

$B_s \rightarrow f_0(980)$ form factors and B_s decays into $f_0(980)$

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(Received 19 February 2010; published 1 April 2010)

We compute the $B_s \rightarrow f_0(980)$ transition form factors using light-cone QCD sum rules at leading order in the strong coupling constant, and also including an estimate of next-to-leading order corrections. We use the results to predict the branching fractions of the rare decay modes $B_s \rightarrow f_0 \ell^+ \ell^-$ and $B_s \rightarrow f_0 \nu \bar{\nu}$, which turn out to be $\mathcal{O}(10^{-7})$ ($B_s \rightarrow f_0(980) \ell^+ \ell^-$, with $\ell = e, \mu$), $\mathcal{O}(10^{-8})$ ($B_s \rightarrow f_0(980) \tau^+ \tau^-$) and $\mathcal{O}(10^{-6})$ ($B_s \rightarrow f_0(980) \nu \bar{\nu}$). We also predict the branching ratio of $B_s \rightarrow J/\psi f_0(980)$ decay under the factorization assumption, and discuss the role of this channel for the determination of the B_s mixing phase compared to the golden mode $B_s \rightarrow J/\psi \phi$. As a last application, we consider $D_s \rightarrow f_0$ form factors, providing a determination of the branching ratio of $D_s \rightarrow f_0 e^+ \nu_e$.

DOI: 10.1103/PhysRevD.81.074001

PACS numbers: 13.20.He, 13.20.Fc, 13.25.Hw

I. INTRODUCTION

Theoretical and experimental efforts aimed at disclosing physics beyond the standard model (SM) proceed in several directions. Among these, there is the study of rare processes which are induced only at loop level in the SM and are therefore sensitive to new physics (NP) contributions which may potentially enhance their small ($< 10^{-5}$) branching ratios [1]. Another testing ground is the precise study of CP violation. It has been realized that the amount of CP violation within the SM is too small to explain the observed baryon asymmetry of the Universe [2], a conclusion confirmed by recent analyses [3]. Since the only source of CP violation in the SM is the complex phase of the Cabibbo Kobayashi Maskawa (CKM) mixing matrix, the determination of the elements of this matrix and of their relative phases is of primary importance, in order to disentangle sources of additional contributions to CP violation. As is well known, the task is afforded through the study of the so-called unitarity triangles, the graphical representations of the conditions stemming from unitarity of the CKM matrix. The most studied triangle is the one which relates the CKM elements involved in B decays. Direct and indirect determinations of its sides and angles lead to a picture of CP violation coherent with the SM description. Also in this case, investigation of effects predicted to be small in the SM is a promising strategy to reveal new physics.

B_s mesons provide the possibility to search for new physics scenarios exploiting both the strategies outlined above. On the one hand, rare B_s decays induced by $b \rightarrow s$ transition are suppressed in the SM, as with all decay modes governed by such a transition, and new physics effects may enhance their branching fractions. For example, it has been shown that, in presence of a single universal extra dimension compactified on a circle with radius R , the rates of $B_s \rightarrow \phi \nu \bar{\nu}$, $B_s \rightarrow \eta^{(\prime)} \ell^+ \ell^-$ and $B_s \rightarrow \eta^{(\prime)} \nu \bar{\nu}$ are enhanced when R^{-1} decreases, while the opposite happens in the case of $B_s \rightarrow \phi \gamma$, which has a smaller

branching fraction with respect to SM for small values of R^{-1} [4].

On the other hand, the analysis of the unitarity triangle of CKM elements relevant for B_s decays is an important test of the SM description of CP violation. The triangle is defined by the relation

$$V_{us} V_{ub}^* + V_{cs} V_{cb}^* + V_{ts} V_{tb}^* = 0. \quad (1)$$

One of its angles, β_s , defined as $\beta_s = \text{Arg}[-\frac{V_{ts} V_{tb}^*}{V_{cs} V_{cb}^*}]$, is half of the phase of $B_s - \bar{B}_s$ mixing, and is expected to be tiny in the SM: $\beta_s \simeq 0.019$ rad. Recent data obtained by the CDF [5] and D0 [6] Collaborations, based on the angular analysis of $B_s \rightarrow J/\psi \phi$, indicate much larger values, although with sizable uncertainties, so that the precise measurement of β_s represents one of the priorities in the physics programs at the hadron colliders and at the B factories operating at the $\Upsilon(5S)$ peak [7].

In this paper we consider B_s decays in both respects. We compute the $B_s \rightarrow f_0(980)$ form factors using light-cone QCD sum rules (LCSR) at the leading order in the strong coupling constant (Sec. II and III A) and including an estimate of next-to-leading (NLO) corrections (Sec. III B). In Sec. IV A, we use the results to predict the branching fractions of the rare decay modes $B_s \rightarrow f_0 \ell^+ \ell^-$ and $B_s \rightarrow f_0 \nu \bar{\nu}$ in the SM. The form factors are also a necessary ingredient to study the nonleptonic mode $B_s \rightarrow J/\psi f_0$ which, together with $B_s \rightarrow J/\psi \phi$, permits one to access the phase β_s [8]. Our predictions for this mode are collected in Sec. IV B. As a by-product of the calculation, we explore the $D_s \rightarrow f_0 e^+ \nu_e$ decay channel, the branching ratio of which has been recently measured by the CLEO Collaboration [9,10]. Conclusions are presented in the last section.

¹Hereafter, we use f_0 to denote the $f_0(980)$ meson.

II. LIGHT-CONE QCD SUM RULE CALCULATION OF $B_s \rightarrow f_0$ FORM FACTORS

The matrix elements involved in $B_s \rightarrow f_0$ transitions can be parameterized in terms of form factors as

$$\begin{aligned} \langle f_0(p_{f_0}) | \bar{s} \gamma_\mu \gamma_5 b | \bar{B}_s(p_{B_s}) \rangle \\ = -i \{ F_1(q^2) \left[P_\mu - \frac{m_{B_s}^2 - m_{f_0}^2}{q^2} q_\mu \right] \\ + F_0(q^2) \frac{m_{B_s}^2 - m_{f_0}^2}{q^2} q_\mu \}, \end{aligned} \quad (2)$$

$$\begin{aligned} \langle f_0(p_{f_0}) | \bar{s} \sigma_{\mu\nu} \gamma_5 q^\nu b | \bar{B}_s(p_{B_s}) \rangle \\ = - \frac{F_T(q^2)}{m_{B_s} + m_{f_0}} [q^2 P_\mu - (m_{B_s}^2 - m_{f_0}^2) q_\mu], \end{aligned} \quad (3)$$

where $P = p_{B_s} + p_{f_0}$ and $q = p_{B_s} - p_{f_0}$. In this section we describe the calculation of the three functions F_1 , F_0 and F_T using the method of light-cone QCD sum rules. For the sake of the calculation, it is convenient to define the auxiliary form factors f_+ and f_- ,

$$\langle f_0(p_{f_0}) | \bar{s} \gamma_\mu \gamma_5 b | \bar{B}_s(p_{B_s}) \rangle = -i \{ f_+(q^2) P_\mu + f_-(q^2) q_\mu \} \quad (4)$$

in terms of which F_1 and F_0 read

$$\begin{aligned} F_1(q^2) &= f_+(q^2), \\ F_0(q^2) &= f_+(q^2) + \frac{q^2}{m_{B_s}^2 - m_{f_0}^2} f_-(q^2). \end{aligned} \quad (5)$$

As a reconciliation of the original QCD sum rule approach [11] and the application of perturbation theory to hard processes, LCSR [12] present several advantages in the calculation of quantities such as the heavy-to-light meson form factors. The method includes both hard scattering and soft contributions. In the hard scattering region the operator product expansion (OPE) near the light-cone is appli-

cable. Based on the light-cone OPE, hadronic quantities, like form factors, are expressed as a convolution of light-cone distribution amplitudes (LCDA) with a perturbatively calculable hard kernel. The leading twist and a few sub-leading twist LCDA give the dominant contribution, while higher twist terms are power suppressed. The LCSR approach has been successfully applied to compute the hadronic parameters involved in many different processes [13].

The starting point for a LCSR evaluation of form factors is the correlation function of suitably chosen quark currents. Here we consider the correlation function

$$\Pi(p_{f_0}, q) = i \int d^4x e^{iq \cdot x} \langle f_0(p_{f_0}) | T \{ j_{\Gamma_1}(x), j_{\Gamma_2}(0) \} | 0 \rangle, \quad (6)$$

where j_{Γ_1} is one of the currents appearing in the matrix elements (2) and (3) defining the form factors: $j_{\Gamma_1} = J_\mu^5 = \bar{s} \gamma_\mu \gamma_5 b$ for F_1 and F_0 , and $j_{\Gamma_1} = J_\mu^{5T} = \bar{s} \sigma_{\mu\nu} \gamma_5 q^\nu b$ for F_T . The current j_{Γ_2} interpolates the B_s meson: we choose $j_{\Gamma_2} = \bar{b} i \gamma_5 s$. Its matrix element between the vacuum and B_s is given in terms of the decay constant f_{B_s} ,

$$\langle \bar{B}_s(p_{B_s}) | \bar{b} i \gamma_5 s | 0 \rangle = \frac{m_{B_s}^2}{m_b + m_s} f_{B_s}. \quad (7)$$

We also introduce the $f_0(980)$ decay constant \bar{f}_{f_0} ,

$$\langle f_0(p_{f_0}) | \bar{s} s | 0 \rangle = m_{f_0} \bar{f}_{f_0} \quad (8)$$

needed in the following; \bar{f}_{f_0} has been evaluated by several groups [14–17].

The LCSR method consists in evaluating the correlation function Eq. (6) both at the hadronic level and in QCD. Equating the two representations provides one with a sum rule suitable to derive the form factors.

The hadronic representation of the correlation function in (6)

$$\Pi^{\text{HAD}}(p_{f_0}, q) = \frac{\langle f_0(p_{f_0}) | j_{\Gamma_1} | \bar{B}_s(p_{f_0} + q) \rangle \langle \bar{B}_s(p_{f_0} + q) | j_{\Gamma_2} | 0 \rangle}{m_{B_s}^2 - (p_{f_0} + q)^2} + \sum_h \frac{\langle f_0(p_{f_0}) | j_{\Gamma_1} | h(p_{f_0} + q) \rangle \langle h(p_{f_0} + q) | j_{\Gamma_2} | 0 \rangle}{m_h^2 - (p_{f_0} + q)^2} \quad (9)$$

consists in the contribution of the \bar{B}_s meson and of the higher resonances and the continuum of states h . In a one-resonance + continuum representation, the correlation function can be written as

$$\Pi^{\text{HAD}}(p_{f_0}, q) = \frac{\langle f_0(p_{f_0}) | j_{\Gamma_1} | \bar{B}_s(p_{f_0} + q) \rangle \langle \bar{B}_s(p_{f_0} + q) | j_{\Gamma_2} | 0 \rangle}{m_{B_s}^2 - (p_{f_0} + q)^2} + \int_{s_0}^{\infty} ds \frac{\rho^h(s, q^2)}{s - (p_{f_0} + q)^2}, \quad (10)$$

where higher resonances and the continuum of states are described in terms of the spectral function $\rho^h(s, q^2)$, which contributes starting from a threshold s_0 .

At the quark level, the correlation function can be evaluated in QCD, writing it as

$$\Pi^{\text{QCD}}(p_{f_0}, q) = \frac{1}{\pi} \int_{(m_b + m_s)^2}^{\infty} ds \frac{\text{Im} \Pi^{\text{QCD}}(s, q^2)}{s - (p_{f_0} + q)^2}. \quad (11)$$

This step is afforded expanding the T-product in Eq. (6) on the light-cone, a procedure which provides us with a series of

operators, ordered by increasing twist, the matrix elements of which between the vacuum and the f_0 [required to evaluate Eq. (6)] are expressed in terms of f_0 LCDAs. The equality of the two representations of the correlation function, Eqs. (10) and (11), does not yet allow us to derive the $B_s \rightarrow f_0$ form factors appearing in the first term on the right-hand side of (10), since the hadronic spectral function ρ^h is unknown. For this reason, we invoke global quark-hadron duality above the threshold s_0 [18], which amounts to identify integrals of the spectral function ρ^h with corresponding integrals of $\rho^{\text{QCD}} = \frac{1}{\pi} \text{Im}\Pi^{\text{QCD}}$, in particular

$$\int_{s_0}^{\infty} ds \frac{\rho^h(s, q^2)}{s - (p_{f_0} + q)^2} = \frac{1}{\pi} \int_{s_0}^{\infty} ds \frac{\text{Im}\Pi^{\text{QCD}}(s, q^2)}{s - (p_{f_0} + q)^2}. \quad (12)$$

Using duality, together with the equality $\Pi^{\text{HAD}}(p_{f_0}, q) = \Pi^{\text{QCD}}(p_{f_0}, q)$, we obtain from Eqs. (10) and (11):

$$\frac{\langle f_0(p_{f_0}) | j_{\Gamma_1} | \bar{B}_s(p_{f_0} + q) \rangle \langle \bar{B}_s(p_{f_0} + q) | j_{\Gamma_2} | 0 \rangle}{m_{B_s}^2 - (p_{f_0} + q)^2} = \frac{1}{\pi} \int_{(m_b+m_s)^2}^{s_0} ds \frac{\text{Im}\Pi^{\text{QCD}}(s, q^2)}{s - (p_{f_0} + q)^2}. \quad (13)$$

Following the usual QCD sum rule procedure, we carry out a Borel transformation of the hadronic and of the QCD expressions of the correlation function, consequently of the two sides in (13). This is defined as

$$\mathcal{B}[\mathcal{F}(Q^2)] = \lim_{Q^2 \rightarrow \infty, n \rightarrow \infty, (Q^2/n) = M^2} \frac{1}{(n-1)!} (-Q^2)^n \times \left(\frac{d}{dQ^2} \right)^n \mathcal{F}(Q^2), \quad (14)$$

where \mathcal{F} is a function of $Q^2 = -q^2$ and M^2 is the Borel parameter, so that

$$\mathcal{B}\left[\frac{1}{(s+Q^2)^n}\right] = \frac{\exp(-s/M^2)}{(M^2)^n (n-1)!}. \quad (15)$$

This operation improves the convergence of the OPE series

by factorials of the power n , and for suitably chosen values of M^2 enhances the contribution of the low lying states to the hadronic expression of the correlation function.

Applying the transformation to both Π^{HAD} and Π^{QCD} we obtain

$$\langle f_0(p_{f_0}) | j_{\Gamma_1} | \bar{B}_s(p_{B_s}) \rangle \langle \bar{B}_s(p_{B_s}) | j_{\Gamma_2} | 0 \rangle \exp\left[-\frac{m_{B_s}^2}{M^2}\right] = \frac{1}{\pi} \int_{(m_b+m_s)^2}^{s_0} ds \exp[-s/M^2] \text{Im}\Pi^{\text{QCD}}(s, q^2), \quad (16)$$

where $p_{B_s} = p_{f_0} + q$. Equation (16) allows to derive the sum rules for f_+ , f_- and F_T , choosing either the current $j_{\Gamma_1} = J_{\mu}^5$ or the current $j_{\Gamma_1} = J_{\mu}^{5T}$.

The calculation of Π^{QCD} is based on the expansion of the T-product in (6) near the light-cone, which produces matrix elements of nonlocal quark-gluon operators. In the description of f_0 as a $s\bar{s}$ state modified by some hadronic dressing [14], these can be defined in terms of f_0 light-cone distribution amplitudes of increasing twist:

$$\begin{aligned} \langle f_0(p_{f_0}) | \bar{s}(x) \gamma_{\mu} s(0) | 0 \rangle &= \bar{f}_{f_0} p_{f_0 \mu} \int_0^1 du e^{iup_{f_0} \cdot x} \Phi_{f_0}(u), \\ \langle f_0(p_{f_0}) | \bar{s}(x) s(0) | 0 \rangle &= m_{f_0} \bar{f}_{f_0} \int_0^1 du e^{iup_{f_0} \cdot x} \Phi_{f_0}^s(u), \\ \langle f_0(p_{f_0}) | \bar{s}(x) \sigma_{\mu\nu} s(0) | 0 \rangle &= -\frac{m_{f_0}}{6} \bar{f}_{f_0} (p_{f_0 \mu} x_{\nu} - p_{f_0 \nu} x_{\mu}) \\ &\quad \times \int_0^1 du e^{iup_{f_0} \cdot x} \Phi_{f_0}^{\sigma}(u), \end{aligned} \quad (17)$$

where the LCDA Φ_{f_0} is twist-2, and the other two are twist-3, and are normalized as

$$\int_0^1 du \Phi_{f_0}(u) = 0, \quad \int_0^1 du \Phi_{f_0}^s(u) = \int_0^1 du \Phi_{f_0}^{\sigma}(u) = 1. \quad (18)$$

In terms of these LCDA, the sum rules for the three form factors read

$$\begin{aligned} f_+(q^2) &= \frac{m_b + m_s}{2m_{B_s}^2 f_{B_s}} \bar{f}_{f_0} \exp\left[\frac{m_{B_s}^2}{M^2}\right] \left\{ \int_{u_0}^1 \frac{du}{u} \exp\left[-\frac{m_b^2 + u\bar{u}m_{f_0}^2 - \bar{u}q^2}{uM^2}\right] \left[-m_b \Phi_{f_0}(u) + um_{f_0} \Phi_{f_0}^s(u) + \frac{1}{3} m_{f_0} \Phi_{f_0}^{\sigma}(u) \right] \right. \\ &\quad \left. + \frac{m_b^2 + q^2 - u^2 m_{f_0}^2}{uM^2} \frac{m_{f_0} \Phi_{f_0}^{\sigma}(u)}{6} \right\} + \exp[-s_0/M^2] \frac{m_{f_0} \Phi_{f_0}^{\sigma}(u_0)}{6} \frac{m_b^2 - u_0^2 m_{f_0}^2 + q^2}{m_b^2 + u_0^2 m_{f_0}^2 - q^2}, \end{aligned} \quad (19)$$

$$\begin{aligned} f_-(q^2) &= \frac{m_b + m_s}{2m_{B_s}^2 f_{B_s}} \bar{f}_{f_0} \exp\left[\frac{m_{B_s}^2}{M^2}\right] \left\{ \int_{u_0}^1 \frac{du}{u} \exp\left[-\frac{m_b^2 + u\bar{u}m_{f_0}^2 - \bar{u}q^2}{uM^2}\right] \left[m_b \Phi_{f_0}(u) + (2-u)m_{f_0} \Phi_{f_0}^s(u) \right. \right. \\ &\quad \left. \left. + \frac{1-u}{3u} m_{f_0} \Phi_{f_0}^{\sigma}(u) - \frac{u(m_b^2 + q^2 - u^2 m_{f_0}^2) + 2(m_b^2 - q^2 + u^2 m_{f_0}^2)}{u^2 M^2} \frac{m_{f_0} \Phi_{f_0}^{\sigma}(u)}{6} \right] \right. \\ &\quad \left. - \frac{u_0(m_b^2 + q^2 - u_0^2 m_{f_0}^2) + 2(m_b^2 - q^2 + u_0^2 m_{f_0}^2)}{u_0(m_b^2 + u_0^2 m_{f_0}^2 - q^2)} \exp[-s_0/M^2] \frac{m_{f_0} \Phi_{f_0}^{\sigma}(u_0)}{6} \right\}, \end{aligned} \quad (20)$$

$$F_T(q^2) = \frac{(m_{B_s} + m_{f_0})(m_b + m_s)}{m_{B_s}^2 f_{B_s}} \bar{f}_{f_0} \exp\left[\frac{m_{B_s}^2}{M^2}\right] \left\{ \int_{u_0}^1 \frac{du}{u} \exp\left[-\frac{(m_b^2 - \bar{u}q^2 + u\bar{u}m_{f_0}^2)}{uM^2}\right] \left[-\frac{\Phi_{f_0}(u)}{2} + m_b \frac{m_{f_0} \Phi_{f_0}^\sigma(u)}{6uM^2} \right] + m_b \frac{m_{f_0} \Phi_{f_0}^\sigma(u_0)}{6} \frac{\exp[-s_0/M^2]}{m_b^2 - q^2 + u_0^2 m_{f_0}^2} \right\} \quad (21)$$

where

$$u_0 = \frac{m_{f_0}^2 + q^2 - s_0 + \sqrt{(m_{f_0}^2 + q^2 - s_0)^2 + 4m_{f_0}^2(m_b^2 - q^2)}}{2m_{f_0}^2}. \quad (22)$$

Our formulas can be compared to the ones for the B -to-scalar meson form factors in Ref. [19], where the case of the meson a_0 is considered. We find differences in the expression of the form factor f_+ .

The expressions (19)–(21) are obtained considering that the weak currents relevant for the processes we are studying probe the $\bar{s}s$ content of $f_0(980)$. If the flavor wave function of f_0 has other light quark components (the only ones one could include in a straightforward way in our method of calculation), such expressions are modified by coefficients involving the cosine of the $f_0 - \sigma$ mixing angle, the value of which is uncertain, not uniquely determined by experiment and close to 0° or 180° [20]. These coefficients leave the ratios of form factors at $q^2 = 0$ and the q^2 dependences unchanged, and only affect the overall normalization at zero recoil, a systematic uncertainty in our numerical results.

III. NUMERICAL RESULTS AND DISCUSSIONS

A. Leading order results

Based on the conformal spin invariance, the LCDA can be expanded in terms of Gegenbauer polynomials $C_n^{3/2}$. The expansion of the twist-2 LCDA $\Phi_{f_0}(u)$ reads:

$$\Phi_{f_0}(u) = 6u(1-u) \left\{ B_0 + \sum_{n=1} B_n C_n^{3/2}(2u-1) \right\}. \quad (23)$$

Because of the charge conjugation invariance, all even Gegenbauer moments of $\Phi_{f_0}(u)$ vanish, so that $B_{2m} = 0$ for $m = 0, 1, \dots$ in (23); as for the odd moments, we include only the first one, using the value of the coefficient $B_1 = -0.78 \pm 0.08$ fixed in Ref. [17]. For the twist-3 LCDA, due to the lack of knowledge about their moments, we use the asymptotic form, i.e. the first term of the Gegenbauer expansion,

$$\Phi_{f_0}^s(u) = 1, \quad \Phi_{f_0}^\sigma(u) = 6u(1-u). \quad (24)$$

Let us quote the numerical values of the other physical parameters. The meson masses are fixed to the PDG values $m_{B_s} = 5.366$ GeV and $m_{f_0} = 0.98$ GeV [21], while for quark masses we use $m_b = 4.8$ GeV and $m_s = 0.14$ GeV [21,22]. As for the decay constants, we use $f_{B_s} = (0.231 \pm$

$0.015)$ GeV [23] and $\bar{f}_{f_0} = (0.18 \pm 0.015)$ GeV [14].² The threshold s_0 is fixed to $s_0 = (34 \pm 2)$ GeV², which should correspond to the mass squared of the first radial excitation of B_s .

With these numerical inputs, the sum rules (19)–(21) provide us with the form factors for each value of q^2 as a function of the Borel parameter. The result is obtained requiring stability against variations of M^2 .

In Fig. 1 we show the dependence of the form factors at $q^2 = 0$ on the Borel parameter M^2 . We observe stability when $M^2 > 6$ GeV², and we fix $M^2 = (8 \pm 2)$ GeV², values that are smaller than in other B to light meson transitions [24,25].

To describe the form factors in the whole kinematically accessible q^2 region, we adopt the parameterization

$$F_i(q^2) = \frac{F_i(0)}{1 - a_i q^2/m_{B_s}^2 + b_i (q^2/m_{B_s}^2)^2}, \quad (25)$$

where F_i denotes any function among $F_{1,0,T}$. The parameters $F_i(0)$, a_i and b_i are obtained through fitting the form factors computed numerically in the small q^2 region (we choose $0 < q^2 < 15$ GeV²); the results for $F_i(0)$, a_i and b_i are collected in Table I, and the q^2 dependence is depicted in Fig. 2. The uncertainties in the results reflect those of the input parameters, the threshold s_0 , the Borel mass M^2 , the Gegenbauer moments and the decay constants. In Table I we also report the values of the form factors at zero-recoil (q_{\max}^2) which are derived using the expression in Eq. (25).

The results in Table I show that the parameters a_i and b_i determining the q^2 dependence are close to each other in the case of F_1 and F_T . The reason is the following. In the heavy-quark limit and in the large energy (LE) limit of the recoiled meson, the three $B_s \rightarrow f_0$ form factors can be related to a single universal function ξ_{f_0} which is specific for f_0 and does not depend on the Dirac structure of the current appearing in the definition of the various matrix elements, such as those in Eqs. (2) and (3) [26]. When the energy E of the light meson in the final state is large, such relations read as

²In Ref. [17] a larger result is reported: $\bar{f}_{f_0} = (0.37 \pm 0.02)$ GeV.

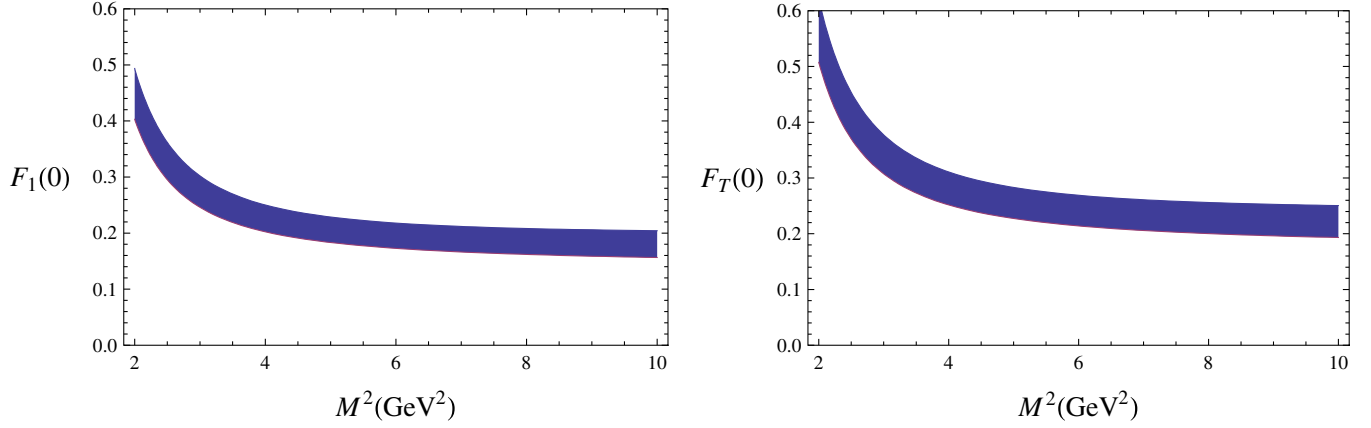


FIG. 1 (color online). Dependence on the Borel parameter M^2 of the $B_s \rightarrow f_0$ form factors at $q^2 = 0$: $F_1(0) = F_0(0)$ (left panel) and $F_T(0)$ (right panel).

$$\frac{m_{B_s}}{m_{B_s} + m_{f_0}} F_T(q^2) = F_1(q^2) = \frac{m_{B_s}}{2E} F_0(q^2), \quad (26)$$

where, neglecting $m_{f_0}^2$ but keeping m_{f_0} in the kinematical factors, E is related to q^2 :

$$q^2 = m_{B_s}^2 - 2m_{B_s}E. \quad (27)$$

The first equality in Eq. (26) shows that the large energy limit predicts that F_1 and F_T have the same q^2 dependence. For the shape parameters of F_0 , one can obtain two relations through the second equality:

$$a_0 = -1 + a_1, \quad b_0 = 1 - a_1 + b_1. \quad (28)$$

Using the results for a_1 and b_1 , we find from (28) that $a_0^{(LE)} \simeq 0.44 \pm 0.1$ and $b_0^{(LE)} \simeq 0.15 \pm 0.12$; therefore, the first relation in (28) is well respected in our calculation, while not much can be said about the second relation due to the uncertainty affecting b_0 .

B. Estimate of the next-to-leading order corrections

In order to provide an estimate of next-to-leading order effects in the determination of the $B_s \rightarrow f_0$ form factors, it is worth comparing this case to the calculation of $B \rightarrow \pi$ form factors. In $B \rightarrow \pi$ transition, both the light quarks and the light π meson have small masses which can be safely neglected, while the strange quark and the scalar meson f_0 masses may induce sizable effects. Another observation is that, neglecting the quark masses, the Lorentz structures of

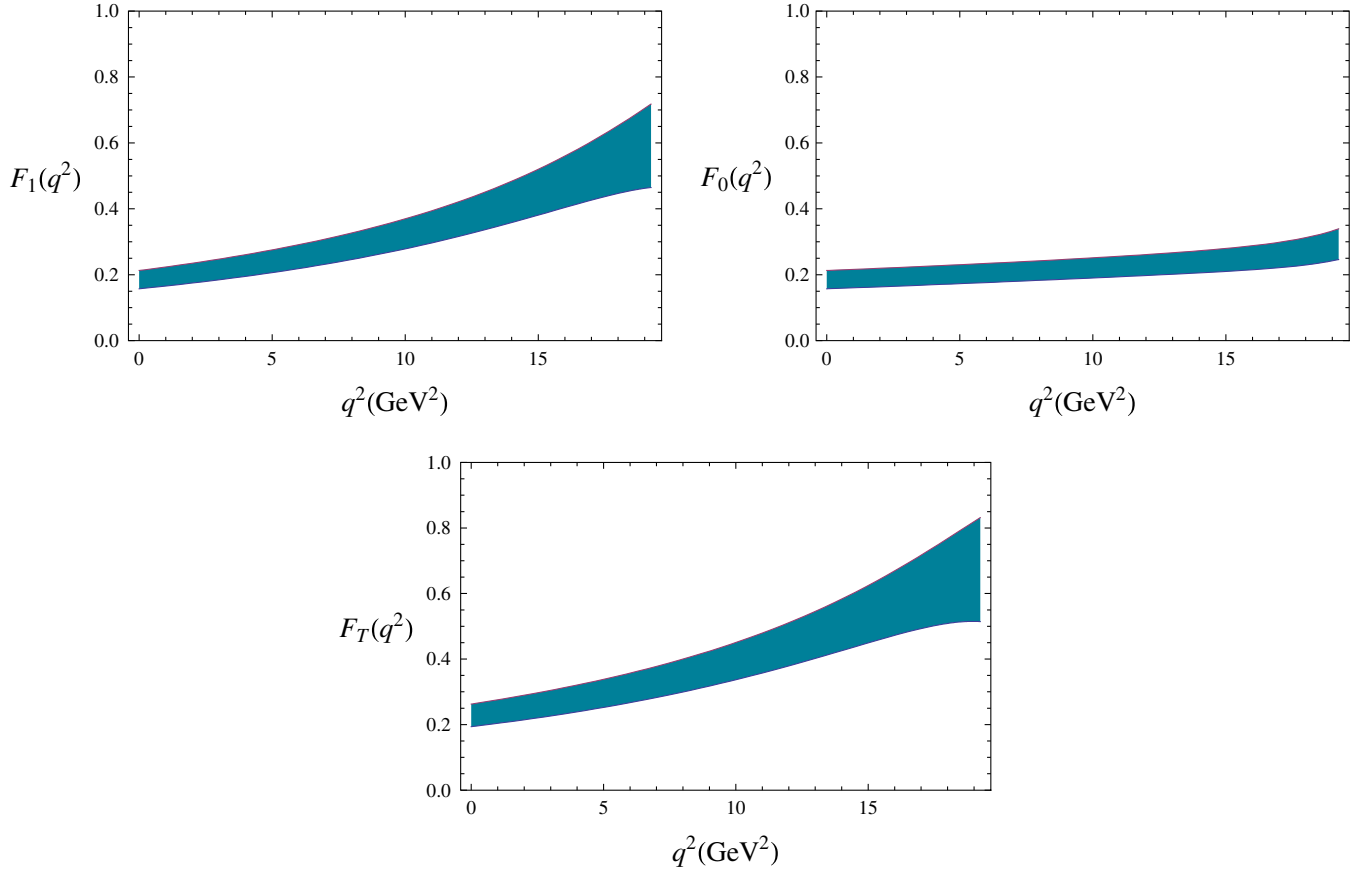
pion and f_0 matrix elements differ by a minus sign in terms proportional to the twist-2 LCDA. Finally, contributions from the twist-3 LCDAs in $B \rightarrow \pi$ transition are characterized by the chiral scale parameter μ_π , while in $B_s \rightarrow f_0$ they are proportional to the mass of f_0 .

In LCSR, NLO corrections to $B \rightarrow \pi$ form factors have been studied by two groups [24,25], while the complete expressions for the NLO corrections to $B_s \rightarrow f_0$ form factors are not known at present. The expressions relevant for $B \rightarrow \pi$ form factors given in Ref. [25] can be used to estimate the radiative corrections in the case $B_s \rightarrow f_0$, keeping in mind the three differences above. We first consider the changes to the leading order result due to the different treatment of quark and hadron masses. Setting the quark mass m_s to zero, the values of the form factors are reduced by about 3%. The mass of f_0 , the analogous of the pion mass m_π and the chiral scale parameter μ_π , cannot be put to zero, as this would smear all terms from twist-3 LCDA: we set the mass square of f_0 to be zero keeping the linear terms in the form factors, obtaining an enhancement of the form factors by about 3%. After that, evolving all the scale-dependent parameters to a scale of about the Borel mass, $\mu \simeq 3$ GeV (as done, e.g. in [24,25]), we find that the leading order contributions are furtherly enhanced, obtaining the central values $F_1(0) = F_0(0) = 0.216$, $a_1 = 1.50$, $b_1 = 0.58$, $a_0 = 0.216$, $b_0 = 0.53$ and $F_T(0) = 0.262$, $a_t = 1.46$, $b_t = 0.58$. Then, radiative corrections to twist-2 and twist-3 LCDA are also found to be rather small, the $B_s \rightarrow f_0$ form factors being changed to $F_1(0) = F_0(0) = 0.238$ and $F_T(0) = 0.308$. The resulting values, with the inclusion the uncertainty due to the input parameters, are collected in Table II; they are also used in the phenomenological analysis, keeping in mind, however, that the procedure used in their determination must be considered as only approximate.

Before closing this section, it is worth mentioning that the $B_s \rightarrow f_0(980)$ form factors have been computed by

TABLE I. Parameters of the $B_s \rightarrow f_0$ form factors by LCSR at the leading order. The values of $F_i(q_{\max}^2)$ are evaluated through Eq. (25).

	$F_i(q^2 = 0)$	a_i	b_i	$F_i(q_{\max}^2)$
F_1	0.185 ± 0.029	$1.44^{+0.13}_{-0.09}$	$0.59^{+0.07}_{-0.05}$	$0.614^{+0.158}_{-0.102}$
F_0	0.185 ± 0.029	$0.47^{+0.12}_{-0.09}$	$0.01^{+0.08}_{-0.09}$	$0.268^{+0.055}_{-0.038}$
F_T	0.228 ± 0.036	$1.42^{+0.13}_{-0.10}$	$0.60^{+0.06}_{-0.05}$	$0.714^{+0.197}_{-0.126}$

FIG. 2 (color online). q^2 dependence of the $B_s \rightarrow f_0$ form factors.

other approaches: the method based on covariant light-front dynamics (CLFD) and dispersion relation (DR) [27], the perturbative QCD approach (PQCD) [28], short-distance QCD sum rules (QCDSR) [29]. The results are

TABLE II. $B_s \rightarrow f_0(980)$ transition form factors obtained including an estimate of next-to-leading order corrections (see text).

	$F_i(q^2 = 0)$	a_i	b_i
F_1	0.238 ± 0.036	$1.50^{+0.13}_{-0.09}$	$0.58^{+0.09}_{-0.07}$
F_0	0.238 ± 0.036	$0.53^{+0.14}_{-0.10}$	$-0.36^{+0.09}_{-0.08}$
F_T	0.308 ± 0.049	$1.46^{+0.14}_{-0.10}$	$0.58^{+0.09}_{-0.07}$

TABLE III. $B_s \rightarrow f_0(980)$ form factors at $q^2 = 0$. Results evaluated by CLFD/DR [27], PQCD [28] and QCDSR [29] approaches are collected for a comparison.

	CLFD/DR	PQCD	QCDSR	This work
$F_1(0)$	0.40/0.29 ^a	$0.35^{+0.09}_{-0.07}$ ^b	0.12 ± 0.03 ^c	0.185 ± 0.029
$F_T(0)$		$0.40^{+0.10}_{-0.08}$ ^b	-0.08 ± 0.02 ^c	0.228 ± 0.036

^aUsing $f_{B_s} = 0.259$ GeV.

^bUsing $\bar{f}_{f_0} = 0.37$ GeV.

^cUsing $\bar{f}_{f_0} = 0.37$ GeV and $f_{B_s} = 0.209$ GeV.

collected in Table III. The form factors by PQCD are proportional to the f_0 decay constant, while those by short-distance QCD sum rules are proportional to the inverse of this constant. Thus, a larger decay constant, $\bar{f}_{f_0} = 0.37$ GeV as reported and used in [17], gives larger form factors in the PQCD approach and smaller ones in QCDSR with respect to ours. Taking into account the difference in the decay constant, the results in Refs. [28,29] are consistent with ours, while the two results in Ref. [27] are sensibly larger.

IV. PHENOMENOLOGICAL APPLICATIONS

A. Semileptonic $\bar{B}_s \rightarrow f_0 \ell^+ \ell^-$ and $\bar{B}_s \rightarrow f_0 \nu \bar{\nu}$ decays

As a first application of our study, we predict the branching ratios of the decays $\bar{B}_s \rightarrow f_0 \ell^+ \ell^-$ and $\bar{B}_s \rightarrow f_0 \nu \bar{\nu}$, processes which, being induced by the flavor-changing neutral current transition $b \rightarrow s$, are potentially important for detecting new physics effects.

The SM $\Delta B = 1$, $\Delta S = -1$ effective Hamiltonian describing the transition $b \rightarrow s \ell^+ \ell^-$ can be expressed in terms of a set of local operators:

$$H_{b \rightarrow s \ell^+ \ell^-} = -4 \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu) O_i(\mu), \quad (29)$$

$G_F = 1.166 \times 10^{-5} \text{ GeV}^{-2}$ being the Fermi constant and V_{ij} the elements of the CKM mixing matrix [since the ratio $|\frac{V_{ub}V_{us}^*}{V_{tb}V_{ts}^*}|$ is $\mathcal{O}(10^{-2})$, we neglect terms proportional to $V_{ub}V_{us}^*$]. The operators O_i are written in terms of quark and gluon fields:

$$\begin{aligned} O_1 &= (\bar{s}_{L\alpha}\gamma^\mu b_{L\alpha})(\bar{c}_{L\beta}\gamma_\mu c_{L\beta}), & O_2 &= (\bar{s}_{L\alpha}\gamma^\mu b_{L\beta})(\bar{c}_{L\beta}\gamma_\mu c_{L\alpha}), \\ O_3 &= (\bar{s}_{L\alpha}\gamma^\mu b_{L\alpha})[(\bar{u}_{L\beta}\gamma_\mu u_{L\beta}) + \dots + (\bar{b}_{L\beta}\gamma_\mu b_{L\beta})], & O_4 &= (\bar{s}_{L\alpha}\gamma^\mu b_{L\beta})[(\bar{u}_{L\beta}\gamma_\mu u_{L\alpha}) + \dots + (\bar{b}_{L\beta}\gamma_\mu b_{L\alpha})], \\ O_5 &= (\bar{s}_{L\alpha}\gamma^\mu b_{L\alpha})[(\bar{u}_{R\beta}\gamma_\mu u_{R\beta}) + \dots + (\bar{b}_{R\beta}\gamma_\mu b_{R\beta})], & O_6 &= (\bar{s}_{L\alpha}\gamma^\mu b_{L\beta})[(\bar{u}_{R\beta}\gamma_\mu u_{R\alpha}) + \dots + (\bar{b}_{R\beta}\gamma_\mu b_{R\alpha})], \\ O_7 &= \frac{e}{16\pi^2}(m_b\bar{s}_{L\alpha}\sigma^{\mu\nu}b_{R\alpha} + m_s\bar{s}_{R\alpha}\sigma^{\mu\nu}b_{L\alpha})F_{\mu\nu}, & O_8 &= \frac{g_s}{16\pi^2}m_b\left[\bar{s}_{L\alpha}\sigma^{\mu\nu}\left(\frac{\lambda^a}{2}\right)_{\alpha\beta}b_{R\beta}\right]G_{\mu\nu}^a, \\ O_9 &= \frac{e^2}{16\pi^2}(\bar{s}_{L\alpha}\gamma^\mu b_{L\alpha})\bar{\ell}\gamma_\mu\ell, & O_{10} &= \frac{e^2}{16\pi^2}(\bar{s}_{L\alpha}\gamma^\mu b_{L\alpha})\bar{\ell}\gamma_\mu\gamma_5\ell, \end{aligned} \quad (30)$$

with α, β color indices, $b_{R,L} = \frac{1\pm\gamma_5}{2}b$, and $\sigma^{\mu\nu} = \frac{i}{2} \times [\gamma^\mu, \gamma^\nu]$; e and g_s are the electromagnetic and the strong coupling constant, respectively, and $F_{\mu\nu}$ and $G_{\mu\nu}^a$ in O_7 and O_8 denote the electromagnetic and the gluonic field strength tensor. O_1 and O_2 are current-current operators, O_3, \dots, O_6 QCD penguin operators, O_7 and O_8 magnetic penguin operators, O_9 and O_{10} semileptonic electroweak penguin operators. The Wilson coefficients in (29) are known at next-to-next-to-leading order (NNLO) in the standard model [30]. The operators O_1 and O_2 contribute to the final state with a lepton pair through a $\bar{c}c$ contribution that can give rise to charmonium resonances $J/\psi, \psi(2S), \dots$, resonant term which can be subtracted by appropriate kinematical cuts around the resonance masses. The Wilson coefficients $C_3 - C_6$ are small, hence the contribution of only the operators O_7, O_9 and O_{10} can be kept for the description of the $b \rightarrow s\ell^+\ell^-$ transition. In our study we use a modification of the Wilson coefficient C_7 : C_7^{eff} , which is a renormalization scheme independent combination of C_7, C_8 and C_2 , given by a formula that can be found, e.g., in [31].

The \bar{B}_s and f_0 matrix elements of the operators in (30) can be written in terms of form factors, so that the differential decay width of $\bar{B}_s \rightarrow f_0\ell^+\ell^-$ reads

$$\begin{aligned} \frac{d\Gamma(\bar{B}_s \rightarrow f_0\ell^+\ell^-)}{dq^2} &= \frac{G_F^2\alpha_{\text{em}}^2|V_{tb}|^2|V_{ts}^*|^2\sqrt{\lambda}}{512m_{B_s}^3\pi^5}\sqrt{\frac{q^2-4m_\ell^2}{q^2}} \\ &\times \frac{1}{3q^2} \times \left[6m_\ell^2|C_{10}|^2(m_{B_s}^2 - m_{f_0}^2)^2 \right. \\ &\times F_0^2(q^2) + (q^2 + 2m_\ell^2)\lambda \\ &\times \left| C_9F_1(q^2) + \frac{2C_7(m_b - m_s)F_T(q^2)}{m_{B_s} + m_{f_0}} \right|^2 \\ &\left. + |C_{10}|^2(q^2 - 4m_\ell^2)\lambda F_1^2(q^2) \right], \end{aligned} \quad (31)$$

with $\lambda = \lambda(m_{B_s}^2, m_{f_0}^2, q^2) = (m_{B_s}^2 - q^2 - m_{f_0}^2)^2 - 4m_{f_0}^2q^2$, $\alpha_{\text{em}} = 1/137$ the fine structure constant and m_ℓ the lepton mass.

Analogously, the SM effective Hamiltonian for $b \rightarrow s\nu\bar{\nu}$,

$$H_{b \rightarrow s\nu\bar{\nu}} = \frac{G_F}{\sqrt{2}} \frac{\alpha_{\text{em}}}{2\pi\sin^2(\theta_W)} V_{tb}V_{ts}^* \eta_X X(x_t) O_L \equiv C_L O_L, \quad (32)$$

includes the operator

$$O_L = (\bar{s}\gamma^\mu(1 - \gamma_5)b)(\bar{\nu}\gamma_\mu(1 - \gamma_5)\nu). \quad (33)$$

θ_W is the Weinberg angle; the function $X(x_t)$ ($x_t = \frac{m_t^2}{m_W^2}$, with m_t the top quark mass and m_W the W mass) has been computed in [32–34], while the QCD factor η_X is close to 1 [33–35], so that one can use $\eta_X = 1$. From this effective Hamiltonian, the differential decay width

$$\frac{d\Gamma(\bar{B}_s \rightarrow f_0\nu\bar{\nu})}{dq^2} = 3 \frac{|C_L|^2\lambda^{3/2}(m_{B_s}^2, m_{f_0}^2, q^2)}{96m_{B_s}^3\pi^3} |F_1(q^2)|^2$$

can be obtained.

In the numerical calculation we use

$$\begin{aligned} C_7 &= -0.30137, & C_9 &= 4.1696, \\ C_{10} &= -4.46418, & C_L &= 2.62 \times 10^{-9}, \end{aligned} \quad (34)$$

together with $V_{ts} = 0.0387$ and $V_{tb} = 0.999$ [21]. Using these inputs and $\tau(B_s) = 1.47 \text{ ps}$ [21] we find

$$\begin{aligned} \mathcal{BR}(\bar{B}_s \rightarrow f_0\ell^+\ell^-) &= (9.5_{-2.6}^{+3.1}) \times 10^{-8}, \\ \mathcal{BR}(\bar{B}_s \rightarrow f_0\tau^+\tau^-) &= (1.1_{-0.3}^{+0.4}) \times 10^{-8}, \\ \mathcal{BR}(\bar{B}_s \rightarrow f_0\nu\bar{\nu}) &= (8.7_{-2.4}^{+2.8}) \times 10^{-7} \end{aligned} \quad (35)$$

with $\ell = e, \mu$. Our estimate of the NLO effects in the form factors modifies the branching ratios to $\mathcal{BR}(\bar{B}_s \rightarrow f_0\ell^+\ell^-) = (16.7 \pm 6.1) \times 10^{-8}$, $\mathcal{BR}(\bar{B}_s \rightarrow f_0\tau^+\tau^-) = (2.7 \pm 1.3) \times 10^{-8}$, and $\mathcal{BR}(\bar{B}_s \rightarrow f_0\nu\bar{\nu}) = (15.2 \pm 5.6) \times 10^{-7}$. These decay modes are therefore accessible at the LHCb experiment at the CERN Large Hadron Collider and at a Super B factory operating at the $Y(5S)$ peak.

B. Nonleptonic $B_s \rightarrow J/\psi f_0$ transition

The study of CP violation and the measurement of the CKM angles mainly proceed through the measurement of nonleptonic decay modes. In the B_s sector, the channel $B_s \rightarrow J/\psi \phi$ is the golden mode to investigate CP violation, and from the analysis of this mode the CDF [5] and D0 [6] Collaborations at the Fermilab Tevatron have obtained values of the B_s mixing phase $\phi_s = -2\beta_s$ much larger than predicted in the SM, modulo a large experimental uncertainty. If confirmed, this measurement would indicate physics beyond SM. It is of prime importance to consider other processes allowing to access β_s , namely, $B_s \rightarrow J/\psi \eta$, $J/\psi \eta'$ and $J/\psi f_0(980)$ in which the final state is a CP eigenstate and no angular analysis is required to disentangle the various CP components, as needed for $B_s \rightarrow J/\psi \phi$. However, the reconstruction of B_s modes into η and η' is experimentally challenging, since the subsequent η or η' decays involve photons in the final state. The case of f_0 seems feasible, since $f_0(980)$ essentially decays to $\pi^+ \pi^-$ and to $2\pi^0$ (the decay to $K^+ K^-$ has also been seen) [21]. Predictions of $B_s \rightarrow J/\psi f_0(980)$ are therefore of great importance.

The quantitative description of nonleptonic decays is very challenging. The theoretical framework to study such decays is based on the operator product expansion and renormalization group methods, which allow to write an effective Hamiltonian as in the case of the modes considered in the previous section. However, now one has to consider hadronic matrix elements $\langle J/\psi f_0 | O_i | B_s \rangle$ with O_i four-quark operators, the calculation of which is a nontrivial task. One of the strategies which has been exploited is the naive factorization [36], in which such quantities are replaced by products of matrix elements of the weak currents appearing in each one of the operators of the effective hamiltonian relative to the considered process. These are expressed in terms of meson decay constants and hadronic form factors. Such a procedure is affected by several drawbacks, and various refinements have been proposed [37–39], starting from the observation that a theoretical justification of naive factorization in the case of B decays can be found in the heavy-quark limit $m_b \rightarrow \infty$ only in a limited class of processes. However, improved approaches, namely, those based on QCD factorization [37], have not been applied to the B_s decay mode of interest here. In order to estimate the size of the $B_s \rightarrow f_0 J/\psi$ decay rate, we consider the so called generalized factorization approach, in which the Wilson coefficients (or appropriate combinations of them) appearing in the factorized amplitudes are regarded as effective parameters

to be fixed from experiment. Using this factorization ansatz, the decay amplitude $\bar{B}_s(p_{B_s}) \rightarrow J/\psi(p_\psi, \epsilon) f_0(p_{f_0})$ (ϵ being the J/ψ polarization vector, p_{B_s} , p_ψ , p_{f_0} the momenta of the three particles) is given as

$$\mathcal{A}(\bar{B}_s \rightarrow J/\psi f_0) = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* a_2 m_\psi f_{J/\psi} F_1^{B_s \rightarrow f_0} \times (m_{J/\psi}^2) 2(\epsilon^* \cdot p_{B_s}); \quad (36)$$

$f_{J/\psi}$ is the J/ψ decay constant, determined from the $J/\psi \rightarrow e^+ e^-$ decay width [21]: $f_{J/\psi} = (416.3 \pm 5.3)$ MeV. The factor a_2 is a combination of Wilson coefficients which can be extracted from the $B \rightarrow J/\psi K$ decays, under the assumption that a_2 is the same in the two processes. For these decays the branching ratios are known [21]:

$$\begin{aligned} \mathcal{B}\mathcal{R}(B^- \rightarrow J/\psi K^-) &= (1.007 \pm 0.035) \times 10^{-3}, \\ \mathcal{B}\mathcal{R}(B^0 \rightarrow J/\psi K^0) &= (8.71 \pm 0.32) \times 10^{-4}. \end{aligned} \quad (37)$$

In order to extract a_2 , the form factor $F_1^{B \rightarrow K}$ is required. We use two different parameterizations, obtained by short-distance (CDSS) [40] and light-cone QCD sum rules (BZ) [24]. The result is different for the two sets of form factors, while there is almost no difference whether we use the charged or the neutral channel:

$$\begin{aligned} |a_2^{B \rightarrow J/\psi K, (CDSS)}| &= 0.394_{-0.041}^{+0.053}, \\ |a_2^{B \rightarrow J/\psi K, (BZ)}| &= 0.25 \pm 0.03. \end{aligned} \quad (38)$$

To be conservative with the hadronic uncertainty, we use the average value $a_2 = 0.32 \pm 0.11$ of the two values to compute $\mathcal{B}\mathcal{R}(\bar{B}_s \rightarrow J/\psi f_0)$. Using $V_{cb} = 0.0412$, $V_{cs} = 0.997$ [21] and our LO prediction for the $B_s \rightarrow f_0$ form factors, we obtain

$$\mathcal{B}\mathcal{R}(\bar{B}_s \rightarrow J/\psi f_0) = (3.1 \pm 2.4) \times 10^{-4} \quad (39)$$

while, including our estimate of NLO corrections, the branching fraction is $\mathcal{B}\mathcal{R}(\bar{B}_s \rightarrow J/\psi f_0) = (5.3 \pm 3.9) \times 10^{-4}$. The rate of $B_s \rightarrow J/\psi f_0$ is large enough to permit a measurement; notice that the branching fraction of $B_s \rightarrow J/\psi \phi$ is $\mathcal{B}\mathcal{R}(B_s \rightarrow J/\psi \phi) = (1.3 \pm 0.4) \times 10^{-3}$ [21].

To gain a better insight on this point, it is interesting to compare these results to the branching fraction of $B_s \rightarrow J/\psi_L \phi_L$ (L denotes a longitudinally polarized meson) computed in the factorization approach. Neglecting the mass difference between ϕ and f_0 in the phase space, the ratio of the branching fractions of the two modes can be written in terms of form factor combinations:

$$R_{f_0/\phi}^{B_s} = \frac{\mathcal{B}\mathcal{R}(B_s \rightarrow J/\psi f_0)}{\mathcal{B}\mathcal{R}(B_s \rightarrow J/\psi_L \phi_L)} \simeq \frac{[F_1^{B_s \rightarrow f_0}(m_\psi^2)]^2 \lambda(m_{B_s}^2, m_\psi^2, m_{f_0}^2)}{[A_1^{B_s \rightarrow \phi}(m_\psi^2)(m_{B_s} + m_\phi) \frac{(m_{B_s}^2 - m_\psi^2 - m_\phi^2)}{2m_\phi} - A_2^{B_s \rightarrow \phi}(m_\psi^2) \frac{\lambda(m_{B_s}^2, m_\psi^2, m_\phi^2)}{2m_\phi(m_{B_s} + m_\phi)}]^2} = \begin{cases} 0.13 \pm 0.06 \\ 0.22 \pm 0.10 \end{cases}, \quad (40)$$

where the two results correspond to the $B_s \rightarrow f_0$ form factor evaluated at the leading order or not. $A_1^{B_s \rightarrow \phi}$ $A_2^{B_s \rightarrow \phi}$ are among the $B_s \rightarrow \phi$ transition form factors and are taken from Ref. [41]. In Ref. [8] it was suggested that the ratio $R_{f_0/\phi}^{B_s}$ can be inferred from the ratio of D_s decay widths to $f_0\pi^+$ and $\phi\pi^+$, obtaining $R_{f_0/\phi}^{B_s} \simeq 0.2 - 0.3$, which is compatible with our result (40).

Another relation has been also proposed in [8] connecting $R_{f_0/\phi}^{B_s}$ to a different observable in D_s decays:

$$R_{f_0/\phi}^{B_s} \simeq R_{f_0/\phi}^{D_s} = \frac{\frac{d\Gamma}{dq^2}(D_s^+ \rightarrow f_0 e^+ \nu, f_0 \rightarrow \pi^+ \pi^-)|_{q^2=0}}{\frac{d\Gamma}{dq^2}(D_s^+ \rightarrow \phi e^+ \nu, \phi \rightarrow K^+ K^-)|_{q^2=0}}. \quad (41)$$

For this quantity the CLEO Collaboration has recently provided a measurement: $R_{f_0/\phi}^{D_s} = (0.42 \pm 0.11)$ [10] which is larger than our (40).

All the above considerations show that the mode $B_s \rightarrow J/\psi f_0$ must be used, together with the golden mode $B_s \rightarrow J/\psi \phi$, to measure the B_s mixing phase, mainly because it provides us with a large number of events and does not require an angular analysis to separate different CP components of the final state. This is also the case of modes in which J/ψ is replaced by a spin 0 charmonium state, such as χ_{c0} , modulo the difficulty of the χ_{c0} reconstruction. $B_s \rightarrow \chi_{c0}\phi$ will provide a side-check when the number of accumulated data will increase. Although $B_s \rightarrow \chi_{c0}\phi$ is a suppressed channel in naive factorization, its branching fraction may not be small due to the intermediate rescattering mechanism [42] or because of the contribution of nonfactorizable diagrams [43] as in $B \rightarrow \chi_{c0}K$. Analogously for $B_s \rightarrow \chi_{c0}\phi$, the branching ratio of $B \rightarrow \chi_{c0}K^*$ has been measured [44],

$$\begin{aligned} \mathcal{BR}(\bar{B}^0 \rightarrow \chi_{c0}\bar{K}^{*0}) &= (1.7 \pm 0.3 \pm 0.2) \times 10^{-4}, \\ \mathcal{BR}(B^- \rightarrow \chi_{c0}K^{*-}) &= (1.4 \pm 0.5 \pm 0.2) \times 10^{-4} \\ &< 2.1 \times 10^{-4} (90\% \text{C.L.}) \end{aligned} \quad (42)$$

and, on the basis of $SU(3)_F$ symmetry, the branching fraction of $B_s \rightarrow \chi_{c0}\phi$ should be similar.

V. DECAY $D_s \rightarrow f_0 e^+ \nu$

By a suitable change of parameters in the sum rules in Sec. II, also the $D_s \rightarrow f_0$ form factors can be computed and the branching ratio of the semileptonic decay $D_s \rightarrow f_0 e^+ \nu$ can be predicted. We use $m_c = 1.4$ GeV and $\tau(D_s) = 0.5$ ps [21]; the threshold parameter is fixed to $s_0^{D_s} = (6.5 \pm 1.0)$ GeV². For the D_s decay constant we use the value quoted by the Heavy Flavor Averaging Group: $f_{D_s} = (256.9 \pm 6.8)$ MeV [45]. The Borel parameter can be fixed requiring stability of the sum rule result with respect to M^2 variations. In Fig. 3 we plot $F_1^{D_s \rightarrow f_0}(0)$ versus M^2 ; the stability window is selected in the range $M^2 = (5 \pm 1)$ GeV². We find

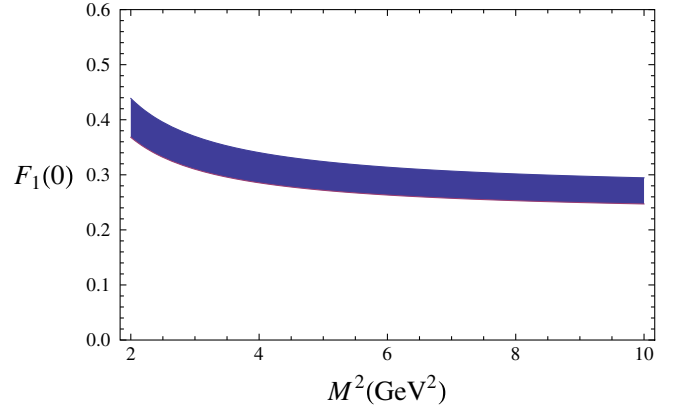


FIG. 3 (color online). Dependence of the $D_s \rightarrow f_0$ form factors at $q^2 = 0$ $F_1(0) = F_0(0)$ on the Borel parameter M^2 .

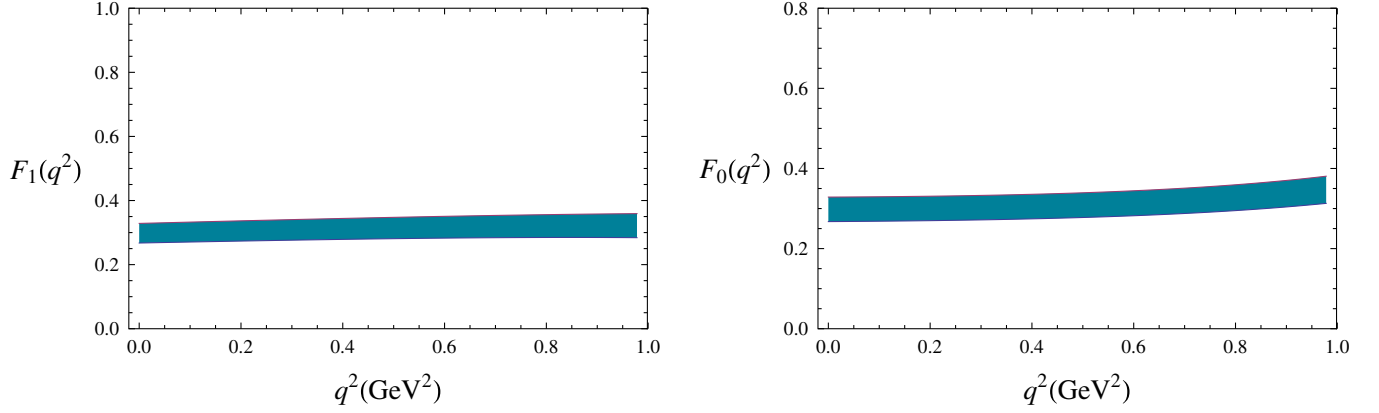
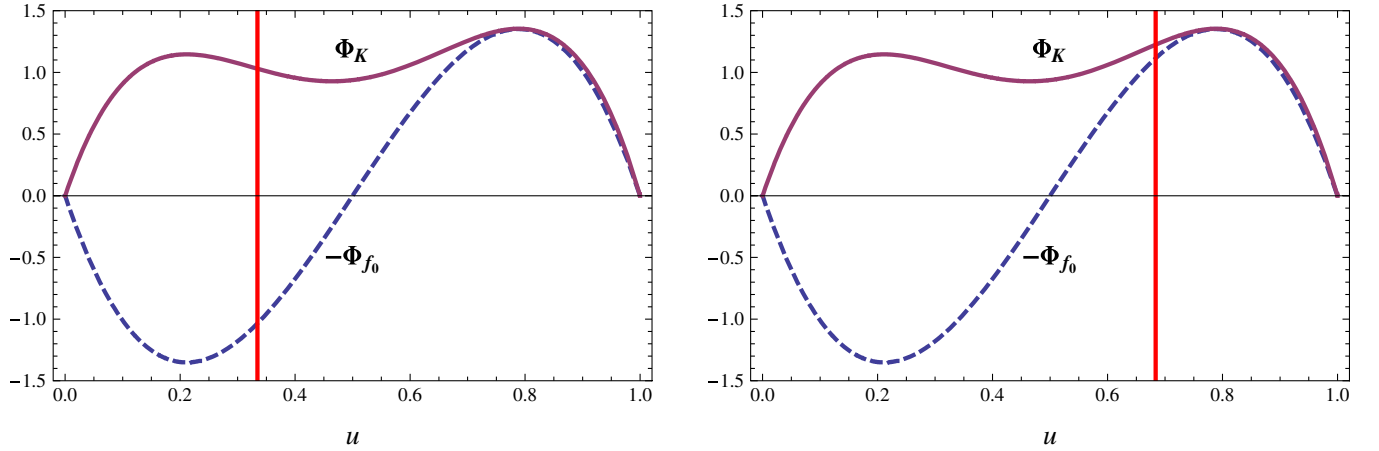
$$F_1^{D_s \rightarrow f_0}(0) = F_0^{D_s \rightarrow f_0}(0) = 0.30 \pm 0.03. \quad (43)$$

The q^2 dependence of the two form factors is displayed in Fig. 4. The value of (43) is much smaller than in the $D \rightarrow K$ case, for which the light-cone sum rule prediction is $F_1^{D \rightarrow K}(0) = 0.75_{-0.08}^{+0.11}$ [46]. We can understand this difference noticing that contribution of the f_0 twist-2 LCDA in $D_s \rightarrow f_0$ transition is small due to the different shape of the twist-2 f_0 distribution amplitude with respect to the case of K . The two LCDA are plotted in Fig. 5, where the position of the parameter u_0 , defined in Eq. (22), is also displayed (left panel). The situation can be compared to the $B_s \rightarrow f_0$ case, shown in the right panel of the figure. Since the LCDA is integrated in the range $[u_0, 1]$, one can see that, in the case of $f_0(980)$, the integral of the distribution amplitude gets two opposite contributions which tend to cancel each other, due to the presence of a zero in the DA. The zero is not present in the kaon DA, so that the integrated DA gives a much larger contribution. In the case of B_s , the position of the parameter u_0 is such that the zero of the DA is not included in the integration region, so that no sizable difference is expected between the f_0 and the kaon cases. This argument explains also why, compared with the results of other approaches, our outcomes are smaller. This can be noticed in Table IV, where we compare our results for the $D_s \rightarrow f_0$ form factors with other estimates [27,47,48].

The form factor $F_1(q^2)$ enters in the expression of the differential decay rate

$$\frac{d\Gamma(D_s \rightarrow f_0 e^+ \nu)}{dq^2} = \frac{G_F^2 V_{cs}^2 \lambda^{3/2}(m_{D_s}^2, m_{f_0}^2, q^2)}{192 m_{D_s}^3 \pi^3} |F_1(q^2)|^2, \quad (44)$$

where the lepton mass is neglected. Since in $D_s \rightarrow f_0 e^+ \nu$ the kinematically accessible q^2 range is limited, the applicable region for LCSR is narrow. One can fit the form factors in the spacelike region, for example -2 GeV² $< q^2 < 0$, and then extrapolate to the timelike region.


 FIG. 4 (color online). q^2 dependence of the $D_s \rightarrow f_0$ form factors.

 FIG. 5 (color online). Shape of the twist-2 LCDA: $-\Phi_{f_0} = -6u(1-u)B_1C_1^{3/2}(2u-1)$ (dashed line) and Φ_K (solid line) taken from [50]. In the left panel, the vertical (red) line denotes the position of $u_0^{D_s} = 0.334$ fixed for the $D_s \rightarrow f_0$ transition, while in the right panel the vertical (red) line corresponds to the position of $u_0^{B_s} = 0.684$ at $q^2 = 0$ in $B_s \rightarrow f_0$ transition.

However, the result of the extrapolation strongly depends on the choice of the fitting region. Moreover, looking at Fig. 4, one can notice that the q^2 dependence of F_1 and F_0 is mild. In view of this, we use a constant form factor $F_1(q^2) = F_1(0)$ to compute the branching ratio of $D_s \rightarrow f_0 e^+ \nu$; the result varies less than 10% including the q^2 dependence according to different fitting formulae. The obtained branching fraction is

$$\mathcal{BR}(D_s \rightarrow f_0 e^+ \nu) = (2.0_{-0.4}^{+0.5}) \times 10^{-3}. \quad (45)$$

The modification due to radiative corrections can be estimated as in the case of $B_s \rightarrow f_0$, finding $F_1^{D_s \rightarrow f_0}(0) = 0.29_{-0.04}^{+0.05}$.

Let us consider the available experimental data. The CLEO Collaboration has measured the product of branching fractions [10]

$$\begin{aligned} \mathcal{BR}(D_s \rightarrow f_0(980) e^+ \nu) \times \mathcal{BR}(f_0 \rightarrow \pi^+ \pi^-) \\ = (0.20 \pm 0.03 \pm 0.01) \times 10^{-2}, \end{aligned} \quad (46)$$

updating a previous determination [9]

$$\begin{aligned} \mathcal{BR}(D_s \rightarrow f_0 e^+ \nu) \times \mathcal{BR}(f_0 \rightarrow \pi^+ \pi^-) \\ = (0.13 \pm 0.04 \pm 0.01) \times 10^{-2}. \end{aligned} \quad (47)$$

Using experimental data provided by the BES Collaboration studying the processes $\chi_{c0} \rightarrow f_0(980) f_0(980) \rightarrow \pi^+ \pi^- K^+ K^-$ and $\chi_{c0} \rightarrow f_0(980) f_0(980) \rightarrow K^+ K^- K^+ K^-$ [49], CLEO quotes

 TABLE IV. $D_s \rightarrow f_0(980)$ form factor at $q^2 = 0$, together with the results obtained by CLFD/DR [27], QCDSR [47] and CLFQM [48] approaches.

	CLFD/DR	QCDSR	CLFQM	This work
$F_1(0)$	0.45/0.46 ^a	1.7(0.27 ± 0.02) ^b	0.434	0.30 ± 0.03

^aUsing $f_{D_s} = 0.274$ GeV.

^bUsing $f_{D_s} = 0.22 \pm 0.02$ GeV; by using different input parameters two results are obtained, the first one in parentheses, the second one 1.7 times larger.

$$\mathcal{BR}(f_0 \rightarrow \pi^+ \pi^-) = (50_{-9}^{+7}) \times 10^{-2} \quad (48)$$

which, combined with (46), gives

$$\mathcal{BR}(D_s \rightarrow f_0 e^+ \nu) = (4.0 \pm 0.6 \pm 0.6) \times 10^{-3}, \quad (49)$$

marginally consistent with our (45).

VI. CONCLUSIONS

We have computed the $B_s \rightarrow f_0$ transition form factors using light-cone QCD sum rules at leading order in the strong coupling constant, and also estimating the size of NLO corrections. The resulting form factors permit to predict the rates of $B_s \rightarrow f_0 \ell^+ \ell^-$ and $B_s \rightarrow f_0 \nu \bar{\nu}$ decays, finding branching ratios accessible at future machines, like

a Super B factory, and at the LHCb experiment at CERN. The branching ratio of $B_s \rightarrow J/\psi f_0$ can be predicted under the factorization assumption: we find $\mathcal{BR}(B_s \rightarrow J/\psi f_0)/\mathcal{BR}(B_s \rightarrow J/\psi \phi) \sim 0.2 - 0.3$, thus the $B_s \rightarrow J/\psi f_0$ channel can be considered another promising mode to access the $B_s - \bar{B}_s$ mixing phase. We have also investigated the $D_s \rightarrow f_0 e^+ \nu$ decay channel by the LCSR approach and compared the results to recent measurements.

ACKNOWLEDGMENTS

W. W. thanks Yu-Ming Wang for useful discussions. This work was supported in part by the EU Contract No. MRTN-CT-2006-035482, FLAVIANet.

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