-7}.

We need to emphasize that the ratios of BR($\chi_{c1} \to \phi\phi$)/BR($\chi_{c1} \to \omega\omega$), BR($\chi_{c1} \to \omega\phi$)/BR($\chi_{c1} \to \omega\omega$), and BR($\chi_{c1} \to \omega\phi$)/BR($\chi_{c1} \to \phi\phi$) are not sensitive to the parameter α corresponding to the $\omega - \phi$ mixing. Furthermore, these two ratios should be independent of the coupling constants, thus the measurement on their values may provide an ideal opportunity to test the $\omega - \phi$ mixing.

IV. DISCUSSION AND SUMMARY

To summarize, in this work, we study the mesonic loop contributions and the $\omega - \phi$ mixing effect to the branching ratios of $\chi_{c1} \to \omega\omega$, $\phi\phi$ and $\omega\phi$. From the results, one can note that the $\omega - \phi$ mixing plays an important role in the understanding of the clear signal for $\chi_{c1} \to \omega\phi$ observed in experiments. Our results also indicate that accurate measurements on the ratios BR($\chi_{c1} \to \omega\phi$)/BR($\chi_{c1} \to \omega\omega$) and BR($\chi_{c1} \to \omega\phi$)/BR($\chi_{c1} \to \phi\phi$) are very helpful for checking mesonic loop contributions and the $\omega - \phi$ mixing effect.

It is noted from the figures we presented in the text that the uncertainties in the theoretical computations originate from the errors of the data; therefore, more accurate measurements are necessary for further studies. Fortunately, a large database on such rare decay modes will be available at BES-III, which will help to draw more solid conclusions.

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Notes added.—Very recently, a similar work [31] appeared in the arXiv submitted by X. H. Liu and Q. Zhao when this manuscript was close to completion. In Ref. [31], the authors calculated $\chi_{c1} \to VV$ and $\chi_{c2} \to VP$ processes by taking hadronic loop effect into account. Then, the branching ratios of $\chi_{c1} \to \rho\rho$, $\omega\omega$, $\phi\phi$ are obtained. In our work, we mainly focused on $\chi_{c1} \to \omega\omega$, $\phi\phi$, $\omega\phi$ channels, where we also consider the hadronic loop effect. Our discussion is based on the recent preliminary results of $\chi_{cJ} \to VV$ presented by the BES Collaboration at the Hadron 2009 conference, especially the first observation of $\chi_{c1} \to \omega\phi$ [2]. Furthermore, we proposed that the $\omega - \phi$ mixing can be tested via $\chi_{c1} \to \phi\phi$, $\omega\omega$, $\omega\phi$ decays, which is different from the idea in Ref. [31] to some extent.

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