

Ramsey's method of separated oscillating fields and its application to gravitationally induced quantum phase shifts

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We propose to apply Ramsey's method of separated oscillating fields to the spectroscopy of the quantum states in the gravity potential above a horizontal mirror. This method allows a precise measurement of quantum mechanical phaseshifts of a Schrödinger wave packet bouncing off a hard surface in the gravitational field of the Earth. Measurements with ultracold neutrons will offer a sensitivity to Newton's law or hypothetical short-ranged interactions, which is about 21 orders of magnitude below the energy scale of electromagnetism.

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I. INTRODUCTION

The system of a Schrödinger quantum particle with mass m bouncing in a linear gravitational field is known as the quantum bouncer [1–5], and a description of “quantum wave packet revivals” can be found in [6]. Gravity tests with neutrons as quantum objects or within the classical limit are reviewed in [7].

Above a horizontal mirror, the linear gravity potential leads to discrete energy eigenstates of a bouncing quantum particle. The lowest energy eigenvalues E_n , ($n = 1, 2, 3, 4, 5$), are 1.41 peV, 2.46 peV, 3.32 peV, 4.09 peV, and 4.78 peV. The energy levels together with the neutron density distribution are shown in Fig. 1. The idea of observing quantum effects in such a gravitational cavity was discussed with neutrons [8] or atoms [9]. Such quantum states have been demonstrated at the Institut Laue-Langevin with ultracold neutrons in a previous collaboration [10–13]. The follow-up experiment qBOUNCE realized the quantum bouncing ball with neutrons and evidence for the time evolution of a superposition of quantum states in the gravity potential has been observed [4,5]. In the neutron whispering gallery, quantum states are caused by a bounding well formed by the centrifugal potential [14]. A quantum mechanical phaseshift of neutrons caused by the interaction with Earth's gravitational field was observed [15] with the interferometers originally developed for x-rays and adapted for thermal neutrons [16].

An important feature of the quantum bouncing ball—in contrast to the harmonic oscillator problem—is the fact that levels are not equidistant in energy. A combination of any two states can therefore be treated as a two-level system. The energy eigenstates in the gravity potential can be coupled to a mechanical or magnetic oscillator field. Transitions between quantum states in the gravitational field of the earth i.e. a change of the state occupation can therefore be induced similar to magnetic transitions, which occur when the oscillator frequency equals one of the Bohr frequencies of the system. This magnetic resonance method was in the original conception for measurements

of nuclear magnetic moments [17,18], but soon it became a very general technique for radio frequency spectroscopy [19]. Ramsey developed his method of separated oscillating fields in which the oscillatory field is confined to a region at the beginning and a region at the end with no oscillating field in between [20]. Variations of Ramsey's method is inherently connected with precision measurements ranging from atomic clocks [21] to atom interferometry [22], from NMR [23] to quantum metrology [24], or the related spin-echo technique [25]. That method has also been used to measure the precession frequency of atoms, molecules or neutrons in a weak magnetic field, for example, in a search for permanent atomic or neutron

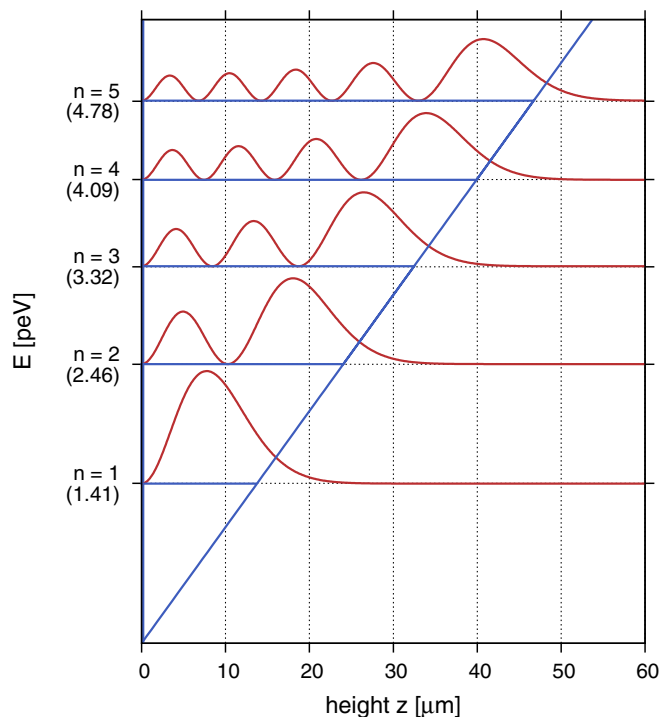


FIG. 1 (color online). Energy eigenvalues and neutron density distributions for level one to five.

electric-dipole moments and in constructions of sensitive magnetometers. The sensitivity is extremely high, because a quantum mechanical phase shift is converted into a frequency measurement. The sensitivity reached so far [26] in a search for the electric-dipole moment of the neutron is 6.8×10^{-22} eV, or one Bohr rotation within 6 days. Regeneration experiments are also a nice application of this method e.g. in the neutral kaon system, where the complex regeneration amplitude interferes with the complex CP -violating parameter.

In analogy to these examples from electrodynamics, we discuss here an application of Ramsey's method to probe the eigenstates in the gravity potential. Such a technique should open a new way to precision gravity experiments, and we propose to apply it to quantum states of neutrons or atoms in the gravitational field of the Earth. Here, we are sensitive to energy shifts of a Schrödinger wave packet bouncing off a hard surface. Such energy shifts are expected from hypothetical gravitylike forces in the light of recent theoretical developments in higher-dimensional field theory and will allow searches for pseudoscalar coupling of axions in the previously experimentally inaccessible astrophysical axion window [27,28]; see Sec. III. Our method will allow a precise measurement of energy differences with a precision similar to the magnetic resonance technique. With a four-layer mu-metal shield, the coupling of residual fluctuations of the magnetic field to the magnetic moment of the neutron can be suppressed to the 10^{-22} eV level as it has been demonstrated in a search for an electric-dipole moment [26]. Other electromagnetic effects are extremely suppressed compared to gravity [27]. A prerequisite of this method is the coherent superposition of bound quantum states over the full length scale of the experiment. In the qBOUNCE experiment, the time evolution of the spatial probability distribution of such a superposition was studied over a length of 6 cm and quantum interference was observed [4,5].

II. RAMSEY'S METHOD AND ITS APPLICATION TO GRAVITY POTENTIALS

A quantum mechanical system that is described by two states can be understood in analogy to a spin 1/2 system (assuming two states of a fictitious spin in the multiplet, similarly to spin up and spin down states). The time development of such systems is described by the Bloch equations. In magnetic resonance of a standard spin 1/2 system, the energy splitting results in the precession of the related magnetic moment in the magnetic field. Transitions between the two states are driven by a transverse magnetic radio frequency field. Similar concepts can be applied to any driven two-level system, e.g. in optical transitions with light fields. Here, we apply this picture to quantum states in the gravity field.

We start with a short description of Rabi's method [17] to measure the energy difference between a two-level

system with a coupled oscillating field. With ω_{pq} , the frequency difference between the two states, ω , the frequency of the driving field, Ω_R , the Rabi frequency and the time t , the Hamiltonian H is given by

$$H = \begin{pmatrix} \frac{\hbar\omega_{pq}}{2} & \frac{1}{2}\hbar\Omega_R e^{-i\omega t} \\ \frac{1}{2}\hbar\Omega_R e^{i\omega t} & -\frac{\hbar\omega_{pq}}{2} \end{pmatrix}. \quad (1)$$

The probability of being found in the excited state as a function of time is

$$P(t) = \left(\frac{\Omega_R}{\Omega'_R}\right)^2 \sin^2\left(\frac{\Omega'_R}{2}t\right), \quad (2)$$

where the effective Rabi frequency is

$$\Omega'_R = \sqrt{\Omega_R^2 + (\omega_{pq} - \omega)^2} = \sqrt{\Omega_R^2 + \delta^2}, \quad (3)$$

with detuning δ from resonance. The sinusoidal population transfer is referred to as Rabi flopping. It has been proposed to measure the energy levels of a neutron in the gravitational field of the Earth with this method (GRANIT experiment [29,30]). The periodic drive is given by neutrons moving through a spatially oscillating magnetic field created by horizontal conducting wires.

As we will show below, one can drive transitions between quantum states in gravity above the mirror by vibrating the mirror surface.

Lets consider the motion of ultracold neutrons in the gravitational field above a mirror. We assume the gravitational force to act in z direction, while the mirror is aligned with the xy plane, vibrating with amplitude a in the z direction. The motion in the x and y directions is free and completely decouples from that in z direction. It suffices therefore to consider the time-dependent Schrödinger equation restricted to the z direction

$$\left\{ -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial z^2} + mgz + V_0 \Theta(-z + a \sin \omega t) \right\} \Psi = i\hbar \frac{\partial \Psi}{\partial t}. \quad (4)$$

Here, g is the acceleration of gravity, m is the mass of the neutron, and Θ is the Heaviside step function. The potential $V_0 \approx 100$ neV associated with the substance of the mirror is repulsive and much larger than eigenenergies of the lowest quantum states in the gravitational field. Therefore, Eq. (4) must be solved with the boundary condition $\Psi(z = a \sin \omega t, t) = 0$. For further considerations, it is preferable to introduce $\tilde{z} = z - a \sin \omega t$ and to transform Eq. (4) into the rest frame of the mirror,

$$\{H_0 + W(\tilde{z}, t)\} \tilde{\Psi} = i\hbar \frac{\partial \tilde{\Psi}}{\partial t}, \quad (5)$$

where

$$H_0 = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial \tilde{z}^2} + mg\tilde{z} + V_0 \Theta(-\tilde{z}), \quad (6)$$

$$W(\tilde{z}, t) = a \left[mg \sin \omega t + i \hbar \omega \cos \omega t \frac{\partial}{\partial \tilde{z}} \right], \quad (7)$$

and $\tilde{\Psi}(\tilde{z}, t) = \Psi(z, t)$.

The Hamiltonian H_0 describes the neutron in the gravitational field above a mirror at rest. The corresponding stationary Schrödinger equation is conveniently scaled by the characteristic gravitational quantum length scale [9] of the bouncing neutron

$$z_0 = \left(\frac{\hbar^2}{2m^2g} \right)^{1/3} = 5.88 \mu\text{m}, \quad (8)$$

resulting in

$$\psi''(\zeta) - (\zeta - \zeta_E)\psi(\zeta) = 0, \quad (9)$$

where $\zeta = z/z_0$, $\zeta_E = z_E/z_0$, and $z_E = E/mg$.

The second term $W(\tilde{z}, t)$ accounts for the vibration of the mirror surface.

The solution of Eq. (5) can be expressed in terms of the eigenfunctions $\psi_n(z)$ of H_0

$$\Psi(z, t) = \sum_n C_n(t) e^{-iE_n t/\hbar} \psi_n(z) \quad (10)$$

with time-dependent coefficients $C_n(t)$.

Using this ansatz, projection of Eq. (5) on the eigenstates of H_0 yields a system of differential equations for the coefficients $C_n(t)$

$$i\hbar \cdot \frac{d}{dt} C_n(t) = \sum_k \langle \psi_n | W | \psi_k \rangle \cdot C_k(t) \cdot e^{i\omega_{nk}t}. \quad (11)$$

The transitions between different quantum states is governed by the matrix elements of $W(\tilde{z}, t)$ defined in (7)

$$\langle \psi_n | W | \psi_k \rangle = a [mg \delta_{n,k} \sin \omega t + i \hbar \omega Q_{n,k} \cos \omega t], \quad (12)$$

with

$$Q_{n,k} = \int_0^\infty dz \psi_n(z) \frac{d}{dz} \psi_k(z). \quad (13)$$

The relevant overlap integrals $Q_{n,k}$ for the transitions between the lowest eigenstates in the gravitational field are given in Table I.

The physics behind the transitions between the energy eigenstates of the quantum bouncer caused by a vibrating mirror or an oscillating potential is related to earlier studies

TABLE I. Relevant overlap integrals $Q_{n,k}$ defined in Eq. (13) for the five lowest eigenstates in the gravitational field in μm^{-1} .

	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$
$n = 1$	0.00000	0.09742	-0.05355	0.03831	-0.03040
$n = 2$	-0.09742	0.00000	0.11894	-0.06314	0.04419
$n = 3$	0.05355	-0.11894	0.00000	0.13458	-0.07031
$n = 4$	-0.03831	0.06314	-0.13458	0.00000	0.14724
$n = 5$	0.03040	-0.04419	0.07031	-0.14724	0.00000

of energy transfer when matter waves bounce of a vibrating mirror [31–34] or on a time-dependent crystal [35–37]. In the later cases, the transitions are between continuum states, in the quantum bouncer between discrete eigenstates. Most interesting for our proposal to drive transitions between eigenstates of the quantum bouncer with a vibrating mirror is the physics of reflection of a neutron by an oscillating potential step as has been investigated at the research reactors Munich and Geesthacht [33], however in a different energy regime.

Applying Ramsey's resonance method with separated oscillating fields will allow a careful measurement of the energy eigenstates states of the quantum bouncer [38]. We propose to implement it with neutrons by traversing five regions as shown in Fig. 2. The horizontal direction in space is considered as free motion, while the vertical one is described by a one-dimensional time-dependent Schrödinger equation [see e.g. Eq. (4)].

To implement Ramsey's method, one has to realize (1) a state selector, (2) a region, where one applies a $\pi/2$ pulse creating the superposition of the two states, whose energy difference should be measured, (3) a region, where the phase evolves, (4) a second region to read the relative phase by applying a second $\pi/2$ pulse, and finally, (5) a state detector.

In the following, we will describe all these components as they are shown in Fig. 2.

In region one, neutrons are prepared in a specific quantum state $|p\rangle$ in the gravity potential following the procedure demonstrated in [10]. A polished mirror on bottom and a rough absorbing scatterer on top at a height of about $20 \mu\text{m}$ is a realization of a state selector. It prepares neutrons into the ground state. Neutrons in higher, unwanted states are scattered out of the system and absorbed i.e. $C_1 = 1$ and $C_n = 0$ for $n > 1$. A quantum mechanical description of such a system can be found in [13]. The neutron passage through a mirror-scatterer system has also been studied in a frame, where the rough scatterer surface

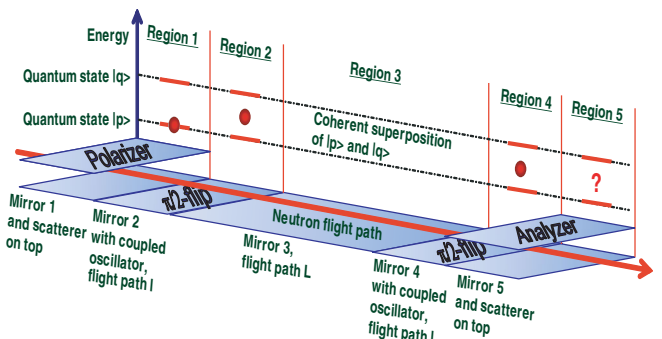


FIG. 2 (color online). Sketch of the proposal. Region 1: Preparation in a specific quantum state, e.g. state one with polarizer. Region 2: Application of first $\pi/2$ -flip. Region 3: Flight path with length L . Region 4: Application of second $\pi/2$ flip. Region 5: State analyzer.

has been treated as a time-dependent variation of the scatterer position [39].

In region two of length l , the first of two identical oscillators is installed. Here, transitions between quantum states $|p\rangle$ and $|q\rangle$ are induced within a time τ according to Eq. (2). The oscillator frequency at resonance for a transition between states with energies E_q and E_p is

$$\omega_{pq} = \frac{(E_q - E_p)}{\hbar}. \quad (14)$$

The squared ratio of Ω_R and Ω'_R as a function of the driving field ω for transitions $|1\rangle \rightarrow |3\rangle$, $|1\rangle \rightarrow |2\rangle$ and $|2\rangle \rightarrow |3\rangle$ is shown in Fig. 3. For the $|1\rangle \rightarrow |2\rangle$ transition, which we chose as an example for transitions in a two-level system, $\omega_{12} = \omega_2 - \omega_1 = 2\pi \times 254 \text{ s}^{-1}$.

Driven on resonance ($\omega = \omega_{pq}$), this oscillator drives the system into a coherent superposition of state $|p\rangle$ and $|q\rangle$. A $\pi/2$ pulse, that is one with pulse area $\Omega_R \tau = \pi/2$, creates an equal superposition between state $|p\rangle$ and $|q\rangle$. This can be done by using oscillating magnetic gradient fields or by vibrating mirrors i.e. a modulation of the mirror potential in height.

In the intermediate region three, a nonoscillating mirror with a neutron flight path of L and flight time T follows. It might be convenient to place a second mirror on top of the bottom mirror at a certain height h . It allows us to tune the resonance frequency between $|a\rangle$ and $|b\rangle$ due to the additional potential, and it provides an effective doubling of sensitivity in a search for hypothetical axion induced phase shifts or other fifth forces; see Sec. III.

Subsequently, in region four a second oscillator in phase with the oscillator in region two is placed. If the oscillating ω is equal to ω_{pq} , then the system is at resonance and we have a complete reversal of the state occupation between $|p\rangle$ and $|q\rangle$. There is no change in the relative phase of the oscillator and the quantum state of the neutron independent of the neutron velocity. In the other cases for $\omega \neq \omega_{pq}$, a velocity dependent relative phase shift builds up, since a

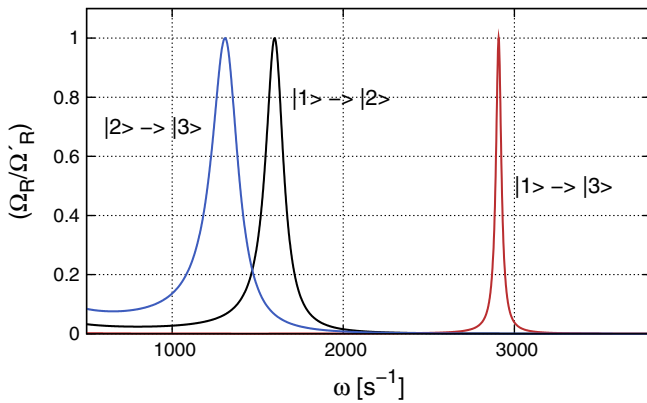


FIG. 3 (color online). Ratio $(\frac{\Omega_R}{\Omega'_R})^2$ as a function of $\omega [s^{-1}]$ for transition $|1\rangle \rightarrow |3\rangle$, $|1\rangle \rightarrow |2\rangle$, and $|2\rangle \rightarrow |3\rangle$. The strength of the vibration was set to 1.0 m/s^2 .

slower neutron is in the region three longer and experiences a greater shift than a faster neutron.

Afterwards in section five, such a phase shift can be measured by transmission through a second state selector.

This method can be realized with some modifications to the previous setup in the following way: Neutrons are taken from the ultracold neutron installation PF2 at Institut Laue-Langevin with a measured horizontal velocity $v = 3.2 \text{ m/s} < v < 20 \text{ m/s}$. At the entrance of the experiment, a collimator absorber system limits the transversal velocity to an energy in the pico-eV range. The experiment itself is mounted on a polished plane granite stone with an active and passive antivibration table underneath. This stone is leveled using piezo translators [40]. Inclinometers together with the piezo translators in a closed loop circuit guarantee leveling with a precision better than $1 \mu\text{rad}$ [41]. A solid block with dimensions $10 \text{ cm} \times 15 \text{ cm} \times 3 \text{ cm}$ composed of optical glass serves as a mirror for neutron reflection. The neutrons see a surface that is essentially flat. In region one, an absorber/scatterer that is a rough mirror with a surface roughness of about $0.4 \mu\text{m}$ is placed above the first mirror at a height of $27 \mu\text{m}$ in order to select the first quantum state. The other states are efficiently removed, except for the second state, which is still present with a contribution of a few percent. In region two, a second mirror is placed after the first one. Piezo elements attached underneath induce a fast modulation of the surface height with amplitude a according to Eq. (4).

As an example, we consider transitions between state $|1\rangle \rightarrow |2\rangle$ for the most probable velocity at the PF2/UCN beam position, 6 m/s . The length $l = 15 \text{ cm}$ of this mirror is chosen in such a way to provide a neutron in a superposition of these two quantum states after $\tau = 25.0 \text{ ms}$. Region three has a flight path of $L = 80 \text{ cm}$ on a single mirror between the two oscillators in region two and, identical to region two, in region four. In region five, a state selector as an analyzer is placed, identical to the

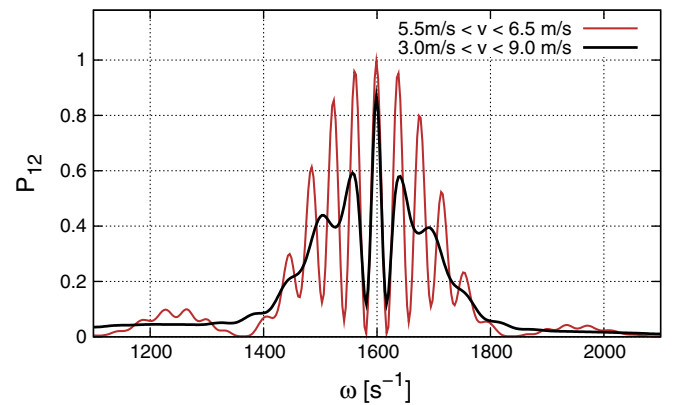


FIG. 4 (color online). Transition probability for a neutron velocity $5.5 \text{ m/s} < v < 6.5 \text{ m/s}$ and $3.0 \text{ m/s} < v < 9.0 \text{ m/s}$ as a function of $\omega [s^{-1}]$. The strength of the vibration was set to 1.0 m/s^2 .

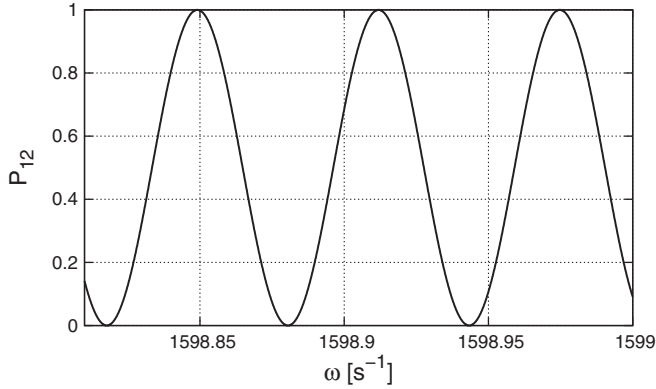


FIG. 5. Theoretical Ramsey signal for a neutron storage time of 100 s. Again, the strength of the vibration was set to 1.0 m/s^2 .

selector in region one but with a neutron detector behind for counting the transmitted neutrons. Calculated transition probabilities [20] for $|p\rangle$ and $|q\rangle$ as a function of ω are shown in Fig. 4 for different parts of the measured velocity spectrum.

This method can also be applied to stored ultracold neutrons. Figure 5 shows the theoretical Ramsey signal for a neutron storage time of 100 s. The appeal of a neutron storage lies in a very narrow resonance line. A search for phase shifts are suggested in the next section.

III. PHASE SHIFTS FROM HYPOTHETICAL GRAVITY-LIKE FIFTH FORCES

Theoretical considerations arising from higher-dimensional gravity, gauge forces, or massive scalar fields suggest that the Newtonian gravitational potential for masses m_i and m_j and distance r should be replaced by a more general expression including a Yukawa term,

$$V(r) = -G \frac{m_i \cdot m_j}{r} (1 - \alpha \cdot e^{-r/\lambda}), \quad (15)$$

where λ is the Yukawa distance over which the corresponding force acts, and α is a strength factor in units of Newtonian gravity. G is the gravitational constant. Most interesting, from the experimental point of view, are scenarios, where the strength of the new force is expected to be many orders of magnitude stronger than Newtonian gravitation. Such forces are possible via Abelian gauge fields in the bulk [42–45], (see also [46,47] for explicit realizations in string theory). The strength of the new force would be $10^6 < \alpha < 10^{12}$ stronger than gravity, independent of the number of extra dimensions n [44]. The observation of quantum states already tests speculations of this kind on large extra dimensions of submillimeter size of space-time [27,48,49]. Most recent theoretical developments support the original proposal of large extra dimensions with bulk gauge fields and more specific predictions for a high interaction strength can be made. One proposal of Callin *et al.* [50] predicts deviations from Newton's law

on the micron scale on the basis of supersymmetric large extra dimensions. The basic idea behind this proposal is to modify gravity at small distances in such a way as to explain the smallness of the observed cosmological constant. The hope is to achieve this without changing non-gravitational physics (which may be possible because of the small gravitational response of the vacuum in specific models of supersymmetric large extra dimensions) and to link the size of the extra dimensions to the energy density 10^{10} eV/m^3 governing the observed dark energy component of the Universe. In the concrete constructions presently under discussion, a radius r of 10 microns, as well as the necessary interaction strength up to $\alpha = 10^6$, may turn out to be well motivated. Best limits at short distances are derived from neutron-scattering experiments [51–54]. Other experimental limits on extra forces are derived from mechanical experiments and can be found, e.g., in [55–61]. Probing submicron forces by interferometry of Bose-Einstein condensated atoms has been proposed by [62]. In practice, the experimental data are subject to corrections, which can be orders of magnitude larger than the effects actually searched for. It is therefore important to stress the completely different nature of possible systematic effects inherent to these micro-mechanical experiments as compared to those in neutron experiments. In the former case, gravitational interactions are studied in the presence of large van der Waals and Casimir forces, which depend strongly on the geometry of the experiment, and the theoretical treatment of the Casimir effect is a difficult task. Currently, atomic force microscopes' measurements using functional tips determine the limits on non-Newtonian gravitation below $10 \text{ }\mu\text{m}$. The best experimental data available, obtained with atomic force microscopes, are claimed to be at the same level of accuracy (1%–2%) as the numerical calculations of the Casimir force. The major obstacle for improvement in the theoretical calculation is the fact that it is very hard to take the boundary conditions of the tip and its functionality properly into account.

Our approach of probing Newtonian gravity at the micron scale with the help of Ramsey's method of separated oscillating fields is advantageous because of its small systematic effects. In contrast to atoms, the electrical polarizability of neutrons [63] inducing such Casimir effects or van der Waals forces is extremely low. Together with its electric neutrality, the neutron provides the key to a sensitivity of more than 10 orders of magnitude below the background strength of atoms.

The dynamics of such a quantum mechanical wave packet combines quantum theory with aspects of Newtonian mechanics at short distances. When a neutron with mass m approaches the mirror, the mass of this extended source might modify the earth acceleration g , when strong non-Newtonian forces with range λ and strength α are present. For small neutron distances z from the mirror, say several micrometers, we consider

the mirror as an infinite half-space with mass density ρ . By replacing the source mass m_i by dm_i and integrating over dm_i , the modified Newtonian potential $\Delta V(z)$ has the form

$$\begin{aligned}\Delta V(z) &= 2\pi m\rho\alpha\lambda^2 G e^{-|z|/\lambda} \\ &= 8.5 \times 10^{-14} \alpha \lambda^2 e^{-|z|/\lambda} \text{ peV}\end{aligned}\quad (16)$$

with $\rho = 19.32 \text{ g/cm}^3$ (gold or tungsten coating) and λ given in μm . Taking these gravitylike forces into account, first order perturbation theory predicts a shift of the n th energy eigenvalue [27],

$$\Delta E_n = \langle \psi_n | \Delta V(z) | \psi_n \rangle. \quad (17)$$

They differ from state to state in the range of interest. The sensitivity can be seen in Fig. 6, where the energy shift as a function of range λ is plotted for a fixed $\alpha = 10^{12}$.

Newtonian gravity and hypothetical fifth forces evolve with different phase information in the nonoscillating region. The following examples consider transitions between state $|1\rangle \rightarrow |3\rangle$, which are energetically well separated; see Fig. 3. We expect the following sensitivity for 50 days of beam time at the PF2/UCN beam position at the Institut Laue-Langevin: With a count rate of 0.1 s^{-1} for neutrons in the ground state, we will have $N = 430\,000$ registered neutrons. Because of the uncertainty principle $\Delta\Phi\Delta N \geq 2\pi$, we estimate a minimal detectable phase shift of 1×10^{-2} radians. For an estimate of $T = 130 \text{ ms}$ interrogation time (flight path between the oscillators), the minimal resolvable energy shift is

$$\Delta E = \Delta\Phi\hbar/T = 5 \times 10^{-17} \text{ eV}. \quad (18)$$

Together with Eqs. (16) and (17), this corresponds to a sensitivity of

$$\alpha = 1 \times 10^8, \quad (19)$$

where the range λ has been set to be equal to the characteristic quantum length scale z_0 . This sensitivity limit for strength α is the projected reach of a first-round Ramsey experiment.

In principle, ultracold neutrons can be stored and the time scale can be increased to $T = 130 \text{ s}$. Therefore, a sensitivity to energy differences of $\Delta E = 5 \times 10^{-20} \text{ eV}$ is feasible. This corresponds to an $\alpha = 1 \times 10^5$. With new neutron sources, which are under development right now, the source strength density is expected to be increased by 2 orders of magnitude. The statistical sensitivity of the new method in a search for hypothetical fifth forces is therefore around

$$\Delta E = 5 \times 10^{-21} \text{ eV}, \quad (20)$$

or

$$\alpha < 1 \times 10^4. \quad (21)$$

Over a wide range of λ , this is orders of magnitude better than existing limits as shown in Fig. 7 (line 1b). The

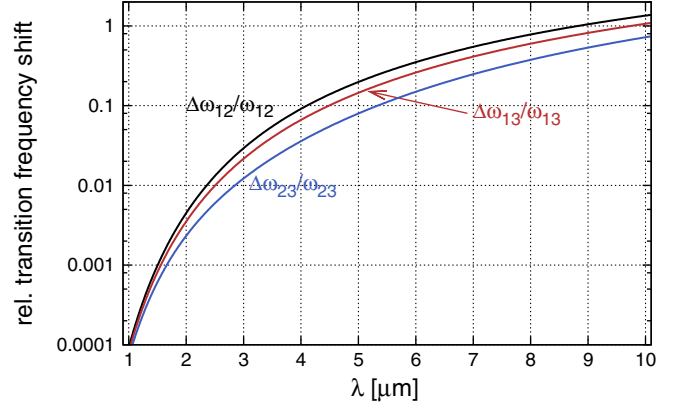


FIG. 6 (color online). The relative frequency shift vs range λ . The strength of the modified Newtonian potential $\Delta V(z)$ of Eq. (16) is set to $\alpha = 10^{12}$.

projected reach of the first round of Ramsey experiments [the limit given by Eq. (19)] is shown as line 1a.

These limits for hypothetical fifth forces can be easily interpreted as bounds of the strength of the matter couplings of axions. Axion interactions with a range within $20 \mu\text{m} < \lambda < 200 \text{ mm}$ (corresponding to axion masses $10^{-6} \text{ eV} < m_a < 10^{-2} \text{ eV}$), the ‘‘axion window,’’ are still allowed by the otherwise stringent constraints posed by cosmological data (see e.g. [64,65]). The CP -violating spin-dependent part in presence of matter given by [66] is

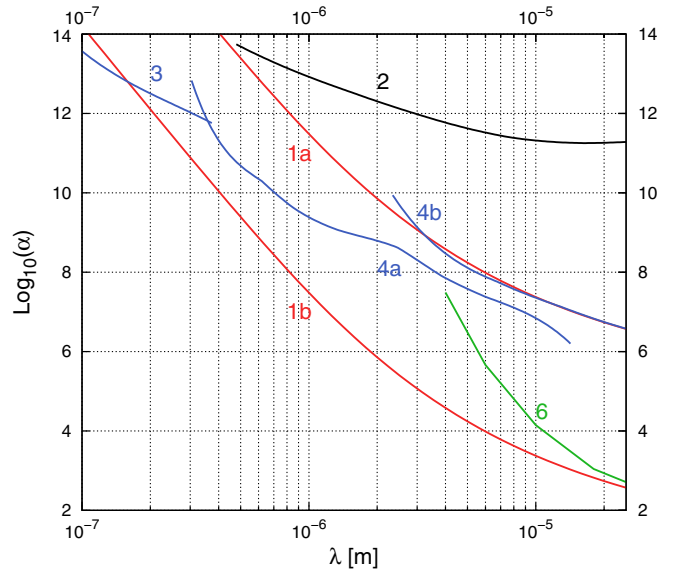


FIG. 7 (color online). Bounds on new forces with coupling strength α from a potential of the form $V(r) = -G \frac{m_i m_j}{r} (1 - \alpha \cdot e^{-r/\lambda})$. 1a: Projected reach of a first round of Ramsey experiments based on Eq. (19). 1b: The lower line 1b indicates a sensitivity based on Eq. (21). 2: Existing limits based on neutron quantum states [27]. 3, 4: Existing limits based on other experiments according to [67] and updated with [60].

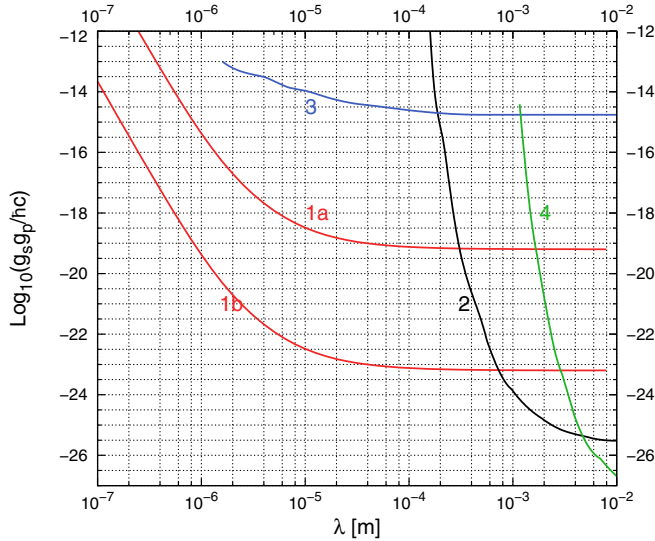


FIG. 8 (color online). Limits for the axionlike coupling strength $\frac{g_s g_p}{\hbar c}$ from a potential of the form $V(\vec{r}) = \hbar g_p g_s \frac{\vec{\sigma} \cdot \vec{n}}{8\pi m c} \times (\frac{1}{\lambda r} + \frac{1}{r^2}) e^{-r/\lambda}$. 1a: The line 1a is the projected reach of a first round of Ramsey experiments based on Eq. (18). 1b: The lower line 1b indicates a sensitivity based on Eq. (20). 2: The only existing limit in this range λ based on neutron quantum states [27]. 3, 4: Existing limits for larger λ based on other experiments [68,69].

$$V(\vec{r}) = \hbar g_p g_s \frac{\vec{\sigma} \cdot \vec{n}}{8\pi m c} \left(\frac{1}{\lambda r} + \frac{1}{r^2} \right) e^{-r/\lambda}. \quad (22)$$

Here, $\vec{\sigma}$ denotes the neutron spin, and \vec{n} is a unit vector related to the geometry of the macroscopic matter configuration.

Integrating this potential over the geometry of region three [27], and using the sensitivity given in Eq. (20), we reach at range $\lambda = z_0 = 5.88 \mu\text{m}$ an experimental bound

$$\frac{g_s g_p}{\hbar c} \leq 9 \cdot 10^{-23} \quad (23)$$

for the dimensionless axion coupling strength; see Fig. 8

(line 1b). This is 8 orders of magnitude better than the only existing limit in the axion window, which has been derived from the previous experiment with bound quantum states of neutrons [27,28]. The projected reach of the first-round Ramsey experiment based on Eq. (18) is shown as the upper line 1a.

IV. SUMMARY

In conclusion, we discussed an application of Ramsey's method of oscillating fields to the quantum bouncer. It will allow high precision spectroscopy of the energy eigenstates of a neutron bouncing on a flat horizontal surface. Such Ramsey-type interference measurements will improve the sensitivity for neutron's coupling to gravity, to hypothetical short-ranged forces or the influence of the cosmological constant. A sensitivity of more than 21 orders of magnitude below the strength of electromagnetism is found, when the energy $\Delta E = 5 \times 10^{-21}$ eV of Eq. (20) is compared with the Rydberg energy of 13.6 eV, which is the energy scale of electromagnetically bound quantum systems. Such an energy change corresponds to a strength of $\alpha \sim 1 \times 10^4$ compared to gravity or to $\frac{g_s g_p}{\hbar c} \sim 9 \cdot 10^{-23}$, the axion coupling strength, at a range $\lambda = z_0$; see Fig. 8.

The new method profits from small systematic effects in such systems, mainly due to the fact that in contrast to atoms, the electrical polarizability of neutrons is extremely low. Neutrons are not disturbed by short range electric forces such as van der Waals or Casimir forces. Together with its neutrality, this provides the key to a sensitivity of several orders of magnitude below the strength of electromagnetism.

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