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Hyperfine splittings in bottomonium and the $B_q(q = n, s, c)$ mesons

A. M. Badalian*

Institute of Theoretical and Experimental Physics, Moscow, Russia

B. L. G. Bakker†

Department of Physics and Astronomy, Vrije Universiteit, Amsterdam, The Netherlands

I. V. Danilkin‡

Institute of Theoretical and Experimental Physics, Moscow, Russia
and Gesellschaft fur Schwerionenforschung (GSI), Planck Strasse 1, 64291 Darmstadt, Germany

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A universal description of the hyperfine splittings (HFS) in bottomonium and the B_q ($q = n$, s, c) mesons is obtained with a universal strong coupling constant $\alpha_s(\mu) = 0.310$ in a spin-spin potential. Other characteristics are calculated within the field correlator method, taking the freezing value of the strong coupling independent of n_f. The HFS $M(B^*) - M(B) = 45.5(1)$ MeV, $M(B_s^*) - M(B_s) = 46.2(1)$ MeV are obtained in full agreement with experiment both for $n_f = 3$ and $n_f = 4$. In bottomonium, $M(\Upsilon(9460)) - M(\eta_b) = 71.1$ MeV for $n_f = 5$ agrees with the *BABAR* data, while a smaller HFS, equal to 63.4(5) MeV, is obtained for $n_f = 4$. We predict HFS $M(\Upsilon(2S)) - M(\eta_b(2S)) = 36(1)$ MeV, $M(\Upsilon(3S)) - M(\eta(3S)) = 28(1)$ MeV, and the mass difference $M(B_c^*) - M(B_c) = 58(1)$ MeV, which gives $M(B_c^*) = 6334(1)$ MeV. If a coupling with open channels is neglected, for higher states the masses $M(B_c(2^1S_0)) = 6865(5)$ MeV and $M(B_c^*(2S^3S_1)) = 6901(5)$ MeV are calculated.

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I. INTRODUCTION

Recently, $\eta_b(1S)$ has been discovered by the *BABAR* Collaboration in the radiative decays $\Upsilon(3S) \to \gamma\eta_b(1S)$ [1] and $\Upsilon(2S) \to \gamma\eta_b(1S)$ [2], with a mass (averaged over two results) $M(\eta_b) = 9391.1 \pm 3.1$ MeV. It gives a rather large hyperfine splitting (HFS), $\Delta(b\bar{b}) = M(\Upsilon(1S)) - M(\eta_b(1S)) = 69.9 \pm 3.1$ MeV. Later this mass was confirmed by the CLEO Collaboration also in the radiative $\Upsilon(3S) \to \gamma\eta_b(1S)$ decay [3]. This important new information allows one to test again our understanding of the hyperfine (HF) interaction in QCD.

A spin-spin potential between heavy quarks was used in numerous studies. The parameters defining this potential significantly differ in different models, therefore theoretical predictions for the mass difference $\Delta(b\bar{b}) = M(\Upsilon(9460)) - M(\eta_b(1S))$ vary in a wide range, 35–90 MeV [4–9], and in most cases they are smaller than the experimental number. The spin-spin potential $V_{ss}(\text{lat})$ has been studied in quenched QCD (see [10] and references therein). This lattice potential is compatible with zero at distances $r \gtrsim 0.30$ fm and (for unknown reasons) has negative sign at smaller r (with a large magnitude); it does not contradict the Fermi-Breit potential with $\delta^3(\boldsymbol{r})$ [11], although the behavior of the spin-spin potential at $r \leq 0.3$ fm remains uncertain.

*badalian@itep.ru
†blg.bakker@few.vu.nl
‡danilkin@itep.ru

A detailed phenomenological analysis given in Ref. [4] has demonstrated the importance of the smearing of the $\delta^3(\boldsymbol{r})$ function, from which one may expect that for heavy mesons, containing a b quark, the use of $\delta^3(\boldsymbol{r})$ may be a good approximation. For lighter mesons, like D, D_s, and charmonium, the nonperturbative spin-spin potential also may give a contribution $\leq 5\%$ [12]. Here we concentrate on bottomonium and the B_q ($q = n$, s, c) mesons, for which nonperturbative contributions are small, and neglect the smearing effect, in this way avoiding having to introduce several unknown parameters.

Our main goal here is the extraction of the strong coupling $\alpha_{\rm HF}(\mu)$ from known HFS. In theoretical models two typical choices of $\alpha_{\rm HF}$ are used:

(1) In the first one, "a universal" $\alpha_{\rm HF}$ is used. For example, in Ref. [5] $\alpha_{\rm HF} = 0.36$ was taken, obtained from a fit to the mass difference $M(J/\psi) - M(\eta_c(1S)) = 117$ MeV, but their HFS, $M(\Upsilon(9460)) - M(\eta_b) = 87$ MeV, is $\sim$25% larger than the experimental number. In Ref. [9], using a smaller $\alpha_{\rm HF} = 0.339$ a good description of the HFS of the B and B_s mesons was obtained. However, a comparison of their and our results is difficult, because a large string tension, $\sigma = 0.257$ GeV^2, was taken in [9], while here and in Ref. [4] the conventional value $\sigma = 0.18$ GeV^2 is used.

(2) The second choice, with a scale μ dependent on the quark mass, is mostly used in pQCD, where $\alpha_{\rm HF}(m_b) \sim 0.18$ and $\alpha_{\rm HF}(m_c) \sim 0.26$. Owing to this small value of $\alpha_{\rm HF}(m_b)$, taken in bottomonium,

small HFS were obtained in Ref. [12], although their wave functions (w.f.) at the origin gave excellent descriptions of the dielectron widths for $\Upsilon(nS)$ ($n = 1, 2, 3$) [13].

Here we use instead of the Fermi-Breit potential a spin-spin potential derived using the field correlator method (FCM) [14], where relativistic corrections are taken into account and with the mass of a light quark $m_n = 5$ MeV ($n = u, d$) and $m_s = 200$ MeV for an s quark, the B, B_s mesons can be considered on the same footing as the B_c mesons.

The HFS are sensitive to the value of $\Lambda_{\overline{\rm MS}}(n_f)$ taken. Since $\Lambda_{\overline{\rm MS}}$ is known only for $n_f = 5$ and $\Lambda_{\overline{\rm MS}}$, used for $n_f = 3, 4$, varies in a wide range, we make here the assumption, already used in [4], that the freezing value of the vector coupling constant [denoted as $\alpha_{\rm crit}(n_f)$] is the same for $n_f = 3, 4, 5$. Then it appears possible to obtain a good description of the HFS for the B_q mesons ($q = n, s, c$) and bottomonium, taking a universal $\alpha_{\rm HF}(\mu) = 0.310$, which is smaller than in Refs. [5,9].

We also predict the HFS of the as yet undiscovered $\eta_b(2S)$ and $\eta_b(3S)$, the masses of $B_c^*(1S)$ and $B_c(nS)$ ($n = 1, 2$).

II. THE HF POTENTIAL IN THE FIELD CORRELATOR METHOD

In heavy quarkonia the Fermi-Breit potential is widely used,

$$\hat{V}_{\rm ss}(r) = \boldsymbol{s}_1 \cdot \boldsymbol{s}_2 \frac{32\pi}{9} \frac{\alpha_{\rm HF}(\mu)}{\tilde{m}_1 \tilde{m}_2} \delta^3(\boldsymbol{r}), \tag{1}$$

which contains the constituent quark masses $\tilde{m}_1$ and $\tilde{m}_2$, which are very much model dependent. In Eq. (1) the strong coupling constant $\alpha_{\rm HF}(\mu)$ may differ from $\alpha_s(\mu)$ (in the $\overline{\rm MS}$ renormalization scheme) due to higher order perturbative corrections, e.g. with one-loop corrections

$$\alpha_{\rm HF}(\mu) = \alpha_s(\mu)\left[1 + \frac{\alpha_s(\mu)}{\pi}\rho(n_f)\right]. \tag{2}$$

However, the factor ρ is known only in the cases when both masses m_1, m_2 are large enough. In heavy quarkonia with $m_1 = m_2$ one-loop corrections appear to be small, $\sim 3\%$ for $n_f = 4$ and $\leq 0.1\%$ for $n_f = 5$, as it is seen from the explicit expression for ρ from [15]:

$$\rho = \frac{5}{12}\beta_0 - \frac{8}{3} - \frac{3}{4}\ln 2, \tag{3}$$

so that in bottomonium with $n_f = 5$ the coupling constants $\alpha_{\rm HF}$ and $\alpha_s(\mu)$ coincide. For heavy-light mesons a relation between these couplings remains unknown and we use here only an effective coupling $\alpha_{\rm HF}$.

The important role of relativistic corrections, even for the B_c meson, has already been underlined in Refs. [4,9], and recently in the lattice calculations of the B_c^* mass [16,17]. We take them into account using the spin-spin potential (without smearing), derived in the FCM [14,18]:

$$\hat{V}_{\rm ss}(r) = \boldsymbol{s}_1 \cdot \boldsymbol{s}_2 \frac{32\pi}{9} \frac{\alpha_{\rm HF}(\mu)}{\omega_1 \omega_2} \delta^3(\boldsymbol{r}), \tag{4}$$

for which the HFS is

$$\Delta_{\rm HF}(nS) = \frac{8}{9} \frac{\alpha_{\rm HF}(\mu)}{\omega_1 \omega_2} |R_n(0)|^2. \tag{5}$$

In Eqs. (4) and (5) the variables $\omega_1(nS)$, $\omega_2(nS)$ are the averaged kinetic energies of a quark 1 and an antiquark 2, which play a role of the dynamical masses:

$$\omega_1(nS) = \langle\sqrt{\boldsymbol{p}^2 + m_1^2}\rangle_{nS}, \qquad \omega_2(nS) = \langle\sqrt{\boldsymbol{p}^2 + m_2^2}\rangle_{nS}. \tag{6}$$

The important point is that in Eq. (6) the masses m_1 and m_2 are well defined; they are the pole masses of c and b quarks (known now with an accuracy of ~ 70 MeV for a b quark and ~ 100 MeV for a c quark [19]). We take here $m_1 = m_n = 5$ MeV for a light quark ($n = u, d$); $m_s = 200$ MeV for an s quark; the pole mass $m_c = 1.41$ GeV; $m_b = 4.79$ GeV for $n_f = 3$ and $m_b = 4.82$ GeV for $n_f = 4, 5$.

The quantities ω_i and the w.f. are calculated with the use of the relativistic string Hamiltonian (RSH), also derived in the FCM [20],

$$H_0 = \frac{\omega_1}{2} + \frac{\omega_2}{2} + \frac{m_1^2}{2\omega_1} + \frac{m_2^2}{2\omega_2} + \frac{\boldsymbol{p}^2}{2\omega_{\rm red}} + V_{\rm B}(r), \tag{7}$$

where the variables ω_i enter as the kinetic energy operators. However, if one uses an einbein approximation [18,21] and considers the spin-dependent potential as a perturbation, then ω_i should be replaced by its matrix elements (6).

A simple expression for the spin-averaged mass $M(nS)$ follows from the RSH [21]:

$$M(nS) = \frac{\omega_1}{2} + \frac{\omega_2}{2} + \frac{m_1^2}{2\omega_1} + \frac{m_2^2}{2\omega_1} + E_{nS}(\omega_{\rm red}). \tag{8}$$

Here, the excitation energy $E_{nS}(\omega_{\rm red})$ depends on the reduced mass: $\omega_{\rm red} = \frac{\omega_1\omega_2}{\omega_1+\omega_2}$. The mass formula (8) does not contain any additive constant in the case of bottomonium, while for the B and B_s mesons a negative (not small) self-energy term, proportional to $(\omega_q)^{-1}$ ($q = n, s$), has to be added to their masses [22].

Then the variables $\omega_i(nS)$, the excitation energy $E_{nS}(\omega_{\rm red})$, and the w.f. are calculated from the Hamiltonian (7) and two extremum conditions, $\partial M(nS)/\partial\omega_i = 0$ ($i = 1, 2$), which are put on the mass $M(nS)$ [18]:

$$\left[\frac{\omega_1}{2} + \frac{\omega_2}{2} + \frac{m_1^2}{2\omega_1} + \frac{m_2^2}{2\omega_2} + \frac{\boldsymbol{p}^2}{2\omega_{\rm red}} + V_{\rm B}(r)\right]\varphi_{nS}(r) = M(nS)\varphi_{nS}, \tag{9}$$

$$\omega_i^2(nS) = m_i^2 - 2\omega_i^2(nS)\frac{\partial E_{nS}(\omega_{\rm red})}{\partial\omega_i(nS)}, \qquad (i = 1, 2). \tag{10}$$

In a Hamiltonian approach the choice of the static potential $V_B(r)$ is of great importance; we take it as a sum of a linear confining term and the one gluon exchange-type term: this property of additivity is well established now in analytical studies [14] and on the lattice [23,24]:

$$V_B(r) = \sigma r + \frac{4\alpha_B(r)}{3r}. \tag{11}$$

For the string tension a conventional value, $\sigma = 0.18$ GeV^2, is used here for all mesons. The main uncertainty comes from the vector coupling $\alpha_V(r)$, which is taken here from Refs. [25,26] and denoted as $\alpha_B(r)$. Two important conditions have to be put on the vector coupling:

(i) As in perturbative QCD, it must possess the property of asymptotic freedom; precisely owing to this property the static interaction depends on the number of flavors.

(ii) The vector coupling freezes at large distances. The property of freezing was widely used in phenomenology [4–7,27] and observed in lattice calculations [23,24].

Unfortunately, one cannot use the static potential and the freezing (critical) constant from lattice studies, where the latter is found to be significantly smaller than in phenomenology and background perturbation theory. There $\alpha_B(\text{crit}) = 0.58$–$0.60$ is used (these numbers are close to the value from [4]). On the lattice, $\alpha_{\text{crit}}(\text{lat}) \sim 0.30$ in full QCD ($n_f = 3$) [24] and $\alpha_{\text{crit}}(\text{lat}) \sim 0.22$ in quenched calculations [10,23] were obtained.

In Eq. (11) the vector coupling $\alpha_B(r)$ is defined via the vector coupling $\alpha_B(q^2)$ in momentum space [26]:

$$\alpha_B(r) = \frac{2}{\pi}\int_0^\infty dq \frac{\sin(qr)}{q}\alpha_B(q), \tag{12}$$

where the vector coupling $\alpha_B(q^2)$ is taken in two-loop approximation,

$$\alpha_B(q) = \frac{4\pi}{\beta_0 t_B}\left(1 - \frac{\beta_1}{\beta_0^2}\frac{\ln t_B}{t_B}\right), \qquad t_B = \frac{q^2 + M_B^2}{\Lambda_B^2}, \tag{13}$$

with the logarithm containing the vector constant $\Lambda_B(n_f)$, which differs from the QCD constant $\Lambda_{\overline{MS}}(n_f)$. The relation between them has been established in Ref. [28]:

$$\Lambda_B(n_f) = \Lambda_{\overline{MS}} \exp\left(-\frac{a_1}{2\beta_0}\right), \tag{14}$$

with $\beta_0 = 11 - \frac{2}{3}n_f$ and $a_1 = \frac{31}{3} - \frac{10}{9}n_f$. Therefore the constant Λ_B is always larger than $\Lambda_{\overline{MS}}$,

$$\begin{aligned} \Lambda_B^{(5)} &= 1.3656\Lambda_{\overline{MS}}^{(5)} \quad (n_f = 5),\\ \Lambda_B^{(4)} &= 1.4238\Lambda_{\overline{MS}}^{(4)} \quad (n_f = 4),\\ \Lambda_B^{(3)} &= 1.4753\Lambda_{\overline{MS}}^{(3)} \quad (n_f = 3). \end{aligned} \tag{15}$$

At present, only the QCD constant $\Lambda_{\overline{MS}}(n_f = 5)$ is known with a good accuracy, while for $n_f = 3$, 4 it is defined with an accuracy $\sim 10\%$ [19]. For a given $\Lambda_{\overline{MS}}(n_f = 5)$ one can define $\alpha_{\text{crit}}(n_f = 5)$. Then, assuming that the freezing constant is the same for $n_f = 3$, 4, the QCD constant, $\Lambda_{\overline{MS}}$ for $n_f = 3$, 4, can be calculated.

The mass M_B under the logarithm t_B is proportional to $\sqrt{\sigma}$, $M_B = 1.0 \pm 0.05$ GeV [29].

Our calculations give small relativistic corrections for bottomonium: $\omega_b(1S) - m_b \sim 190$ MeV ($\sim 4\%$) and $\sim 7\%$ for the $2S$ and $3S$ states. It is of interest to notice that the relativistic correction to the b-quark mass is even smaller in the B_c meson, $\omega_b(1S) - m_b \sim 83$ MeV ($\sim 2\%$), while for a c quark the difference $\omega_c(1S) - m_c \sim 250$ MeV is already $\sim 20\%$. The values of $\omega_q(1S) - m_q$ are given in Table I for the B_q mesons ($q = n, s, c$).

It is convenient to introduce the ratio g_{B_q},

$$g_{B_q}(nS) = \frac{|R_n(0)|^2}{\omega_1(nS)\omega_2(nS)}, \tag{16}$$

which directly enters the HFS (5) and appears to be weakly dependent on small variations of the masses m_1 and m_2 (see Table II).

The w.f. at the origin are sensitive to the values of n_f through $\Lambda_B(n_f)$. If one takes the same freezing value of the coupling constant for $n_f = 3$, 4, 5, then in bottomonium $R_{1S}(0)$ for $n_f = 3$, 4 is by 10%, 6%, respectively, smaller than in the case with $n_f = 5$. Taking also into account small changes in the value of ω_b—the kinetic energy of a b quark—we have obtained the following numbers for the factor $g_b(1S) = g(b\bar{b})$, defined as in Eq. (16):

$$g_b(n_f = 3) = 0.213, \qquad g_b(n_f = 4) = 0.289, \qquad g_b(n_f = 5) = 0.258. \tag{17}$$

As a result, the HFS for $n_f = 3$, 4 appears to be smaller than for $n_f = 5$. For $n_f = 3$ our value $\Delta_{\text{HF}}(b\bar{b}) = 58(1)$ MeV, which is in good agreement with lattice calculations performed with $n_f = 3$: in [30] the value is $\Delta_{\text{HF}}(b\bar{b}) = 61 \pm 13 \pm 4$ MeV and in the recent paper [31] the value $\Delta_{\text{HF}}(b\bar{b}) = 54 \pm 12$ MeV was obtained.

We use here two values for α_{crit}: $\alpha_{\text{crit}} = 0.580$ and 0.604, for which corresponding values of $\Lambda_B(n_f)$, $\Lambda_{\overline{MS}}(n_f)$ are given in Eq. (18).

In bottomonium the difference between g_b for $n_f = 4$ and $n_f = 5$ appears to be larger, $\sim 10\%$ (see Table III), where in both cases $\alpha_{\text{crit}} = 0.604$ is used.

TABLE I. The kinetic energies $\omega_q(1S)$ ($q = n$, s, c) and $\omega_b(1S)$ (in MeV) for the static potential $V_B(r)$ (11) with $n_f = 4$ ($\alpha_{\text{crit}} = 0.605$) and $m_b = 4.79$ GeV.

Meson	B	B_s	B_c
m_q	5	200	1410
$\omega_q - m_q$	634	486	276
ω_b	4832	4836	4879
$\omega_b - m_b$	42	46	89

TABLE II. The ratios g_{B_q} (16) (in GeV) and $|R_1(0)|^2$ (in GeV^3) for the $B_q(1S)$ mesons ($\alpha_{\text{crit}} = 0.605$, $n_f = 4$).

	B	B_s	B_c
$\vert R(0)\vert^2$	0.509	0.558	1.742
g_{B_q}	0.165	0.168	0.212

For the values of $\alpha_{\text{crit}}(n_f) = 0.604(0.58)$ and $\Lambda_{\overline{\text{MS}}}(n_f = 5) = 0.245(0.236)$ GeV, the following vector constants $\Lambda_{\text{B}}(n_f = 3) = 0.40(0.389)$ GeV, $\Lambda_{\text{B}}(n_f = 4) = 0.372(0.360)$ GeV, and $\Lambda_{\text{B}}(n_f = 5) = 0.335(0.323)$ GeV are obtained. Then from the relation (14) we have

$$\begin{aligned} \Lambda_{\overline{\text{MS}}}(n_f = 3) &= 271(264)\ \text{MeV}, \\ \Lambda_{\overline{\text{MS}}}(n_f = 4) &= 261(253)\ \text{MeV}, \\ \lambda_{\overline{\text{MS}}}(n_f = 5) &= 245(236)\ \text{MeV}. \end{aligned} \tag{18}$$

For $n_f = 5$ it gives $\alpha_s(M_Z)(\text{two loop}) = 0.1194$ for $\alpha_{\text{crit}} = 0.604$ and $\alpha_s(M_Z) = 0.1188$ for $\alpha_{\text{crit}} = 0.58$; both numbers agree with the world averaged value, $\alpha_s(M_Z) = 0.1176 \pm 0.0020$ [19], within its error bar.

III. RESULTS

The experimental error in the HFS

$$\Delta_{\text{HF}}(b\bar{b}) = M(\Upsilon(9460)) - M(\eta_b) = 69.9 \pm 3.1\ \text{MeV} \tag{19}$$

is small, ± 3 MeV, and it is even smaller, ≤ 1 MeV, for the mass differences $M(B^*) - M(B)$, $M(B_s^*) - M(B_s)$ [19]. Therefore, we expect that the coupling constant α_{HF} can be extracted with a good accuracy from these data; its value, equal to 0.310, is used in our analysis.

For $g_b = 0.230$ GeV ($n_f = 4$) and $g_b = 0.258$ GeV ($n_f = 5$) (see Table III) and $\alpha_{\text{HF}}(n_f = 5) = \alpha_s(\mu) = 0.310$ we obtain $\Delta_{\text{HF}}(b\bar{b}) = 71.1$ MeV ($n_f = 5$) and 63.4 MeV for $n_f = 4$. The difference between them, $\sim$10%, is not small and one may conclude that the HFS in bottomonium is in full agreement with the *BABAR* HFS data Eq. (19) only for $n_f = 5$.

For the $2S$ and $3S$ bottomonium states the difference between HFS for $n_f = 4$ and $n_f = 5$ is small; they coincide within 2 MeV, being equal to 36(1) and 28(1) MeV, respectively, for the $3S$ and $2S$ states.

For the B_q mesons, both for $n_f = 3$, 4 agreement with experiment can be reached, if for $n_f = 4$ the coupling $\alpha_{\text{HF}} = 0.310$ (as in bottomonium) and for $n_f = 3$ a larger value, $\alpha_{\text{HF}} = 0.324$ are taken (see Table IV). Since for B and B_s a preferable number of flavors cannot be fixed, using only data on the HFS and the spectrum, one may speak of a universal coupling α_{HF} only within about 5%. Therefore, for them some additional information, like decay constants, is needed to fix n_f.

In our calculations of the mass difference the value $M(B_c^*) - M(B_c) = 57.8(6)$ MeV is obtained, which is close to the lattice result from Ref. [16], where the value of this HFS quantity is equal to 53 ± 7 MeV.

From our HFS calculation the following masses of the triplet and singlet $c\bar{b}(2S)$ states, $M(B_c^*(2^3S_1)) = 6902(5)$ MeV and $M(B_c(2^1S_0)) = 6865(5)$ MeV, are predicted.

The value of the extracted coupling, $\alpha_{\text{HF}}(\mu) = 0.310$ is smaller than the one used in Refs. [5,9]. The renormalization scale, $\mu \sim 1.6$ GeV, corresponding to this coupling, is rather large and agrees with the existing interpretation of the spin-spin potential as dominantly a short-range perturbative one.

IV. CONCLUSION

Our study of the HFS is performed assuming that the freezing value of the coupling constant is the same for $n_f = 3$, 4, 5 and considering $\alpha_{\text{crit}} = 0.58$ and 0.60.

The HFS of the B and B_s mesons are obtained in good agreement with experiment for both freezing constants, if for $n_f = 3$ $\alpha_{\text{HF}} = 0.324$ is taken, while for $n_f = 4$ the value $\alpha_{\text{HF}} = 0.310$, as in bottomonium, is used.

The HFS, averaged over two results, are $M(B^*) - M(B) = 45.5(1)$ MeV, $M(B_s^*) - M(B_s) = 46.2(1)$ MeV, and $M(B_c^*) - M(B_c) = 57.8(6)$ MeV. The latter number gives the mass of the as yet unobserved B_c^* meson, $M(B_c^*) = 6.334(1)$ GeV. For excited $B_c(2S)$ states we predict the masses $M(B_c^*(2S) = 6902(5)$ MeV and $M(B_c(2S) = 6865(5)$ MeV, which are calculated neglecting open channels.

In bottomonium for $n_f = 5$ the HFS $\Delta_{\text{HF}}(b\bar{b}) = 71.1$ MeV, is in full agreement with experiment, and this case may be considered preferable, since for $n_f = 4$ a smaller value, $\sim$64 MeV, is obtained. For the $2S$ and $3S$

TABLE III. The ratios $g_b(nS)$ (in GeV), $|R_n(0)|^2$ (in GeV^3), and HFS Δ_{HF} (in MeV) for the $1S$, $2S$, and $3S$ bottomonium states with $\alpha_{\text{crit}} = 0.604$ and for $n_f = 4$, 5.

	$1S$	$2S$	$3S$
$\vert R_n(0)\vert^2(n_f = 5)$	6.476	3.398	2.682
$g_b(n_f = 5)$	0.258	0.134	0.105
$\Delta_{\text{HF}}(n_f = 5)$	71.1	36.9	28.9
$\vert R_n(0)\vert^2(n_f = 4)$	5.668	3.126	2.508
$g_b(n_f = 4)$	0.230	0.127	0.100
$\Delta_{\text{HF}}(n_f = 4)$	63.4	35.0	27.6

TABLE IV. The HFS (in MeV) of the B_q mesons with $\alpha_{\text{HF}}(n_f = 4) = 0.310$ and $\alpha_{\text{HF}}(n_f = 3) = 0.324$.

	B	B_s	$B_c(1S)$	$B_c(2S)$
$\Delta_{\text{HF}}(n_f = 4)$	45.6	46.3	58.4	37.3
$\Delta_{\text{HF}}(n_f = 3)$	45.4	46.1	57.2	37.4
$\Delta_{\text{HF}}(\text{exp})$	45.78 ± 0.35	46.5 ± 1.25	Absent	Absent

bottomonium states, the calculated HFS are 36(1) and 28(1) MeV, respectively.

The extracted coupling, $\alpha_{\mathrm{HF}}(\mu) = 0.310$, is smaller than in many other analyses, where a universal coupling of the HF interaction is used; it determines the characteristic scale of the spin-spin interaction, $\mu \sim 1.6$ GeV. Knowledge of this scale can help to better understand the behavior of the spin-spin potential at small distances.

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