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AdS_5 solutions in Einstein–Yang-Mills–Chern-Simons theory

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We investigate static, spherically symmetric solutions of an Einstein-Yang-Mills-Chern-Simons system with negative cosmological constant, for an $SO(6)$ gauge group. For a particular value of the Chern-Simons coefficient, this model can be viewed as a truncation of the five-dimensional maximal gauged supergravity and we expect that the basic properties of the solutions in the full model to persist in this truncation. Both globally regular, particlelike solutions and black holes are considered. In contrast with the Abelian case, the contribution of the Chern-Simons term is nontrivial already in the static, spherically symmetric limit. We find two types of solutions: the *generic* configurations whose magnetic gauge field does not vanish fast enough at infinity (although the spacetime is asymptotically AdS), whose mass function is divergent, and the *special* configurations, whose existence depends on the Chern–Simons term, which are endowed with finite mass. In the case of the *generic* configurations, we argue that the divergent mass implies a nonvanishing trace for the stress tensor of the dual $d = 4$ theory.

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I. INTRODUCTION

It was originally found in $d = 4$ spacetime dimensions [1,2], that a variety of well-known features of asymptotically flat self-gravitating non-Abelian solutions are not shared by their anti–de Sitter (AdS) counterparts. In the presence of a negative cosmological constant $\Lambda < 0$, the Einstein-Yang-Mills (EYM) theory possesses a continuous spectrum of regular and black hole non-Abelian solutions in terms of the adjustable parameters that specify the initial conditions at the origin or at the event horizon, rather than at discrete values of these parameters. The gauge field of generic solutions does not vanish asymptotically, resulting in a nonzero magnetic flux at infinity. Moreover, in contrast with the $\Lambda = 0$ case, some of the AdS configurations are stable against linear perturbations [3].

As found in [4,5], some of these features are shared by higher dimensional EYM solutions with AdS asymptotics. Since gauged supergravity theories generically contain non-Abelian matter fields in the bulk, these configurations are relevant in an AdS/CFT context, offering the possibility of studying some aspects of the nonperturbative structure of a CFT in a background gauge field [6]. On the CFT side, the boundary non-Abelian fields correspond to external source currents coupled to various operators.

Given its relevance in the conjectured AdS/CFT correspondence [7,8], the case of $\mathcal{N} = 8$, $d = 5$ gauged supergravity [9,10] is of particular interest. The bosonic sector of this theory consists of the metric, twenty scalars and fifteen $SO(6)$ Yang-Mills gauge fields.[1] Apart from the usual F^2 term, the Yang-Mills (YM) fields have in this case a non-Abelian Chern-Simons (CS) term in the action, which unlike in the Abelian case does not vanish when subjected to spherical symmetry.

Solutions of this model have been considered by several authors for various consistent truncations, with subgroups of $SO(6)$ (see e.g. the recent work [11] and the references therein). However, to our knowledge, no attempt has been made to construct non-Abelian solutions for the general case of the full $SO(6)$ gauge group. In particular, the effects resulting from the introduction of the CS term have so far not been studied.

This paper is aimed as a first step in this direction, by taking a truncation of the $\mathcal{N} = 8$, $d = 5$ model corresponding to a pure Einstein-Yang-Mills-Chern-Simons (EYMCS) theory (*i.e.* with a negative cosmological constant, but with no scalar fields). We propose an Ansatz for a spherically symmetric $SO(6)$ gauge group and investigate the basic properties of both the black hole, and, particlelike globally regular solutions. Special attention is paid to the new features induced by the CS term.

As originally found in [4,5,12], a generic property of higher dimensional EYM solutions is that their masses and actions, as defined in the usual way, diverge. (For a recent review of these solutions, see [13].) This can be understood heuristically by noting that the Derrick scaling requirement is not fulfilled in spacetimes for dimension five and higher. To our knowledge, the only mechanism for regularizing the mass of the $d > 4$ asymptotically flat or (A)dS non-Abelian gravitating solutions, proposed so far in the literature, is to include higher order terms in the YM hierarchy [5,14,15] and the corresponding

[1]Note that the field content of the full $\mathcal{N} = 8$, $d = 5$ gauged supergravity is richer. However, a number of bosonic fields can be consistently set to zero [10].

YM–Higgs terms [16].[2] These are the YM counterparts of the Lovelock gravities (or the hierarchy of Einstein systems), and occur in the low energy effective action of string theory [18,19].

One of the main features of the present work is the introduction of the CS term in $4+1$ dimensions, as an *alternative* to the higher order curvature terms of the YM hierarchy employed previously for regularizing the mass. It turns out that this prescription *does* result in finite mass solutions, but in addition to these, we find solutions with divergent mass in what we have termed as the *generic case*. These last are characterized by a continuum of values of the shooting parameters, which is a typical feature of EYM system with a negative cosmological constant. The finite mass solutions on the other hand, termed as the *special case*, are special in that they occur only for a discrete set of values of the shooting parameters. Of course, in the absence of the CS term the *only* solutions that exist are ones with divergent mass.

Although the spacetime still approaches asymptotically the maximally symmetric AdS background, the mass and the total action of a *generic* solution present a logarithmically divergent part. The coefficient of the divergent term is fixed by the square of the induced non-Abelian fields on the boundary at infinity. We shall argue that the logarithmic divergence of the non-Abelian AdS_5 configurations does not signal a problem with these solutions, but rather provides a consistency check of the AdS/CFT conjecture, the coefficient of the divergent term in the action being related in this case to the trace anomaly of the dual CFT defined in a background non-Abelian field. Moreover, one can define a mass and action for the generic solutions by using the counterterm prescription of [20]. The counterterms here depend not only on the boundary metric but also on the induced non-Abelian fields on the boundary.

However, perhaps the most interesting feature of the EYMCS model is the existence of a set of solutions with finite mass. In the case of these solutions, as for the well known $d=4$ Bartnick-McKinnon solitons [21], they exist only for discrete values of the shooting parameters (associated with the initial values of the gauge fields). As can be seen by using a simple Derrick-type argument, they are supported by the contribution of the non-Abelian CS term, a prescription which can be exploited only in odd dimensional spacetimes where a CS term is defined.

The paper is structured as follows: in Sec. II we present the general framework and analyze the field equations and boundary conditions. We present the numerical results in Sec. III, special attention being paid to solutions with a finite mass. The computation of the mass and electric charge of the solutions is addressed in Sec. IV. We conclude with Sec. V where the significance of, and further consequences arising from, the solutions we have constructed are briefly discussed.

II. THE MODEL

A. The action

We consider the following action

$$S=\int_{\mathcal{M}} d^5x\sqrt{-g}\left(\frac{1}{16\pi G}(R-2\Lambda)-\mathcal{L}_{\rm YM}\right)-\int_{\mathcal{M}} d^5x\,\mathcal{L}_{\rm CS}, \tag{1}$$

where

$$\mathcal{L}_{\rm YM}=\frac{1}{4}\,{\rm Tr}\{F_{\mu\nu}F^{\mu\nu}\}, \tag{2}$$

is the usual Yang-Mills Lagrangian for a gauge group $SO(6)$ (with $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu+e[A_\mu,A_\nu]$ the gauge field strength tensor), and

$$\mathcal{L}_{\rm CS}=i\kappa\varepsilon_{\mu\nu\rho\sigma\tau}\,{\rm Tr}\left\{A^\tau\left[F^{\mu\nu}F^{\rho\sigma}-eF^{\mu\nu}A^\rho A^\sigma+\frac{2}{5}e^2A^\mu A^\nu A^\rho A^\sigma\right]\right\}, \tag{3}$$

is the CS term[3] (with κ the CS coupling constant), while $\Lambda=-6/\ell^2$ is the cosmological constant and e is the gauge coupling constant.

These are the basic pieces which enter the bosonic action of the $d=5$, $\mathcal{N}=8$ gauged supergravity [9], [10], the CS coefficient being $\kappa=1/8$ in this case. However, the full $\mathcal{N}=8$ system contains an additional twenty scalars, which are represented by a symmetric unimodular tensor. These scalars have a nontrivial potential approaching a constant negative value at infinity which fixes the value of the effective cosmological constant. Although ignoring the scalar sector is not a consistent truncation of the general $\mathcal{N}=8$ model, we expect that the basic properties of our solutions hold also in that case.[4]

The field equations are obtained by varying the action (1) with respect to the field variables $g_{\mu\nu}$, A_μ

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R+\Lambda g_{\mu\nu}=8\pi GT_{\mu\nu},$$
$$\frac{1}{\sqrt{-g}}D_\mu(\sqrt{-g}F^{\mu\tau})+3\kappa\varepsilon^{\mu\nu\rho\sigma\tau}F_{\mu\nu}F_{\rho\sigma}=0. \tag{4}$$

where the energy momentum tensor is defined by

[2] It is in principle possible to supply such higher scaling terms by employing only Higgs kinetic terms, or, the kinetic terms of suitably gauged higher dimensional sigma models [17], but these have not been attempted.

[3] The factor of i appears in (3) because we are using an anti-Hermitian representation for the $SO(6)$ algebra matrices.

[4] This is the situation for $d=4$ EYM solutions. As discussed *e.g.* in [22], the properties of the EYM-dilaton solutions (the dilaton field possessing a nontrivial potential approaching a constant negative value at infinity) are quite similar to those of the pure EYM-AdS case.

$$T_{\mu\nu} = \mathrm{Tr}\left\{F_{\mu\alpha}F_{\nu\beta}g^{\alpha\beta} - \frac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}\right\}. \quad (5)$$

One can show that this tensor is covariantly conserved (*i.e.* $\nabla_\mu T^{\mu\nu} = 0$) for solutions of the YM-CS equations.

B. The spherically symmetric ansatz

In this work we shall restrict to the simplest case of static, spherically symmetric solutions. Thus we consider a metric Ansatz in terms of two metric functions $N(r)$ and $\sigma(r)$

$$ds^2 = \frac{dr^2}{N(r)} + r^2 d\Omega_3^2 - N(r)\sigma^2(r)dt^2, \quad (6)$$

where we have found convenient to define

$$N(r) = 1 - \frac{m(r)}{r^2} + \frac{r^2}{\ell^2}, \quad (7)$$

the function $m(r)$ being related to the local mass-energy density (as defined in the standard way) up to some factor. r and t are the radial and time coordinates, while $d\Omega_3^2$ is the metric on the round three-sphere.

The static, spherically symmetric $SO(6)$ YM fields are taken in one of the two chiral representations of $SO(6)$, such that the spherically symmetric Ansatz is expressed in terms of the representation matrices,

$$\Sigma_{\alpha\beta} = -\frac{1}{4}\Sigma_{[\alpha}\tilde{\Sigma}_{\beta]}, \quad (8)$$

where $\Sigma_i = -\tilde{\Sigma}_i = i\gamma_i$, $\Sigma_5 = -\tilde{\Sigma}_5 = i\gamma_5$, $\Sigma_6 = +\tilde{\Sigma}_6 = 1\mathrm{I}$, are defined in terms of the usual Dirac gamma matrices γ_i.

Our spherically symmetric Ansatz for the $SO(6)$ YM connection $A_\mu = (A_t, A_i)$ is a variant of Witten's Ansatz for the axially symmetric instanton [23]. Also, this Ansatz is one of the two $SU(4)$ Ansätze proposed in [24], namely, the one employing Dirac gamma matrices as opposed the one employing the Gell-Mann matrices.[5] It is expressed as

$$\begin{aligned} A_t &= \frac{1}{e}(-(\varepsilon\chi(r))^M\hat{x}_j\Sigma_{jM} - \chi^7(r)\Sigma_{56}), \\ A_i &= \frac{1}{e}\left(\left(\frac{\phi^7(r)+1}{r}\right)\Sigma_{ij}\hat{x}_j + \left[\left(\frac{\phi^M(r)}{r}\right)(\delta_{ij} - \hat{x}_i\hat{x}_j)\right.\right. \\ &\quad \left.\left. + (\varepsilon A_r(r))^M\hat{x}_i\hat{x}_j\right]\Sigma_{jM} + A_r^7(r)\hat{x}_i\Sigma_{56}\right), \end{aligned} \quad (9)$$

where $\hat{x}_i = x_i/r$ (with x^i the usual Cartesian coordinates on R^4 and $x_i x_i = r^2$). In the above relations $i, j = 1, 2, 3, 4$ and the index M runs over 5, 6. Also, ε is the two dimensional Levi-Civita symbol.

[5]This distinction is important since the Dirac gamma matrix Ansatz cannot be contracted to a $SU(2)$ subalgebra, while clearly Gell-Mann matrix Ansatz does have a $SU(2)$ subalgebra.

After taking the traces over the spin matrices, it is convenient to relabel the triplets of radial function as $\vec{\phi} \equiv (\phi^M, \phi^3)$, $\vec{\chi} \equiv (\chi^M, \chi^3)$ and $\vec{A}_r \equiv (A_r^M, A_r^3)$, with $M = 1, 2$ now.

Substituting (9) and (10) in the YM Lagrangian density we find the compact expression for the reduced one dimensional reduced YM action density[6]

$$\begin{aligned} \mathcal{L}_{\mathrm{YM}}\sqrt{-g} = \frac{1}{e^2}\left[\frac{3}{2}r\sigma\left(N|D_r\phi^a|^2 + \frac{1}{r^2}(|\phi^a|^2 - 1)^2\right)\right. \\ \left. - \frac{1}{2}\frac{r^3}{\sigma}\left(|D_r\chi^a|^2 + \frac{3}{Nr^2}(\varepsilon^{abc}\phi^b\chi^c)^2\right)\right]. \end{aligned} \quad (10)$$

The calculation of the reduced CS action density is rather more tedious since unlike (10), this term is not gauge invariant. It can be expressed in a compact way as

$$\begin{aligned} \mathcal{L}_{\mathrm{CS}} = \kappa\frac{3}{4e^3}[&12(|\phi^d|^2 - 1)A^a(\varepsilon^{abc}\chi^b\phi^c) \\ &+ 3|\phi^b|^2(\phi^a D_r\chi^a) + 6(\phi^b\chi^b)(\phi^a D_r\phi^a) \\ &- 9|\phi^b|^2(\chi^a D_r\phi^a) - (2\phi^3 + 5)(\phi^a D_r\chi^a) \\ &+ (3|\phi^a|^2 - 2\phi^3 - 5)D_r\chi^3 \\ &+ (6\chi^3\phi^a + 7\chi^a - 2\phi^3\chi^a)D_r\phi^a \\ &- 2(\chi^a\phi^a + \chi^3)D_r\phi^3], \end{aligned} \quad (11)$$

which of course does not feature the metric functions. Note that (11) is not a scalar after contraction of the indices (a, b, c). This is a consequence of the *gauge variance* of the CS density.

In both (10) and (11) we have used the notation

$$\begin{aligned} D_r\phi^a &= \partial_r\phi^a + \varepsilon^{abc}A_r^b\phi^c, \\ D_r\chi^a &= \partial_r\chi^a + \varepsilon^{abc}A_r^b\chi^c, \end{aligned} \quad (12)$$

which are $SO(3)$ covariant derivatives of the two triplets $\vec{\phi} \equiv \phi^a = (\phi^M, \phi^3)$, and $\vec{\chi} \equiv \chi^a = (\chi^M, \chi^3)$, with respect to the $SO(3)$ gauge connection $\vec{A}_r \equiv A_r^a$. But $\vec{A}_r$ is really a *pure-gauge* since in one dimension there is no curvature. As such, it can be consistently set equal to zero. But more importantly, taking the variations $\delta\vec{A}_r$ leads to the constraint equations, which are first integrals of the equations for $\vec{\phi}$ and $\vec{\chi}$, and which play an important technical role in the numerical integrations. We will return to these below. The occurrence of constraint equations in a system supporting what are basically *sphaleron* solutions is completely expected, as the solutions we construct are indeed *sphalerons*, just like the familiar Bartnik-McKinnon solutions. Needless to say, the consistency of the Ansatz used has been verified, so it is sufficient to work with the reduced one dimensional Lagrangian (10) and (11).

[6]Here we ignore the (irrelevant) angular part in $\sqrt{-g}$.

Finding solutions within the general YM Ansatz (9), which after setting $\vec{A}_r = 0$ still features six independent functions, is technically a difficult task. A further consistent truncation of the general Ansatz is $\phi^2 = \chi^2 = 0$, leading to an EYMCS system with six unknown functions, four of them being gauge potentials parametrizing the gauge field, and, two metric functions. Indeed, the two gauge functions suppressed are redundant and would only be excited in an eventual stability analysis of our *sphalerons*.

To connect with notations used in the previous work [4,12,14] on $d = 5$ EYM solutions, we adopt the notation

$$\begin{aligned} \phi^1(r) &= \tilde{w}(r), \qquad \phi^3(r) = w(r), \\ \chi^1(r) &= \tilde{V}(r), \qquad \chi^3(r) = V(r). \end{aligned} \tag{13}$$

The resulting system has some residual symmetry under a rotation of the 'doublets' $w(r)$, $\tilde{w}(r)$, and $V(r)$, $\tilde{V}(r)$ with the same constant angle u (*e.g.* $w \to w\cos u + \tilde{w}\sin u$ etc.) One can use this symmetry to consistently set $\tilde{w}(r) = \tilde{V}(r) = 0$ (or $w(r) = V(r) = 0$) which results in a *particular truncation* of the system, which we shall exploit in Sec. III B. Note that for configurations with $\tilde{w}(r) = \tilde{V}(r) = 0$ the gauge potentials are invariant under the "chiral" transformations generated by Σ_5. The configurations with $w(r) = V(r) = 0$ instead change just by a sign under the same transformations. Also, this Ansatz is invariant under the parity reflections transformation $\phi^a \to -\phi^a$, $\chi^a \to -\chi^a$. The asymmetry between (w, V) and $(\tilde{w}, \tilde{V})$ is manifested by the different set of boundary conditions they satisfy.

C. The equations and boundary conditions

Inserting this Ansatz into the action (1), the EYMCS field Eqs. (4) reduce to (to simplify the notation we denote $\alpha^2 = 16\pi G/(3e^2)$ and absorb a factor of $1/e$ in the expression of κ):

$$\begin{aligned} m' &= \frac{1}{2}\alpha^2\left(3r\left(N(w'^2 + \tilde{w}'^2) + \frac{(w^2 + \tilde{w}^2 - 1)^2}{r^2}\right) + \frac{r^3}{\sigma^2}\left(V'^2 + \tilde{V}'^2 + \frac{3}{r^2 N}(\tilde{V}w - V\tilde{w})^2\right)\right), \\ \frac{\sigma'}{\sigma} &= \frac{3\alpha^2}{2r}\left(w'^2 + \tilde{w}'^2 + \frac{1}{N^2\sigma^2}(\tilde{V}w - V\tilde{w})^2\right), \\ (r\sigma N w')' &= r\sigma\left(\frac{2w(w^2 + \tilde{w}^2 - 1)}{r^2} + \frac{\tilde{V}}{\sigma^2 N}(V\tilde{w} - \tilde{V}w)\right) + 4\kappa(V'(w^2 + \tilde{w}^2 - 1) + 2\tilde{w}'(V\tilde{w} - \tilde{V}w)), \\ (r\sigma N \tilde{w}')' &= r\sigma\left(\frac{2\tilde{w}(w^2 + \tilde{w}^2 - 1)}{r^2} + \frac{V}{\sigma^2 N}(\tilde{V}w - V\tilde{w})\right) + 4\kappa(\tilde{V}'(w^2 + \tilde{w}^2 - 1) + 2w'(\tilde{V}w - V\tilde{w})), \\ \left(\frac{r^3 V'}{\sigma}\right)' &= \frac{3r}{\sigma N}\tilde{w}(V\tilde{w} - \tilde{V}w) + 12\kappa(w^2 + \tilde{w}^2 - 1)w', \qquad \left(\frac{r^3\tilde{V}'}{\sigma}\right)' = \frac{3r}{\sigma N}w(\tilde{V}w - V\tilde{w}) + 12\kappa(w^2 + \tilde{w}^2 - 1)\tilde{w}', \end{aligned} \tag{14}$$

together with the constraint equation

$$\begin{aligned} &\frac{r^3}{\sigma}(\tilde{V}V' - V\tilde{V}') + 3rN\sigma(w\tilde{w}' - \tilde{w}w') \\ &\quad - 12\kappa(\tilde{V}w - V\tilde{w})(w^2 + \tilde{w}^2 - 1) = 0, \end{aligned} \tag{15}$$

which originates from the variational equation for $\delta\vec{A}_r$ (where a prime denotes a derivative with respect to r).

These equations support both globally regular and black hole solutions. The only known closed form solutions of these equations are discussed in the next subsection and are trivial in some sense, since the magnetic gauge potentials do not feature any dependence on r. However, it is conceivable that non-Abelian analytic solutions can be found by studying the first order Bogomol'nyi equations of the full $\mathcal{N} = 8$ gauged supergravity model, with all scalar functions included. This was the case of other gauged supergravity theories, the most famous example being the Chamseddine-Volkov solution [25] of the $\mathcal{N} = 4$, $d = 4$ Freedman-Schwarz model [26]. One might therefore expect the full $\mathcal{N} = 8$ model to support BPS solutions describing also non-Abelian globally regular solitons, which actually contrasts with the case of an Abelian truncation. However, given the large number of matter functions, even finding the explicit form of the first order Bogomol'nyi equations of the $\mathcal{N} = 8$ supergravity model with non-Abelian fields is bound to be a very difficult task, which has not been addressed so far in the literature. This is not surprising since the analogous task in the construction of the Chamseddine-Volkov solution involves simply a $SU(2)$ gauge field and a single dilaton field.

However, one can analyze the properties of the solutions of the system (14) by using a combination of analytic and numerical methods, which is sufficient for most purposes.

The globally regular configurations are nontrivial deformations of the AdS_5 and have the following expansions near the origin $r = 0$:

$$
\begin{aligned}
w(r) &= 1 - br^2 + O(r^4),\\
\tilde{w}(r) &= \frac{g_1(V(0) - 24\kappa b\sigma_0)}{9\sigma_0^2} r^3 + O(r^5),\\
V(r) &= V(0) + 6b^2\kappa\sigma_0 r^2 + O(r^4),\\
\tilde{V}(r) &= g_1 r + O(r^3),\\
m(r) &= \frac{\alpha^2(g_1^2 + 6b^2\sigma_0^2)}{2\sigma_0^2} r^4 + O(r^6),\\
\sigma(r) &= \sigma_0 + \frac{3\alpha^2(g_1^2 + 4b^2\sigma_0^2)}{4\sigma_0} r^2 + O(r^4).
\end{aligned}
\tag{16}
$$

The free parameters are $b = -\frac{1}{2}w''(0)$, $V(0)$, $g_1 = \tilde{V}'(0)$ and $\sigma_0 = \sigma(0)$. The coefficients of all higher order terms in the $r \to 0$ expansion are fixed by these parameters.

We are also interested in solutions having a regular event horizon at $r = r_h > 0$ and representing non-Abelian generalizations of the Reissner-Nordström-AdS$_5$ (RNAdS) black hole. To simplify the general picture we shall consider mainly nonextremal black holes, in which case $N(r)$ has a single zero at $r = r_h$ and $\sigma(r_h) > 0$. We expect that extremal black holes also exist for the full $SO(6)$ theory, but we have restricted their numerical construction only to the *particular truncation* ($\tilde{w} = \tilde{V} = 0$) of the system, alluded to in the previous subsection, which is of course, a consistent truncation. We have made this restriction simply due to our desire to render the numerical task easier. From our study of this *particular* truncation of the system, we deduce that it is likely extremal black holes exist also for the full $SO(6)$ theory.

For the nonextremal case, the field equations imply the following behavior as $r \to r_h$ in terms of five parameters $w_h = w(r_h)$, $\tilde{w}_h = \tilde{w}(r_h)$, $V_h = V(r_h)$, $V_1 = V'(r_h)$, and $\sigma_h = \sigma(r_h)$:

$$
\begin{aligned}
&w(r) = w_h + w_1(r - r_h) + O(r - r_h)^2, \qquad \tilde{w}(r) = \tilde{w}_h + \tilde{w}_1(r - r_h) + O(r - r_h)^2,\\
&V(r) = V_h + V_1(r - r_h) + O(r - r_h)^2, \qquad \tilde{V}(r) = \frac{\tilde{w}_h V_h}{w_h} + \frac{\tilde{w}_h V_1}{w_h}(r - r_h) + O(r - r_h)^2,\\
&m(r) = r_h^2\left(1 + \frac{r_h^2}{\ell^2}\right) + \frac{\alpha^2}{2}\left(\frac{r_h^3 V_1^2(w_h^2 + \tilde{w}_h^2)}{\sigma_h^2 w_h^2} + \frac{3(w_h^2 + \tilde{w}_h^2 - 1)^2}{r_h}\right)(r - r_h) + O(r - r_h)^2,\\
&\sigma(r) = \sigma_h + \sigma_1(r - r_h) + O(r - r_h)^2,
\end{aligned}
\tag{17}
$$

where

$$
\begin{aligned}
w_1 &= -\frac{4r_h\ell^2\sigma_h w_h^2(2\kappa r_h V_1 + \sigma_h w_h)(w_h^2 + \tilde{w}_h^2 - 1)}{-4r_h^2(2r_h^2 + \ell^2)\sigma_h^2 w_h^2 + \alpha^2\ell^2(3\sigma_h^2 w_h^2(w_h^2 + \tilde{w}_h^2 - 1)^2 + r_h^4 V_1^2(w_h^2 + \tilde{w}_h^2))},\\
\tilde{w}_1 &= -\frac{4r_h\ell^2\sigma_h w_h\tilde{w}_h(2\kappa r_h V_1 + \sigma_h w_h)(w_h^2 + \tilde{w}_h^2 - 1)}{-4r_h^2\ell^2\sigma_h^2 w_h^2 + 3\alpha^2\ell^2\sigma_h^2 w_h^2(w_h^2 + \tilde{w}_h^2 - 1)^2 + r_h^4(-8\sigma_h^2 w_h^2 + \alpha^2\ell^2 V_1^2(w_h^2 + \tilde{w}_h^2))},\\
\sigma_1 &= \frac{24\alpha^2 r_h\ell^4\sigma_h^3 w_h^2(2\kappa r_h V_1 + \sigma_h w_h)^2(w_h^2 + \tilde{w}_h^2 - 1)^2(w_h^2 + \tilde{w}_h^2)}{(4r_h^2\ell^2\sigma_h^2 w_h^2 - 3\alpha^2\ell^2\sigma_h^2 w_h^2(w_h^2 + \tilde{w}_h^2 - 1)^2 + r_h^4(8\sigma_h^2 w_h^2 - \alpha^2\ell^2 V_1^2(w_h^2 + \tilde{w}_h^2)))^2}.
\end{aligned}
\tag{18}
$$

Large r asymptotic expansions

The expansion at infinity of the solutions is more complicated and involves separate analyses in the *generic* and the *special* cases.

In the *generic* case, the potentials parametrizing the non-Abelian gauge field take arbitrary values as $r \to \infty$, with the leading order behavior

$$
\begin{aligned}
&w(r) = w_0 + \frac{w_2}{r^2} + w_c\frac{\log r}{r^2} + \ldots, \qquad \tilde{w}(r) = \tilde{w}_0 + \frac{\tilde{w}_2}{r^2} + \tilde{w}_c\frac{\log r}{r^2} + \ldots, \qquad V(r) = V_0 + \frac{q}{r^2} + V_c\frac{\log r}{r^2} + \ldots,\\
&\tilde{V}(r) = \tilde{V}_0 + \frac{\tilde{q}}{r^2} + \tilde{V}_c\frac{\log r}{r^2} + \ldots, \qquad m(r) = M_0 + \frac{3}{2}\alpha^2((w_0^2 + \tilde{w}_0^2 - 1)^2 + \ell^2(\tilde{V}_0 w_0 - V_0\tilde{w}_0)^2)\log r,\\
&\sigma(r) = 1 - \frac{\alpha^2(w_c^2 + \tilde{w}_c^2)\log^2 r}{r^6} + \ldots,
\end{aligned}
\tag{19}
$$

with

$$w_c = -\frac{\ell^2}{2}(2w_0(w_0^2+\tilde{w}_0^2-1)+\tilde{V}_0\ell^2(V_0\tilde{w}_0-\tilde{V}_0w_0)),$$
$$\tilde{w}_c = -\frac{\ell^2}{2}(2\tilde{w}_0(w_0^2+\tilde{w}_0^2-1)+V_0\ell^2(\tilde{V}_0w_0-V_0\tilde{w}_0)),$$
$$V_c = -\frac{3}{2}\ell^2\tilde{w}_0(V_0\tilde{w}_0-\tilde{V}_0w_0),$$
$$\tilde{V}_c = -\frac{3}{2}\ell^2w_0(\tilde{V}_0w_0-V_0\tilde{w}_0), \tag{20}$$

where w_0, $\tilde{w}_0$, V_0, $\tilde{V}_0$ and w_2, $\tilde{w}_2$, q, $\tilde{q}$ are arbitrary parameters satisfying the constraint

$$3(w_2\tilde{w}_0-\tilde{w}_2w_0)+\ell^2(\tilde{q}V_0-q\tilde{V}_0) + 6\ell^2\kappa(V_0\tilde{w}_0-\tilde{V}_0w_0)(w_0^2+\tilde{w}_0^2-1)=0. \tag{21}$$

Thus, similar to the well known case of a $SO(3)$ gauge group [4], the generic non-Abelian configurations have a nonvanishing magnetic field on the AdS boundary (*i.e.* $F_{\mu\nu}F^{\mu\nu}|_{r\to\infty}\neq 0$). As a result, one can see from the above relations that the mass function $m(r)$, and hence also the action of these solutions, diverge logarithmically.[7]

However, despite the divergence of the mass, the spacetime is still asymptotically AdS, the large r behavior of the metric function $N(r)$ being $N(r)\to r^2/\ell^2+1$. Asymptotically AdS solutions with diverging mass have been considered recently by some authors, mainly for a scalar field in the bulk (see e.g. [30]). In this case it might be possible to relax the standard asymptotic conditions without losing the original symmetries, but modifying the charges in order to take into account the presence of matter fields. In Sec. V of this work we shall argue that this is also the case of the EYMCS configurations with the general asymptotics (19). By using a counterterm approach, one can define a mass for these solutions, which is fixed by the parameter M_0 appearing in (19). (Note that for generic solutions not only $m(r)$ diverges logarithmically as $r\to\infty$ but also the terms $r^3V'(r)$ and $r^3\tilde{V}'(r)$ which, as argued in Sec. IV, fixes the electric charge(s) of the solutions. In the numerics, we have studied mainly the solutions with $V_c=\tilde{V}_c=0$.)

For the *special* configurations, which support finite mass, the required asymptotic behavior at large r is $|\phi_0^a|\to 1$ and $|\vec{\phi}\times\vec{\chi}|\to 0$ (*i.e.* $w_0^2+\tilde{w}_0^2\to 1$ and $w_0\tilde{V}_0-\tilde{w}_0V_0\to 0$). These conditions can be satisfied *only* in the presence of the CS term. Different from the case of a simple EYM theory [4], the existence of finite mass configurations here is not forbidden by the Derrick-type scaling argument. Indeed, the numerics in the following section indicate the existence of a subset solutions with finite mass, with the following expansion at infinity

$$w(r)=\sin\alpha+\frac{w_2}{r^2}+O(1/r^4),$$
$$\tilde{w}(r)=\cos\alpha+\frac{\tilde{w}_2}{r^2}+O(1/r^4),$$
$$V(r)=\Phi\sin\alpha+\frac{q}{r^2}+O(1/r^4),$$
$$\tilde{V}(r)=\Phi\cos\alpha+\frac{\tilde{q}}{r^2}+O(1/r^4),$$
$$m(r)=M-\frac{\alpha^2(q^2+\tilde{q}^2)\ell^2+3(w_2^2+\tilde{w}_2^2))}{r^2\ell^2}+O(1/r^4),$$
$$\sigma(r)=1-\frac{\alpha^2(w_2^2+\tilde{w}_2^2)}{r^6}+O(1/r^8), \tag{22}$$

where the amplitude of the electric potential at infinity is fixed by

$$\Phi=\frac{3(w_2\cos\alpha-\tilde{w}_2\sin\alpha)}{\ell^2(q\cos\alpha-\tilde{q}\sin\alpha)}. \tag{23}$$

Thus the free parameters in the far field expansion of this special set of solutions are M_0, $\alpha=\arctan(V(\infty)/\tilde{V}(\infty))$ and the coefficients q, $\tilde{q}$, w_2, $\tilde{w}_2$ of the $1/r^2$ decaying terms in the non-Abelian potentials.

D. Particular cases

The simplest solution of the field equations has pure gauge fields ($F_{\mu\nu}=0$) and corresponds to the Schwarzschild-AdS$_5$ black hole

$$N(r)=1+\frac{r^2}{\ell^2}-\frac{M}{r^2},\qquad \sigma(r)=1,\qquad w(r)=\sin\alpha,$$
$$\tilde{w}(r)=\cos\alpha,\qquad V(r)=\Phi\sin\alpha,\qquad \tilde{V}(r)=\Phi\cos\alpha, \tag{24}$$

where Φ, α are arbitrary constant.

The embedding of the RNAdS Abelian solution is recovered for

$$N(r)=1+\frac{r^2}{\ell^2}-\frac{M}{r^2}+\frac{\alpha^2q^2}{r^4},\qquad \sigma(r)=1,$$
$$\tilde{w}(r)=\tilde{V}(r)=0,\qquad w(r)=\pm 1,\qquad V(r)=\Phi+\frac{q}{r^2}. \tag{25}$$

The only AdS exact solutions with nontrivial non-Abelian fields known so far is:

$$N(r)=1+\frac{r^2}{\ell^2}-\frac{M_0+\frac{3\alpha^2}{2}\log r}{r^2},\qquad \sigma(r)=1,$$
$$w(r)=\tilde{w}(r)=0,\qquad V(r)=\tilde{V}(r)=0, \tag{26}$$

(with M_0 an arbitrary positive constant). This solution was obtained in [4] and describes a Reissner-Nordström type

[7] The existence of a logarithmic divergence in the action is a known property of some classes of AdS$_5$ solutions which are endowed with a special boundary geometry [27]. The coefficients of the divergent terms there are related to the conformal Weyl anomaly in the dual theory [28,29]. However, this is not the case for the non-Abelian AdS$_5$ configurations here, which have the same boundary metric as the Schwarzschild-AdS solution and hence feature no Weyl anomaly in the dual CFT.

geometry in EYM theory with a gauge group $SO(3)$ (note that the mass function is logarithmically divergent in this case). Its embedding in the $SO(6)$ gauged supergravity model and the extremal limit has been discussed in the recent work [11].

A particularly interesting model is found by taking $\tilde{w}(r) = \tilde{V}(r) = 0$ (or equivalently $w(r) = V(r) = 0$). This is our *particular truncation* of the full $SO(6)$ model. The resulting solutions are those of the $SU(2) \times U(1)$ truncation of the model[8] parametrized in terms of the representations of the algebra of $SU(4)$ instead of $SO(6)$. (Of course in that case both gauge groups have the same gauge coupling constant.) One can see that the CS term is still nontrivial in this case

$$\begin{aligned}\mathcal{L}_{\mathrm{CS}} &= i\kappa V(r)\varepsilon_{a_1a_2a_3a_4}\,\mathrm{Tr}\{F^{a_1a_2}F^{a_3a_4}\}\\ &= 12\kappa V(r)w'(r)(w^2(r)-1),\end{aligned} \tag{27}$$

(with $a_i = 1, \ldots 4$). As we shall argue in the next section, the solutions of this particular truncation contain already the basic features of the full model. However, they are much easier to study numerically.

The YM-CS equations in this case admit the first integral

$$V'(r) = \frac{\sigma}{r^3}(K + 4\kappa w(w^2-3)), \tag{28}$$

with K being an integration constant. One can easily see that the solutions are regular at the origin, $r = 0$, only if $K = 8\kappa$. The value of K is not fixed *a priori* for black hole solutions.

The asymptotics of the $SU(2) \times U(1)$ solutions can easily be read from the general relations (16), (19), and (22). At infinity, the finite mass solutions have $\alpha = \pm\pi/2$ in (22), with the gauge potentials

$$\begin{aligned}V(r) &= \Phi - \frac{8\kappa \mp K}{2r^2} + O(1/r^4),\\ w(r) &= \pm 1 + \frac{w_2}{r^2} + O(1/r^4).\end{aligned} \tag{29}$$

Thus, for the case $w(\infty) = -1$ studied in this work, the parameter q fixing the Abelian electric charge of these solutions is $q = -(4\kappa + K/2)$ for black holes and $q = -8\kappa$ for globally regular solutions.

III. NUMERICAL RESULTS

We start by noticing that the Eqs. (14) are not affected by the transformation:

$$\begin{aligned}&r \to \lambda r, \qquad m \to \lambda^2 m, \qquad \ell \to \lambda\ell, \qquad V \to V/\lambda,\\ &\tilde{V} \to \tilde{V}/\lambda, \qquad \alpha \to \lambda\alpha, \qquad \kappa \to \lambda\kappa,\end{aligned} \tag{30}$$

while w, $\tilde{w}$ and σ remain unchanged. It follows that one can always take an arbitrary positive value for α. The usual choice is $\alpha = 1$, which fixes the EYM length scale $L = \sqrt{8\pi G/(3e^2)}$, while the mass scale is fixed by $\mathcal{M} = 8\pi/(3e^2)$. All other quantities get multiplied with suitable factors of L. However, in this Section, to avoid cluttering our expressions with a complicated dependence on (G, e), we fix the value of α at $\alpha = 1$, and ignore the extra factors of e and G in the expressions of various global quantities.

Therefore the remaining input parameters are the AdS length scale ℓ and the CS coupling constant κ. Determining the pattern of the solutions in the parameter space represents a very complex task which is outside the scope of this paper. Instead, we analyzed in detail a few particular classes of solutions which, hopefully, reflect all relevant properties of the general pattern. For definiteness we set $\ell = 1$ in our numerical analysis, although we have found nontrivial solutions also for other values of the cosmological constant.[9]

Instead we have looked for the dependence of the solutions on the value of the CS coefficient κ, which has not been fixed *a priori*. (This investigation has been partly motivated by the study in [32] of the Einstein-Maxwell-CS system, which revealed a nontrivial dependence of the properties of the solutions on the value of κ. We shall see that this is also the case for the solutions constructed in this work, which feature a critical value of CS coefficient.)

The resulting set of six ordinary differential equations[10] is solved with suitable boundary conditions which result from (16), (17), (19), and (22). The numerics employs a collocation method for boundary-value ordinary differential equations equipped with an adaptive mesh selection procedure [33]. Typical mesh sizes include 10^3–10^4 points. The solutions have a relative accuracy of 10^{-7}. In addition to employing this algorithm, some solutions were also constructed by using a standard Runge-Kutta ordinary differential equation solver. In this approach we evaluate the initial conditions at $r = 10^{-5}$ (or $r = r_h + 10^{-5}$), for global tolerance 10^{-12}, adjusting for shooting parameters and integrating towards $r \to \infty$. We have confirmed that there is good agreement between the results obtained with these two different methods.

[8]This truncation of the full $SO(6)$ model shares a number of common features with the five-dimensional $N = 4$ gauged $SU(2) \times U(1)$ supergravity model considered in [31]. For example, a first integral similar to (28) appears there also. However, the solutions in [31] have an extra dilaton field with a Liouville potential and thus are not asymptotically AdS.

[9]In particular, the finite energy solutions survive in the limit $\Lambda \to 0$, being supported by the CS term (this contrasts strongly with the case of a pure EYM theory [12]). A discussion of the asymptotically flat EYMCS solutions will be presented elsewhere.

[10]Although we have solved the second order YM equations in (14), we have also monitored the constraint (15), which was always satisfied with very good accuracy. Also, the Eq. (15) has been used to construct the asymptotic expansions (16), (17), (19), and (22).

The properties of the solutions depend on the input parameters, but it is rather difficult to find a general pattern. However, a feature shared by all asymptotically AdS solutions is that the metric functions $m(r)$, $\sigma(r)$ monotonically approach their asymptotic values, which can easily be seen from the corresponding field equations. Also, so far we could not find solutions where the electric potentials $V(r)$, $\tilde{V}(r)$ present oscillations, even though such solutions are allowed.

A. The generic solutions

The generic solutions studied here are for the $d = 4 + 1$ dimensional model, which features a CSterm. But solutions with similar properties are found also in the EYM model with no CS term. All these solutions bear a qualitative similarity to those of the more familiar $d = 3 + 1$ EYM model [1,2]. These solutions can also be seen as higher gauge group generalizations of the EYM solutions in [4], but unlike the latter they feature a nontrivial electric potential, made possible by the larger gauge group. (The electric potential necessarily vanishes for a $d = 5$ static, spherically symmetric $SU(2)$ gauge field).

Considering first the case of globally regular configurations, one finds that solutions approaching asymptotically the AdS_5 background exist for compact intervals of the initial parameters $w''(0)$, $V(0)$, $V'(0)$ and $\sigma(0)$. The values of the parameters w_0, $\tilde{w}_0$, V_0, $\tilde{V}_0$, w_2, $\tilde{w}_2$, and q, which enter the asymptotics of the solutions are fixed by the numerics. (There are also branches of solutions with a different asymptotic behavior, which stop to exist for finite values of r. To study these configurations, one needs to employ a metric Ansatz different from (6). Such solutions, being not asymptotically AdS, are of no interest here.)

Of the full set of solutions with AdS asymptotics, we have paid special attention to the physically more interesting case of configurations with a $1/r^2$ decay of the electric potentials at infinity (*i.e.* $V_c = \tilde{V}_c = 0$ and $\tilde{V}_0 w_0 - V_0 \tilde{w}_0 = 0$), this being the only case reported in this Section. These solutions have a finite electric charge, although their mass functions will diverge asymptotically since $|\vec{\phi}| = \sqrt{w_0^2 + \tilde{w}_0^2} \neq 1$ here.

A typical configuration with a regular origin is presented in Fig. 1 (left), for $\kappa = 1$. One can see that the mass function diverges logarithmically while $\sigma(r)$, $w(r)$, $\tilde{w}(r)$ and $V(r)$, $\tilde{V}(r)$ asymptotically approach some finite values. Solutions with nodes in $w(r)$, $\tilde{w}(r)$ were also found.

In Fig. 2 we plot a number of relevant parameters as a function of the coefficient b in the initial data at $r = 0$ ($b = -w''(0)/2$), for a family of asymptotically AdS solutions. (One of the parameters there is M_0 appearing in (19), which in Sec. V we argue that it can be taken as the renormalized mass of the solutions; note that M_0 may take also negative values). This branch ends for some finite values of b, where $-g_{tt}(0) = \sigma(0) \to 0$ while V_0, q diverge. The condition $|\vec{\phi} \times \vec{\chi}| \to 0$ as $r \to \infty$ has been enforced by treating $V(0)$ as a shooting parameter. Then $\tilde{V}'(0)$ is a free parameter while $\sigma(0)$ results from the numerics (the solutions in Fig. 2 have $V'(0)/\sigma_0 = 0.15$).

The results in Fig. 2 show that the for generic solutions $w^2(\infty) + \tilde{w}^2(\infty) \neq 1$. From (19), this leads to a divergent mass-energy as defined in the usual way. However, one can see that the condition $|\vec{\phi}| \to 1$ is satisfied for a discrete set of the parameter b (*e.g.* $b \simeq 0.3219 \times 10^{-3}$ and $b \simeq 0.6125 \times 10^{-5}$ for the data in Fig. 2). This suggests the existence of several branches of finite mass solutions parametrized by $\tilde{V}'(0)$ (or, equivalently, $V(0)$), which is confirmed by the results in the next subsection.

Black hole solutions have been found as well, presenting the same general features. Here also we find a continuum

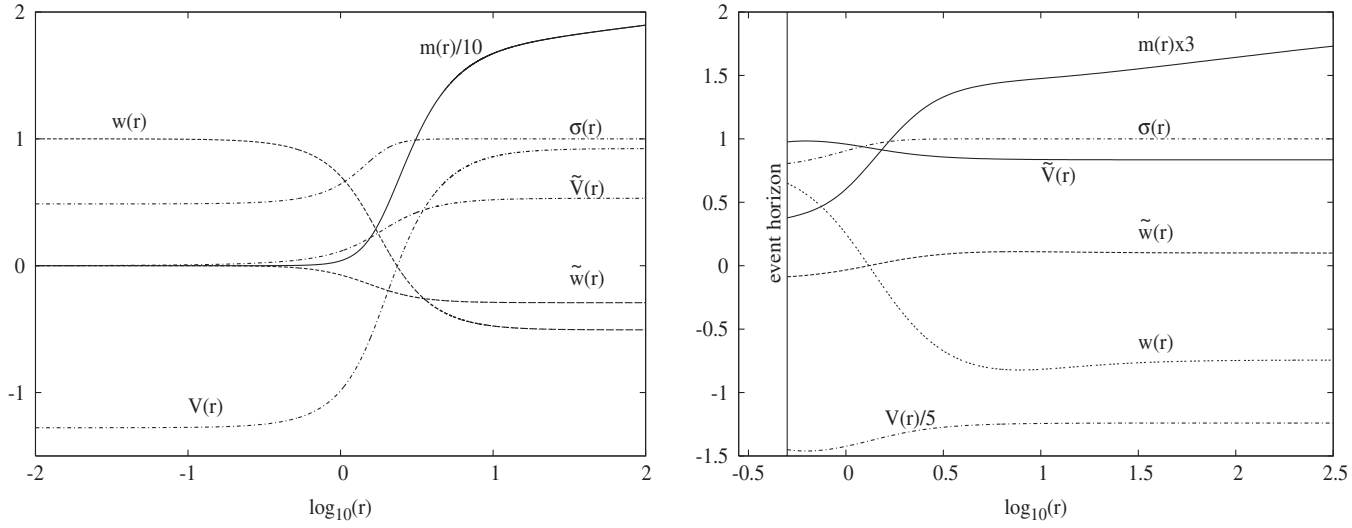

FIG. 1. The profiles of a typical globally regular solution (left) and a black hole solution (right) of the EYMCS equations are presented as a function of the radial coordinate r. For these solutions, the mass function $m(r)$ diverges logarithmically as $r \to \infty$.

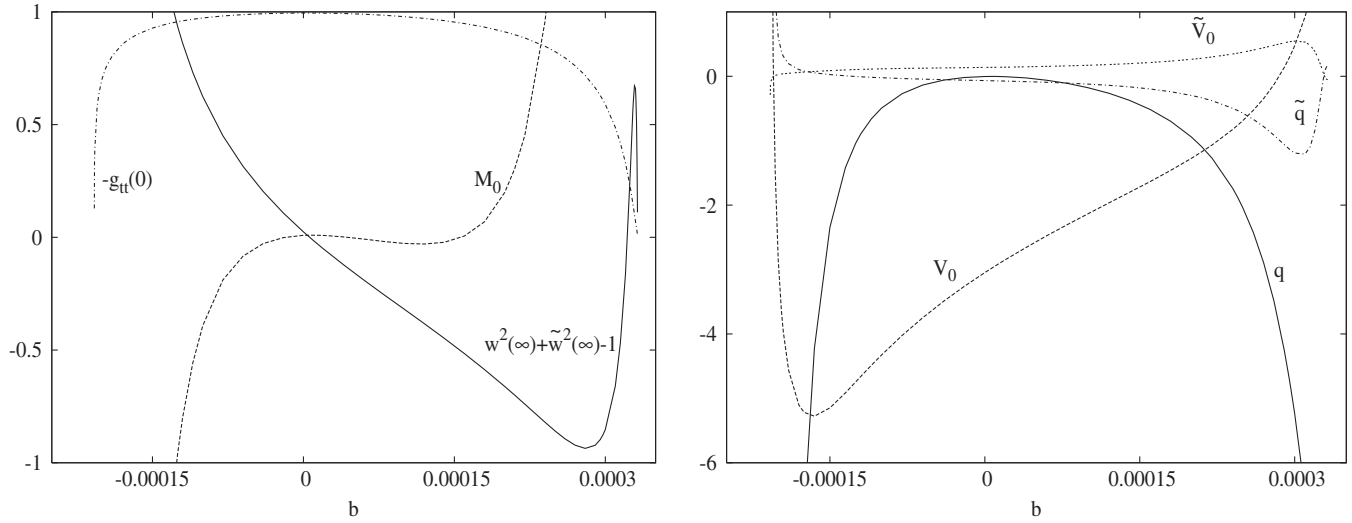

FIG. 2. A number of relevant parameters are plotted as functions of the coefficient b for globally regular solutions of the EYMCS model with $\ell = 1$, $\kappa = 1$. These solutions have finite electric charges, which are fixed by q and $\tilde{q}$.

of solutions with arbitrary values of gauge potentials at infinity, the relevant parameters being the values of the gauge potentials at the event horizon as given by (17). Again, finite mass black holes are found only for specific values of the gauge potentials on the horizon.

As a general remark, we note that the presence of the CS term is not crucial for the existence of the generic solutions (*i.e.* with a divergent mass). We have found solutions with rather similar properties also for $\kappa = 0$. Thus, the role of the CS term is indispensable only for the construction of *special*, finite mass, solutions to be presented in the next subsection.

B. Finite mass solutions

1. Regular configurations

In the numerics, special attention has been paid to solutions with a finite mass. As noted above, in this case, three of the four parameters in the data at $r = 0$ are fixed. The remaining free parameter was chosen to be $V(0)$ (similar results are found when $\tilde{V}(0)$ is chosen instead).

Remembering the invariance under the parity reflection $\vec{\phi} \to -\vec{\phi}$, $\vec{\chi} \to -\vec{\chi}$ it turns out to be sufficient to consider $V(0) \geq 0$ (or $\tilde{V}(0) \geq 0$).

Thus, corresponding to a choice of the coupling constant κ and of the cosmological constant Λ, there exist in principle a family of finite mass charged EYMCS solutions labeled by the value at the origin of one of the electric potentials, in this case, $V(0)$. The pattern of these solutions turns out to be extremely rich, with some unexpected features.

In order to illustrate this, we first fix $V(0) = 0.3$ and study the solutions as functions of the CS parameter κ. (Qualitatively, the same results have been found when considering other values of $V(0)$). The numerical results show that a branch of solutions with $\tilde{w}(r) = \tilde{V}(r) = 0$ always exists for sufficiently large values of κ, for instance, $\kappa \geq \kappa_0 \simeq 0.07$ in the present case. These are the solutions of the reduced $SU(2) \times U(1)$ model with the asymptotic angle $\alpha = \pi/2$. For convenience, we will refer to this branch as the *main branch*. In the limit $\kappa \to \kappa_0$ the metric function $\sigma(r)$ vanishes at the origin and the solution becomes singular. For all values of the other parameters, no finite mass solutions have been found for $\kappa < \kappa_0$.

The interesting feature is that new branches of solutions with nontrivial functions $\tilde{w}(r)$, $\tilde{V}(r)$ emerge from the main branch at critical values of κ. In our cases, the first branch of excited solution appears $\kappa \simeq 0.0953$ and a second branch at $\kappa \simeq 0.1875$. This behavior is illustrated in Fig. 3, where a number of relevant global parameters are plotted a functions of the CS coupling constant κ. It should be noted that, for a fixed κ, the excited solutions have larger masses than the solutions on the *main branch*.

The profiles of the two first excited solutions corresponding to $\kappa = 0.2$ is presented in Fig. 4. One can see that the functions w, $\tilde{w}$ parametrizing the magnetic field of the excited solutions develop more pronounced oscillations before becoming constant in the asymptotic region.

It is also natural to study the spectrum of solutions in terms of one of the charges, say $\tilde{q}$ for a fixed value of κ. The second electric charge q is determined from numerics. The results of our analysis for the value $\kappa = 0.2$ are summarized in Fig. 5. In this case, two excited solutions are available. Several relevant parameters are plotted there *versus* $\tilde{q}$ (see Eqn. (22)) for two excited solutions. Fixing the parity symmetry by means of $\tilde{V}'(0) \geq 0$, the numerical analysis reveals that the solutions of the branch "1" (respectively "2") are characterized by negative (respectively positive) values of $\tilde{q}$. Also, they exist up to a minimal (respectively maximal) value, say $\tilde{q} = \tilde{q}_{cr}$. For $\kappa = 0.2$

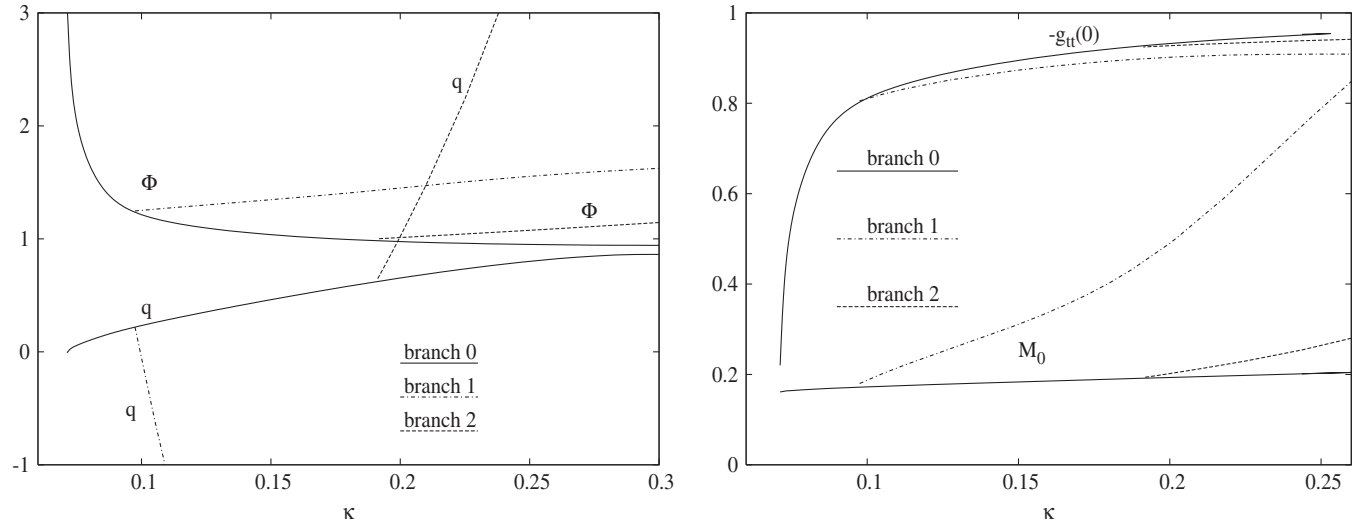

FIG. 3. A number of relevant parameters are plotted as functions of the Chern-Simons coupling κ for finite mass, globally regular solutions of the EYMCS model. One can see that new branches of solutions emerge as κ increases.

we have $\tilde{q}_{cr} \approx -7.8$ and $\tilde{q}_{cr} \approx 2.5$ respectively, for the first and second branches. In the limit $\tilde{q}_{cr} \to 0$, the excited solutions converge to the main solution. The ending of the branches at $\tilde{q} = \tilde{q}_{cr}$ is more subtle. Indeed, our numerical results show that another family of solutions (with larger mass) exists in the region $|\tilde{q}| < |\tilde{q}_{cr}|$, backbending from the branch coming directly from the main solution. These new branches are shown on Fig. 5; however, we have not attempted to construct further branches in this region, although they are likely exist.

2. Black holes

The EYMCS system presents also black hole solutions which were constructed using similar techniques. In contrast with the regular solutions presented above, the main thrust here is confined to the solutions of our *particular truncation* of the full $SO(6)$ model. This restriction is made to simplify an otherwise very complex numerical task.

The Hawking temperature and the entropy of the black holes are given by

$$T_H = \frac{\sigma(r_h)N'(r_h)}{4\pi}, \qquad S = \frac{A_H}{4G}, \quad \text{with} \quad A_H = V_3 r_h^3, \tag{31}$$

where $V_3 = 2\pi^2$ is the area of S^3. An interesting feature here is that the finite mass black hole solutions have two free parameters in the event horizon initial data, which were taken to be $V(r_h)$ and $\tilde{V}(r_h)$. As a result, and in

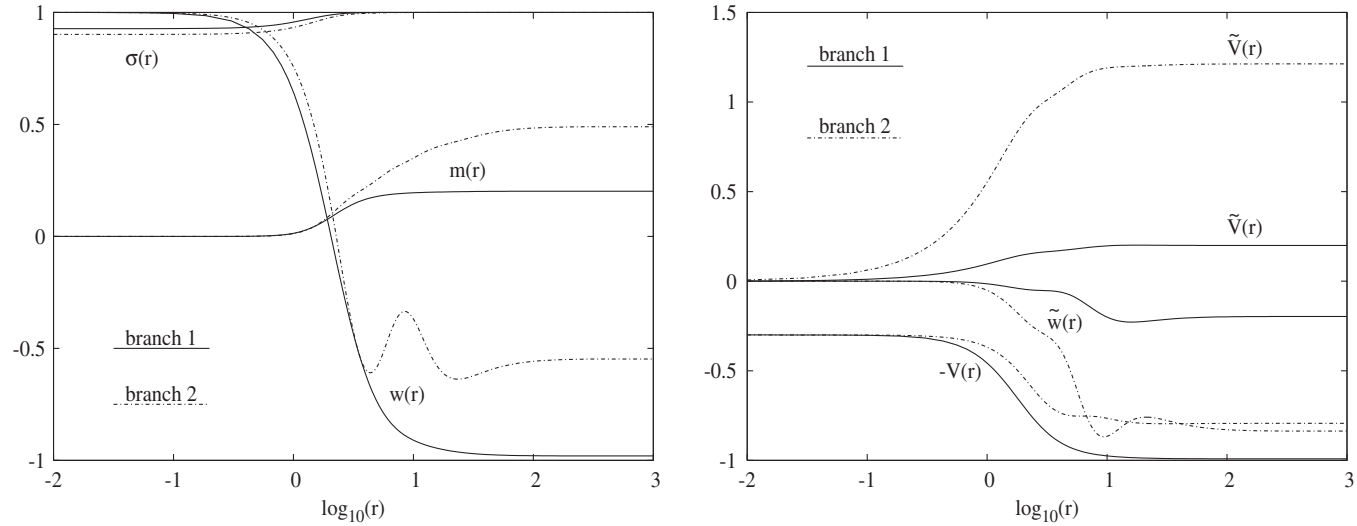

FIG. 4. The profiles of two typical globally regular EYMS solutions are presented as function of the radial coordinate r. $m(r)$ and $\sigma(r)$ are metric functions, which $w(r)$, $\tilde{w}(r)$ and $V(r)$, $\tilde{V}(r)$ are non-Abelian potentials.

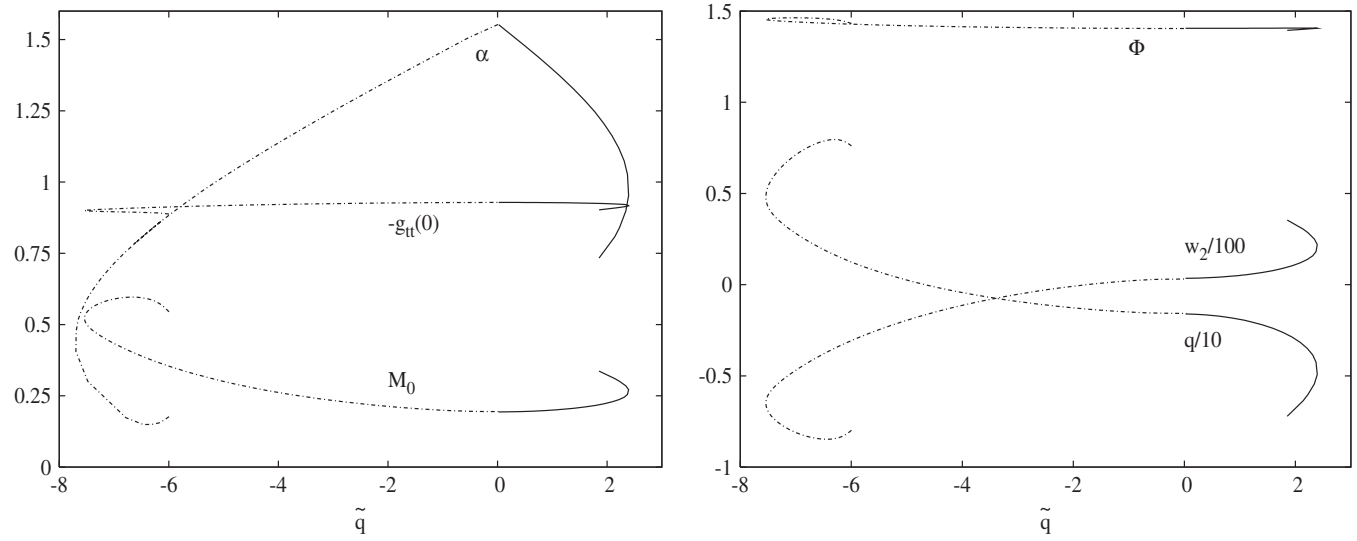

FIG. 5. A number of relevant parameters are plotted as functions of the electric charge $\tilde{q}$ for finite mass, globally regular solutions of the EYMCS model with a Chern-Simons coefficient $\kappa = 0.2$. The dashed and solid curves denote different branches of solutions.

contrast to the case of globally regular solutions, the two electric charges q and $\tilde{q}$ are independent for black holes. This leads to a much richer parameter space of solutions.

Our numerical results provide evidence for the existence of finite mass black hole solutions of the EYMCS system with a set of four nontrivial gauge functions (*i.e.* for the full group $SO(6)$). The profile of a generic black hole corresponding to $\kappa = 0.2$ and $r_h = 0.5$ is presented in Fig. 6 (left) for $\tilde{q} = 0.5$, $q = -1.5$.

However, given the large number of free input parameters, we did not attempt a systematic study of these solutions, concentrating instead on the simpler case of the $SU(2) \times U(1)$ truncated model. An interesting feature here is that solutions with AdS asymptotics exist only for the limited interval $\kappa_{\min} \leq \kappa \leq \kappa_{\max}$ of κ. The limits of this interval depend on the values of r_h and K. Clearly, the range of the electric charge of these solutions is also bounded. These features are illustrated in Fig. 7 (left), for black hole solutions with a fixed horizon radius $r_h = 1$. One can see that as $\kappa \to \kappa_{\min}$ the value at the horizon of metric function $\sigma(r)$ tends to zero, as does also the Hawking temperature, while other quantities stay finite. The behavior of solutions for large κ is less clear, the accuracy decreasing with κ. However, the numerical results seem to indicate that this branch ends in a critical solution with $\sigma(r_h)$ close to 1 and a finite nonzero value of T_H. Unfortunately, the study of solutions for $\kappa \to \kappa_{\max}$ is a difficult task and the general picture may be much more

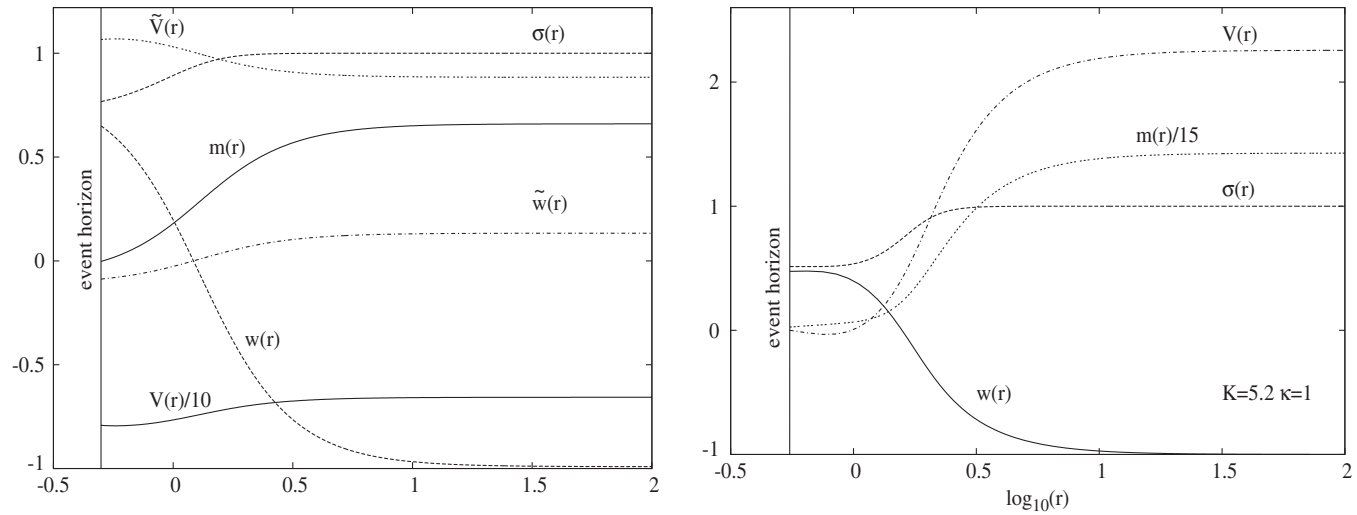

FIG. 6. *Left:* The profiles of a typical non extremal black hole solution of the $SO(6)$ model is presented as a function of the radial coordinate r. *Right:* An extremal black hole solution for our *particular truncation* of the full model.

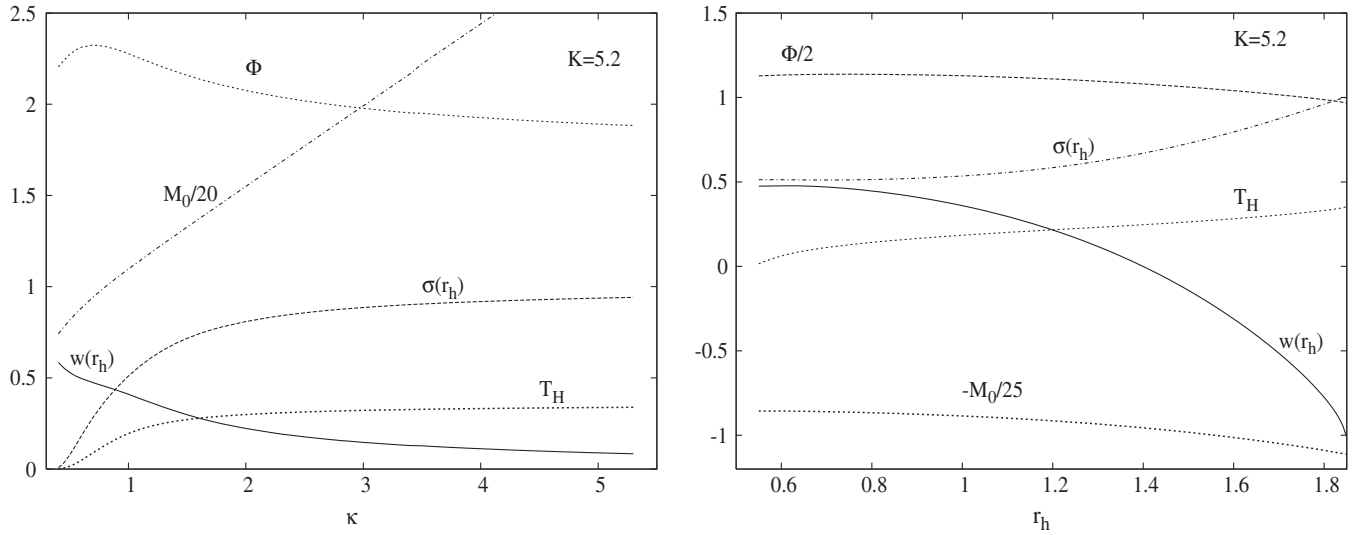

FIG. 7. The mass parameter M_0, the value at the horizon of the metric function $\sigma(r)$ and the magnetic gauge potential $w(r)$, the electrostatic potential Φ and the Hawking temperature are plot as a functions of the CS coupling constant κ (left) and of the event horizon radius r_h (right).

complicated. For example, we have noticed the existence there of a secondary branch of solutions, which are close to a finite mass extremal configuration with nontrivial gauge fields. A systematic study of these aspects would require a different parametrization of the metric line element than (6), and is beyond the scope of this work.

One can also keep κ and K fixed and vary the value of the event horizon radius. As noted above, choosing the values of κ, K fixes also the electric charge, *i.e.*, these black holes are in a canonical ensemble. Our numerics indicate that for any K, the value of the gauge field potential at the horizon decreases with r_h. For large enough values of the event horizon radius, the solutions become essentially RNAdS black holes, with $w(r)$ being close to the value -1 everywhere, with the non-Abelian magnetic field vanishing.

However, the picture for small enough values of the horizon radius depends crucially on the value of the integration constant K in the V-Eq. (29) (*i.e.* on the Abelian electric charge). Starting with the special value $K = 8\kappa$, we plot in Fig. 8 a number of relevant features of the solutions, for three different values of the CS coefficient κ. One can see that these non-Abelian black holes behave in a similar way to the vacuum AdS solutions. The small black holes are thermally unstable, the entropy being a decreasing function of the Hawking temperature. They become thermally stable for large enough values of the event horizon radius. In the limit $r_h \to 0$, these black holes approach the set of globally regular particlelike solutions with $\tilde{V}(r) = \tilde{w}(r) = 0$, discussed above.

The picture is very different when choosing instead $K \neq 8\kappa$ [see Fig. 7 (right)]. As an interesting new feature, here an extremal black hole solution is approached for a critical value of r_h. In this case the horizon is degenerate (*i.e.*, $N(r)$ has a double root: $N(r_h) = N'(r_h) = 0$) and the near horizon geometry is $\mathrm{AdS}_2 \times S^3$.

As $r \to r_h$ one finds the approximate form of the solution in the near horizon region:

$$N(r) = N_2(r - r_h)^2 + O(r - r_h)^3,$$
$$\sigma(r) = \sigma_h - \frac{3\sigma_h w_1^2}{2r_h}(r - r_h) + O(r - r_h)^2, \tag{32}$$

$$w(r) = w_h + w_1(r - r_h) + O(r - r_h)^2,$$
$$V(r) = V_h - \frac{\sigma_h w_h}{2\kappa r_h}(r - r_h) + O(r - r_h)^2, \tag{33}$$

where

$$w_1 = \frac{32\kappa^2 r_h \ell^2 w_h (1 - w_h^2)}{-3\alpha^2 r_h^2 \ell^2 w_h^2 + 384\kappa^4 \ell^2 (1 - w_h^2)^2 - 4\kappa^2 (24 r_h^4 - 4 r_h^2 \ell^2 (w_h^2 - 2) + 3\alpha^2 \ell^2 (1 - w_h^2)^2)}$$
$$N_2 = \frac{24 r_h^4 + 8 r_h^2 \ell^2 - 3\alpha^2 \ell^2 (1 - w_h^2)^2)}{2 r_h^4 \ell^2}. \tag{34}$$

The parameters r_h and w_h in the above relation are solutions of the equations

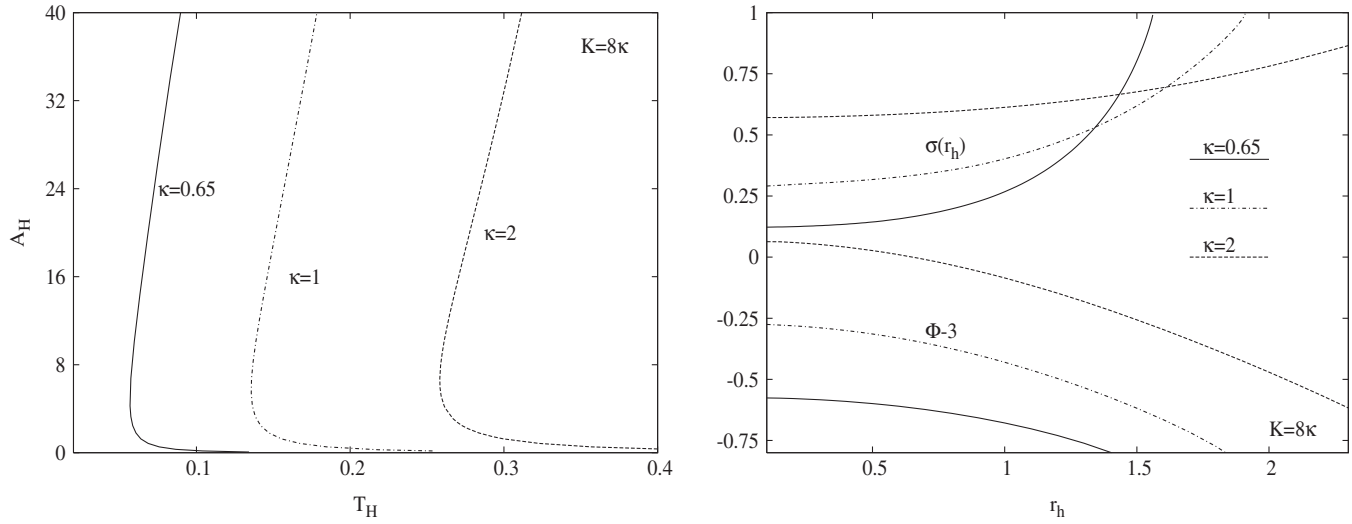

FIG. 8. The temperature-entropy diagram (left) and the value at the horizon of the metric function $\sigma(r)$ and of the electrostatic potential Φ (right) are ploted as functions of the event horizon radius r_h for three different values of the Chern-Simons coupling constant κ. These configurations have an integration constant $K = 8\kappa$, and approach the globally regular particlelike solutions as $r_h \to 0$.

$$\frac{32r_h^4}{\ell^2} - 12\alpha^2(1 - w_h^2)^2 + r_h^2\left(16 - \frac{\alpha^2 w_h^2}{\kappa^2}\right) = 0,$$
$$2K\kappa + w_h(r_h^2 + 8\kappa^2(w_h^2 - 3)) = 0. \tag{35}$$

Recalling that $K = 2(q - 4\kappa)$, it follows that all event horizon boundary data (except $\sigma(r_h)$) are fixed by the κ, ℓ and the electric charge q. (Note the analogy with the extremal Abelian solution case.) This extremal solution differs from the RNAdS one, presenting non-Abelian magnetic hair and a nontrivial metric function $\sigma(r)$ [see Fig. 6 (right)].

As expected, the near horizon structure of the extremal solutions can be extended to a full $AdS_2 \times S^3$ solution of the field equations. This configuration has a line element

$$ds^2 = \frac{dr^2}{1 - \frac{\Lambda_1 r^2}{6}} + r_0^2 d\Omega_3^2 - \left(1 - \frac{\Lambda_1 r^2}{6}\right)dt^2, \tag{36}$$

and the matter fields

$$w = w_0, \qquad V(r) = V_0 - \frac{w_0}{2\kappa r_0}(r - r_h). \tag{37}$$

For $w_0^2 \neq 1$, this is a non-Abelian solution, with the gauge field living on the three-sphere. The parameters Λ, w_0 and the radius r_0 of the S^3 are constrained by the relation

$$\Lambda = \frac{3}{r_0^2} - \frac{\pi G}{e^2}\frac{r_0^2 w_0^2 + 12\kappa^2(w_0^2 - 1)^2}{\kappa^2 r_0^4}, \tag{38}$$

the value of the AdS_2 cosmological constant Λ_1 in (36) being

$$\Lambda_1 = 6\left(\Lambda - \frac{1}{r_0^2} - \frac{\pi G}{e^2}\frac{r_0^2 w_0^2 + 4\kappa^2(1 - w_0^2)^2}{\kappa^2 r_0^4}\right). \tag{39}$$

IV. GLOBAL CHARGES

A. The mass and boundary stress tensor

The action and mass of these AdS_5 non-Abelian configurations is computed by using a boundary counterterm prescription. As found in [20], the following counterterms are sufficient to cancel divergences in five dimensions, for Schwarzschild-AdS black hole solution (in this section we restore the $8\pi G$ and e factors in the expressions):

$$I_{\rm ct} = -\frac{1}{8\pi G}\int_{\partial\mathcal{M}} d^4x\sqrt{-h}\left[\frac{3}{\ell} + \frac{\ell}{4}\mathrm{R}\right], \tag{40}$$

with R the Ricci scalar for the boundary metric h.

(Note also that, as usual, to ensure well-defined Euler-Lagrange field equations, one adds to the action (1), the Gibbons-Hawking surface term [34] $I_{\rm surf} = -\frac{1}{8\pi G}\times \int_{\partial\mathcal{M}} d^4x\sqrt{-h}K$, where K is the trace of the extrinsic curvature for the boundary $\partial\mathcal{M}$.) However, in the presence of matter fields, additional counterterms may be needed to regulate the action [35], which is also the case for the generic non-Abelian solutions discussed in the previous sections.[11]

This divergence is cancelled by a supplementary counterterm of the form (with a, b boundary indices):

$$I_{\rm ct}^{\rm YM} = -\log\left(\frac{r}{\ell}\right)\int_{\partial\mathcal{M}} d^4x\sqrt{-h}\,\frac{\ell}{2e^2}\,\mathrm{Tr}\{F_{ab}F^{ab}\}. \tag{41}$$

[11]The geometric counterterm (40) regularises also the action of the RNAdS$_5$ black hole solution. However, this does not hold for any $d = 5$ solutions of the Einstein-Maxwell-Λ system. An interesting example here are the AdS black strings with a magnetic $U(1)$ field [36], in which case one has to consider an additional matter counterterm on the form (41).

Note that this term is identically zero for the solutions with $w^2 + \tilde{w}^2 \to 1$, $\tilde{V}w - V\tilde{w} \to 0$.

Using these counterterms and the Gibbons-Hawking boundary term, one can construct a divergence-free boundary stress tensor T_{ab}

$$\mathrm{T}_{ab} = \frac{1}{8\pi G}\left(K_{ab} - Kh_{ab} - \frac{3}{\ell}h_{ab} + \frac{\ell}{2}E_{ab}\right) - \frac{2\ell}{e^2} \times \log\left(\frac{r}{\ell}\right)\mathrm{Tr}\left\{F_{ac}F_{bd}h^{cd} - \frac{1}{4}h_{ab}F_{cd}F^{cd}\right\}, \quad (42)$$

where E_{ab} and K are the Einstein tensor and the trace of the extrinsic curvature K_{ab} for the induced metric of the boundary, respectively. In this approach, the mass M of the solutions is the conserved charge associated with the Killing vector $\partial/\partial t$ [20]. A straightforward computation leads to the following simple result for the mass of the generic EYMCS solutions:

$$\mathrm{M} = \frac{3V_3M_0}{16\pi G} + M_c, \quad \text{with} \quad M_c = \frac{3V_3\ell^2}{64\pi G}. \quad (43)$$

For the case of black hole solutions of the truncated $SU(2) \times U(1)$ model, we have found that M coincides within the numerical accuracy with the mass computed from the first law of thermodynamics, up to the constant term M_c which is usually interpreted as the mass of the pure global AdS_5.

From the AdS/CFT correspondence, we expect the non-Abelian hairy black holes to be described by some thermal states in a dual theory formulated in a metric background given by

$$\gamma_{ab}dx^a dx^b = -dt^2 + \ell^2(d\psi^2 + \sin^2\psi(d\theta^2 + \sin^2\theta d\varphi^2)), \quad (44)$$

where ψ, θ, φ are the usual polar angles parametrizing S^3.

The matter fields in the dual CFT would interact with a background non-Abelian field, whose expression, as read from (9) and (19) is

$$\begin{aligned}
A_t &= \frac{1}{e}(\tilde{V}_0[\sin\psi\sin\theta(\Sigma_{16}\cos\varphi + \Sigma_{26}\sin\varphi) + \sin\psi\cos\theta\Sigma_{36} + \cos\psi\Sigma_{46}] - V_0\Sigma_{56}),\\
A_\psi &= \frac{1}{e}((1+w_0)[\sin\theta(\Sigma_{14}\cos\varphi + \Sigma_{24}\sin\varphi) + \cos\theta\Sigma_{34}] + \tilde{w}_0[\cos\psi\sin\theta(\Sigma_{15}\cos\varphi + \Sigma_{25}\sin\varphi)\\
&\quad + \cos\psi\cos\theta\Sigma_{35} - \sin\psi\Sigma_{45}]),\\
A_\theta &= \frac{1}{e}((1+w_0)\sin\psi[\sin\psi(\Sigma_{13}\cos\varphi + \Sigma_{23}\sin\varphi) - \sin\psi\cos\theta\Sigma_{14}\cos\varphi + \cos\psi\cos\theta\Sigma_{24}\sin\varphi + \cos\psi\sin\theta\Sigma_{34}]\\
&\quad + \tilde{w}_0\sin\psi[\cos\theta(\Sigma_{15}\cos\varphi + \Sigma_{25}\sin\varphi) - \sin\theta\Sigma_{35}]),\\
A_\varphi &= \frac{1}{e}(-(1+w_0)\sin\psi[\sin\psi\sin^2\theta\Sigma_{12} + \sin\psi\sin\theta\cos\theta(\Sigma_{13}\sin\varphi - \Sigma_{23}\cos\varphi) + \cos\psi\sin\theta(\Sigma_{14}\sin\varphi - \Sigma_{24}\cos\varphi)]\\
&\quad - \tilde{w}_0\sin\psi\sin\theta(\Sigma_{15}\sin\varphi - \Sigma_{25}\cos\varphi)),
\end{aligned} \quad (45)$$

We note that this is still fully an $SO(6)$ gauge field.

The expectation value $\langle\tau^a_b\rangle$ of the dual CFT stress tensor can be calculated using the relation [37]

$$\sqrt{-\gamma}\gamma^{ab}\langle\tau_{bc}\rangle = \lim_{r\to\infty}\sqrt{-h}\,h^{ab}\mathrm{T}_{bc}. \quad (46)$$

Employing also (42), we find the finite and covariantly conserved stress tensor (with $x^1 = \psi$, $x^2 = \theta$, $x^3 = \varphi$, $x^4 = t$)

$$8\pi G\langle\tau^a_b\rangle = \frac{1}{2\ell}\left(\frac{M_0}{\ell^2} + \frac{1}{4}\right)\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&-3\end{pmatrix} - \frac{4\pi G((w_0^2 + \tilde{w}_0^2 - 1)^2 + \ell^2(\tilde{V}_0w_0 - V_0\tilde{w}_0)^2)}{e^2\ell^3} \times \begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&0\end{pmatrix}. \quad (47)$$

Different *e.g.* from the case of Reissner-Nordström-AdS Abelian solutions, this stress tensor has a nonvanishing trace. Moreover, for the physically relevant case of solutions with a finite electric charge (*i.e.* $|\vec{\phi}\times\vec{\chi}| \to 0$ asymptotically) one finds that $\langle\tau^a_a\rangle = \mathcal{A}_{\mathrm{YM}} = -3(w_0^2 + \tilde{w}_0^2 - 1)^2/(2\ell^2e^2)$. This agrees with the general results [35,38,39] on the trace anomaly in the presence of an external gauge field, $\mathcal{A}_{\mathrm{YM}} = \mathcal{R}F^2_{(0)}$, the coefficient $\mathcal{R}$ being related to the charges of the fundamental constituent fields in the dual CFT.

B. Electric charge(s)

For the solutions with $|\vec{\phi}\times\vec{\chi}| \to 0$ (*i.e.* $(\tilde{V}_0w_0 - V_0\tilde{w}_0)|_{r\to\infty} = 0$) the coefficients of the $1/r^2$ terms in the asymptotic expansion of the electric potentials are finite. Thus, from the Gauss flux theorem one can formally define the electric charge

$$Q_E = \oint_\infty dS_k\sqrt{-g}F^{kt} = \frac{4\pi^2}{e}(q\Sigma_{56} + \tilde{q}\Sigma_{12}), \quad (48)$$

which is clearly not gauge invariant. (This is a generic problem for the definition of the non-Abelian charges in the absence of a Higgs field, see *e.g.* [40]).

Perhaps a more proper definition can be given following the reasoning in [41]. In this approach one starts by evaluating the quantity $\mathrm{Tr}\{F_{it}F^{it}\}$,

$$\begin{aligned}\sqrt{-g}\,\mathrm{Tr}\{F_{it}F^{it}\} &= \sqrt{-g}\,\mathrm{Tr}\{D_iA_tF^{it}\}\\ &= \partial_i(\sqrt{-g}\,\mathrm{Tr}\{A_tF^{it}\})\\ &\quad - \sqrt{-g}\,\mathrm{Tr}\{A_tD_iF^{it}\}.\end{aligned}$$

Using the Gauss' law equation, we find that the contribution of the electric field to the total mass then is

$$\begin{aligned}E_e = &-\frac{1}{e^2}\int dS_k\sqrt{-g}\vec{\chi}\cdot D_r\vec{\chi}\\ &+ 4V_3\frac{\kappa}{e^2}\int dr(|\vec{\phi}|^2-1)\vec{\chi}\cdot D_r\vec{\phi}g^{tt}.\end{aligned}\tag{49}$$

Subject to the truncations this expression simplifies; the integral in the second term must be evaluated using the numerical solution, while the surface integral in the first term can be evaluate using only the asymptotic values of the functions.

In the absence of the Chern-Simons term, *i.e.*, when $\kappa = 0$, the contribution of the electric field to the total mass can be written as

$$E_e = -\oint_\infty dS_k\sqrt{-g}\,\mathrm{Tr}\{A_tF^{kt}\} = \mathbf{Q_E}\mathbf{\Phi},$$

where

$$\begin{aligned}\mathbf{\Phi} &= \sqrt{\mathrm{Tr}\{A_tA_t\}} = \sqrt{V_0^2+\tilde{V}_0^2},\\ \mathbf{Q_E} &= \frac{4\pi^2}{e}\frac{V_0q+\tilde{V}_0\tilde{q}}{\sqrt{V^2+\tilde{V}^2}},\end{aligned}\tag{50}$$

are the electrostatic potential and the electric charge, respectively. However, one can extend this definition of $\mathbf{\Phi}$ and $\mathbf{Q_E}$ to solutions of the EYMCS system. This applies to both *generic* and *special* configurations (note that $\mathbf{\Phi} = \Phi$, $\mathbf{Q_E} = 4\pi^2(\sin\alpha q + \cos\alpha\tilde{q})/e$ for finite mass solutions).

For finite mass solutions, following [21], one can also define an effective non-Abelian charge Q_{eff} by the asymptotic behavior of the metric function $N(r)$, which, to order $1/r^4$ is similar to that of the RNAdS solution:

$$N(r) = 1 + \frac{r^2}{\ell^2} - \frac{M_0}{r^2} + \frac{Q_{\mathrm{eff}}^2}{r^4} + \ldots,\tag{51}$$

i.e.

$$Q_{\mathrm{eff}} = \frac{\alpha\sqrt{(q^2+\tilde{q}^2)\ell^2 + 3(w_2^2+\tilde{w}_2^2))}}{\ell}.\tag{52}$$

Concerning a definition of a "magnetic" flux, the only natural quantity we have at our disposal for this purpose is the Chern–Pontryagin density, which we know is the leading and sole contributing term to the topological charge ("magnetic" flux) of the monopole in 4 + 1 dimensions [42]. There however the gauge group is $SO(4)$ and the model features an iso-four-vector Higgs field.

This quantity can be calculated easily for the $SO(6)$ Ansatz employed in this paper

$$\begin{aligned}\varepsilon_{ijkl}\,\mathrm{Tr}\{F^{ij}F^{kl}\} = &-\frac{4!}{r^3}(|\vec{\phi}|^2-1)\,\mathrm{Tr}(D_r\phi^3\Sigma_{56}\\ &+ (D_r\phi\varepsilon)^M\Sigma_{M4}),\end{aligned}\tag{53}$$

which vanishes, *i.e.*, the candidate for a magnetic charge for the solutions found in the present work equals zero identically.

V. FURTHER REMARKS

On general grounds, one expects that extending the known classes of solutions of the $d = 5$ supergravity to a non-Abelian gauge group would lead to a variety of new physical effects.

This work has been aimed as a first step towards constructing the non-Abelian solutions of the maximal $d = 5$ gauged supergravity. Restricting to the simplest case of static, spherically symmetric solutions, we have proposed a suitable Ansatz for the gauge fields and presented numerical evidence for the existence of both particlelike and black hole solutions. Our systematic description of the black holes is restricted to a certain truncation of the full $SO(6)$ model, with the sole purpose of rendering the numerics practicable. In this limited context, we have also found extremal black holes.

As a consequence of the presence of a negative cosmological constant in the model, we have recovered the qualitative properties of the solutions to the usual EYM-Λ model in 3 + 1 dimensions [1,2]. Notably, some of our solutions to which we have referred as *generic*, are characterized by arbitrary asymptotic values of the potentials parametrizing the gauge field. Also, these solutions share another property with those of the 3 + 1 dimensional EYM model, namely, that the shooting parameters involved take on a continuum of values. Unlike the latter however, their masses turn out to be divergent in our 4 + 1 dimensional case. This is expected on the basis of the Derrick-type scaling argument. However, we have proposed a regularization procedure for the mass of these solutions, in the context of the AdS/CFT correspondence. As far as these *generic* solutions are concerned, the presence of the Chern–Simons term makes no qualitative difference.

Perhaps the most interesting feature of the EYMCS model is the existence of finite mass solutions. We have referred to these as *special* solutions and they contrast with the *generic* ones in that the shooting parameters involved take on a discrete set of values. The *special* solutions exist only when the non-Abelian Chern–Simons term is present.

Concerning the physical context of our results, it is in order to make several remarks on the issue of the more general solutions of the $d = 5$, $\mathcal{N} = 8$ gauged supergravity. This model contains an additional 20 scalars, which are represented by a symmetric unimodular tensor. These scalars have a nontrivial potential approaching a constant negative value at infinity which fixes the value of the effective cosmological constant. No obvious consistent truncation of this sector seems to exist for a gauge group $SO(6)$ and strictly speaking one should work with the full set of scalars. In principle, at least when this general model is subjected to spherical symmetry, the one dimensional subsystem resulting from the application of the Ansatz here can be studied using the same methods as in this paper, *i.e.*, the solutions can be constructed by solving a boundary-value problem. One can in that case find the approximate expressions at the origin or event horizon and at infinity. The only obstacle to this task that we see at this moment is the huge complexity of the ensuing equations. Based on the results in this paper, we expect the parameter space of the full $SO(6)$ solutions of the $d = 5$, $\mathcal{N} = 8$ model to be very rich. Inclusion of the scalar sector will lead to many new free parameters in the asymptotics and will make any attempt to classify the solutions difficult (involving numerous different ways of approaching a constant negative value at infinity for the scalar potential).

The *generic* non-Abelian solutions on the other hand will always present a nonvanishing magnetic gauge field on the boundary which appears as a background for the dual theory. (This feature is independent of the presence or absence of scalars.) Thus the expectation value of the dual CFT stress tensor will contain a part which is similar to (47). Configurations with vanishing non-Abelian magnetic field on the boundary, and finite mass, should exist as well, being supported by the CS term. Also, similar to the case of four dimensional EYM non-Abelian solutions in [6], the existence of both spherically symmetric globally regular and hairy black hole solutions with the same set of data at infinity raises the question as to how the dual CFT is able to distinguish between these different bulk configurations.

Concerning future developments of this work, we expect a much richer structure of the non-Abelian solutions to be found when relaxing the spacetime symmetries. For example, one may envisage the existence of asymptotically AdS_5 solutions which are static and nonspherically symmetric, generalizing the configurations in [43]. Particularly interesting would be to approach the issue of EYMCS rotating solutions. To our knowledge, the only $d > 4$ rotating solutions with non-Abelian fields known so far in the literature are the $d = 5$ EYM-$SU(2)$ black holes in [44]. However, the mass of these solutions diverges logarithmically. It is likely that the inclusion of a CS term will lead to finite mass solutions also in that case.

Another very natural direction to be explored is the case of zero cosmological constant, which would be outside the context of $\mathcal{N} = 8$ supergravity but would nonetheless be technically very interesting. For example, this would afford a comparison with the corresponding $3 + 1$ dimensional asymptotically flat EYM solutions in [21], which involve only a discrete spectrum of the shooting parameters.

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