

Scalar-scalar, scalar-tensor, and tensor-tensor correlators from anisotropic inflation

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(Received 31 January 2010; published 22 March 2010)

We compute the phenomenological signatures of a model [Watanabe *et al.* Phys. Rev. Lett. **102**, 191302 (2009)] of anisotropic inflation driven by a scalar and a vector field. The action for the vector is U(1) invariant, and the model is free of ghost instabilities. A suitable coupling of the scalar to the kinetic term of the vector allows for a slow roll evolution of the vector vacuum expectation value, and hence for a prolonged anisotropic expansion; this provides a counter example to the cosmic no hair conjecture. We compute the nonvanishing two point correlation functions between physical modes of the system, and express them in terms of power spectra with angular dependence. The anisotropy parameter g_* for the scalar-scalar spectrum (defined as in the Ackerman *et al.* [Phys. Rev. D **75**, 0835002 (2007)] parametrization) turns out to be negative in the simplest realization of the model, which, therefore, cannot account for the angular dependence emerged in some analyses of the Wilkinson Microwave Anisotropy Probe data. A g_* of order -0.1 is achieved when the energy of the vector is about 6–7 orders of magnitude smaller than that of the scalar during inflation. For such values of the parameters, the scalar-tensor correlation (which is in principle a distinctive signature of anisotropic spaces) is smaller than the tensor-tensor correlation.

DOI: 10.1103/PhysRevD.81.063528

PACS numbers: 98.80.Cq

I. INTRODUCTION

The Wilkinson Microwave Anisotropy Probe (WMAP) data on the cosmic microwave background (CMB) anisotropies [1] have strongly improved our knowledge of the Universe, and are in overall excellent agreement with the theory of inflation. However, the unprecedented quality of the data has driven a number of analysis on finer effects, which seem to be hard to reconcile with the simplest inflationary models. These so-called “anomalies” include the low power in the quadrupole moment [2–4], the alignment of the lowest multipoles [5], a $\sim 5^\circ$ cold spot with suppressed power [6], an asymmetry in power between the northern and southern ecliptic hemispheres [7], and broken rotational invariance [8]. Most of these effects (and, particularly, those appearing at the largest scales) can be generally thought of as violation of statistical isotropy of the CMB. It has been suggested in [9] that a period of anisotropic expansion during inflation may explain some of these features. Specifically, Ref. [9] pointed out that the anisotropic expansion would provide a primordial power spectrum with a dependence on the directionality of the modes, and computed the corresponding correlation matrix $\langle a_{\ell m} a_{\ell' m'}^* \rangle \neq \delta_{\ell\ell'} \delta_{mm'}$.

Reference [10] introduced the power spectrum parametrization

$$P(\mathbf{k}) = P(k)[1 + g_*(k)\xi^2] \quad (1)$$

where ξ is the cosine of the angle between \mathbf{k} and a given fixed direction, and $k = |\mathbf{k}|$, and computed the corresponding CMB correlation matrix as an expansion series in g_* . Although the specific inflationary model proposed in [10] to obtain this power spectrum was later found to be unstable [11,12], the Ackerman-Carroll-Wise (ACW) study

based on (1) is extremely useful, since it provides a general reference result for generic breaking of rotational invariance during inflation.¹ The ACW parametrization was tested against the WMAP data in [8], where a 3.8σ evidence was obtained for nonvanishing g_* (an upper limit on the anisotropy was instead obtained in [15]). The study assumed a constant g_* , and can likely be applied to slow roll inflationary backgrounds, in which a mild scale dependence may be expected. This analysis was then refined in two more recent works [16,17], which include a $(-i)^{\ell-\ell'}$ factor in the covariance matrix which was neglected in the first version of [10], and in the analysis of [8]. The inclusion of this factor has two important effects. First, it substantially increased the significance of a nonvanishing asymmetry: $g_* = 0.29 \pm 0.031$ [17] (the analysis includes multipoles up to $\ell = 400$ in the W band; a smaller evidence emerges from the V and Q bands). Second, it shifted the privileged direction very close to the ecliptic poles [16,17]. Although this near coincidence suggests that the asymmetry may not be cosmological, a systematical cause for it has not yet been determined. Specifically, Ref. [17] ruled out that the effect may be due to asymmetric beams, misestimated noise, or Zodiacal lights.

In the absence of a definite answer, it will be important to see whether the asymmetry will be also present in the forthcoming Planck data. In the meantime, it is also interesting to study whether there exist simple inflationary

¹Reference [13] studied instead the power spectrum for violation of translational invariance. A general framework for directional dependence of the power spectrum was studied in [14], where it was forecast that a quadrupole modulation of the power spectrum as small as 2% of the total anisotropic signal can be detected by the Planck satellite.

models which can (i) account for the asymmetry, and, possibly, (ii) provide other measurable predictions. As a first step towards this, Ref. [18] provided the formalism for computing cosmological perturbations on backgrounds with a privileged direction (Bianchi-I background, with a residual 2D symmetry).² The formalism was then applied to the simplest case of inflation driven by a slow rolling scalar field, assuming that the expansion rate of one direction is different from that of the other two as an initial condition at the onset of inflation. One of the gravity wave polarizations experiences a large growth during the initial anisotropic era (this is intimately related to the instability of Kasner spaces), which may result in a large B signal in the CMB [21]. Inflation however rapidly removes the background anisotropy. The modes that leave the horizon well after the Universe has isotropized were deep inside the horizon while the Universe was anisotropic, and one recovers a standard power spectrum at the corresponding scales. The signature of the earlier anisotropic stage are therefore present only on relatively large scales, which were comparable to the horizon when the Universe was anisotropic. These scales can be visible today only if the duration of inflation is limited to a minimal amount.

To avoid this tuning, one can obtain prolonged anisotropic inflationary solutions by introducing some ingredients that violate the premises of Wald's theorem [22] on the rapid isotropization of Bianchi universes. This has been realized through the addition of quadratic curvature invariants to the gravity action [23], with the use of the Kalb-Ramond axion [24],³ or of vector fields [26]. A number of recent works focuses on this last possibility, which is probably technically simpler than the other two.⁴ The underlying idea is that the vector has a nonvanishing expectation value along one spatial direction, causing that direction to expand differently from the other two.⁵ In the standard case, i.e. for a minimal kinetic term ($-F^2/4$) and no potential term for the vector, the vector vacuum expectation value (VEV) rapidly decreases with the expansion of the Universe, leading to a rapid isotropization. Therefore, the action for the vector needs to be carefully arranged. To our knowledge, four different possibilities have been explored. The first three possibilities are characterized by

(i) a potential $V(A^2)$ for the vector [26], (ii) a fixed spatial norm of the vector, enforced by a Lagrange multiplier [10], or (iii) a nonminimal coupling of the vector to the scalar curvature [30].⁶ All these proposals break the U(1) symmetry which is present for a minimal vector action. This introduces an additional degree of freedom (the longitudinal vector mode), which, for all of these models, turns out to be a ghost [11,12,32]. This leads to instabilities of these models both at the linear (the linearized solutions for the perturbations diverge at a finite time close to horizon crossing) and nonlinear level (vacuum decay, with a UV divergent rate, into ghost-nonghost excitations).⁷

A completely different model was proposed in [36]. It is characterized by a scalar inflaton field, with a flat potential, and by a vector whose kinetic term is multiplied by a function of the scalar, $-f(\phi)^2 F^2/4$.⁸ As shown in [36], for a suitable choice of f the VEV of the vector evolves slowly during inflation, and the model therefore supports a prolonged anisotropic stage.⁹ Moreover, since the model is U(1) invariant, the problematic longitudinal vector is absent. One therefore expects this model to be stable. This was indeed shown to be the case in Ref. [41].¹⁰ We believe that this makes the model of [36] particularly interesting, since it is a complete counter example to the Wald's no hair theorem [22], which is proven to be free of instabilities. In the present work, we study the phenomenology of this model. We compute the full spectrum of perturbations, and the resulting two point correlation functions. We particularly focus on two phenomenological signatures: the angular dependence of the scalar-scalar correlator, to see whether it can reproduce the value for g_* reported in [17], and the scalar-tensor correlator, which is a distinctive prediction for this class of models (since it vanishes in the standard case). While, to our knowledge, the one obtained here is the first definite prediction for an anisotropic inflationary evolution, we show that the specific model proposed in [36] does not reproduce the observed signal.

The plan of this paper is the following. In Sec. II we review the model of [36] and we study the background

²Reference [19] provided the computation for general Bianchi-I backgrounds; the results of [18,19] agree in the limit of 2D isotropy, and reduce to the standard computation [20] in the limit of 3D isotropy.

³For later studies of inflationary models with p forms, see [25].

⁴Several recent works [27] also discuss the possible non-Gaussian signature from vector fields during inflation (with or without anisotropic expansion), as this can be a way to differentiate them from scalar fields. See also [28] for an earlier study of the cosmological curvature perturbations generated by a vector field.

⁵While we focus on primordial inflation, vector fields with nonvanishing spatial VEV have also been employed as sources of the late time acceleration [29].

⁶Vectors with nonminimal coupling to the curvature on an isotropic inflationary background were studied in [31].

⁷References [33,34] argued against this instability for models of type (iii). We however believe that the arguments and the calculations presented in [11,12,32] are robust. For other stability studies, see [35].

⁸Reference [37] studied the generation of primordial magnetic fields in the case in which the vector is the electromagnetic potential. An analogous study was performed in [38].

⁹A similar idea is also proposed in [39], where the function multiplying the kinetic term is taken to be an external function of time. For other works with vector fields with nonstandard kinetic terms during inflation, see [40].

¹⁰Reference [41] also presented the power spectrum of a subset of the perturbations of the model, including one of the gravity waves polarization, for which the computation is technically simpler than the complete one given here.

evolution. In Sec. II we instead perform the computation of the perturbations and we show the resulting power spectra. The two main steps are the computation of the quadratic action for the perturbations (from which we obtain the linearized evolution equations for the modes), and the quantization of that action (which is needed for computing the initial conditions for the modes, and the correlators). For the latter, we employ the formalism of [42] for the quantization of a system of coupled bosonic fields. Our results are discussed in the concluding Sec. IV.

II. THE MODEL AND THE BACKGROUND EVOLUTION

We study the background solution in the model of [36], which is characterized by the action

$$S = \int d^3x \sqrt{-g} \left[\frac{M_p^2}{2} R - \frac{1}{2} (\partial\phi)^2 - V(\phi) - \frac{1}{4} f(\phi)^2 F^2 \right]. \quad (2)$$

Namely, there is a scalar field ϕ with potential $V(\phi)$, which is taken sufficiently flat to allow for a slow evolution of the scalar. There is also a vector field, which enters only with its kinetic term ($F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$), so that the action is U(1) invariant. This kinetic term is multiplied by a function of the scalar, which causes a nonstandard evolution for the vector VEV. For an appropriate choice of $f(\phi)$ the energy density of the vector also evolves slowly, and the model admits a prolonged anisotropic inflationary solution [36].

We assume a Bianchi-I background with a residual isotropy in the plane perpendicular to the vector VEV,

$$ds^2 = -dt^2 + a(t)^2 dx^2 + b(t)^2 [dy^2 + dz^2],$$

$$\langle A_\mu \rangle = (0, A_1(t), 0, 0)$$

and we parametrize the two scale factors by

$$a = e^{\alpha-2\sigma}, \quad b = e^{\alpha+\sigma} \quad (3)$$

(so that $e^\alpha = a^{1/3} b^{2/3}$, $e^\sigma = b^{1/3} a^{-1/3}$). Namely, α parametrizes the overall volume expansion, while σ controls the amount of anisotropy. More accurately, the normalization of the two scale factors is unphysical, and one can shift the values of α and σ by a constant factor without changing the physics of the system. Therefore, the ‘‘initial’’ (i.e., at the initial time of a simulation) values α_{in} and σ_{in} can be set to any arbitrary value. The degree of anisotropy is related to $\dot{\sigma}/\dot{\alpha}$: a constant σ corresponds to a Friedmann-Robertson-Walker (FRW) flat geometry, with the Hubble rate given by $\dot{\alpha}$.

The background equation of motion for the vector VEV is

$$\ddot{A}_1 + \left[\dot{\alpha} + 4\dot{\sigma} + 2 \frac{\dot{\phi} f'(\phi)}{f(\phi)} \right] \dot{A}_1 = 0 \quad (4)$$

where the dots denote differentiation with respect to time, while f' —and, later, V' —denotes the derivative of that

function with respect to the scalar field. This equation is solved by

$$\dot{A}_1 = p_A e^{-\alpha-4\sigma} / f(\phi)^2 \quad (5)$$

where p_A is constant. We insert this solution in the other nontrivial background equations following from (2). These equations then rewrite

$$3\dot{\alpha}^2 - 3\dot{\sigma}^2 = \frac{1}{M_p^2} \left[\frac{\dot{\phi}^2}{2} + V(\phi) + \frac{\tilde{p}_A^2}{2f(\phi)^2} \right]$$

$$2\ddot{\alpha} + 3\dot{\alpha}^2 + 3\dot{\sigma}^2 = \frac{1}{M_p^2} \left[-\frac{\dot{\phi}^2}{2} + V(\phi) - \frac{\tilde{p}_A^2}{6f(\phi)^2} \right]$$

$$\ddot{\sigma} + 3\dot{\alpha} \dot{\sigma} = \frac{\tilde{p}_A^2}{3M_p^2 f(\phi)^2}$$

$$\ddot{\phi} + 3\dot{\alpha} \dot{\phi} + V'(\phi) = \frac{\tilde{p}_A^2 f'(\phi)}{f(\phi)^3} \quad (6)$$

where we have defined

$$\tilde{p}_A(t) \equiv e^{-2\alpha-2\sigma} p_A. \quad (7)$$

The first of (6) is the tt Einstein equation for the system, and, for $\dot{\sigma} = \tilde{p}_A = 0$, it reduces to the standard Friedmann equation. The last term in this equation is the energy density of the vector field. Since the normalization of the scale factors is unphysical, the quantity \tilde{p}_A must be independent of it. Therefore, if one wishes to change the initial values of the two scale factors, the integration constant p_A must also be changed according to $p_A \propto e^{2\alpha_{\text{in}}+2\sigma_{\text{in}}}$. The second and fourth equation in (6) are a combination of the spatial xx and $yy = zz$ Einstein equations, while the fourth equation is the equation for the scalar field. One of these three equations can be obtained from the other two, and from the tt Einstein equation, as a consequence of a non-trivial Bianchi identity.

We are interested in an inflationary background solution of (6) in a regime of slow roll and small anisotropy. The first and last of (6) can be approximated as in the standard case, and combined to give $\alpha(\phi) \approx -\int^\phi \frac{V}{M_p^2 V'} d\phi$. We can also find how f and V need to be related to each other to have a prolonged anisotropic stage. Namely, we require that the ratio between the energy densities of the vector and the scalar field remains approximately constant during inflation. In the slow roll regime, and for small anisotropy, we have

$$\frac{\rho_A}{\rho_\phi} \approx \frac{p_A^2 e^{-4\sigma}}{2V(\phi)} \left(\frac{e^{-2\alpha}}{f(\phi)} \right)^2 \quad (8)$$

and, given that the first factor is very slowly evolving in this regime, we require [36] that $f \approx \exp[-2\alpha] \approx \exp[\int^\phi \frac{2V}{M_p^2 V'} d\phi]$. For definiteness, we will consider the simplest chaotic inflationary potential

$$V(\phi) = \frac{1}{2} m^2 \phi^2, \quad f(\phi) = \exp\left(\frac{c\phi^2}{2M_p^2}\right) \quad (9)$$

where c is a numerical constant close to 1.

We expect that, for $c > 1$, f decreases more quickly than $e^{-2\alpha}$ during inflation, so that the anisotropy actually increases; to verify this, we study the first and last of (6) in more details. Namely, we disregard the terms proportional to $\dot{\sigma}$ and \tilde{p}_A in the first equation, and the term proportional to $\ddot{\phi}$ in the last one (it is consistent to keep the term proportional to \tilde{p}_A in the last equation, and disregard it in the first one, provided that $\phi \gg M_p/\sqrt{c}$, which is indeed a good approximation during inflation). These two equations can be then integrated to give [36]

$$e^{c(\phi^2/M_p^2)+4\alpha} \approx \frac{c^2}{c-1} \frac{P_A^2}{m^2 M_p^2} + D e^{-4(c-1)\alpha} \quad (10)$$

where D is an integration constant. For $c < 1$, the second term on the right-hand side of this equation increases over the first one during inflation. If this term is dominant, then Eq. (10) reduces to $\alpha \approx -\phi^2/(4M_p^2)$ (plus an unphysical constant), which is the standard isotropic result. On the contrary, for $c > 1$, the first term of (10) increases over the second term over time. If this term is dominant, Eq. (10) reduces to $\alpha \approx -c\phi^2/(4M_p^2)$. Combining this with the approximate tt Einstein equation, $6\dot{\alpha}^2 \approx m^2\phi^2/M_p^2$, gives [36]

$$3\dot{\alpha}\dot{\phi} \approx -\frac{m^2\phi}{c} \quad (c > 1) \quad (11)$$

which gives a value for $\dot{\phi}$ about $1/c$ times the standard result. To quantify the anisotropy, we neglect the $\ddot{\sigma}$ term in the third of (6). Combining the resulting expression with (10), in a regime in which the term proportional to D can be disregarded, we find

$$\frac{\dot{\sigma}}{\dot{\alpha}} \approx \frac{2}{3} \frac{c-1}{c^2} \frac{M_p^2}{\phi^2} \quad (c > 1) \quad (12)$$

which indeed confirms that the anisotropy increases during inflation.

This quantity is approximately equal the ratio between the energy densities of the vector and the scalar. Indeed, combining Eqs. (8) and (10), in a regime in which the term proportional to D can be disregarded, we also find

$$\frac{\rho_A}{\rho_\phi} \approx \frac{c-1}{c^2} \frac{M_p^2}{\phi^2} \quad (c > 1). \quad (13)$$

Namely, for $c < 1$ the system evolves towards isotropy, while for $c > 1$ the system evolves towards isotropy during inflation. In both cases the solution is an attractive one. One can decide to take $c < 1$ and start away from the isotropic attractor solution. If c is sufficiently close to 1, the anisotropy will decrease very slowly, and still give some observable nonstandard signature. The result will however be sensitive on the initial conditions, and not only on the model. On the other hand, for $c > 1$ we can start in the attractor anisotropic solution characterized by (11). The underlying idea is that inflation lasted much more than the observable last 60 e-folds, and that the solution converged to the attractor one during that time. In this case, the phenomenological signatures of the model are insensitive

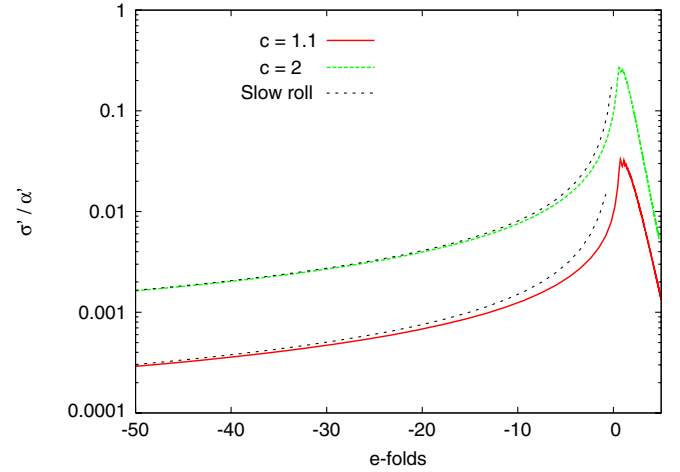


FIG. 1 (color online). Evolution of the anisotropy factor $\dot{\sigma}/\dot{\alpha}$ for two different values of c in the function (9), as a function of the number of e-folds. $N = 0$ corresponds to the end of inflation.

on the initial conditions, precisely as in the standard inflationary case.¹¹ For this reason, we only study the $c > 1$ case in this work.

We conclude this section with two remarks. First, we note that the anisotropy is proportional to $c - 1$. The anisotropic attractor solution is continuously connected to a FRW solution in the $c \rightarrow 1$ limit. We expect standard results for the perturbations in this limit, as the computations presented in the next sections confirm. Second, while the anisotropy increases during inflation, it decreases after inflation. Indeed, after inflation ϕ oscillates around zero, with a decreasing amplitude. Then $f \rightarrow 1$, and the mechanism of prolonged anisotropy becomes ineffective. The amplitude of the vector rapidly decreases, and the background evolution becomes isotropic.

For illustrative purposes, we show in Fig. 1 the evolution of the anisotropy $\dot{\sigma}/\dot{\alpha}$ as a function of the number of e-folds $N \equiv \alpha$, normalized to zero at the end of inflation.¹² We show the evolution for two different values of c , starting from the slow roll anisotropic initial condition.¹³ We

¹¹Even in the standard case, one can assume that the inflaton was not yet in the attractor solution when the largest observed multipoles left the horizon. For instance, a fast roll evolution at that stage results in a suppression of the CMB quadrupole [43]. This signal is however dependent on the assumed initial conditions.

¹²Since $\dot{\sigma} \ll \dot{\alpha}$, we define the number of e-folds N and the end of inflation as in the FRW case. This gives $N = \alpha$, and the end of inflation occurs when $\ddot{\alpha} + \dot{\alpha}^2$ becomes negative.

¹³Specifically, we use the last three equations of (6) in our numerical evolutions. We also satisfy the first of (6) through the initial conditions (if this equation holds at the initial time, it is preserved by the remaining equations): we relate the initial value of $\dot{\phi}$ and of $\dot{\sigma}$ to that of $\dot{\alpha}$ through Eqs. (11) and (12); we insert these expressions in the first of (6), and we then use this equation to relate $\dot{\alpha}_{\text{in}}$ to the initial value of ϕ . In this way, all the initial conditions are given in terms of ϕ_{in} , which completely specifies any point along the attractor solution.

also show the evolution of the anisotropy parameter given by the slow roll solution, Eq. (12). We see that the slow roll expression is very accurate during most of the inflationary evolution.

III. PERTURBATIONS

This section studies the perturbations of the model [36] discussed in the previous section. The discussion is divided into several subsections. In Subsec. III A we introduce the perturbations, and we classify them according to how they behave with respect to 2D spatial rotations in the isotropic yz plane. We also perform the gauge choices which completely fix the freedom associated with general coordinate transformations, and with the U(1) invariance of the vector. We then integrate out the nondynamical perturbations, and we provide the quadratic action for the dynamical modes. In Subsec. III B we quantize this action, and we define the initial adiabatic vacuum, valid during inflation in the sub-horizon regime. In Subsec. III C we introduce the two point correlation functions for anisotropic backgrounds, and we express them in terms of angular dependent power spectra. In Subsec. III D we write more explicitly the initial conditions for the modes (coming from the adiabatic vacuum obtained in III B) and their evolution equations (coming from the action obtained in III A). Finally, in Subsec. III E we discuss which combination of perturbations reduce to the standard one as the Universe isotropizes, and we provide their power spectra.

Both the computation of the quadratic action, and its quantization, follow formalisms that have been developed elsewhere. Although we made an effort to keep the present discussion self-contained, some details have been omitted here for brevity. The interested reader can refer to Appendix B of [18] and to Sec. III of [32] for more details on the computation of the quadratic action, and to Sec. II of [42] for the quantization of the perturbations based on this action.

A. Classification, gauge choice, and quadratic action for the dynamical modes

The most general set of perturbations about the anisotropic background solution discussed in the previous section is given by

$$\delta g_{\mu\nu} = \begin{pmatrix} -2\Phi & a\partial_x\chi & b(\partial_i B + B_i) \\ & -2a^2\Psi & ab\partial_x(\partial_i \tilde{B} + \tilde{B}_i) \\ & & b^2[-2\sum\delta_{ij} + 2E_{,ij} + E_{(i,j)}] \end{pmatrix},$$

$$\delta A_\mu = (\delta A_0, \delta A_1, \partial_i \delta A + \delta A_i), \quad \delta\phi \quad (14)$$

where the index $i = 2, 3$ spans the isotropic yz plane, and $E_{(i,j)} \equiv \partial_i E_j + \partial_j E_i$. With this choice, the perturbations are decomposed in scalar and vector with respect to rotations in the isotropic plane. This is a convenient procedure since the two different types of modes are decoupled from

each other at the linearized level, and can be studied separately [18].¹⁴ Specifically, the modes $B_i, \tilde{B}_i, E_i, \delta A_i$ are 2D vector (2dv) modes, satisfying $\partial_i B_i = \dots = 0$, and they encode 1 degree of freedom each. The remaining modes are 2D scalars (2ds), which also encode 1 degree of freedom each. Altogether, we have 15 degrees of freedom in the perturbations (14).

We Fourier transform the perturbations,

$$\delta(t, \mathbf{x}) = \int \frac{d^3k}{(2\pi)^{3/2}} e^{i\mathbf{k}\cdot\mathbf{x}} \delta(t, \mathbf{k}) \quad (15)$$

where δ denotes any of the perturbations, and we use the same symbol for a perturbation in real and in momentum space. The reality of $\delta(\mathbf{x})$ imposes the condition $\delta^\dagger(\mathbf{k}) = \delta(-\mathbf{k})$ (we use Hermitian conjugate, and not simply charge conjugate, since in the next section the modes are treated as operators, in order to quantize them). At the linearized level, all modes are decoupled from each other, and can be studied separately. We denote the comoving momentum of the mode as $\mathbf{k} = (k_L, k_{T2}, k_{T3})$. Because of the symmetry in the y - z plane, physical results depend on $k_T \equiv \sqrt{k_{T2}^2 + k_{T3}^2}$ rather than on k_{T2} and k_{T3} separately.¹⁵ We denote the physical momentum of the mode by $\mathbf{p} = (p_L, p_{T2}, p_{T3})$, where $p_L = k_L/a$ and $p_{Ti} = k_{Ti}/b$. As for the comoving momentum, we define $p_T = \sqrt{p_{T2}^2 + p_{T3}^2}$.

It is convenient to write explicitly the single degree of freedom encoded in the 2D vector modes. In momentum space, we have

$$\begin{aligned} B_i &\equiv i\epsilon_{ij}k_{Tj}B_v, & \tilde{B}_i &\equiv i\epsilon_{ij}k_{Tj}\tilde{B}_v, \\ E_i &\equiv i\epsilon_{ij}k_{Tj}E_v, & \delta A_i &\equiv i\epsilon_{ij}k_{Tj}\delta A_v \end{aligned} \quad (16)$$

where ϵ_{ij} is antisymmetric, and $\epsilon_{12} = 1$.

To proceed, we need to fix the gauge freedoms of the system. We start from the freedom associated with general coordinate transformations. To do so, we can either choose a gauge that completely removes this freedom, as done in [18,21], or we can rewrite the action and the equations for

¹⁴This strategy is mutated from the one adopted in the standard 3 dimensional case, in which the modes are classified in scalar, vector, and tensors with respect to 3D rotations. Two dimensional tensor modes do not exist, since imposing the transversality and traceless conditions eliminates all the degrees of freedom of a 2×2 symmetric tensor.

¹⁵This was exploited in [18], where k_{T3} was set to zero. As a consequence, the perturbations in real space do not depend on z , and the decomposition (14) was simpler. Here we adopt the more general formulation used in [21], where both k_{T2} and k_{T3} can be nonvanishing. We also point out that spatial derivatives are introduced in the parametrization (14) for algebraic convenience. In practice, we assume that both k_L and k_T are nonvanishing. This restriction does not affect the computation of any observable, since modes with $k_L = 0$ and/or $k_T = 0$ constitute a subset of zero measure when one integrates over the momentum to obtain results in real space.

the perturbations in terms of gauge invariant modes, as done in [12,32]. The two procedures are equivalent. In the present work, we choose the first one, which is algebraically simpler. Specifically, we set

$$\delta g_{1i,2ds} = \delta g_{ij} = 0 \quad (17)$$

which, in the parametrization (14), gives $\tilde{B} = \Sigma = E = E_i = 0$. One can check (see Appendix B.1 of [18]) that indeed (i) these choices can be always made, and (ii) it fixes completely the freedom associated with the coordinate transformations. There is also the U(1) gauge associated with the transformations $A_\mu \rightarrow A_\mu + \partial_\mu \xi$. We fix this by setting $\delta A = 0$.

This gauge fixing leaves us with seven 2ds modes ($\Phi, \chi, B, \Psi, \delta A_0, \delta A_1, \delta \phi$) and three 2dv modes ($B_i, \tilde{B}_i, \delta A_i$). Not all these modes correspond to physically propagating degrees of freedom. Indeed the modes δA_0 and $\delta g_{0\mu}$ enter in the quadratic action of the perturbations without time derivatives [18]. As a consequence, the equations of motion for the perturbations are algebraic in them, and their value is specified in terms of the values of the other modes, without introducing independent degrees of freedom. Our gauge choice is motivated by the fact that it preserves the δA_0 and $\delta g_{0\mu}$ perturbations, so that the identification of the nondynamical modes is immediate (in other gauges, the nondynamical modes correspond to more complicated linear combinations of the perturbations which are preserved in those gauges). The nondynamical modes need to be integrated out of the action.¹⁶ Namely, we express them in terms of the dynamical modes (through the corresponding Einstein equations), and we insert these expressions back into the action. In this way, we are left with an action in terms of the dynamical modes only¹⁷

$$S_{\text{perts}}^{(2)} = S_{2ds}^{(2)}[\Psi, \delta \phi, \delta A_1] + S_{2dv}^{(2)}[\tilde{B}_i, \delta A_i]. \quad (18)$$

The fields entering in these actions are not canonically normalized. The canonically normalized fields are obtained through the redefinitions

¹⁶This is what is also done in the standard computations [20]. For instance, in the case of a single scalar inflaton on a FRW background, the number of perturbations is 11 (initial perturbations in the metric and in the scalar) -4 (after gauge fixing) -4 (after eliminating the nondynamical modes) $= 3$ (namely, the scalar density contrast, and the two gravity waves polarization). Our procedure extends this computation to the more general background we are studying. In Appendix B we verify that our computation reduces to the standard one in the limit of isotropic background.

¹⁷The procedure of integrating out the nondynamical modes is described in details in Sec. III of [32].

$$\begin{aligned} \delta \phi &\equiv e^{-(3/2)\alpha} \left(V_+ - \frac{\dot{\phi}}{\sqrt{2}M_p(\dot{\alpha} + \dot{\sigma})} H_+ \right) \\ \Psi &\equiv e^{-(3/2)\alpha} \frac{2p^2 \dot{\alpha} + (2p_L^2 - p_T^2) \dot{\sigma}}{\sqrt{2}M_p p_T^2 (\dot{\alpha} + \dot{\sigma})} H_+ \end{aligned} \quad (19)$$

$$\begin{aligned} \delta A_1 &\equiv e^{-(1/2)\alpha - 2\sigma} \left[\frac{p}{f(\phi) p_T} \Delta_+ \right. \\ &\quad \left. - \frac{\tilde{p}_A}{\sqrt{2}M_p f(\phi)^2 (\dot{\alpha} + \dot{\sigma})} H_+ \right] \end{aligned}$$

in the 2ds sector, and

$$\tilde{B}_i \equiv -\sqrt{2} \epsilon_{ij} k_{Tj} e^{-(3\alpha/2)} \frac{\sqrt{e^{6\sigma} k_L^2 + k_T^2}}{M_p k_L k_T} H_\times \quad (20)$$

$$\delta A_i \equiv i \epsilon_{ij} \frac{k_{Tj}}{k_T} \frac{e^{\sigma - (\alpha/2)}}{f(\phi)} \Delta_\times$$

in the 2dv sector (where ϵ_{ij} is antisymmetric, and $\epsilon_{12} = 1$).

Once expressed in terms of these fields, the two actions in (18) rewrite

$$\begin{aligned} S_{2ds}^{(2)} &= \frac{1}{2} \int dt d^3 k [\dot{Y}_s^\dagger \dot{Y}_s + \dot{Y}^\dagger K_s Y_s - Y_s^\dagger K_s \dot{Y}_s - Y^\dagger \Omega_s^2 Y_s] \\ S_{2dv}^{(2)} &= \frac{1}{2} \int dt d^3 k [\dot{Y}_v^\dagger \dot{Y}_v + \dot{Y}_v^\dagger K_v Y_v \\ &\quad - Y_v^\dagger K_v \dot{Y}_v - Y_v^\dagger \Omega_v^2 Y_v] \end{aligned} \quad (21)$$

where

$$Y_s \equiv \begin{pmatrix} V_+ \\ H_+ \\ \Delta_+ \end{pmatrix}, \quad Y_v \equiv \begin{pmatrix} H_\times \\ \Delta_\times \end{pmatrix} \quad (22)$$

and where the explicit expressions for the matrices $K_{s,v}$ and $\Omega_{s,v}^2$ are given in Appendix A.

B. Quantization and initial adiabatic vacuum

We need to quantize the two actions (21) in order to provide the initial conditions for the modes and the expressions for the correlators. We do not need to discuss the two systems separately, since the two actions are formally the same, and the fields and the matrices entering in them have identical properties. In both cases, the actions are of the type

$$S = \frac{1}{2} \int dt d^3 k [\dot{Y}^\dagger \dot{Y} + \dot{Y}^\dagger K Y - Y^\dagger K \dot{Y} - Y^\dagger \Omega^2 Y] \quad (23)$$

where Y is an array of fields, K a real and antisymmetric matrix, and Ω^2 a real and symmetric matrix. These matrices are unchanged under the parity transformation $\mathbf{k} \rightarrow -\mathbf{k}$ (as can be seen from the explicit expressions given in Appendix A). It can be checked from the reality condition stated after Eq. (15), and from the definitions (19) and (20) of the canonical modes, that, for both systems, any of the Y_i fields entering in the array Y satisfies $Y_i^\dagger(\mathbf{k}) = Y_i(-\mathbf{k})$.

To remove the mixed terms proportional to the matrix K , we first perform the field redefinition

$$\psi \equiv RY \quad (24)$$

where R is an orthogonal matrix (so that $\dot{Y}^\dagger \dot{Y} = \dot{\psi}^\dagger \dot{\psi}$), satisfying

$$\dot{R} = RK, \quad R_{\text{late}} = \mathbb{1} \quad (25)$$

where the second condition states that R should reduce to the identity at late times, when the Universe becomes isotropic, and $K \rightarrow 0$ [in which case, the rotation (24) is no longer needed]. The first condition can be also written as $K = R^T \dot{R}$; we also note that $R(-\mathbf{k}) = R(\mathbf{k})$, since this property is also satisfied by K . As a consequence, each of the fields entering in the array ψ satisfies $\psi_i^\dagger(\mathbf{k}) = \psi_i(-\mathbf{k})$. In terms of the fields ψ , the action (23) rewrites

$$S = \frac{1}{2} \int dt d^3k [\dot{\psi}^\dagger \dot{\psi} - \psi^\dagger \tilde{\Omega}^2 \psi], \quad (26)$$

$$\tilde{\Omega}^2 \equiv R(\Omega^2 + K^T K)R^T$$

where we note that $\tilde{\Omega}^2$ is real, symmetric, and invariant under $\mathbf{k} \rightarrow -\mathbf{k}$.

We can also define the real space fields $\psi(t, x)$ as in (15). These fields are real, and their action is formally identical to the action for the coupled bosonic system quantized in [42]. Therefore, we quantize the fields ψ_i as done in that work.

We introduce the matrix C satisfying

$$C^T \tilde{\Omega}^2 C = \text{diag}(\omega_1^2, \dots, \omega_N^2) \equiv \omega^2 \quad C_{\text{end}} = \mathbb{1} \quad (27)$$

where the second condition follows from the fact that, in the late time isotropic limit $\tilde{\Omega}^2 = \Omega^2$ is already diagonal. We note that C is orthogonal, and unchanged under $\mathbf{k} \rightarrow -\mathbf{k}$. We then define [42]

$$\psi_i(\mathbf{k}) = C_{ij} [h_{ji}(\mathbf{k}) \hat{a}_i(\mathbf{k}) + h_{ji}^*(\mathbf{k}) \hat{a}_i^\dagger(-\mathbf{k})]$$

$$\pi_i(\mathbf{k}) = \dot{\psi}_i(\mathbf{k}) = C_{ij} [\tilde{h}_{ji}(\mathbf{k}) \hat{a}_i(\mathbf{k}) + \tilde{h}_{ji}^*(\mathbf{k}) \hat{a}_i^\dagger(-\mathbf{k})] \quad (28)$$

where \hat{a} and \hat{a}^\dagger are annihilation and creation operators, respectively, satisfying

$$[a_i(\mathbf{k}), a_j^\dagger(\mathbf{k}')] = \delta^3(\mathbf{k} - \mathbf{k}') \delta_{ij}. \quad (29)$$

From the equations of motion following from (26), and from the fact that $\pi_i = \dot{\psi}_i$, we find that the coefficients h_{ij} and \tilde{h}_{ij} , obey the evolution equations (in matrix form)

$$\dot{h} = \tilde{h} - \Gamma h, \quad \dot{\tilde{h}} = -\Gamma \tilde{h} - \omega^2 h, \quad \Gamma \equiv C^T \dot{C}. \quad (30)$$

From the parity properties of the matrices C and ω , and from the initial conditions [which we determine below, see Eq. (38)], we see that h_{ij} and \tilde{h}_{ij} are unchanged under $\mathbf{k} \rightarrow -\mathbf{k}$.¹⁸

¹⁸This is why we wrote $h_{ij}^*(\mathbf{k})$ and $\tilde{h}_{ij}^*(\mathbf{k})$, rather than $h_{ij}^*(-\mathbf{k})$ and $\tilde{h}_{ij}^*(-\mathbf{k})$, in the decompositions (28).

We further define

$$h = \frac{1}{\sqrt{2\omega}}(\alpha + \beta), \quad \tilde{h} = \frac{-i\omega}{\sqrt{2\omega}}(\alpha - \beta). \quad (31)$$

It has been shown in [42] that the normal ordered Hamiltonian for the fields ψ_i can be then cast in the form

$$\hat{H} = \int d^3k \omega_i \hat{b}_i^\dagger(\mathbf{k}) \hat{b}_i(\mathbf{k}) \quad (32)$$

where \hat{b}_i and \hat{b}_i^\dagger are new annihilation and creation operators, related to those defined in (28) by (notice that the matrices α and β are unchanged under $\mathbf{k} \rightarrow -\mathbf{k}$)

$$\begin{pmatrix} \hat{b}(t, \mathbf{k}) \\ \hat{b}^\dagger(t, -\mathbf{k}) \end{pmatrix} \equiv \begin{pmatrix} \alpha & \beta^* \\ \beta & \alpha^* \end{pmatrix}_{t, \mathbf{k}} \begin{pmatrix} \hat{a}(\mathbf{k}) \\ \hat{a}^\dagger(-\mathbf{k}) \end{pmatrix}. \quad (33)$$

We see that the Hamiltonian is diagonal in the $\hat{b}_i, \hat{b}_i^\dagger$ basis, so that these operators annihilate and create quanta of the (time-dependent) physical eigenstates of the system. The matrices α and β generalize to a system of N coupled fields the Bogolyubov coefficients that are needed for the quantization of a field with time-dependent frequency. As shown in [42], the canonical quantization of the ψ_i fields imposes the conditions

$$\alpha \alpha^\dagger - \beta^* \beta^T = \mathbb{1}, \quad \alpha \beta^\dagger - \beta^* \alpha^T = 0. \quad (34)$$

Moreover, from the evolution equations (30), and the definitions (31), one finds that α and β obey the evolution equations [42]

$$\dot{\alpha} = -i\omega \alpha + \frac{\dot{\omega}}{2\omega} \beta - I\alpha - J\beta$$

$$\dot{\beta} = i\omega \beta + \frac{\dot{\omega}}{2\omega} \alpha - I\beta - J\alpha \quad (35)$$

where

$$I = \frac{1}{2} \left(\sqrt{\omega} \Gamma \frac{1}{\sqrt{\omega}} + \frac{1}{\sqrt{\omega}} \Gamma \sqrt{\omega} \right),$$

$$J = \frac{1}{2} \left(\sqrt{\omega} \Gamma \frac{1}{\sqrt{\omega}} - \frac{1}{\sqrt{\omega}} \Gamma \sqrt{\omega} \right). \quad (36)$$

An inspection of the initial matrices K and Ω^2 shows that, at early times (when the mode is deeply inside the horizon) $\Omega^2 = p^2 \mathbb{1} + \mathcal{O}(H)$ and $K = \mathcal{O}(\sqrt{c-1}H)$ (in these expressions, $H \equiv \dot{a}/a \simeq \dot{b}/b$ under the assumption of small anisotropy that we are making in this work). As a consequence,

$$\omega_i^2 \simeq p^2 + \mathcal{O}(H^2), \quad \Gamma, I, J, \frac{\dot{\omega}}{\omega} = \mathcal{O}(H) \quad (37)$$

in this early time regime. Therefore, we can disregard all

but the first term in both of the right-hand sides of (35). This leads us to the initial adiabatic vacuum solutions¹⁹

$$\alpha_{\text{early}} = e^{-i \int^t dt \omega}, \quad \beta_{\text{early}} = 0. \quad (38)$$

This solution obeys (35) in the early time regime, satisfies the quantization conditions (34), and corresponds to an initial empty vacuum, if we impose that the vacuum state is annihilated by the operators \hat{a}_i entering in (28).

C. Two point correlation functions

By combining the various redefinitions given in the above section, we can write the canonically normalized fields in real space as

$$Y_i(t, \mathbf{x}) = \int \frac{d^3 k}{(2\pi)^{3/2}} e^{i\mathbf{k}\cdot\mathbf{x}} [Y_{ij}(t, \mathbf{k}) \hat{a}_j(\mathbf{k}) + Y_{ij}^*(t, \mathbf{k}) \hat{a}_j^\dagger(-\mathbf{k})] \quad (39)$$

where

$$Y_{ij}(\mathbf{k}) \equiv (R^T C h)_{ij} \quad (40)$$

(notice that h and Y coincide at late times).

The (statistically averaged) two point correlation function can be expressed as the quantum expectation value

$$C_{ij}(\mathbf{x}, \mathbf{y}) \equiv \frac{1}{2} \langle Y_i(t, \mathbf{x}) Y_j(t, \mathbf{y}) + Y_j(t, \mathbf{y}) Y_i(t, \mathbf{x}) \rangle \quad (41)$$

where the symmetrization is required since the statistical average is a classical operation, independent of the ordering chosen [when computing any correlation with real data, $C_{ij}(\mathbf{x}, \mathbf{y}) = C_{ji}(\mathbf{y}, \mathbf{x})$].

We insert (39) into (41). The resulting expression can be then simplified using the commutation relations (29) and the fact that the vacuum state is annihilated by \hat{a}_i at all times (since the quantization of the previous subsection is performed in the Heisenberg picture). After some algebra, we find

¹⁹Notice that α_{early} is diagonal; the allowed initial conditions are actually more general than (38), since one can multiply each diagonal entry of α_{early} by a constant, and arbitrary, phase factor $e^{i\gamma_i}$. This amounts in changing the matrices α and β given here by a matrix multiplication from the right, $\alpha \rightarrow \alpha P$, $\beta \rightarrow \beta P$, where $P \equiv \text{diag}(e^{i\gamma_1}, \dots, e^{i\gamma_N})$. The equations of motion (35) are unchanged by this multiplication. The same is true in terms of the matrices h and \tilde{h} . Since h enters in the observable two point correlation function through the combination $h h^\dagger$, see Eq. (42), the matrix P drops from the observable result. This confirms that the arbitrary phases contained in P are unphysical, and can be set to any value. We use this freedom to set all the phases to zero at the initial time of our numerical simulations.

$$C_{ij}(\mathbf{x}, \mathbf{y}) = \int \frac{d^3 k}{(2\pi)^3} e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})} \text{Re}[(Y Y^\dagger)_{ij}]. \quad (42)$$

We now define the power spectra associated with these correlators. All of them are of the type

$$C_{\mathcal{F}} = \int \frac{d^3 k}{(2\pi)^3} e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})} \mathcal{F}(\mathbf{k}) \quad (43)$$

where the function \mathcal{F} is real and it depends only on the absolute values of the components of \mathbf{k} along the anisotropic x direction (which we denoted by $|k_L|$), and on the y - z plane (denoted by $k_T = \sqrt{k_{T2}^2 + k_{T3}^2}$). Thanks to this property (which follows from the symmetry of the background under rotations in the yz plane, and under parity), for any two points \mathbf{x} and \mathbf{y} , we can always choose the y and z axes of the system such that the third component of $\mathbf{x}-\mathbf{y}$ vanishes (without changing \mathcal{F}). We therefore set

$$\mathbf{r} \equiv \mathbf{x}-\mathbf{y} \equiv (r_L, r_T, 0) \quad (44)$$

and $\mathbf{k} = k(\xi, \sqrt{1-\xi^2} \cos\phi_k, \sqrt{1-\xi^2} \sin\phi_k)$ in the integral (43), where ξ is the cosine of the angle between the x axis and \mathbf{k} . The function \mathcal{F} does not depend on ϕ_k and is even in ξ . The integral over ϕ_k then gives

$$C_{\mathcal{F}} = \int \frac{dk}{k} \int_0^1 d\xi \cos(k\xi r_L) J_0(k\sqrt{1-\xi^2} r_T) P_{\mathcal{F}} \quad (45)$$

where J is the Bessel function of the first kind, and where we have introduced the power spectrum

$$P_{\mathcal{F}} \equiv \frac{k^3}{2\pi^2} \mathcal{F}(k, \xi). \quad (46)$$

In the case at hand, the power spectra depend both on the magnitude of the momentum of the modes, and on the angle between the momentum and the anisotropic direction.

On anisotropic backgrounds, the power spectrum is isotropic, and Eq. (45) reduces to the standard expression

$$C_{\mathcal{F}} = \int \frac{dk}{k} \frac{\sin(kr)}{kr} \mathcal{P}_{\mathcal{F}} \quad \text{isotropy.} \quad (47)$$

D. Evolution of the perturbations and initial conditions

The equations of motion for the dynamical perturbations follow from (21). We expressed the action in momentum space, by Fourier transforming the starting modes as in (15), and by introducing the canonically normalized fields in (19) and (20). The equations of motion for the coefficient of the Fourier transforms of the canonical fields are

$$\begin{aligned} \ddot{Y}_s + 2K_s \dot{Y}_s + (\Omega_s^2 + \dot{K}_s) Y_s &= 0, \\ \ddot{Y}_v + 2K_v \dot{Y}_v + (\Omega_v^2 + \dot{K}_v) Y_v &= 0 \end{aligned} \quad (48)$$

where the matrices $K_{s,v}$ and $\Omega_{s,v}^2$ are given in Appendix A.

We stress that these equations are a closed subset of the linearized Einstein equations for the perturbations. Specifically, all the perturbations of the model can be divided in dynamical and nondynamical ones. The nondynamical ones enter in the Einstein equations without time derivatives. One can solve the Einstein equations for these perturbations, and express the latter in terms of the dynamical ones. One then inserts these expressions into the remaining Einstein equations. The resulting expressions coincide with Eqs. (48).²⁰

We need to exactly specify what the coefficients $Y_{s,i}$ and $Y_{v,i}$ exactly are. The standard way to compute the generation of perturbations during inflation is a semiclassical computation, in which the perturbations are quantum fields on a classical background [20]. We take this approach in this work. In most systems, the canonical perturbations are decoupled from each other, and can be quantized separately. This is not the case for the system we are studying, and the quantization had to be done accordingly. In Subsec. III B, we introduced an array of annihilation/creation operators, and we saw that the Fourier coefficients of (15) are actually linear combinations of these operators, cf. Eq. (39):

$$Y_i(t, \mathbf{k}) = Y_{ij}(t, \mathbf{k}) \hat{a}_j(\mathbf{k}) + Y_{ij}^*(t, \mathbf{k}) \hat{a}_j^\dagger(-\mathbf{k}). \quad (49)$$

Inserting this decomposition into (48), we find

$$\begin{aligned} [\ddot{Y}_{ij} + 2K_{il} \dot{Y}_{lj} + (\Omega^2 + \dot{K})_{il} Y_{lj}] a_j + [\ddot{Y}_{ij}^* + 2K_{il} \dot{Y}_{lj}^* \\ + (\Omega^2 + \dot{K})_{il} Y_{lj}^*] a_j^\dagger = 0 \end{aligned} \quad (50)$$

both in the 2D scalar and 2D vector sector. The linear combinations multiplying different annihilation and creation operators need to cancel separately; therefore

$$\ddot{Y}_{ij} + 2K_{il} \dot{Y}_{lj} + (\Omega^2 + \dot{K})_{il} Y_{lj} = 0. \quad (51)$$

These equations also guarantee that the linear combinations multiplying the annihilation operators vanish, since the matrices K and Ω^2 are real.

Having determined the evolution equations obeyed by Y_{ij} , we now turn to the determination of their initial condition. We find

$$Y = R^T C h, \quad \dot{Y} = -K R^T C h + R^T C \tilde{h}. \quad (52)$$

The first expression is simply the definition (40). The second expression is obtained by differentiating the first one, and by using the first and third of (30), as well as $K = -\dot{R}^T R$.

The initial values of $R^T C$ and ω are obtained by the diagonalization of $\Omega^2 + K^T K$ at the initial time. Indeed $R^T C$ is defined as the matrix that diagonalizes $\Omega^2 + K^T K$, while ω^2 is the diagonal matrix formed by the eigenvalues of $\Omega^2 + K^T K$; see Eqs. (26) and (27). The initial values of h and \tilde{h} follow instead from Eqs. (31), and from the initial conditions for α and β according to the adiabatic vacuum prescription, Eq. (38): $\alpha_{\text{in}} = \mathbb{1}$, $\beta_{\text{in}} = 0$.

E. Power spectra after isotropization

As shown in Appendix B, as the Universe becomes isotropic after inflation, the canonical perturbations that we have introduced in III A become the standard scalar and tensor modes of FRW cosmology. More precisely, we find that, in this regime,

$$\begin{aligned} \mathcal{R} &= \frac{H}{a^{3/2} \dot{\phi}} V_+, & h_+ &= -\frac{\sqrt{2}}{a^{3/2} M_p} H_+, \\ h_\times &= \frac{i\sqrt{2}}{a^{3/2} M_p} H_\times \end{aligned} \quad (53)$$

where \mathcal{R} is the standard (scalar) comoving curvature perturbation, and h_+ and h_\times are the two standard gravity wave polarizations.

We also show in Appendix B that, in the isotropic regime, our formalism reproduces the standard evolution equations for these modes. These equations are decoupled, so that the modes evolve independently from each other (at the linearized level). However, the two 2D scalar modes V_+ and H_+ are coupled to each other, and to the mode Δ_+ during inflation, when the background is anisotropic. The mode H_\times is always decoupled from these three modes, but it is coupled to the mode Δ_\times during inflation. The coupling modifies the diagonal correlation functions $\langle V_+^2 \rangle$, $\langle H_+^2 \rangle$, $\langle \Delta_+^2 \rangle$, $\langle H_\times^2 \rangle$, $\langle \Delta_\times^2 \rangle$ with respect to the standard inflationary results, and it introduces the nondiagonal correlations $\langle V_+ H_+ \rangle$, $\langle V_+ \Delta_+ \rangle$, $\langle H_+ \Delta_+ \rangle$, $\langle H_\times \Delta_\times \rangle$, which are absent in standard inflation.

As the Universe isotropizes, the two modes Δ_+ and Δ_\times become the two transverse polarizations of the vector field. These modes rapidly decrease after inflation (when the evolution of the vector becomes standard). Therefore, all correlators involving these modes become negligible at late times, and we disregard them in the remainder of this work. The remaining correlators can be written as in Eq. (45), in terms of the power spectra

²⁰The explicit proof of this is given in Sec. III of [32] for details.

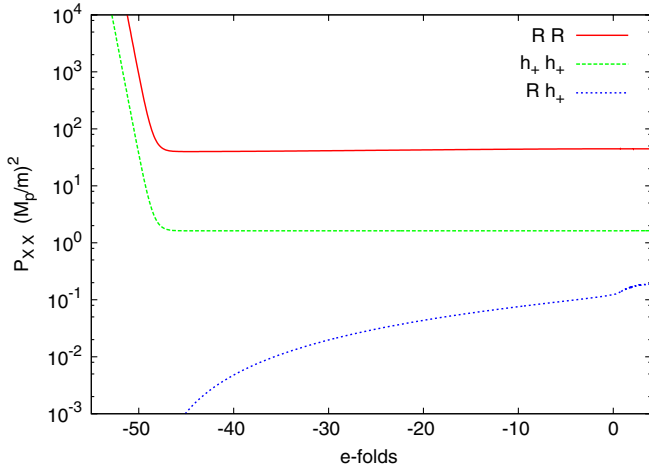


FIG. 2 (color online). Time evolution for the power of a specific mode, on a nearly isotropic background ($c - 1 = 10^{-5}$). The number of e-folds α is used as a “time” variable, and it is normalized to 0 at the end of inflation. The mode shown leaves the horizon about 50 e-folds before the end of inflation. See the main text for details.

$$\begin{aligned}
 P_{\mathcal{R}\mathcal{R}} &= \frac{1}{2\pi^2} \frac{H^2}{\dot{\phi}^2} p^3 (Y_s Y_s^\dagger)_{11} \\
 P_{\mathcal{R}h_+} &= -\frac{1}{\sqrt{2}\pi^2} \frac{H}{\dot{\phi}M_p} p^3 \text{Re}[(Y_s Y_s^\dagger)_{12}] \\
 P_{h_+h_+} &= \frac{1}{\pi^2 M_p^2} p^3 (Y_s Y_s^\dagger)_{22} \\
 P_{h_\times h_\times} &= \frac{1}{\pi^2 M_p^2} p^3 (Y_v Y_v^\dagger)_{11}.
 \end{aligned} \tag{54}$$

To evaluate the power spectra, we impose the initial conditions on Y as discussed at the end of the previous subsection. We then evolve Y through their equations of motion (51). We remark that the resulting power spectra are dimensionless, and are written in terms of only physical quantities (namely, they are insensitive to the normalization of the scale factors; this is because only physical quantities appear in the initial conditions and the evolution equations for Y).

Our results are shown in Figs. 2–4. Figure 2 shows the evolution of the power for the specific value of $c - 1 = 10^{-5}$ in the kinetic function (9), and for a specific mode: we denote by k_* the magnitude of the momentum of a mode which barely exits the horizon at the end of inflation; we choose a mode with momentum $k = 10^{-20} k_*$, and we choose the cosine of the angle between the momentum and the privileged direction to be $\xi = 1/2$ (such values have no particular meaning, and are just chosen for illustrative purposes; different values of k and ξ lead to the same qualitative behavior); this mode is initially deeply inside the horizon, and it leaves the horizon about 50 e-

folds before the end of inflation.²¹ The power in the scalar-scalar and tensor-tensor correlations behaves analogously to the isotropic case. The power in the scalar-tensor cross correlation is instead very small initially, and slightly increases during and immediately after inflation. The power becomes constant (at the value seen in the latest time shown in the figure) as the Universe isotropizes after inflation.

In Fig. 3 we show the power spectra for the specific value of $c - 1 = 10^{-5}$ in the kinetic function (9). The smallest momentum shown corresponds to modes that exited the horizon about 60 e-folds before the end of inflation. For the scalar-scalar and tensor-tensor case, the standard result is also shown for comparison. The scalar-scalar power spectrum (upper left panel) is slightly greater than in the isotropic case; this can be compensated by decreasing the scalar field mass (for this reason, the ratio m/M_p has been kept as a free parameter in the figure). The angular dependence of $P_{\mathcal{R}\mathcal{R}}$ is of $O(10^{-1})$ at the largest scales, while it slowly decreases at greater scales. The tensor-tensor power spectra (bottom panels) are closer to the standard result, and they exhibit a much milder angular dependence [we found g_* , defined in Eq. (1), for the scalar spectra is suppressed with respect to g_* in the tensor spectrum by approximately the ratio between the power of the tensor and the scalar spectra]. Moreover, the results for the two polarizations are nearly identical. The scalar-tensor cross correlation (upper right panel) shows a stronger angular dependence, but it is smaller than the other two spectra.

In Fig. 4 we show the angular dependence g_* for the scalar-scalar power spectrum, for different values of c in Eq. (9). We remark that all values of c provide a negative g_* (in Fig. 4 we actually show $|g_*| = -g_*$ in logarithmic scale). The value of g_* shown in the plots is obtained by comparing, for each value of $k = |\mathbf{k}|$, the power at $\xi = 0.1$ and at $\xi = 0.9$. We have however verified that the ACW parametrization (1) is very accurate, in the sense that, once g_* and $P(k)$ are obtained from the results at $\xi = 0.1$ and $\xi = 0.9$, also other values of ξ are fitted very well by (1).

The significance of the results summarized in these figures is discussed in the next concluding section.

IV. DISCUSSION

In this work we compute the precise phenomenological signatures for the model of [36]. This model admits an anisotropic inflationary background evolution, supported by the combined presence of a scalar and a vector field. This solution is (mathematically) continuously connected to an isotropic solution, in the limit in which the energy

²¹Since this choice of $c \approx 1$ corresponds to a very small anisotropy, we define the horizon, and the number of e-folds, only through the evolution of the “average” scale factor e^α , as in FRW cosmology.

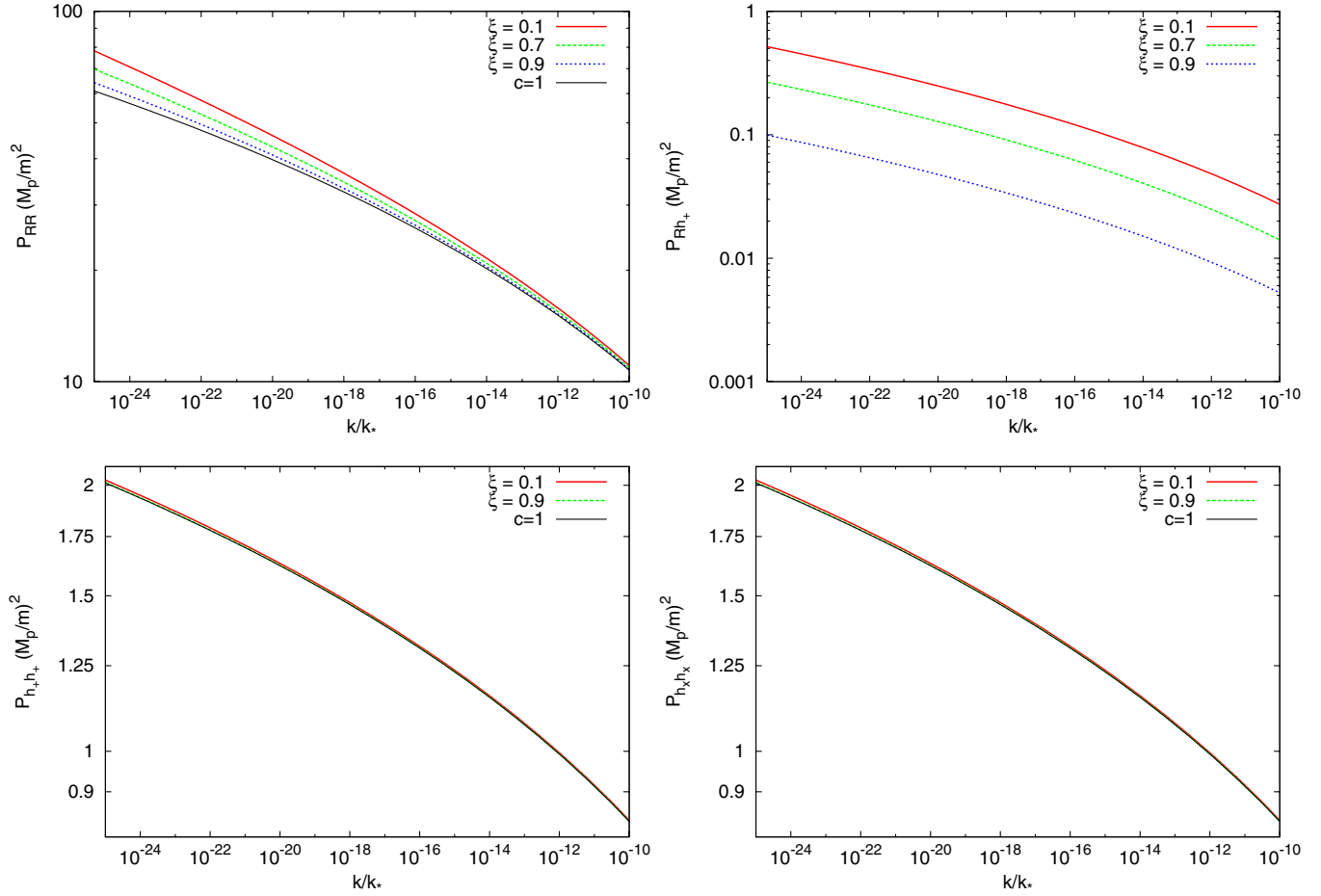


FIG. 3 (color online). Scalar-scalar, scalar-tensor, and tensor-tensor power spectra for a nearly isotropic background ($c - 1 = 10^{-5}$). Notice that the scale on the y axis is different for the different panels. For the scalar-scalar, and the tensor-tensor spectra, we also show the standard result for comparison (in the present model, the isotropic limit is reached for $c = 1$).

density associated to the vector is sent to zero. Moreover, the model is free of the ghost instabilities that plague other models with vector fields during inflation, due to the U(1) invariance of its action under a shift of the vector. Therefore, it offers a complete, and stable counterexample to Wald’s no hair theorem on the isotropization of Bianchi spaces [22].

We studied the simplest realization of the idea of [36], in which a single vector is present, and the potential of the scalar is taken to be that of massive chaotic inflation. We found that the scalar-scalar correlation function exhibits an angular dependence which is however of the wrong sign to account for the breaking of rotational invariance seen in the data: the model gives a negative value for g_* , while the analysis of [17] indicates that $g_* = 0.29 \pm 0.031$ in the WMAP W band. We also found that the amount of anisotropy in the spectrum (the order of magnitude of $|g_*|$) is not of the same order of magnitude as the amount of anisotropy in the expansion (the order of magnitude of $\Delta H/H \sim \dot{\sigma}/\dot{\alpha}$, where H is the average expansion rate, and ΔH the difference between the expansion rate in the different

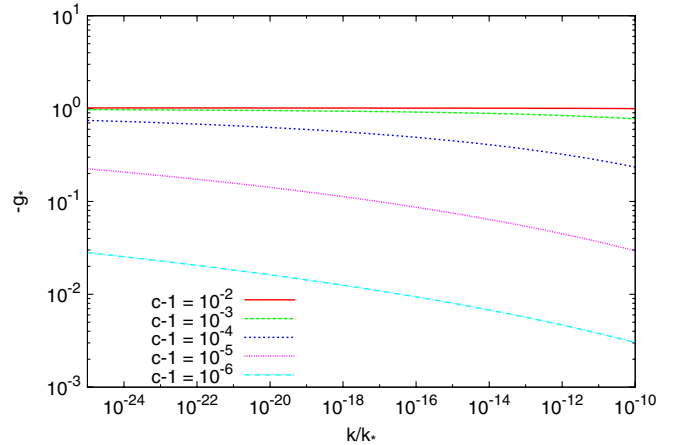


FIG. 4 (color online). Angular dependence (expressed through the ACW g_* factor) of the scalar-scalar power spectrum for different choices of c .

coordinates). A $|g_*| = O(10^{-1})$ is obtained when the anisotropy in the expansion is of $O(10^{-7}-10^{-6})$ during inflation. It is also worth noting that the anisotropy in the tensor-tensor spectrum is much milder than that in the scalar-scalar spectrum (we find that the suppression is approximately proportional to the ratio between the two power spectra).

A characteristic feature of the model (and, of anisotropic spaces in general) is a nonvanishing scalar-tensor correlation, which, if sufficiently high, may give a detectable distinctive (from the scalar-scalar one) contribution to the T-T and T-E correlations. A naive estimate actually suggests that the scalar-tensor correlation could be higher than the tensor-tensor one. Indeed the amplitude of a tensor mode is approximately multiplied by $2\sqrt{\epsilon}$ with respect to that of a scalar mode, where $\epsilon \equiv -\dot{H}/H^2$ is a slow roll parameter.²² The tensor-tensor correlator is suppressed by one more power of this factor than the tensor-scalar correlator. However, the scalar-tensor correlator vanishes for an isotropic space, and hence must be suppressed by some factor related to the asymmetry. If one naively assumes that this factor is $|g_*|$, one would get the prediction that $P_{\mathcal{R}h}/P_{hh} \simeq |g_*|/(2\sqrt{\epsilon})$. One would then get $P_{\mathcal{R}h}/P_{hh} \simeq 6|g_*|$ for chaotic inflation, and a greater value for other inflationary models, characterized by a smaller value of ϵ . Our explicit results show that this estimate is reasonable, but not exact. For the case of $c - 1 = 10^{-5}$ shown in Fig. 3 (giving $g_* \simeq -0.23$), the estimate gives $P_{\mathcal{R}h}/P_{hh} \simeq 1.4$, while the actual spectra give $P_{\mathcal{R}h}/P_{hh} \simeq 0.05-0.25$, depending on the orientation of the mode (the value of ξ).

To conclude, while the simplest realization of [36] cannot explain the breaking of rotational invariance seen in the data, and, most likely, it does not give rise to an interesting scalar-tensor correlation, this model is, to our knowledge, the first complete and stable model of anisotropic inflation for which the phenomenological predictions strictly follow from the action (and not from arbitrary initial conditions), and have been computed. As the observed breaking of rotational invariance awaits for a confirmation, or a refutation, from Planck, our work provides the tools for studying different models, to see whether they can reproduce the WMAP feature, and perhaps lead to new predictions.

ACKNOWLEDGMENTS

The work of A. E. G. and of M. P. was partially supported by the DOE Grant No. DE-FG02-94ER-40823. The work of B. H. was supported by the Graduate School at the University of Minnesota.

Note added.—After the completion of the analysis reported here, and as we were finalizing the preparation of the present manuscript for submission, Ref. [45] appeared on the preprint archive, which also computed some of the correlation functions computed here. While we obtained the expressions for the correlators analytically, and we then evaluated them numerically, Ref. [45] performs a fully analytical computation, treating the anisotropy as a small perturbation. The relation between g_* and $\dot{\sigma}/\dot{\alpha}$ found in [45] and in the present work are in very good agreement with each other. The two studies also agree on the fact that the tensor-tensor correlator shows a smaller angular dependence than the scalar-scalar one. The scalar-tensor correlator has not been computed in [45].

APPENDIX A: EXPLICIT QUADRATIC ACTION OF THE PERTURBATIONS

We provide here the explicit expressions for the matrices entering in the action and the evolution equations of the canonical modes, Eqs. (21) and (48).

For the 2D scalar modes we have

$$K_s = \begin{pmatrix} 0 & 0 & K_{13,s} \\ 0 & 0 & K_{23,s} \\ -K_{13,s} & -K_{23,s} & 0 \end{pmatrix}, \quad (A1)$$

$$\Omega_s^2 = \begin{pmatrix} \Omega_{11,s}^2 & \Omega_{12,s}^2 & \Omega_{13,s}^2 \\ \Omega_{12,s}^2 & \Omega_{22,s}^2 & \Omega_{23,s}^2 \\ \Omega_{13,s}^2 & \Omega_{23,s}^2 & \Omega_{33,s}^2 \end{pmatrix}$$

where

²²Since we are considering a small anisotropy, we can use the FRW computation in this estimate; then, comparing the two equations in (B9) we see that $|V_+| \simeq |H_{+, \times}|$. This gives $|h_{+, \times}/\mathcal{R}| \simeq \sqrt{2}\dot{\phi}/(M_p H) \simeq 2\sqrt{\epsilon}$.

$$\begin{aligned}
K_{13,s} &= -\frac{\tilde{p}_A p_T f'(\phi)}{p f(\phi)^2}, & K_{23,s} &= -\frac{\tilde{p}_A p_T}{\sqrt{2} M_p p f(\phi)} \\
\Omega_{11,s}^2 &= p^2 - \frac{9}{4} \dot{\alpha}^2 + \frac{15 \dot{\phi}^2}{4 M_p^2} + \frac{9}{2} \dot{\sigma}^2 + \frac{2 \dot{\phi} V'(\phi)}{M_p^2 \dot{\alpha}} + V''(\phi) - 2 \frac{p^4}{\mathcal{D}^2} \frac{\dot{\phi}^4}{M_p^4} + \frac{2(p_T^2 - 2p_L^2) V'(\phi)}{\mathcal{D}} \frac{\dot{\phi}}{M_p^2 \dot{\alpha}} \dot{\sigma} \\
&\quad - 3 \left(4 \frac{(2p_L^2 - p_T^2) p^2}{\mathcal{D}^2} \dot{\alpha} + \frac{8p_L^4 - 8p_L^2 p_T^2 + 5p_T^4}{\mathcal{D}^2} \dot{\sigma} \right) \frac{\dot{\phi}^2}{M_p^2} \dot{\sigma} + \frac{\tilde{p}_A^2}{2M_p^2 f(\phi)^2} \left[1 + \frac{2M_p^2 (3p_L^2 - p_T^2) f'(\phi)^2}{p^2 f(\phi)^2} \right. \\
&\quad \left. - 4 \frac{p_T^2 p^2}{\mathcal{D}^2} \frac{\dot{\phi}^2}{M_p^2} - 8 \frac{p_L^2}{\mathcal{D}} \frac{f'(\phi)}{f(\phi)} \dot{\phi} - 2M_p^2 \frac{f''(\phi)}{f(\phi)} \right] \\
\Omega_{12,s}^2 &= -\frac{3\sqrt{2} p_T^2 p^2 \dot{\sigma}}{\mathcal{D}^2} \left[\frac{\dot{\phi}^3}{M_p^3} - 6 \frac{\dot{\phi}}{M_p} (\dot{\alpha} + \dot{\sigma}) \left(\dot{\alpha} + \frac{p_L^2 - p_T^2}{p^2} \dot{\sigma} \right) - \frac{V'(\phi)}{M_p} \left(2\dot{\alpha} + \frac{2p_L^2 - p_T^2}{p^2} \dot{\sigma} \right) \right] \\
&\quad - \frac{\sqrt{2} \tilde{p}_A^2 p_T^2}{M_p^2 f(\phi)^2 \mathcal{D}^2} \left[3p_T^2 \frac{\dot{\phi}}{M_p} \dot{\sigma} + M_p \frac{f'(\phi)}{f(\phi)} \left(4p^2 \dot{\alpha}^2 + 2(7p_L^2 - 2p_T^2) \dot{\alpha} \dot{\sigma} + \frac{(2p_L^2 - p_T^2)(5p_L^2 - p_T^2)}{p^2} \dot{\sigma}^2 \right) \right] \\
\Omega_{13,s}^2 &= \frac{\tilde{p}_A p_T}{p f(\phi)} \left\{ -\frac{2\tilde{p}_A^2 p^2}{M_p^2 f(\phi)^2 \mathcal{D}^2} \left[\frac{\dot{\phi}}{M_p^2} p_T^2 + \frac{f'(\phi)}{f(\phi)} p_L^2 \left(2\dot{\alpha} + \frac{2p_L^2 - p_T^2}{p^2} \dot{\sigma} \right) \right] - 2 \frac{p^4}{\mathcal{D}^2 M_p} \left[\frac{\dot{\phi}^3}{M_p^3} - 6 \frac{\dot{\phi}}{M_p} (\dot{\alpha} + \dot{\sigma}) \right. \right. \\
&\quad \left. \left. \times \left(\dot{\alpha} + \frac{p_L^2 - p_T^2}{p^2} \dot{\sigma} \right) \right] + 2 \frac{p^4}{\mathcal{D}^2 M_p} \frac{V'(\phi)}{M_p} \left(2\dot{\alpha} + \frac{2p_L^2 - p_T^2}{p^2} \dot{\sigma} \right) - \frac{f'(\phi)}{f(\phi)} \left(\dot{\alpha} + \frac{7p_L^2 - 2p_T^2}{p^2} \dot{\sigma} \right) + \dot{\phi} \frac{f''(\phi)}{f(\phi)} \right\} \\
\Omega_{22,s}^2 &= p^2 - \frac{1}{4} \dot{\alpha}^2 + \frac{3\dot{\phi}^2}{4M_p^2} - 8 \frac{p^4}{\mathcal{D}^2} \dot{\alpha}^4 - \frac{9p_L^4 \dot{\sigma}^2}{\mathcal{D}^2} \frac{\dot{\phi}^2}{M_p^2} - 8 \frac{(2p_L^2 - p_T^2) p^2}{\mathcal{D}^2} \dot{\alpha}^3 \dot{\sigma} + 2 \frac{5p_L^4 + 58p_L^2 p_T^2 + 35p_T^4}{\mathcal{D}^2} \dot{\alpha}^2 \dot{\sigma}^2 \\
&\quad + 18 \frac{2p_L^4 + 9p_L^2 p_T^2 - p_T^4}{\mathcal{D}^2} \dot{\alpha} \dot{\sigma}^3 + \frac{9}{2} \frac{4p_L^4 + 12p_L^2 p_T^2 - 11p_T^4}{\mathcal{D}^2} \dot{\sigma}^4 + \frac{\tilde{p}_A^2}{2M_p^2 f(\phi)^2} \frac{p_L^2 - p_T^2}{p^2} - \frac{9\tilde{p}_A^2}{M_p^2 f(\phi)^2} \frac{p_T^6}{p^2 \mathcal{D}^2} \dot{\sigma}^2 \\
\Omega_{23,s}^2 &= \frac{\tilde{p}_A p_T}{\sqrt{2} M_p p f(\phi)} \left\{ -\frac{6\tilde{p}_A^2}{M_p^2 f(\phi)^2} \frac{p_T^4}{\mathcal{D}^2} \dot{\sigma} + \frac{f'(\phi)}{f(\phi)} \dot{\phi} - 4 \frac{p^4}{\mathcal{D}^2} \dot{\alpha}^3 - 6 \frac{p_T^2 p^2}{\mathcal{D}^2} \left(\frac{\dot{\phi}^2}{M_p^2} - 2 \frac{p_L^2 + 4p_T^2}{p_T^2} \dot{\alpha}^2 \right) \dot{\sigma} \right. \\
&\quad \left. + 9 \frac{4p_L^4 + 16p_L^2 p_T^2 - p_T^4}{\mathcal{D}^2} \dot{\alpha} \dot{\sigma}^2 + \frac{20p_L^6 + 96p_L^4 p_T^2 - 39p_L^2 p_T^4 - 34p_T^6}{p^2 \mathcal{D}^2} \dot{\sigma}^3 \right\} \\
\Omega_{33,s}^2 &= p^2 + \frac{\dot{\phi}^2}{4M_p^2} - \frac{1}{4} \dot{\alpha}^2 - \frac{p_L^2 - 2p_T^2}{2p^2} \left(4\dot{\alpha} - \frac{p_L^4 + 50p_L^2 p_T^2 - 5p_T^4}{p^2 (p_L^2 - 2p_T^2)} \dot{\sigma} \right) \dot{\sigma} + \left[V'(\phi) + 2\dot{\phi} \dot{\alpha} + 2 \frac{p_L^2 - 2p_T^2}{p^2} \dot{\phi} \dot{\sigma} \right] \frac{f'(\phi)}{f(\phi)} \\
&\quad - \dot{\phi}^2 \frac{f''(\phi)}{f(\phi)} - \frac{\tilde{p}_A^2 p^2 p_T^2}{2M_p^2 f(\phi)^2 \mathcal{D}^2} \left[\frac{4\tilde{p}_A^2}{M_p^2 f(\phi)^2} \frac{p_T^2}{p^2} + \frac{2M_p^2 \mathcal{D}^2}{p^2 p_T^2} \frac{f'(\phi)^2}{f(\phi)^2} + \frac{4\dot{\phi}^2}{M_p^2} - 20 \frac{p^2}{p_T^2} \dot{\alpha}^2 \right. \\
&\quad \left. - 4 \frac{10p_L^4 + 17p_L^2 p_T^2 + p_T^4}{p^2 p_T^2} \dot{\alpha} \dot{\sigma} - \frac{(2p_L^2 + 5p_T^2)(10p_L^4 - p_L^2 p_T^2 - 5p_T^4)}{p^4 p_T^2} \dot{\sigma}^2 \right] \tag{A2}
\end{aligned}$$

and where we have defined

$$\mathcal{D} \equiv 2p^2 \dot{\alpha} + (2p_L^2 - p_T^2) \dot{\sigma}. \tag{A3}$$

For the 2D vector modes we have instead

$$K_\nu = \begin{pmatrix} 0 & K_{12,\nu} \\ -K_{12,\nu} & 0 \end{pmatrix}, \quad \Omega_\nu^2 = \begin{pmatrix} \Omega_{11,\nu}^2 & \Omega_{12,\nu}^2 \\ \Omega_{12,\nu}^2 & \Omega_{22,\nu}^2 \end{pmatrix} \tag{A4}$$

where

$$\begin{aligned}
K_{12,v} &= -\frac{\tilde{p}_A |p_T|}{\sqrt{2} M_p f(\phi) p} & \Omega_{11,v}^2 &= p^2 - \frac{9}{4} \dot{\alpha}^2 - \dot{\sigma}^2 \left[\frac{9}{2} - 36 \frac{p_L^2}{p^2} + 27 \frac{p_L^4}{p^4} \right] + \frac{3\phi^2}{4M_p^2} - \frac{\tilde{p}_A^2}{2M_p^2 f(\phi)^2} \left[1 - 2 \frac{p_L^2}{p^2} \right] \\
\Omega_{22,v}^2 &= p^2 - \frac{\dot{\alpha}^2}{4} - 2\dot{\alpha}\dot{\sigma} + \frac{\dot{\sigma}^2}{2} + \frac{\phi^2}{4M_p^2} + \frac{\tilde{p}_A^2}{2M_p^2 f(\phi)^2} \left[1 + 4 \frac{p_L^2}{p^2} \right] + [V'(\phi) + 2(\dot{\alpha} + \dot{\sigma})\dot{\phi}] \\
&\quad \times \frac{f'(\phi)}{f(\phi)} - \tilde{p}_A^2 \frac{f'(\phi)^2}{f(\phi)^4} - \dot{\phi}^2 \frac{f''(\phi)}{f(\phi)} \\
\Omega_{12,v}^2 &= \frac{\tilde{p}_A |p_T|}{\sqrt{2} M_p f(\phi) p} \left[-\dot{\alpha} + \left(9 \frac{p_L^2}{p^2} - 4 \right) \dot{\sigma} + \frac{\dot{\phi} f'(\phi)}{f(\phi)} \right].
\end{aligned} \tag{A5}$$

The physical momenta entering in these expressions are related to the comoving ones given in the main text by

$$p_L \equiv \frac{k_L}{a(t)}, \quad p_T \equiv \frac{k_T}{b(t)}, \quad p \equiv \sqrt{p_L^2 + p_T^2}. \tag{A6}$$

APPENDIX B: GAUGE INVARIANT PERTURBATIONS IN TERMS OF PERTURBATIONS IN OUR GAUGE, AND LATE TIME ISOTROPIC LIMIT

In this appendix we discuss the late time interpretation of the perturbations in the gauge chosen in the main text. This interpretation is done when the background has become isotropic, and the VEV of the vector has gone to zero. In this case, the Fourier coefficients of our metric perturbations read

$$\delta g_{\mu\nu}(k) = \begin{pmatrix} -2\Phi & ia k_L \chi & ia(k_{T2} B + k_{T3} B_v) & ia(k_{T3} B - k_{T2} B_v) \\ -2a^2 \Psi & & -a^2 k_L k_{T3} \tilde{B}_v & a^2 k_L k_{T2} \tilde{B}_v \\ 0 & & 0 & 0 \\ 0 & & 0 & 0 \end{pmatrix}, \tag{B1}$$

as can be seen by imposing the gauge (17) on the parametrization given by (14) and (16). We recall that (k_L, k_{T2}, k_{T3}) denotes the comoving momentum of the mode we are studying.²³ We perform a rotation to a coordinate system for which the momentum is along the third direction, $k_\mu \rightarrow \tilde{k}_\mu = R_\mu^\nu k_\nu = (0, 0, \sqrt{k_L^2 + k_{T2}^2 + k_{T3}^2})$. The explicit form of the rotation matrix is

$$R_\nu^\mu = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -\frac{k_T}{k} & \frac{k_L k_{T2}}{k_T k} & \frac{k_L k_{T3}}{k_T k} \\ 0 & 0 & \frac{k_{T3}}{k_T} & -\frac{k_{T2}}{k_T} \\ 0 & \frac{k_L}{k} & \frac{k_{T2}}{k} & \frac{k_{T3}}{k} \end{pmatrix} \tag{B2}$$

where $k_T = \sqrt{k_{T2}^2 + k_{T3}^2}$, and the metric transforms as

$$g_{\mu\nu} \rightarrow \tilde{g}_{\mu\nu} = R_\mu^\alpha g_{\alpha\beta} R_\nu^\beta. \tag{B3}$$

Although the gauge in which Eq. (B1) appears is non-standard, we can combine the metric perturbations (B1) into the gauge invariant expressions that are commonly used. These gauge invariant combinations are usually expressed starting from the most general metric perturba-

tions, classified as scalar, vector, or tensor, with respect to 3D spatial rotations:

$$\begin{aligned}
\delta g_{00} &= -2\Phi_*, & \delta g_{0i} &= 2a(B_{*i} + \partial_i B_*), \\
\delta g_{ij} &= a^2[-2\Psi_* \delta_{ij} + 2E_{*,ij} + E_{*(i,j)} + h_{*ij}]
\end{aligned} \tag{B4}$$

where $i = 1, 2, 3$. The (3D) vectors B_{*i} and E_{*i} are transverse, while the tensor mode h_{*ij} is transverse and traceless. The remaining modes are scalar, and are coupled also to the perturbation of the scalar field $\delta\phi_*$. Out of these modes, we are interested in the gauge invariant scalar combination [20,44]

$$\mathcal{R} \equiv \Psi_* + \frac{H}{\dot{\phi}} \delta\phi_* \tag{B5}$$

and in the two (gauge invariant) tensor mode polarizations h_+ and h_\times encoded in h_{*ij} (the vector modes disappear once the Universe becomes isotropic).

Expression (B4) gives the metric perturbations before any gauge is chosen. By equating them with our expressions (B1) we find how our modes can be decomposed into 3D scalar, vector, and tensor perturbations. We can then use the resulting expressions to write \mathcal{R} , h_+ , and h_\times in terms of our modes. To make this identification, we should spell out explicitly how the components of the 3D vector

²³In the main text, the suffices L and T refer to ‘‘longitudinal’’ or ‘‘transverse’’ with respect to direction x , which was the anisotropic one during inflation.

and tensor modes enter in (B4), accounting for their transversality, and traceless properties. We do so in the \tilde{x}^μ coordinate system, for which the momentum of the mode is $\tilde{k}_\mu = (0, 0, \sqrt{k_L^2 + k_{T2}^2 + k_{T3}^2})$. In this system, Eqs. (B4) give

$$\tilde{\delta}g_{\mu\nu}(k) = \begin{pmatrix} -2\Phi_* & aB_{*1} & aB_{*2} & aikB_* \\ a^2(-2\Psi_* + h_+) & a^2h_\times & a^2(-2\Psi_* - h_+) & a^2ikE_{*1} \\ a^2(-2\Psi_* - h_+) & a^2h_\times & a^2(-2\Psi_* - h_+) & a^2ikE_{*2} \\ a^2(-2\Psi_* - 2k^2E_*) & & & a^2(-2\Psi_* - 2k^2E_*) \end{pmatrix}. \quad (\text{B6})$$

The entries of this metric can be now identified with those of our metric (B1), transformed according to (B3). We obtain

$$\mathcal{R} = \frac{k_T^2}{2k^2}\Psi + \frac{H}{\dot{\phi}}\delta\phi, \quad h_+ = -\frac{k_T^2}{k^2}\Psi, \quad h_\times = \frac{k_L k_T^2}{k}\tilde{B}_v. \quad (\text{B7})$$

Finally, we rewrite our three variables Ψ , $\delta\phi$, and \tilde{B}_v in terms of the canonically normalized modes introduced in (19) and (20) [see also Eq. (16)]. This leads to

$$\begin{aligned} \mathcal{R} &= \frac{H}{a^{3/2}\dot{\phi}}V_+, & h_+ &= -\frac{\sqrt{2}}{a^{3/2}M_p}H_+, \\ h_\times &= \frac{i\sqrt{2}}{a^{3/2}M_p}H_\times. \end{aligned} \quad (\text{B8})$$

As a check, we can verify that our evolution equations reduce to the standard ones in the limit of isotropic background. Using $\tilde{p}_A = \dot{\sigma} = 0$ in the explicit expressions given in Appendix A, we find that $K_{s,v} = 0$ and $\Omega_{s,v}^2 = 0$

diagonal in this limit. Therefore, all the canonical modes are decoupled. One can then show that, in this limit Eqs. (48) give

$$\begin{aligned} \left(\frac{V_+}{\sqrt{a}}\right)'' + \left[k^2 - \frac{z''}{z}\right]\left(\frac{V_+}{\sqrt{a}}\right) &= 0, & z &\equiv \frac{a^2\dot{\phi}}{a} \\ \left(\frac{H_{+, \times}}{\sqrt{a}}\right)'' + \left[k^2 - \frac{a''}{a}\right]\left(\frac{H_{+, \times}}{\sqrt{a}}\right) &= 0 \end{aligned} \quad (\text{B9})$$

where prime denotes differentiation with respect to conformal time η , related to the physical time t by $d\eta = dt/a$. From these expressions, and from Eqs. (B8) we find

$$\begin{aligned} v'' + \left(k^2 - \frac{z''}{z}\right)v &= 0, \\ (ah_{+, \times})'' + \left(k^2 - \frac{a''}{a}\right)(ah_{+, \times}) &= 0 \end{aligned} \quad (\text{B10})$$

where $v \equiv z\mathcal{R}$. These equations are the standard ones [20], confirming that our formalism reduces to the standard one in the limit of isotropic background.

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