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Casimir-Polder effect for a plane with Chern-Simons interaction

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A novel formalism for the evaluation of the Casimir-Polder potential in an arbitrary gauge of vector potentials is introduced. The ground state energy of a neutral atom in the presence of an infinite two-dimensional plane with Chern-Simons interaction is derived at zero temperature. The essential feature of the result is its dependence on the antisymmetric part of a dipole moment correlation function.

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I. INTRODUCTION

Casimir-Polder effect was predicted theoretically in 1948 [1]. Casimir and Polder found the energy of a neutral point atom in its ground state in the presence of a perfectly conducting infinite plate. In the case of a perfectly conducting plate one can say that the interaction of a fluctuating dipole with the electric field of its image yields the Casimir-Polder potential.

In response to the external electromagnetic field the atom emits the electromagnetic field propagating from the atom. This electromagnetic field propagates to the plate, reflects from the plate so that the boundary conditions on the plate are satisfied, and returns to the atom. The equation for normal modes of the system can be written if one determines the reflection matrices of the plate and the atom. The ground state energy of the system can be defined then as the sum of the eigenfrequencies of the normal modes of the system. This sum can be evaluated by making use of the argument principle if the equation for normal modes of the system is substituted into it [2–4].

An equivalent mathematical description is the use of Green's functions of the system. This technique was first applied to the Casimir effect by Lifshitz [5]. An alternative derivation of the Lifshitz formula in the framework of scattering technique was first given by Renne [6]. Recently, various scattering techniques were applied to the evaluation of the Casimir energy for different geometries (see Refs. [4,7–13] for details).

The Casimir-Polder effect was studied theoretically for various geometries: two parallel plates [14], a wedge [15], a dielectric ball [16], and other geometries. The first experimental measurements of the Casimir-Polder effect were performed in the wedge geometry [17]. One can find a review of the results in [18]. Scattering techniques for the Casimir-Polder effect in terms of reflection matrices were recently developed in the Refs. [19,20].

In this paper we present a novel theoretical formalism for the Casimir-Polder effect by making use of Green's functions technique. In this formalism the atom is described by the neutral point dipole source. We assume that the atom creates a dipole field in response to the external electric field, we do not consider the contribution of higher multipoles. The point source interacts with vector potentials of the electromagnetic field in a gauge invariant way. Our formalism for the Casimir-Polder effect is applicable in an arbitrary gauge of vector potentials.

In the Casimir-Polder effect the correlations of spontaneous dipole moments at different moments of time are important. Because of the fluctuation-dissipation theorem they are related to the polarizability of the atom in an external electric field. In the most general case the polarizability of the atom includes frequency dependent symmetric and antisymmetric parts [21,22]. The contribution of the antisymmetric part of the atomic polarizability to the Casimir-Polder energy was equal zero in the Casimir systems that were considered in literature before [23,24]. In the current paper we present an example of the Casimir-Polder system where the contribution of the antisymmetric part of the atomic polarizability is different from zero and leads to a measurable effect.

Boundary Chern-Simons terms in the Casimir effect were considered in Refs. [25,26]. In this paper we derive the zero temperature formula (29) for the Casimir-Polder interaction of a neutral polarizable atom in the presence of an infinite two-dimensional plane characterized by the Chern-Simons action. Outside the plane the standard action for the potentials of the electromagnetic field is considered. A distinctive feature of our main result (29) is its dependence on the antisymmetric part of the atomic polarizability. The result for the potential may be applied for the experimental studies of the Casimir-Polder effect for two-dimensional materials such as graphene [27,28] or quantum Hall effect systems.

We adopt Heaviside-Lorentz units and put $\hbar = c = 1$.

II. MODEL

In our model the interaction of the plane surface with a quantum electromagnetic field A_μ is described by the action

$$S(A) = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + S_{def}(A), \tag{1}$$

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where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and

$$S_{def}(A) = a \int \epsilon^{\alpha\beta\gamma 3} A_\alpha(x) \partial_\beta A_\gamma(x) \delta(x_3) dx. \quad (2)$$

We will use Latin indices for the components of 4-tensors with numbers 0, 1, 2, and also with the following notations:

$$P^{lm}(\vec{k}) = g^{lm} - k^l k^m/\vec{k}^2, \qquad L^{lm}(\vec{k}) = \epsilon^{lmn3} k_n/|\vec{k}|, \quad \vec{k}^2 = k_0^2 - k_1^2 - k_2^2, \quad (3)$$

where $|\vec{k}| = \sqrt{\vec{k}^2}$, and g—metric tensor. The atom is modeled as a localized electric dipole at the point $(x_1, x_2, x_3) = (0, 0, l)$, which is described by the current $J_\mu(x)$:

$$J_0(x) = \sum_{i=1}^{3} p_i(t) \partial^i \delta(x_1) \delta(x_2) \delta(x_3 - l), \quad (4)$$

$$J_i(x) = -\dot{p}_i(t) \delta(x_1) \delta(x_2) \delta(x_3 - l), \qquad i = 1, 2, 3. \quad (5)$$

The condition of the current conservation holds:

$$\partial_\mu J^\mu = 0,$$

$p_i(t)$ is a function with a zero average and the pair correlator

$$\langle p_j(t_1) p_k(t_2) \rangle = -i \int_{-\infty}^{+\infty} \frac{e^{-i\omega(t_1 - t_2)}}{2\pi} \alpha_{jk}(\omega) d\omega, \quad (6)$$

where $\alpha_{jk}(\omega)$ for $\omega > 0$ coincides with the atomic polarizability. The aim of our paper is to calculate the interaction energy E of the atom with a plane, and we will use the following representation for the energy:

$$E = \frac{i}{T} \left\langle \left\{ \ln \int \exp(iS(A) + JA) DA - \ln \int \exp(iS(A)) DA \right\}_{(a)} \right\rangle, \quad (7)$$

$\{\cdots\}_{(a)}$ means that the $a = 0$ value of the a-dependent function has to be subtracted: $\{f(a)\}_{(a)} \equiv f(a) - f(0)$.

III. PROPAGATOR OF THE ELECTROMAGNETIC FIELD

Integrals in the right-hand side of (7) are gauge invariant, and there are no restrictions on gauge fixing in them. To perform the calculations it is convenient to choose the Coulomb like gauge $\partial_0 A^0 + \partial_1 A^1 + \partial_2 A^2 = 0$. In this gauge the action $S(A)$ can be written as follows [29]:

$$S(A) = \frac{1}{2} \int \vec{A}(x) \mathcal{K} \vec{A}(x) dx = \frac{1}{2} \int \{ \vec{A}^*(\vec{k}, x_3) K^\perp \vec{A}(\vec{k}, x_3) - A_3^*(\vec{k}, x_3) \vec{k}^2 A_3(\vec{k}, x_3) \} d\vec{k} dx_3. \quad (8)$$

Here,

$$K^\perp = -P[\partial_3^2 + \vec{k}^2] - 2i|\vec{k}| a L \delta(x_3), \quad (9)$$

P and L is a brief way of writing P^{lm} and L^{lm}. Our model of a two-dimensional plane is translation invariant along the coordinates x_0, x_1, x_2. This is why for the propagator

$$D(x, y) = i\mathcal{K}^{-1}(x, y)$$

it is convenient to use the Fourier integral representation

$$D(x, y) = \frac{1}{(2\pi)^3} \int D(\vec{k}, x_3, y_3) e^{i\vec{k}(\vec{x} - \vec{y})} d\vec{k}$$

for which $D(\vec{k}, x_3, y_3)$ can be found. We denote $g_q(z)$ the Green's function of a differential operator $\partial_z^2 + q^2$:

$$[\partial_z^2 + q^2] g_q(z - z') = \delta(z - z'), \quad (10)$$

$$g_q(z) \equiv -i \frac{e^{i|q||z|}}{2|q|}. \quad (11)$$

By making use of the tensors $P_{\mu\nu}$, $L_{\mu\nu}$ introduced in (3), we define

$$G_{\mu\nu}(\vec{q}; x_3, y_3) \equiv -i \left[g_{|\vec{q}|}(x_3 - y_3) P_{\mu\nu} - \frac{a^2 P_{\mu\nu} + a L_{\mu\nu}}{1 + a^2} \frac{g_{|\vec{q}|}(x_3) g_{|\vec{q}|}(y_3)}{g_{|\vec{q}|}(0)} \right]. \quad (12)$$

With the help of identities

$$P^2 = P, \qquad L^2 = -P^2, \qquad LP = PL = L \quad (13)$$

it is easy to check the equality

$$K^\perp G(\vec{k}; x_3, y_3) = i\delta(x_3 - y_3) P.$$

Hence, in a selected gauge the propagator $D_{\mu\nu}(\vec{k}, x_3, y_3)$ has the form

$$D_{33}(\vec{k}, x_3, y_3) = \frac{-i\delta(x_3 - y_3)}{|\vec{k}|^2},$$
$$D_{l3}(\vec{k}, x_3, y_3) = D_{3m}(\vec{k}, x_3, y_3) = 0,$$
$$D_{lm}(\vec{k}, x_3, y_3) = G_{lm}(\vec{k}, x_3, y_3) = \frac{P_{lm}(\vec{k}) \mathcal{P}_1(\vec{k}, x_3, y_3) + L_{lm}(\vec{k}) \mathcal{P}_2(\vec{k}, x_3, y_3)}{2|\vec{k}|[1 + a^2]}, \quad (14)$$

where $l, m = 0, 1, 2$, and

$$\mathcal{P}_1(\vec{k}, x_3, y_3) = a^2 e^{i|\vec{k}|(|x_3| + |y_3|)} - (1 + a^2) e^{i|\vec{k}||x_3 - y_3|},$$
$$\mathcal{P}_2(\vec{k}, x_3, y_3) = a e^{i|\vec{k}|(|x_3| + |y_3|)}. \quad (15)$$

After the integration over the photon field we obtain

$$\left\{\ln\left[\frac{\int \exp\{iS(A)+iJA\}DA}{\int \exp\{iS(A)\}DA}\right]\right\}_{(a)} = -\frac{1}{2}J\{D\}_{(a)}J, \quad (16)$$

where $\{D\}_{(a)} = D - D|_{a=0}$. Thus, for the energy E, which is defined by the right-hand side of (7), we obtain the following expression:

$$E = -i\frac{\langle J\{D\}_{(a)}J\rangle}{2T}, \quad (17)$$

which we are planning to evaluate now.

IV. POTENTIAL OF THE INTERACTION

Because of (14) the propagator $\{D\}_{(a)}$ has the form

$$\{D\}^{33}_{(a)}(\vec{k}, x_3, y_3) = \{D\}^{l3}_{(a)}(\vec{k}, x_3, y_3) = \{D\}^{3m}_{(a)}(\vec{k}, x_3, y_3) = 0,$$

$$\{D\}^{lm}_{(a)}(\vec{k}, x_3, y_3) = \frac{P^{lm}(\vec{k})a^2 + L^{lm}a}{2|\vec{k}|[1+a^2]}e^{i|\vec{k}|(|x_3|+|y_3|)},$$
$$l, m = 0, 1, 2. \quad (18)$$

The potential $V(l, a)$ of interaction of the Chern-Simons plane and the electric dipole $p_i(t)$ described by the current defined in (4) and (5) can be written as follows:

$$V(l,a) \equiv -\frac{i}{2}J\{D\}_{(a)}J = -\frac{i}{4(2\pi)^3[1+a^2]} \times \int d\vec{k}dtdt'\frac{e^{i(k_0(t-t')+2l|\vec{k}|)}}{|\vec{k}|}\mathcal{F}(a,\vec{k},t,t'). \quad (19)$$

We introduced the notation

$$\mathcal{F}(a,\vec{k},t,t') = a^2\left(\sum_{i=1}^{2}p_i(t)k^i + p_3(t)|\vec{k}|\right)\left(\sum_{i=1}^{2}p_i(t')k^i - p_3(t')|\vec{k}|\right)P^{00}(\vec{k}) - i\sum_{j=1}^{2}\dot{p}_j(t)\left(\sum_{i=1}^{2}p_i(t')k^i - p_3(t')|\vec{k}|\right)$$
$$\times[a^2P^{j0}(\vec{k}) + aL^{j0}(\vec{k})] + i\left(\sum_{i=1}^{2}p_i(t)k^i - p_3(t)|\vec{k}|\right)\sum_{j=1}^{2}\dot{p}_j(t')[a^2P^{0j}(\vec{k}) + aL^{0j}(\vec{k})]$$
$$+\sum_{i,j=1}^{2}\dot{p}_i(t)\dot{p}_j(t')[a^2P^{ij}(\vec{k}) + aL^{ij}(\vec{k})]. \quad (20)$$

Because of a definition (3)

$$P^{00}(\vec{k}) = -\frac{k_P^2}{\vec{k}^2}, \qquad k_P = \sqrt{k_1^2+k_2^2}, \qquad P^{0i}(\vec{k}) = P^{i0}(\vec{k}) = -\frac{k^0k^i}{\vec{k}^2}, \qquad i \neq 0,$$

so integrating by parts the time t we can make the substitutions $\dot{p}(t) \to -ik_0p(t)$, $\dot{p}(t') \to ik_0p(t')$ in (19), and due to independence of $p(t)$ from the momentum k we can also make the following substitutions in the integral:

$$\sum_{i,j=1}^{2}p_i(t)p_j(t')k^ik^j \to \frac{k_P^2}{2}\sum_{i=1}^{2}p_i(t)p_i(t'), \qquad \left(\sum_{i=1}^{2}p_i(t)k^i\right)p_3(t)|\vec{k}| \to 0.$$

As a result the function $\mathcal{F}(a,\vec{k},t,t')$ (20) in the integral (19) is changed to

$$\mathcal{G}(a,\vec{k},t,t') = -a^2\left(\sum_{i=1}^{2}p_i(t)p_i(t')\frac{k_P^2}{2} - p_3(t)p_3(t')|\vec{k}|^2\right)\frac{k_P^2}{\vec{k}^2} + \frac{a^2k_P^2k_0^2}{\vec{k}^2}\sum_{j=1}^{2}p_j(t)p_j(t') - [\vec{p}(t)\times\vec{p}(t')]_3\frac{ak_P^2k_0}{|\vec{k}|}$$
$$+ a^2k_0^2\sum_{j=1}^{2}p_j(t)p_j(t')\left(-1-\frac{k_P^2}{2\vec{k}^2}\right) + [\vec{p}(t)\times\vec{p}(t')]_3\frac{ak_0^3}{|\vec{k}|}$$
$$= a^2\left[\sum_{i=1}^{2}p_i(t)p_i(t')\left(\frac{k_P^2}{2}-k_0^2\right) + k_P^2p_3(t)p_3(t')\right] + ak_0|\vec{k}|[\vec{p}(t)\times\vec{p}(t')]_3. \quad (21)$$

Now we perform the Fourier transformation of $p_i(t)$, $p_i^*(k_0) = p_i(-k_0)$, then (19) can be written

$$V(l,a) = -\frac{i}{4(2\pi)^3[1+a^2]}\int d\vec{k}\frac{e^{i2l|\vec{k}|}}{|\vec{k}|}\mathcal{H}(a,\vec{k}), \quad (22)$$

where

$$\mathcal{H}(a,\vec{k}) = a^2\Big[\sum_{i=1}^{2} p_i(k_0)p_i^*(k_0)\Big(\frac{k_P^2}{2}-k_0^2\Big) + k_P^2 p_3(k_0)p_3^*(k_0)\Big] + ak_0|\vec{k}|[\vec{p}(k_0)\times\vec{p}^*(k_0)]_3. \quad (23)$$

This result can be simplified after performing in (22) the integration over k_1, k_2. Our task is to integrate the expression of the form $F(k_P^2)$ over k_1, k_2. The following formula is valid:

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} F(k_P^2)dk_1dk_2 = \pi\int_0^\infty F(k)dk.$$

The three integrals need to be evaluated:

$$I_1(\alpha,k_0) \equiv \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\frac{e^{i\alpha|\vec{k}|}k_P^2dk_1dk_2}{|\vec{k}|},$$

$$I_2(\alpha,k_0) \equiv \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\frac{e^{i\alpha|\vec{k}|}dk_1dk_2}{|\vec{k}|},$$

$$I_3(\alpha,k_0) \equiv \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{i\alpha|\vec{k}|}dk_1dk_2.$$

We get

$$I_1(\alpha,k_0) = \pi\int_0^{+\infty}\frac{e^{i\alpha\sqrt{k_0^2-\rho}}\rho d\rho}{\sqrt{k_0^2-\rho}} = \pi|k_0|^3\int_0^{+\infty}\frac{e^{i\alpha|k_0|\sqrt{1-\rho}}\rho d\rho}{\sqrt{1-\rho}}$$
$$= 2\pi\frac{e^{i\alpha|k_0|\sqrt{1-\rho}}(2i+2\alpha|k_0|\sqrt{1-\rho}+i\alpha\rho|k_0|^2)}{\alpha^3}\Big|_0^{+\infty} = -2\pi i\frac{e^{i\alpha|k_0|}2(1-i\alpha|k_0|)}{\alpha^3},$$

$$I_2(\alpha,k_0) = \pi\int_0^{+\infty}\frac{e^{i\alpha\sqrt{k_0^2-\rho}}d\rho}{\sqrt{k_0^2-\rho}} = \pi|k_0|\int_0^{+\infty}\frac{e^{i\alpha|k_0|\sqrt{1-\rho}}\rho d\rho}{\sqrt{1-\rho}} = 2\pi i\frac{e^{i\alpha|k_0|\sqrt{1-\rho}}}{\alpha}\Big|_0^{+\infty} = -2\pi i\frac{e^{i\alpha|k_0|}}{\alpha},$$

$$I_3(\alpha,k_0) = \pi\int_0^{+\infty} e^{i\alpha\sqrt{k_0^2-\rho}}d\rho = \pi|k_0|^2\int_0^{+\infty} e^{i\alpha|k_0|\sqrt{1-\rho}}d\rho = 2\pi i\frac{e^{i\alpha|k_0|\sqrt{1-\rho}}(i+\alpha|k_0|\sqrt{1-\rho})}{\alpha^2}\Big|_0^{+\infty}$$
$$= -2\pi i\frac{e^{i\alpha|k_0|}(i+\alpha|k_0|)}{\alpha^2}.$$

Thus, we derive the potential of interaction of the electric dipole with a plane:

$$V(l,a) = \frac{a^2Q_1(l)+aQ_2(l)}{128\pi^2[1+a^2]l^3}, \quad (24)$$

where

$$Q_1(l) = -\int_{-\infty}^{+\infty} e^{2il|k_0|}\Big[(1-2il|k_0|-4l^2|k_0|^2)\sum_{i=1}^{2}p_i(k_0)p_i^*(k_0) + 2(1-2il|k_0|)p_3(k_0)p_3^*(k_0)\Big]dk_0, \quad (25)$$

$$Q_2(l) = i\int_{-\infty}^{+\infty} e^{2il|k_0|}(1-2il|k_0|)(-2l|k_0|)\varepsilon(k_0)[\vec{p}(k_0)\times\vec{p}^*(k_0)]_3dk_0, \quad (26)$$

and here $\varepsilon(k_0) = k_0/|k_0|$. The functions Q_1, Q_2 can also be written as integrals over the positive frequencies

$$Q_1(l) = -2\int_0^\infty e^{2il\omega}\Big[(1-2il\omega-4l^2\omega^2)\sum_{i=1}^{2}p_i(\omega)p_i^*(\omega) + 2(1-2il\omega)p_3(\omega)p_3^*(\omega)\Big]d\omega, \quad (27)$$

$$Q_2(l) = 2\int_0^\infty e^{2il\omega}(1-2il\omega)(-2il\omega)[\vec{p}(\omega)\times\vec{p}^*(\omega)]_3d\omega. \quad (28)$$

By making use of (6), (17), (19), and (24) and rotating the contour to the imaginary axis we obtain the ground state energy of a neutral atom in the presence of a plane with Chern-Simons interaction:

$$E = -\frac{1}{64\pi^2 l^3}\frac{a^2}{1+a^2}\left(\int_0^{+\infty} d\omega e^{-2\omega l} 2(1+2\omega l)\alpha_{33}(i\omega) + \int_0^{+\infty} d\omega e^{-2\omega l}(1+2\omega l+4\omega^2 l^2)(\alpha_{11}(i\omega)+\alpha_{22}(i\omega))\right) + \frac{1}{64\pi^2 l^2}\frac{a}{1+a^2}\int_0^{+\infty} d\omega e^{-2\omega l} 2\omega(1+2\omega l)(\alpha_{12}(i\omega)-\alpha_{21}(i\omega)). \quad (29)$$

It is worth discussing physical consequences following from (29). The expression (29) yields the well known Casimir-Polder potential [1] in the limit $a \to +\infty$. The part of the formula (29) with diagonal matrix elements of matrix $\alpha_{jk}(i\omega)$ is equal $a^2/(1+a^2)$ times the Casimir-Polder interaction of a neutral atom with a perfectly conducting plane. The last line of the formula (29) is odd in a and contains the antisymmetric combination of off-diagonal elements of the atomic polarizability. When one can neglect the contribution of off-diagonal elements of the atomic polarizability (see a discussion below) the Casimir-Polder interaction of an atom with a Chern-Simons plane is a fraction $a^2/(1+a^2)$ of the corresponding Casimir-Polder interaction with a perfectly conducting plane.

It is interesting to analyze the contribution from the off-diagonal elements of the atomic polarizability to the potential (29) in more detail. The atomic polarizability can be expressed in terms of dipole matrix elements [30]:

$$\alpha_{jk}(\omega) = \sum_n\left(\frac{\langle 0|d_j|n\rangle\langle n|d_k|0\rangle}{\omega_{n0}-\omega-i\epsilon} + \frac{\langle 0|d_k|n\rangle\langle n|d_j|0\rangle}{\omega_{n0}+\omega-i\epsilon}\right), \quad (30)$$

ω_{n0} is a transition energy between the excited state $|n\rangle$ of the atom and its ground state $|0\rangle$, $\vec{d}$ is a dipole moment operator in the Schrodinger representation. The symmetric $\alpha^S_{jk}(\omega)$ and antisymmetric $\alpha^A_{jk}(\omega)$ parts of $\alpha_{jk}(\omega) = \alpha^S_{jk}(\omega) + \alpha^A_{jk}(\omega)$ can be written as follows:

$$\alpha^S_{jk}(\omega) = \sum_n \frac{2\omega_{n0}\,\mathrm{Re}M^n_{jk}}{\omega^2_{n0}-\omega^2} = \alpha^S_{kj}(\omega), \quad (31)$$

$$\alpha^A_{jk}(\omega) = \sum_n \frac{2i\omega\,\mathrm{Im}M^n_{jk}}{\omega^2_{n0}-\omega^2} = -\alpha^A_{kj}(\omega), \quad (32)$$

$$M^n_{jk} \equiv \langle 0|d_j|n\rangle\langle n|d_k|0\rangle. \quad (33)$$

From (32) and (33) it follows that the contribution of $\alpha^A_{jk}(\omega)$ to the potential (29) is different from zero when matrix elements of a dipole moment operator have imaginary parts.

Consider the system with a nonzero $\alpha^A_{jk}(\omega)$ and assume for simplicity the one mode model of the atomic polarizability with a characteristic frequency ω_{10}. Then $\alpha^A_{12}(\omega) = i\omega C_2/(2(\omega^2_{10}-\omega^2))$, where C_2 is a real constant. In the limit of large separations $\omega_{10} l \gg 1$ we obtain from (29)

$$E|_{\omega_{01}l\gg 1} = -\frac{a^2}{1+a^2}\frac{\alpha_{11}(0)+\alpha_{22}(0)+\alpha_{33}(0)}{32\pi^2 l^4} - \frac{a}{1+a^2}\frac{C_2}{32\pi^2\omega^2_{10} l^5}. \quad (34)$$

At large enough separations the first term in (34) always dominates. Assuming for simplicity $\alpha_{11}(0) = \alpha_{22}(0) = \alpha_{33}(0) = C_1/(3\omega_{10})$, C_1 is a positive constant, one can see from (34) that if the condition $\frac{|a|C_1}{|C_2|} < 1$ holds then for separations $l \lesssim \frac{|C_2|}{|a|C_1\omega_{10}}$ the term with off-diagonal elements of the atomic polarizability [the second term in (34)] dominates.

In the limit of short separations ($b \equiv \omega_{10} l \ll 1$) we obtain from (29)

$$E|_{\omega_{01}l\ll 1} = -\frac{1}{64\pi^2 l^3}\frac{a^2}{1+a^2}\int_0^{+\infty} d\omega(\alpha_{11}(i\omega)+\alpha_{22}(i\omega)+2\alpha_{33}(i\omega)) - \frac{C_2}{32\pi^2 l^3}\frac{a}{1+a^2}\left(1-\frac{\pi}{2}b+2b^2-\frac{\pi}{2}b^3+\dots\right) \simeq -\frac{1}{32\pi^2 l^3}\left(\frac{a^2}{1+a^2}C_1\frac{\pi}{3}+\frac{a}{1+a^2}C_2\right) \quad \text{for } b\to 0. \quad (35)$$

From (35) it follows that if the condition $\frac{|a|C_1}{|C_2|}\frac{\pi}{3} < 1$ holds, then the term with off-diagonal elements of the atomic polarizability dominates in (29) in the limit of short separations. Thus, if we consider the one mode model for the atomic polarizability and the criterion $|a| \lesssim \frac{|C_2|}{C_1}$ holds, then the antisymmetric part of the atomic polarizability plays a dominant role in the interaction of the atom with the Chern-Simons plane.

V. CONCLUSIONS

In the framework of quantum electrodynamics we consider a model with the Chern-Simons action on a two-dimensional plane having one dimensionless parameter a, which describes properties of the material. The formula (29) for the energy of interaction of a neutral atom (molecule) with fluctuations of vacuum of the photon field in the presence of a two-dimensional plane with Chern-Simons interaction is derived. In the limiting case $a \rightarrow +\infty$ the result coincides with the Casimir-Polder result for the energy of interaction of a neutral atom with a perfectly conducting plane. The essential feature of the result (29) is the term depending on the antisymmetric part of a dipole correlation function for finite values of the parameter a, we derive a criterion of its dominance in terms of imaginary and real parts of dipole matrix elements of the atom and the parameter a of the Chern-Simons surface term.

We expect quantum Hall effect systems and graphene to be the most promising known materials for the measurements of the potential derived in this paper. The Casimir-Polder effect provides a recipe for direct measurements of the parameter a in such materials, which can be relevant for better understanding of quantum dynamics in these systems. The measurements of the antisymmetric part of the atomic polarizability by means of the Casimir-Polder effect can be an independent possibility for study of antisymmetric parts of atomic polarizabilities in various atomic and molecular systems.

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