

Resonant leptogenesis and a verifiable seesaw model from large extra dimensions

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In the presence of large extra dimensions, the fundamental scale could be as low as a few TeV. This yields leptogenesis and seesaw at a TeV scale. Phenomenologically, two TeV-scale Majorana fermions with a small mass split can realize a resonant leptogenesis, whereas a TeV-scale Higgs triplet with a small trilinear coupling to the standard model Higgs doublet can give a verifiable seesaw. We propose an interesting scenario where the small parameters for the resonant leptogenesis and the type-II seesaw can be simultaneously generated by the propagation of lepton number violation from distant branes to our world.

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In the theory of large extra dimensions [1], the Planck scale of the 4-dimensional theory is related to that of the $(4 + n)$ -dimensional theory by

$$M_{\text{Pl}}^2 \sim R^n M_*^{n+2}, \quad (1)$$

where $M_{\text{Pl}} \simeq 2.4 \times 10^{18}$ GeV is the reduced Planck mass and R is the size of the extra dimensions. Therefore, the fundamental scale M_* of quantum gravity could be as low as a few TeV for solving the hierarchy problem between the electroweak and Planck scales. Clearly, all of other scales in this theory could not be larger than a few TeV. This can give interesting implications on neutrino physics. In this direction, there have been many works [2–6]. For example, ones [6] find the lepton number violation in a distant brane can induce a very small trilinear coupling of the Higgs triplet to the standard model (SM) Higgs doublet in our world. This naturally makes the type-II [7] seesaw [8] model accessible at colliders.

The low fundamental scale also constrains leptogenesis [9–16] for baryon asymmetry. In the leptogenesis scenario, the decaying particles should be very heavy for generating a sizable CP asymmetry unless their masses are quasidegenerate, which is simply input by hand or is induced by radiative correction in some special models [17], so that the CP asymmetry can be resonantly enhanced [11,12]. Remarkably the resonant leptogenesis allows us to produce the baryon asymmetry below the low fundamental scale. The author of [14] has studied the resonant leptogenesis with bulk right-handed neutrinos. In the present work, we shall introduce two right-handed neutrinos with equal but opposite lepton numbers and localize them in our brane. In our model, the lepton number is maximally broken in the distant branes and then is shined to our brane by a bulk scalar. Therefore, the lepton number violation is highly suppressed in our world, where the right-handed neutrinos mix together through a lepton number conserving mass term and both have Yukawa couplings to the bulk scalar so

that they can naturally have a quasidegenerate mass spectrum. The induced Majorana fermions can accommodate a resonant leptogenesis by their decays into the SM lepton and Higgs doublets. Within this context, the small trilinear coupling of the Higgs triplet to the SM Higgs doublet is also ready for the verifiable type-II seesaw.

In our model, besides two right-handed neutrinos $N_{R_{1,2}}$ and the SM fields, there are three scalars: triplet ξ , doublet η , and singlet χ . We assign $L = 1$ for N_{R_1} and $N_{R_2}^c$ while $L = 2$ for ξ^* , η and χ . The lepton number conserving interactions would be

$$\begin{aligned} \mathcal{L} \supset & -y_{\alpha 1} \bar{\psi}_{L_\alpha} \phi N_{R_1} - y_{\alpha 2} \bar{\psi}_{L_\alpha} \eta N_{R_2} - M_N \bar{N}_{R_1}^c N_{R_2} \\ & - \frac{1}{2} h_1 \chi^* \bar{N}_{R_1}^c N_{R_1} - \frac{1}{2} h_2 \chi \bar{N}_{R_2}^c N_{R_2} - \rho \chi \eta^\dagger \phi \\ & - \frac{1}{2} f_{\alpha\beta} \bar{\psi}_{L_\alpha}^c i\tau_2 \xi \psi_{L_\beta} - \kappa_1 \chi \phi^T i\tau_2 \xi \phi \\ & - \kappa_2 \chi^* \eta^T i\tau_2 \xi \eta + \text{H.c.} \end{aligned} \quad (2)$$

where ψ_L and ϕ denote the SM lepton and Higgs doublets, respectively. The right-handed neutrinos $N_{R_{1,2}}$, the triplet ξ , the doublet η and the SM fields are localized in our brane ($x, y^a = 0$) while the singlet χ propagates in the bulk. Here $a = 1, \dots, n$ runs over the extra dimensions. We further introduce a singlet scalar σ with a lepton number $L = 2$. This singlet is localized in a distant brane ($x', y^a = y_*^a$). The transverse distance from the distant brane to our brane is $r = |y_*|$.

The singlet χ can interact with our brane through its couplings to the right-handed neutrinos and the triplet and doublet scalars as shown in Eq. (2). In the distant brane, it can also have lepton number conserving interaction with the singlet σ [2],

$$L \supset \int d^4 x' M_*^2 \sigma(x', y_*^a) \chi^*(x', y_*^a). \quad (3)$$

So, the singlet χ can mediate the communication from the distant brane to ours. In particular, it can carry the lepton number violation to our world through its shined value $\langle \chi \rangle$,

$$\langle \chi(x, y^a = 0) \rangle = \langle \sigma(x, y = y_*^a) \rangle \Delta_n(r). \quad (4)$$

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Here the vacuum expectation value (VEV) $\langle\sigma\rangle$ acts as a point source whereas $\Delta_n(r)$ is the Yukawa potential in the n transverse dimensions [4,6],

$$\Delta_n(r) = \frac{1}{(2\pi)^{n/2}} \left(\frac{m_\chi}{M_*}\right)^{n-2} (m_\chi r)^{-((n-2)/2)} K_{(n-2)/2}(m_\chi r). \quad (5)$$

With the natural choice $r = R$ and $\langle\sigma\rangle \lesssim M_*$, it is easy to read

$$\langle\chi\rangle \sim M_* \Delta_n(R). \quad (6)$$

We will clarify later a heavy mass m_χ is necessary for a successful leptogenesis. However, the modified Bessel function $K_{(n-2)/2}(m_\chi r)$ will exponentially suppress $\langle\chi\rangle$ if $m_\chi \gg 1/r$. This $\langle\chi\rangle$ is too small to generate the desired neutrino masses. Therefore, we consider the brane-lattice crystallization scenario [18] where the bulk is populated with large numbers of branes. One finds the lepton number violation in our brane would be [2]

$$\begin{aligned} \langle\chi\rangle &\sim M_* \int d^n r n_{\text{brane}} \Delta_n(r) = M_* n_{\text{brane}} \left(\frac{m_\chi}{M_*}\right)^{n-2} \frac{1}{m_\chi^n} \\ &= M_* \left(\frac{M_*}{m_\chi}\right)^2 \left(\frac{M_*}{M_{\text{Pl}}}\right)^{4/n} \end{aligned} \quad (7)$$

with the brane density [18]

$$n_{\text{brane}} \sim M_*^n \left(\frac{M_*}{M_{\text{Pl}}}\right)^{4/n}. \quad (8)$$

By taking the natural assumption $m_\chi \lesssim M_*$, the VEV $\langle\chi\rangle$ is power suppressed by the ratio of M_* over M_{Pl} . In the following we will consider

$$\begin{aligned} \langle\chi\rangle &\sim 260 \text{ eV} \quad \text{for } n = 6, \\ M_* &= 3 \text{ TeV} \quad \text{and } m_\chi \lesssim M_*. \end{aligned} \quad (9)$$

Because of the VEV $\langle\chi\rangle$, the masses of the right-handed neutrinos $N_{R_{1,2}}$ would be

$$\mathcal{L} \supset -M_N \bar{N}_{R_1}^c N_{R_2} - \frac{1}{2} m_1 \bar{N}_{R_1}^c N_{R_1} - \frac{1}{2} m_2 \bar{N}_{R_2}^c N_{R_2} + \text{H.c.} \quad (10)$$

with

$$m_{1,2} = h_{1,2} \langle\chi\rangle \ll M_N. \quad (11)$$

We can diagonalize the above mass terms to be

$$\mathcal{L} \supset -\frac{1}{2} M_\pm \bar{X}_R^\pm X_R^\pm + \text{H.c.} \quad (12)$$

by taking the rotations as below,

$$N_{R_1} = cX_R^+ - i sX_R^-, \quad (13a)$$

$$N_{R_2} = sX_R^+ + i cX_R^-. \quad (13b)$$

Here we have take the following notations,

$$c \equiv \cos\vartheta, \quad s \equiv \sin\vartheta \quad \text{for } \vartheta = \frac{1}{2} \arctan \frac{2M_N}{m_2 - m_1} \quad (14)$$

and

$$M_+ = 2scM_N + c^2m_1 + s^2m_2, \quad (15a)$$

$$M_- = 2scM_N - s^2m_1 - c^2m_2. \quad (15b)$$

Without loss of generality we will assume $m_1 < m_2$ so that

$$M_+ > M_- > 0. \quad (16)$$

Actually, the small and large masses (11), i.e. $m_{1,2} \ll M_N$ will induce

$$\vartheta \simeq \frac{\pi}{4}, \quad (17a)$$

$$M_\pm \simeq M_N \pm \frac{1}{2}(m_1 + m_2) \gg M_+ - M_-. \quad (17b)$$

It is convenient to define the Majorana fermions

$$X^+ = X_R^+ + X_R^{+c}, \quad (18a)$$

$$X^- = X_R^- + X_R^{-c}, \quad (18b)$$

as the mass eigenstates,

$$\mathcal{L} \supset -\frac{1}{2} M_\pm \bar{X}^\pm X^\pm. \quad (19)$$

We further assume the doublet scalar η much heavier than the Majorana fermions X^\pm , say $m_\eta^2 \gg M_\pm^2$. In this case, we can integrate out η and then simplify the Yukawa couplings given by the first line of Eq. (2) in a new form,

$$\begin{aligned} \mathcal{L} \supset &-(cy_{\alpha 1} - sy'_{\alpha 2}) \bar{\psi}_{L_\alpha} \phi X^+ - i(sy_{\alpha 1} + cy'_{\alpha 2}) \bar{\psi}_{L_\alpha} \phi X^- \\ &+ \text{H.c.} \end{aligned} \quad (20)$$

with

$$y'_{\alpha 2} = -y_{\alpha 2} \frac{\rho \langle\chi\rangle}{m_\eta^2}. \quad (21)$$

For $M_\pm \gg M_+ - M_-$, we can follow the standard method [12] of the resonant leptogenesis¹ to compute the lepton asymmetry from the decays of per X^\pm ,

¹The right-handed neutrinos for the type-I seesaw will induce the vertex loop besides the self-energy loop in their decays. At the same time, the Higgs triplet for the type-II seesaw, which will be introduced later, will mediate another vertex correction in the decays of the right-handed neutrinos. The right-handed neutrinos and the Higgs triplet are at the TeV scale so that the CP asymmetry constrained by the neutrino masses would be too small unless we take the resonant enhancement into account. However, the resonant effect only exists in the self-energy loop. So, the vertex corrections induced by the Higgs triplet or the right-handed neutrinos would have no significant contributions to the leptogenesis.

$$\begin{aligned} \varepsilon_{X^\pm} &= \frac{\sum_\alpha [\Gamma(X^\pm \rightarrow \psi_{L_\alpha} + \phi^*) - \Gamma(X^\pm \rightarrow \psi_{L_\alpha}^c + \phi)]}{\sum_\alpha [\Gamma(X^\pm \rightarrow \psi_{L_\alpha} + \phi^*) + \Gamma(X^\pm \rightarrow \psi_{L_\alpha}^c + \phi)]} \\ &\simeq \frac{sc \sum_{\alpha\beta} (|y_{\alpha 1}|^2 - |y'_{\alpha 2}|^2)}{4\pi A_{X^\pm}} \text{Im}(c^2 y_{\beta 1}^* y'_{\beta 2} - s^2 y_{\beta 2}^* y_{\beta 1}) \\ &\quad \times \frac{x}{x^2 + \frac{1}{64\pi^2} A_{X^\pm}^2} \end{aligned} \quad (22)$$

with

$$A_{X^+} = \sum_\alpha |c y_{\alpha 1} - s y'_{\alpha 2}|^2, \quad (23a)$$

$$A_{X^-} = \sum_\alpha |s y_{\alpha 1} + c y'_{\alpha 2}|^2, \quad (23b)$$

$$x = \frac{M_+^2 - M_-^2}{M_+ M_-}. \quad (23c)$$

In the weak washout region, i.e.

$$\Gamma_{X^\pm} < H(T)|_{T \simeq M_\pm}, \quad (24)$$

the final baryon asymmetry can be approximately given by [19]

$$\begin{aligned} \frac{n_B}{s} &= \frac{28}{79} \frac{n_B - n_L}{s} = -\frac{28}{79} \frac{n_L}{s} \simeq -\frac{28}{79} \varepsilon_{X^\pm} \frac{n_{X^\pm}^{\text{eq}}}{s} \Big|_{T \simeq M_\pm} \\ &\simeq -\frac{1}{\mathcal{O}(10)} \frac{\varepsilon_{X^\pm}}{g_*}. \end{aligned} \quad (25)$$

Here Γ_{X^\pm} is the decay width

$$\Gamma_{X^\pm} = \frac{1}{8\pi} A_{X^\pm} M_\pm, \quad (26)$$

whereas $H(T)$ is the Hubble constant

$$H(T) = \left(\frac{\pi^2 g_*}{90} \right)^{1/2} \frac{T^2}{M_{\text{Pl}}} \quad (27)$$

with $g_* \simeq 106.75$ being the relativistic degrees of freedom.

We should keep in mind that the right-handed neutrinos $N_{R_{1,2}}$, or equivalently the Majorana fermions X^\pm have Yukawa couplings with the bulk scalar χ . The induced annihilations of X^\pm should go out of equilibrium. This can be achieved if X^\pm is much lighter than the bulk scalar χ and the triplet scalar ξ . Actually, a factor $m_\chi/M_\pm \sim 3-10$ is enough for the decoupling of the t -channel processes $N_{R_{1,2}}^c N_{R_{1,2}} \rightarrow \chi^* \chi$ which is fast Boltzmann suppressed at temperatures below the mass m_χ . In addition, the annihilations of $N_{R_{1,2}}$ to the SM Higgs doublet ϕ (through the s -channel exchange of χ) is highly suppressed by the shined value $\langle \chi \rangle$. Furthermore, the coupling of χ to ξ and ϕ will also lead to annihilations of $N_{R_{1,2}}$. Such processes can be suppressed if ξ is much heavier than X^\pm , say $m_\xi/M_\pm \sim 3-10$. As for the lepton number violating processes mediated by ξ , they will not wash out the produced lepton asymmetry due to the small $\langle \chi \rangle$. As a result of the symmetry breaking of the global lepton number, there would be a massless Goldstone—majoron [20], which is composed of the imaginary parts of the triplet ξ

and the doublet η in our brane, the singlet χ in the bulk and the singlets σ in the distant branes. Because ξ , η and χ with small VEVs only contribute a tiny fraction, this majoron is harmless [21], in particular, it will not significantly affect the annihilation of $N_{R_{1,2}}$ [22].

We now give a numerical estimation. With the input (9), we can obtain

$$M_\pm \simeq 600 \text{ GeV} \quad \text{and} \quad x = \mathcal{O}(10^{-12}) \quad (28)$$

by inserting

$$M_N = 600 \text{ GeV} \quad \text{and} \quad h_{1,2} \sim \mathcal{O}(10^{-3}) \quad (29)$$

to Eqs. (11), (14), (15), and (23c). We further consider

$$\rho = m_\eta \lesssim M_* \quad (30)$$

in Eq. (21) and then get

$$y'_{\alpha 2} \sim \mathcal{O}(10^{-10}) \quad \text{for} \quad y_{\alpha 2} \sim \mathcal{O}(1). \quad (31)$$

We also take

$$|y_{\alpha 1}| = \mathcal{O}(10^{-7}) \quad \text{and} \quad \sin \frac{y_{\alpha 1}}{|y_{\alpha 1}|} = \mathcal{O}(0.1). \quad (32)$$

With the parameter choice (28), (31), and (32), it is easy to derive the CP asymmetry

$$\varepsilon_\pm = \mathcal{O}(10^{-7}) \quad (33)$$

and hence the baryon asymmetry

$$\frac{n_B}{s} = \mathcal{O}(10^{-10}). \quad (34)$$

The left-handed neutrinos can obtain a Majorana mass matrix with two nonzero eigenvalues from the two right-handed neutrinos $N_{R_{1,2}}$ as a result of inverse [23,24] seesaw. However, these masses are too tiny to explain the observed neutrino oscillations because of the smallness of the Yukawa couplings $y_{\alpha 1}$ and $y'_{\alpha 2}$. Alternatively, our model accommodates the type-II seesaw,

$$\mathcal{L} \supset -\frac{1}{2} f_{\alpha\beta} \bar{\psi}_{L_\alpha}^c i\tau_2 \xi \psi_{L_\beta} - \mu \phi^T i\tau_2 \xi \phi + \text{H.c.} \quad (35)$$

Here the trilinear coupling μ of the Higgs triplet ξ to the SM Higgs doublet ϕ is given by the shined value of the bulk field χ ,

$$\mu = \left(\kappa_1 + \kappa_2 \frac{\rho^2 \langle \chi \rangle^2}{m_\eta^4} \right) \langle \chi \rangle. \quad (36)$$

With the previous parameter choice for the leptogenesis, the neutrino masses would be

$$\begin{aligned}
(m_\nu)_{\alpha\beta} &= f_{\alpha\beta} \langle \xi \rangle \simeq -f_{\alpha\beta} \frac{\mu \langle \phi \rangle^2}{m_\xi^2} \\
&\simeq -f_{\alpha\beta} \kappa_1 M_* \frac{\langle \phi \rangle^2}{m_\xi^2} \left(\frac{M_*}{m_\chi} \right)^2 \left(\frac{M_*}{M_{\text{Pl}}} \right)^{4/n} \\
&= -f_{\alpha\beta} \times \frac{\kappa_1}{0.1} \times \left(\frac{M_*}{m_\xi} \right)^2 \times 0.086 \text{ eV}. \quad (37)
\end{aligned}$$

For a natural choice

$$m_\xi \lesssim M_*, \quad (38)$$

the Yukawa couplings $f_{\alpha\beta}$ of the Higgs triplet ξ to the left-handed lepton doublets $\psi_{L_{\alpha\beta}}$ should be sizable to give the expected neutrino masses.

Since the Higgs triplet ξ with the mass $m_\xi \lesssim M_*$ is kinematically accessible at the LHC and at future colliders whereas its sizable Yukawa couplings $f_{\alpha\beta}$ to the SM lepton doublets $\psi_{L_{\alpha\beta}}$ determine the texture of the neutrino mass matrix, the neutrino masses can be verified [25,26] by the decays of $\xi^{\pm\pm}$ into the charged leptons $l_{L_\alpha}^\pm l_{L_\beta}^\pm$. At the same time, the doublet η with the mass $m_\eta \lesssim M_*$ has a Yukawa coupling $y_{\alpha 2} \sim \mathcal{O}(1)$ to the SM lepton doublet ψ_{L_α} and the right-handed neutrino N_{R_2} . Through the detection on the

decays $\eta^- \rightarrow l_{L_\alpha}^- N_{R_2}^c$ and $\eta^+ \rightarrow l_{L_\alpha}^+ N_{R_2}$ and/or on the annihilations $l_{L_\alpha}^+ l_{L_\beta}^- \rightarrow N_{R_2}^c N_{R_2}$, the right-handed neutrino N_{R_2} and then the Majorana fermions X^\pm could be found as a missing energy.

In summary, the theory with the large extra dimensions implies a low fundamental scale of the order of TeV. In this scenario the resonant leptogenesis becomes attractive as it can generate the baryon asymmetry at a TeV scale. The resonant leptogenesis requires a tiny mass difference between the decaying particles. The low fundamental scale also constrains the Higgs triplet for the type-II seesaw at the TeV scale. The neutrino masses can be verified in the presence of a very small trilinear coupling between the triplet and doublet Higgs scalars. We show the small parameters for the resonant leptogenesis and the verifiable type-II seesaw can be simultaneously achieved by the shined lepton number violation from the distant branes to our world. In our model, it is possible to detect the existence of the decaying Majorana fermions for the resonant leptogenesis, besides the neutrino masses from the type-II seesaw with the Higgs triplet.

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