Quasar variability limits on cosmological density of cosmic strings

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We put robust upper limits on the average cosmological density Ω_s of cosmic strings based on the variability properties of a large homogeneous sample of Sloan Digital Sky Survey quasars. We search for an excess of characteristic variations of quasar brightness that are associated with string lensing and use the observed distribution of these variations to constrain the density of strings. The limits obtained do not invoke any clustering of strings. They apply to both open segments and closed loops of strings, usefully extend over a wide range of tensions $10^{-13} < G\mu/c^2 < 10^{-9}$, and reach down to the level of $\Omega_s = 0.01$ and below. Further progress in this direction will depend on better understanding of quasar intrinsic variability rather than a mere increase in the volume of data.

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I. INTRODUCTION

Linear topological defects arise naturally during phase transitions in diverse areas of physics. Various processes in the early Universe could also produce such defects, which are called cosmic strings [1–3]. It is often assumed that phase transitions lead to formation of strings with a characteristic tension of order the squared energy scale of the string-producing theory (in Planck units). After formation, the strings build up an intricate network, combined from open segments of the horizon scale and a multitude of loops that detach during the evolution of the network in interconnections of open strings and smoothing of their small-scale structure. The network evolves perpetually as both long segments and open loops move and oscillate at relativistic velocities.

Strings are believed to be a subdominant species in the matter-energy balance of the Universe. Analytical calculations and numerical simulations indicate that string networks, early in the course of their evolution, can reach a scaling behavior where a typical distance between the strings increases in proportion to the horizon scale d_h [4–10]. This corresponds to the density of strings ρ decreasing with redshift z as $\rho(z) \propto d_h^{-2}(z)$, although one should keep in mind that these results were obtained for radiation- and matter-dominated eras with no contribution from the vacuum energy. In the matter-dominated era this dependence coincides with that for cold matter $\rho(z) \propto (1 + z)^3$, which appears to be a natural behavior for networks dominated by noninteracting loops, whose density would decrease solely due to the universal expansion. For subsequent calculations, we will use both relations, $\rho(z) \propto d_h^{-2}(z)$ and $\rho(z) \propto$ $(1 + z)^3$; the law followed by the strings in the actual Universe is likely to be an interpolation between these two cases.

Despite being a natural prediction of many cosmological theories, cosmic strings have not been observed yet [11–13], which raises an obvious question about the origin of this discrepancy. Attempts to answer this question would benefit from an estimate of the actual density of stings in the real Universe or, in the absence of their detection, an upper limit on this parameter. However, observational estimates of this kind are surprisingly scarce [14–18]. In a recent paper [19], we constrained the local (at ~1 kpc scale) density of light ($10^{-16} < G\mu/c^2 < 10^{-10}$) cosmic string loops using their observational signatures in pulsar timing and precision photometric surveys. These constraints were made possible by significant enhancement in the local density of strings due to clustering of string loops expected in the Galaxy [20]. However, this enhancement is subject to theoretical uncertainties that are hard to quantify at our current level of understanding of cosmic strings.

In the present work, we derive robust observational upper limits on the average cosmological density of cosmic strings that are independent of any clustering effects and apply to both open segments and closed loops of strings. Instead of the local enhancement, we rely on giant distances to and a large number of extragalactic objects, namely, quasars, used in deriving these constraints. Our method is based on the statistical analysis of quasar variability obtained for a large sample of quasars from the Sloan Digital Sky Survey (SDSS) catalogue [21]. Lensing by cosmic strings that are heavy enough ($G\mu/c^2 > 10^{-14}$–10^{-12}, though this is somewhat quasar-model-dependent) would lead to an excess of twofold jumps in the distribution of brightness variation between two obser-

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vational epochs. Hence, the absence of any such features in the observed distribution allows us to infer robust upper limits on the density of strings.

The paper is organized as follows. In Sec. II, we elaborate on the idea above to see how the observed distribution of the variability in an ensemble of quasars can be used to put an upper limit on the probability of string lensing. Section III relates this limit to the density of strings by calculating the probability of lensing as a function of string and source parameters. Then, in Sec. IV we use observational data to obtain actual limits on the density of strings from the SDSS data. Finally, in Sec. V we present our results in Fig. 2 and conclude with a short discussion.

All numerical calculations are made for a standard flat cosmological model with a cosmological constant $\Lambda = 1 - \Omega$, cold matter density $\Omega = 0.27$ [22], and the present-day Hubble constant $H_0 = 71$ km $\cdot$ s^{-1} $\cdot$ Mpc^{-1} [23]; we assume that strings do not contribute appreciably to the energy budget.

II. METHOD: OBSERVATIONAL LIMITS ON PROBABILITY OF FLUX DOUBLING

Cosmic strings produce a distinctive pattern of lensing; for a point source inside a narrow strip along the string, a second positive-parity image appears in a duplicate strip on the other side of the string [24–26]. In the following, we assume that at any given moment the source is crossed by at most one string. This is a sensible approximation given that, otherwise, lensing by cosmic strings would be ubiquitous and would likely have been detected by now. A formal demonstration of the plausibility of this assumption relies on the fact that the strips' intersections cover a fractional area $\propto \tau^2$ in projection, where τ is the optical depth to string lensing. For cosmologically faraway sources ($z_s \sim 1$), the string lensing optical depth, like that for point lenses, is of order the fraction of critical density Ω_s in the strings [19], which is unlikely to be greater than unity.

When an extended source being crossed by the string cannot be resolved, the observer sees a flux increase given by the flux in the part of the source that is momentarily inside the strip. As the source moves into the strip and the duplicate image appears, the total flux rises gradually from a flat unlensed "bottom" of the light curve to some maximum value. Then, as the original source disappears behind the string while the duplicate image leaves the duplicate strip, the total flux goes down towards the same bottom. The exact shape of the light curve depends on the brightness distribution in the source, and the maximum is determined by the size of the source in relation to the strip, which can only be guessed in the case of quasars. However, for small enough sources that fit into the strip completely, the maximum increase is exactly twofold, which is $\Delta m_0 = 2.5 \lg 2 \approx 0.75^m$ in terms of stellar magnitudes. Moreover, string lensing light curves of such sources possess a char-

acteristic extended "plateau" at this level; its width is given by the time it takes the source to traverse the strip.

The distinctive shape of the light curve readily imprints itself in the distribution density of the magnification μ of a small source lensed by a cosmic string. This function consists of a certain smooth component at $1 < \mu < 2$ and a pair of δ functions that correspond to the unlensed case $\mu = 1$ and the maximally lensed case $\mu = 2$; the latter events have a nonzero measure due to the extended nature of the corresponding "bottom" and "plateau" of the light curve. The distribution density of the magnification allows one to calculate the density $p(\Delta m)$ of the magnitude jump Δm in two observations due to a change in the magnification factor between the corresponding epochs. This function is even because of the symmetry between the two epochs and consists of three δ functions at $\Delta m = 0$ and $\Delta m = \pm \Delta m_0$ on top of a smooth component $\bar{p}(\Delta m)$ for $-\Delta m_0 < \Delta m < \Delta m_0$:

$$p(\Delta m) = \bar{p}(\Delta m) + P\delta(\Delta m \mp \Delta m_0) + Q\delta(\Delta m). \quad (1)$$

For sources of an unknown brightness profile, it is not possible to calculate the smooth component of this distribution; however, the amplitudes of δ functions can be calculated rather straightforwardly, as shown in the next section. If lensing by cosmic strings is a rare phenomenon, which we will assume, $2P$ is essentially the optical depth [see Eq. (14)] to lensing by a cosmic string, which is $\tau \ll 1$.

The ensemble variability studies approach the question of the variability of celestial objects by comparing the magnitudes of a large number of individual sources observed at a few (two or more) epochs and by presenting various statistical measures of the individual magnitude change in the ensemble—its mean, variance, distribution density, autocorrelation function, and the like [21,27–33]. It is often "ergodically" assumed that these measures reflect those of individual sources to an extent given by the size and homogeneity of the observational sample and the time span of the variability survey. However, certain statistical measures of the ensemble variability have a value of their own. Of these, the distribution density $f(\Delta m)$ of the observed magnitude change will be particularly important for our study.

If the sources crossed by cosmic strings were not variable, $f(\Delta m)$ would be a direct observational estimate of the underlying density of lensing magnification $p(\Delta m)$. The quasars, on the contrary, are observed to vary at different magnitudes and time scales (e.g., [31,34–36]). Nevertheless, lensing by cosmic strings might still be apparent in $f(\Delta m)$ of these objects, as $\pm \Delta m_0$ inter-epoch changes will be overrepresented in the observed variability sample. This is the essence of our method to constrain the cosmological density of cosmic strings.

Mathematically, this can be expressed as follows. The string-induced variability is extrinsic to quasars, and there-
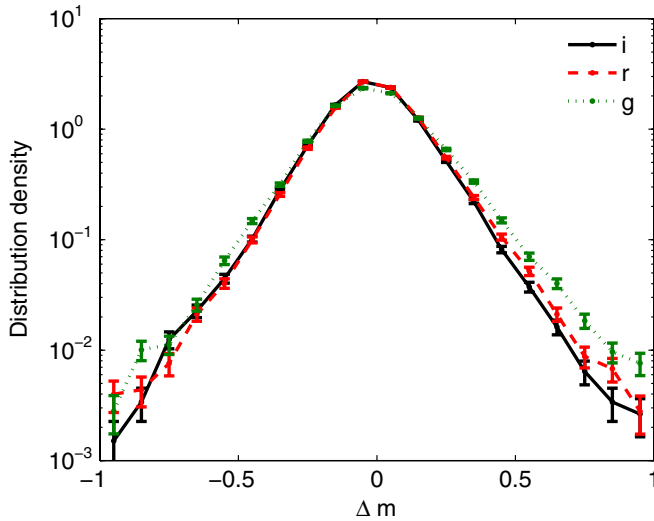
FIG. 1 (color online). The observed distribution densities $f(\Delta m)$ of the brightness variation Δm between photometric and spectroscopic measurements by the SDSS for $>25\,000$ quasars. The distributions in three SDSS passbands, i, r, and g, are derived from the results of [21] using a procedure described in Sec. IV. The error bars shown in the figure correspond to the "Poissonian" square roots from the number of quasars in each Δm bin.

fore the observed magnitude change distribution $f(\Delta m)$ is a convolution of those due to strings $p(\Delta m)$ and those due to intrinsic processes in quasars $s(\Delta m)$[1]; for $p(\Delta m)$ given by (1) the convolution equates to

$$f(\Delta m) = Qs(\Delta m) + Ps(\Delta m \mp \Delta m_0)) + \bar{s}(\Delta m), \quad (2)$$

with $\bar{s}(\Delta m)$ being the convolution of the intrinsic density $s(\Delta m)$ and the (unknown) smooth component of lensing density $\bar{p}(\Delta m)$. Figure 1 shows an example of the observed distribution density $f(\Delta m)$ derived from the data presented in [21] (see Sec. IV for details); it shows little evidence for any excess at $\pm\Delta m_0$, which will be used below to infer an upper limit on the density of cosmic strings.

Equation (2) immediately gives a handle on the parameter P related to the density of cosmic strings:

$$P = \frac{f(\Delta m) - Qs(\Delta m) - \bar{s}(\Delta m)}{s(\Delta m + \Delta m_0) + s(\Delta m - \Delta m_0)}. \quad (3)$$

We do *not* know what the intrinsic variability $s(\Delta m)$ is and therefore cannot extract the string signal from the observed $f(\Delta m)$ directly. However, if we assume that *all* of the variability at a certain level Δm comes from strings, this clearly gives us an upper limit on their contribution to the variability, which can be used to infer robust constraints on the population of strings:

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[1]There is also a distribution of observational uncertainties, but it can be absorbed into the intrinsic variability distribution; we assume that, for most quasars in the sample, this distribution is only weakly dependent on their actual brightness.

$$P \leq \frac{f(\Delta m)}{s(\Delta m + \Delta m_0) + s(\Delta m - \Delta m_0)}; \quad (4)$$

this inequality is valid irrespective of the assumptions on P because both neglected subtrahends in the numerator of the fraction in (3) are non-negative.

To deal with the denominator we assume that lensing by strings is rare; this is a sensible assumption, as discussed above. In this case the amplitude $P \ll 1$, $\bar{s}(\Delta m) \ll s(\Delta m)$, $Q \approx 1$, and the observed variability distribution density $f(\Delta m)$ is very close to the intrinsic one $s(\Delta m)$—except, possibly, at points $\Delta m = \pm\Delta m_0$, where a contribution $Ps(0)$ due to an excess of Δm_0 jumps related to the lensing light curve plateau might be expected. It therefore makes sense to use (4) at one of those points to derive an upper limit $\hat{P}$ on the parameter P. The denominator at these points can be approximated by the observed function $f(\Delta m)$, and one has

$$\hat{P} \approx \frac{f(\pm\Delta m_0)}{f(0) + f(\pm 2\Delta m_0)} \approx \frac{f(\pm\Delta m_0)}{f(0)}; \quad (5)$$

the last step reflects the observational fact that f measured at $\Delta m = \pm 2\Delta m_0$ is orders of magnitude lower than at zero where it peaks (cf. Fig. 1).

The constraints obtained in this way can be further refined and potentially even turned into assertive estimates for the string population properties by including additional information such as dependence of the observed distribution of magnitude change on source parameters or interepoch time lag. This can be accomplished by calculating the probabilities of the observed data given model parameters and using the Bayes theorem to infer the reverse. However, such an endeavor would inevitably require a model for the distribution of the intrinsic variability $s(\Delta m)$ and its dependence on source parameters, time lag, or whatever else is included in the analysis of the overall observed variability. In this study, we will use the simple approach outlined above, which is independent of the authors' ignorance of the intrinsic variability of quasars, though it can only provide upper limits on the density of strings.

III. MODEL: PROBABILITY AS A FUNCTION OF STRINGS POPULATION

The amplitude P is the probability that the magnification Δm jumps by Δm_0 between the two observational epochs t and $t + \Delta t$. Since Δm_0 is the maximum brightness increase due to string lensing, the only configuration that corresponds to this jump is that where the source is completely inside the strip in one of the observations and completely outside the strip in the other. Because of the symmetry between the two epochs, we can assume that the first observation occurs when the source is inside the strip and the second occurs when it is outside, thereby replacing Δt with is absolute value:

$$2P = \mathcal{P}[\Delta m(t) = \Delta m_0 \quad \text{and} \quad \Delta m(t + |\Delta t|) = 0]. \quad (6)$$

To estimate this value we first introduce the angular width Δ of the string lensing strip. According to [24,25], it depends on the tension μ of the string and its local inclination θ to the line of sight:

$$\Delta = 8\pi |\sin\theta| \frac{G\mu}{c^2} \frac{D_{ls}}{D_{os}}, \quad (7)$$

where D_{os} and D_{ls} are the (angular diameter) distances, respectively, from the observer and from the string to the source (along the line of sight); we use the average value of $\langle |\sin\theta| \rangle = \pi/4$. It seems sufficient for our study to assume that the string segment responsible for lensing is long and straight compared to the angular size of the source; for a comprehensive study of lensing by general configurations of strings, see [37,38].

Now let x be the initial epoch position of the projection of the source center onto the lens plane with respect to the strip median line (measured towards the outer edge of the strip such that the string itself is at $x_s = -\Delta/2$). The position of the source in the second epoch is then $x + \beta_\perp c|\Delta t|/(1 + z_l)D_{ol}$, where $\beta_\perp c$ is the orthogonal (to the string) component of transverse (to the line of sight) velocity of the string with respect to the source; the factor $(1 + z_l)^{-1}$ corresponds to the dilation of the observed time lag Δt from the lens plane at redshift z_l. Cosmic strings are expected to move relativistically, $\beta \sim \mathcal{O}(1)$ [1]; following [19,39], we use $\beta_\perp = 0.3$ in subsequent calculations.[2]

Since we assume that the source is lensed by at most one string, conditions in the argument of (6) require that the center of the source is inside the strip by a margin of at least the source size $r_\perp = R_\perp/D_{os}$ ($R_\perp$ is its linear size) in the first observation and outside it by the same margin in the second epoch:

$$|x| \le \Delta/2 - r_\perp,$$
$$x + \beta_\perp c|\Delta t|/(1 + z_l)D_{ol} \ge \Delta/2 + r_\perp. \quad (8)$$

Taken together, they restrict x to a narrow strip of width ξ, which is the lowest of $\Delta - 2r_\perp$ and $\beta_\perp c|\Delta t|/(1 + z_l)D_{ol} - 2r_\perp$, as long as this lowest is positive, and zero otherwise:

$$\xi = \max\left\{0, \min\left[\Delta, \frac{\beta_\perp c|\Delta t|}{(1 + z_l)D_{ol}}\right] - 2r_\perp\right\}. \quad (9)$$

The probability that a randomly placed source will lie within the strip of this width parallel to a string in an infinitesimally thin slice of string network with local number density ρ/μ is

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[2]We assume that $\beta_\perp > 0$, i.e. the source is moving away from the string; this is not restrictive due to the time symmetry mentioned.

$$d\tau = \frac{\rho}{\mu}\xi D_{ol} d\bar{D}_{ol} = \Omega_s \frac{3H_0^2}{8\pi G\mu}\omega(z_l)\xi D_{ol} d\bar{D}_{ol}, \quad (10)$$

where $\bar{D}$ is the proper distance along the line of sight parametrized by the slice redshift z_l.

In the formula above we also introduced the current cosmological density of strings, Ω_s, and its dependence on redshift, $\omega(z_l)$, such that the proper density $\rho(z_l) = \omega(z_l)\Omega_s 3H_0^2/8\pi G$. We use two models for $\omega(z_l)$—that corresponding to scaling solutions,

$$\omega(z) = \left[\frac{d_h(0)}{d_h(z)}\right]^2, \quad (11)$$

and that corresponding to pressureless dust,

$$\omega(z) = (1 + z)^3. \quad (12)$$

As discussed in the Introduction, there is currently no consensus on the relative contribution of closed and open strings to the energy budget of the Universe, but whatever the contributions are, the final result for the density constraints can be obtained by interpolating those derived from the application of (11) and (12).

With Eqs. (9) and (10) we can now write down the probability $2P$ of a twofold magnification jump in any of the infinitesimal slices. It is then given by the integrated optical depth τ along the line of sight to the source,

$$\frac{\tau}{\Omega_s} = \frac{3H_0^2}{8\pi G\mu} \int_0^{z_s} d\bar{D}(z_l)\omega(z_l)D(z_l)\xi(z_l, z_s, \mu, R_\perp, |\Delta t|), \quad (13)$$

according to

$$P = \frac{1}{2}(1 - e^{-\tau}) = \frac{\tau}{2} + \mathcal{O}(\tau^2); \quad (14)$$

this probability is placed symmetrically in $\Delta m = \pm\Delta m_0$, hence the factor $1/2$ in front of the brackets.

IV. APPLICATION: OBSERVATIONAL LIMITS ON STRING DENSITY

In order to put an upper limit $\hat{\Omega}_s$ on the density of cosmic strings, one now can simply equate the observational upper limit $\hat{P}$ on the probability of lensing to its model estimate P given by (14) in the limit $\tau \ll 1$:

$$\hat{\Omega}_s = \frac{2\hat{P}}{\tau/\Omega_s}. \quad (15)$$

The numerator of the fraction above can be estimated using (5) from the distribution density $f(\Delta m)$ of quasar brightness variations, which can be calculated directly from the observational data. In this regard, the SDSS quasar survey [40] provides an invaluable observational sample, where the brightness of tens of thousands of quasars is homogeneously measured in a number of optical passbands and could be compared against an equally homogeneous sam-

ple of brightness estimates derived from the quasar spectra, which are obtained month and years after the photometric observations.

Such an analysis has indeed been done for $N = 25\,710$ SDSS quasars by Vanden Berk *et al.* [21], and we use their data on quasar variability in SDSS passbands g, r, and i to derive constraints on the string population. The authors of the cited study do not explicitly quote estimates on the probability distribution density $f(\Delta m)$, and we do not possess sufficient resources to re-reduce the publicly available SDSS data to derive $f(\Delta m)$ independently. However, it can be readily obtained from Fig. 3 in the PDF version of [21]. To do so, we manually counted the data points corresponding to individual quasar measurements in the scatter plots of the figure in 0.1^m-wide bins for $|\Delta m| \geq 0.4^m$, or summed the heights of the respective histograms for $|\Delta m| < 0.4^m$,[3] and then divided them by the bin width and the total number of quasars in the ensemble.

The distribution densities of quasar variability in three passbands obtained in this way are plotted in Fig. 1, while Table I presents corresponding values $f(\Delta m)$ at points of our interest and an estimate for $\hat{P}$ in each passband. Since lensing is achromatic (as long as the string is heavy enough, such that $\Delta \geq 2r_\perp$ for all passbands) and intrinsic variability is not, we are free to choose the lowest $\hat{P}$ to use in (15); this is $\hat{P} = 3.2 \times 10^{-3}$, which corresponds to passband r.

The denominator of the fraction in (15) is given by the right-hand side of (13), which depends on the source redshift, and we therefore need to take an average with respect to the distribution of the observed quasars. The quasar sample of [21] includes most of the quasars in the SDSS Data Release 1 (DR1) Quasar Catalogue [41] and a substantial fraction of quasars observed by SDSS that were not included in SDSS DR1. The properties of individual quasars in the entire sample used in that study do not appear to have ever been detailed in a publication, and therefore we use the SDSS DR3 QSO Catalogue [42] as a proxy to the statistical properties of the true sample. We have verified numerically that our results do not change significantly if we use either DR1 or DR5 [43] catalogues in averaging τ/Ω_s (numbers do get higher when using more recent, deeper versions of the catalogue, but only by 2%–3%).

Another source of uncertainty is the physical size $2R_\perp$ that produces most of the observed flux. The size of the

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[3]The quality of the figure does not allow us to use a consistent counting approach in the entire domain of Δm—inner regions ($|\Delta m| < 0.4^m$) of scatter plots suffer from considerable confusion of data points, while the linear scale of the histograms makes them hardly readable for $|\Delta m| > 0.6^m$. However, where this comparison is possible, at bins centered at $\pm 0.45^m$ and $\pm 0.55^m$, the numbers agree to within a few percent, which is acceptable given the somewhat low-tech approach employed in the absence of published digital data.

TABLE I. Observational estimates for the distribution density of quasar brightness variation at $\Delta m = 0$, $\pm \Delta m_0$ in three SDSS passbands, and corresponding estimates for $\hat{P}$. For consistency, we calculate the central value of the distribution density $f(0^m) = [f(-0.05^m) + f(0.05^m)]/2$; the value of $\hat{P}$ quoted in the table is the average between values corresponding to $\Delta m = -\Delta m_0$ and $\Delta m = \Delta m_0$ according to (5).

Passband	$f(0^m)$	$f(-0.75^m)$	$f(0.75^m)$	$\hat{P}$
g	2.2	1.1×10^{-2}	1.8×10^{-2}	6.6×10^{-3}
r	2.5	7.6×10^{-3}	8.8×10^{-3}	3.2×10^{-3}
i	2.5	1.2×10^{-2}	6.4×10^{-3}	3.7×10^{-3}

quasar affects our results significantly by limiting the sensitivity of our estimate as a function of string tension μ. Estimates on these quantities vary appreciably in the literature, depending on the method used; reverberation mapping seems to favor sizes in the range $R \sim (10^{16}–10^{17})$ cm [44–48], while microlensing techniques give somewhat smaller values of $R \sim (10^{15}–10^{16})$ cm [49–53]. Neither of the methods is model independent and the estimates they give are expected to correlate with individual properties of QSOs, such as the luminosity or the mass of the central black hole. In the apparent absence of a better option, we perform our calculations using three representative values of $2R_\perp \in \{10^{15}, 10^{16}, 10^{17}\}$ cm, treating 10^{16} cm as a fiducial estimate.

Finally, the value of the time lag Δt between two observational epochs in the observer frame cannot be read from the results of [21] directly, which also introduces some uncertainty. The value of Δt affects our results directly via (9) and therefore needs to be fixed to perform calculations. From the visual inspection of Fig. 4 of [21] it is clear that typical time lag in the source frame $\Delta t/(1 + z_s) \sim (100–200)$ days. The average redshift of SDSS quasars is $z_s \approx 1.5$. We therefore take $\Delta t = 150$ days $\times 2.5 \approx 3.2 \times 10^7$ s.

V. RESULTS AND DISCUSSION

Figure 2 presents the upper limits on the average cosmological density of strings set by the statistics of the observed variability of more than $25\,000$ SDSS quasars as a function of string tension $G\mu/c^2$. Depending on the assumed size of the source, which turns out to be the major parameter of the method developed in this paper, the region where the string density is usefully constrained extends for up to 5 orders of magnitude. At the same time, our constraints are only weakly dependent on the assumed behavior of string density with redshift, mostly because prescriptions (11) and (12) started to diverge from each other relatively recently, when the vacuum density began to dominate in the Universe.

The most stringent constraints are obtained at $G\mu/c^2 \sim (10^{-13}–10^{-11})$, where the upper limits reach below the level of $\Omega_s = 0.01$. We note that these are rather weak
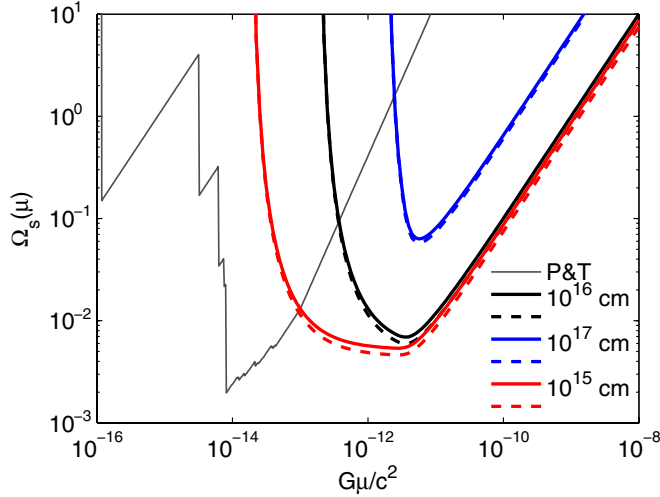
FIG. 2 (color online). Upper limit on the average present-day cosmological density of light cosmic strings as a function of its tension μ based on the quasar variability distribution shown in Fig. 1. Three pairs of curves are shown for the assumed source size $2R_\perp$ of 10^{16} cm (black, middle), 10^{17} cm (blue, top), and 10^{15} cm (red, bottom). Solid lines correspond to "scaling" evolution of string density with redshift according to (11), and dashed ones assume a "dustlike" law (12); they do not differ much. A thin grey line shows the constraints obtained in [19] from the local effects of strings.

limits for open topological cosmic strings because their cosmological density is believed to be of order $\Omega_s \sim (10\text{–}100)G\mu/c^2$ based on the reconnection argument leading to scaling solutions (e.g., [2]). However, this argument might not be applicable to fundamental strings, for which the reconnection probability is highly model dependent and can be significantly lower than unity [54,55]; it is also unclear how it applies to cosmic string loops, which are essentially a product of the mechanism that ensures that the density of open strings has a scaling behavior. Our observational upper limits apply to all kinds of strings, whether topological or fundamental, open or otherwise, and therefore provide strong constraints in the case of string loops and fundamental strings, independent of theoretical uncertainty. Moreover, our results might potentially be useful for constraining the unknown reconnection probability of fundamental strings.

The upper limits obtained here rely on an extensive data set of SDSS quasars and are robust in the sense that they do not depend on any local enhancement of the density expected for string loops in the Galaxy [20] or a particular model of quasar emission, as long as the source remains sufficiently small compared to the lensing strip width. This method is also rather insensitive to the photometric accuracy of observations because 0.75^m-wide jumps in brightness are fairly obvious by any standard. And if follow-up observations could be performed when a survey telescope sees a sudden twofold increase in the brightness of a

quasar, it might even be possible to confirm or rule out the string nature of this increase with confidence.

However, all this robustness also means that upper limits obtained in this paper could not be improved much by simply increasing the number of data points in the sample, which is expected to grow by orders of magnitude with the launch of next generation photometric surveys, such as the Large Synoptic Survey Telescope [56]. Rather, it is the study of quasar intrinsic variability that can bring the most benefit to this cause. The variation distribution density presented in Fig. 1 does not appear to show any notable features around $\Delta m = \pm 0.75^m$ which would make these points stand out from the smooth decrease of $f(\Delta m)$ with $|\Delta m|$. Therefore, the upper limits derived in this paper are most likely a significant overestimation of the true density of strings, because what is conservatively interpreted as a string "signal" here is most likely the quasar variability "noise." Therefore, simply adding more quasars to the data will not reduce this noise. To make further progress here, one needs to learn how to separate two contributions—e.g., using the varying dependence of the two effects on various parameters, such as the source redshift, color, luminosity, and so on. We can easily calculate how the statistics of string lensing effects should depend on these parameters, but at present we are less certain when it comes to quasar intrinsic variability.

Nevertheless, the science of quasar variability is advancing fast and our understanding of it might soon be sufficient for extracting the string effect from the observational data. This approach can also be followed in the analysis of future data sets from the GAIA mission [57] on the variability of stars, which are less distant but photometrically more stable and much more numerous than SDSS quasars. Moreover, at present we do not know the density of cosmic strings in the Universe and cannot predict just when the twin peaks of strings signal at $\Delta m = \pm \Delta m_0$ will begin to show up in the growing data sets of ensemble variability studies.

ACKNOWLEDGMENTS

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