

Stochastic gravitational waves from a new type of modified Chaplygin gasMariam Bouhmadi-López,^{*} Pedro Frazão,[†] and Alfredo B. Henriques[‡]*Centro Multidisciplinar de Astrofísica-CENTRA, Departamento de Física, Instituto Superior Técnico, Av. Rovisco Pais 1, 1049-001 Lisboa, Portugal*

(Received 6 November 2009; published 2 March 2010)

We propose a new scenario for the early Universe where there is a smooth transition between an early de Sitter-like phase and a radiation-dominated era. In this model, the matter content is modeled by a new type of generalized Chaplygin gas [6] for the early Universe, with an underlying scalar field description. We study the gravitational waves generated by the quantum fluctuations. In particular, we calculate the gravitational-wave power spectrum, as it would be measured today, following the method of the Bogoliubov coefficients. We show that the high frequencies region of the spectrum depends strongly on one of the parameters of the model. On the other hand, we use the number of e folds, along with the power spectra and spectral index of the scalar perturbations, to constrain the model observationally.

DOI: 10.1103/PhysRevD.81.063504

PACS numbers: 98.80.Bp, 04.30.-w, 95.36.+x

I. INTRODUCTION

The inflationary paradigm is the most consistent scenario to explain the origin of the large scale structure (LSS) of the Universe, as well as the anisotropies in the cosmic microwave background radiation (CMBR) [1,2]. Originally, an early inflationary era in the Universe was invoked as a mechanism to solve several shortcomings of the big bang theory [3]. Afterwards, it was realized that an early inflationary era of the Universe generates density perturbations as seeds for the structure we see nowadays. In addition, a background of stochastic primordial gravitational waves (GW) is also predicted and it is originated from the vacuum fluctuations. The latter (GW) if ever detected in the future would provide an amusing source of information about the early Universe. On the other hand, and most importantly, the predictions of the inflationary theory have been corroborated by several cosmological observations like the recent WMAP5 data [4].

In this paper we propose a phenomenological model for the inflationary era and the subsequent radiation-dominated epoch of the Universe. In particular, we suggest a way to extend the framework of the generalized Chaplygin gas (GCG) to the first stages of the Universe.¹ The GCG has attracted a lot of attention over the last years [6–10]. It was initially introduced in cosmology as a mean to unify the dark sectors of the Universe, i.e. dark energy and dark matter, and has been contrasted with several cosmological observations [9]. On the other hand, it has been as well shown that some GCG models can be an

excellent framework to analyze dark energy related singularities [7,10].

In this work, we investigate the possible imprints in the power spectrum of the stochastic background of GW, for the model we propose for the transition from the inflationary era to the radiation-dominated epoch.² This transition is far from being well known, and it can be of some interest to explore the signatures associated with it, shedding light on the inflationary model behind the accelerated expansion in the early Universe.

We calculate the GW production using the Bogoliubov coefficients which obey a set of differential equations [12–15]. This method has advantages over the frequently used sudden transition approximation, because, associated with it, there is always an overproduction of gravitons of large frequencies, which is avoided in a natural way by the use of the continuous Bogoliubov coefficients [15]. This is also a very practical method to calculate the full spectrum, from the very low frequencies corresponding to the present cosmological horizon 10^{-17} Hz, until those large, kHz up to GHz, frequencies associated with the transition between the inflationary and the radiation-dominated Universe.

The present letter is organized as follows: In Sec. II, we present our model based on a *new type of generalized Chaplygin gas for the early Universe*. This fluid can be as well represented by a minimally coupled scalar field as shown in Sec. III. In Sec. IV, we constrain our model observationally. In Sec. V, we summarize the methodology used to obtain the spectrum of the stochastic GW, which is based on the Bogoliubov coefficients. We present in Sec. VI the spectrum of the GW for the model introduced

^{*}mariam.bouhmadi@ist.utl.pt[†]pedro.frazao@ist.utl.pt[‡]alfredo@fisica.ist.utl.pt¹An alternative inflationary model inspired on the Chaplygin gas was proposed in Ref. [5] where only the inflationary era is accounted for.²In Ref. [11], the spectrum of GW for a Universe whose dark energy component corresponds to a Chaplygin gas was analyzed. This scenario is different from ours.

in Secs. II and III. Finally, in Sec. VII we present our conclusions.

II. A MODIFIED CHAPLYGIN GAS FOR THE EARLY UNIVERSE

We consider a perfect fluid whose energy density, ρ , scales with the scale factor, a , as

$$\rho = \left(A + \frac{B}{a^{4(1+\alpha)}} \right)^{1/(1+\alpha)}, \quad (2.1)$$

where A , B , and α are constants. This energy density interpolates between a positive cosmological constant at small scale factors and a radiation fluid at large scale factors as long $1 + \alpha$ is negative and A , B are positive.

By assuming the conservation of the energy density of such a fluid [cf. Equation (2.1)], which is a direct consequence of the Bianchi identity, the equation of state satisfied by its pressure, P , and energy density, ρ , reads

$$P = \frac{1}{3}\rho - \frac{4}{3}\frac{A}{\rho^\alpha}. \quad (2.2)$$

Before moving on a few words on the physics behind such a fluid are in order: like a generalized Chaplygin gas, whose equation of state reads $P = -A/\rho^\alpha$, a fluid with the equation of state (2.2) could be formally related to an effective description of a complex scalar field whose action can be written as a generalized Born-Infeld action [6], corresponding to a *perturbed* d -brane in a $(d+1, 1)$ space-time. For the propose of this work it is sufficient to describe such a matter content through the equation of state (2.2) or through a scalar field as it is shown on the next section.

The equation of state (2.2) corresponds to a mixture of a radiation fluid and a generalized Chaplygin gas [6,7] although, it is only the total energy density ρ which is conserved. This type of equation of state has been previously introduced in the context of dark energy models [16–19] (see also [20]). Nevertheless, we would like to stress that it has never been analyzed in the context of the early evolution of the Universe. More precisely, the equation of state (2.2) has been analyzed in Ref. [16] for $0 < \alpha < 1$; i.e. a fluid interpolating between a radiation at early time and a cosmological constant at late time. Here, we will be analyzing a completely different range of the space parameter where $\alpha < -1$.

For $\alpha < -1$ the energy density (2.1) can be approximated as

$$\rho \simeq A^{1/(1+\alpha)} \quad a \ll a_*, \quad (2.3)$$

$$\rho \simeq \frac{B^{1/(1+\alpha)}}{a^4} \quad a \gg a_*, \quad (2.4)$$

where

$$a_* = \left(\frac{B}{A} \right)^{1/(4(1+\alpha))}. \quad (2.5)$$

Therefore, a homogeneous and isotropic Universe with this matter content [cf. Eqs. (2.1) and (2.2)] undergoes a primordial inflationary era; initially described by a de Sitter (dS) expansion [see Eq. (2.3)]. Then, the Universe keeps inflating until the scale factor reaches a_* where inflation ceases; i.e. $\rho + 3P = 0$. Finally, it enters a radiation-dominated epoch [see Eq. (2.4)]. The transition from the inflationary era to the radiation-dominated phase takes place smoothly.

The Friedmann equation

$$H^2 = \frac{\kappa^2}{3}\rho, \quad (2.6)$$

where $\kappa^2 = 8\pi G$ and ρ is defined in Eq. (2.1), can be integrated analytically³ [21,22]

$$\begin{aligned} & \frac{\kappa a_*}{\sqrt{3}} A^{1/(2(1+\alpha))} (\eta - \eta_*) \\ &= 2^r F\left(-r, 1; 1 - \frac{r}{2}; \frac{1}{2}\right) - y^{-(r/2)} (1+y)^r \\ & \times F\left(-r, 1; 1 - \frac{r}{2}; \frac{y}{1+y}\right). \end{aligned} \quad (2.7)$$

In the previous expression η is the conformal time, η_* is a constant, $F(a, b; c; y)$ is a hypergeometric function⁴ and

$$y = \left(\frac{a}{a_*} \right)^{-4(1+\alpha)}, \quad r = -\frac{1}{2(1+\alpha)}. \quad (2.8)$$

At the conformal time η_* the Universe exits the inflationary era as $a = a_*$.

The primordial Universe starts its dS expansion at $\eta \rightarrow -\infty$ where $a \ll a_*$. On the other hand, the Universe gets radiation dominated at $\eta \rightarrow \infty$ where $a_* \ll a$. At this respect, we note that [22]

$$F\left(-r, 1; 1 - \frac{r}{2}; 1\right) = -1. \quad (2.9)$$

By choosing

$$\frac{\kappa a_*}{\sqrt{3}} A^{1/(2(1+\alpha))} \eta_* = -2^r F\left(-r, 1; 1 - \frac{r}{2}; \frac{1}{2}\right), \quad (2.10)$$

the relation between the conformal time, η , and the scale factor given in Eq. (2.7) can be simplified to

³This Friedmann equation can be integrated in two steps by (i) performing it for $0 < 1 + \alpha$ [21] and (ii) performing an analytical continuation of the hypergeometric function to values of α such that $1 + \alpha < 0$ [22].

⁴A hypergeometric series $F(b, c; d; x)$, also called a hypergeometric function, converges at any value x such that $|x| \leq 1$, whenever $b + c - d < 0$. However, if $0 \leq b + c - d < 1$ the series does not converge at $x = 1$. In addition, if $1 \leq b + c - d$, the hypergeometric function blows up at $|x| = 1$ [21].

$$\kappa a_* A^{1/(2(1+\alpha))} \eta = -\sqrt{3} y^{-(r/2)} (1+y)^r \times F\left(-r, 1; 1 - \frac{r}{2}; \frac{y}{1+y}\right). \quad (2.11)$$

We notice that this redefinition of η does not modify the fact that in this model the Universe has an infinite past (in terms of the conformal time), where it is asymptotically dS, and an infinite future (in terms again of the conformal time), where it is radiation dominated.

The whole evolution of the Universe can be described by the modified Chaplygin gas (2.2) at early time and by the Λ CDM model at late time; i.e.

$$\rho = \begin{cases} (A + \frac{B}{a^{4(1+\alpha)}})^{1/(1+\alpha)} & \text{early time} \\ \rho_{r0}(\frac{a_0}{a})^4 + \rho_{m0}(\frac{a_0}{a})^3 + \rho_\Lambda & \text{late time} \end{cases}, \quad (2.12)$$

where a_0 is the current scale factor, ρ_{r0} , ρ_{m0} are ρ_Λ are the current energy densities corresponding to radiation, matter, and the cosmological constant, respectively. On the other hand, at the radiation-dominated epoch the energy densities (2.12) are equal implying

$$B = (\rho_{r0} a_0^4)^{1+\alpha}. \quad (2.13)$$

Finally, we notice that A is related to the scale of inflation [see Eq. (2.3)].

III. THE INFLATIONARY DYNAMICS

The inflationary dynamics of the model presented in Sec. II is better described through a minimally coupled scalar field, ϕ , with a potential, $V(\phi)$, whose energy density and pressure read

$$\rho_\phi = \frac{\phi'^2}{2a^2} + V(\phi), \quad P_\phi = \frac{\phi'^2}{2a^2} - V(\phi). \quad (3.1)$$

In the previous equations the prime stands for derivative respect to the conformal time.

In a Friedmann-Lemaître-Robertson-Walker universe, the scalar field, ϕ , is equivalent to the modified Chaplygin gas introduced in Eqs. (2.1) and (2.2), as long as $\rho_\phi = \rho$ and $P_\phi = P$. The last two equalities imply⁵

$$V(\phi) = \frac{V_0}{3} [\cosh^{2/(1+\alpha)}(\kappa|1+\alpha|\phi) + 2\cosh^{-(2\alpha)/(1+\alpha)}(\kappa|1+\alpha|\phi)], \quad (3.2)$$

where

$$V_0 = A^{1/(1+\alpha)}. \quad (3.3)$$

In addition, ϕ can also be determined analytically in terms of the scale factor

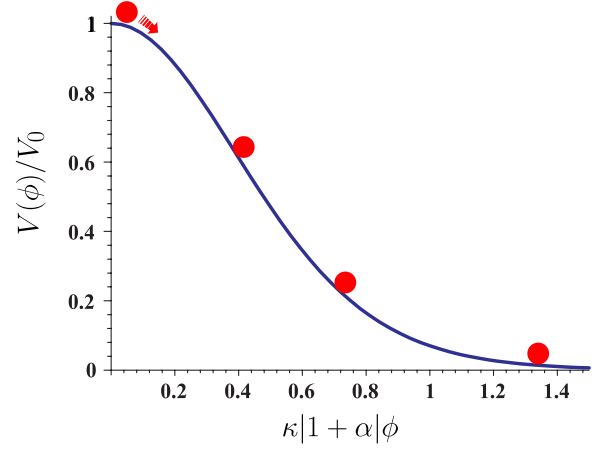


FIG. 1 (color online). Plot of the potential (3.2) as a function of the scalar ϕ given in Eq. (3.4).

$$\phi(a) = \pm \frac{1}{\kappa|1+\alpha|} \operatorname{arcsinh}\left(\sqrt{\frac{B}{A}} a^{-2(1+\alpha)}\right). \quad (3.4)$$

For simplicity, we will restrict to the solution (3.4) with + as our results does not depend on which of the two signs we choose. The Universe is asymptotically dS on the past where the scalar field is almost at the top of the potential; i.e. at $p \approx 0$ (see Fig. 1). Then it starts rolling down the potential, inducing an inflationary era, until it exits the inflationary era when $\phi(a_*) = \phi_*$; i.e.

$$\phi_* = \frac{\ln(1+\sqrt{2})}{\kappa|1+\alpha|}. \quad (3.5)$$

Finally, the radiation-dominated phase starts for large values of the scalar field, i.e. $\phi_* \ll \phi$. For later convenience, we derive as well ρ_ϕ in terms of the scalar field

$$\rho_\phi = V_0 \cosh^{2/(1+\alpha)}(\kappa|1+\alpha|\phi). \quad (3.6)$$

The number of e folds of expansion since a given mode k exits the horizon ($k = aH$) during the inflationary era, at $\phi = \phi_c$, until the end of inflation can be approximated by⁶

$$N_c \approx \kappa^2 \int_{\phi_*}^{\phi_c} V\left(\frac{dV}{d\phi}\right)^{-1} d\phi. \quad (3.7)$$

Substituting the scalar field potential (3.2) on the previous expression, we obtain

$$N_c(\alpha, \phi_c) = \frac{1}{2(1+\alpha)(2\alpha-1)} \ln \left\{ \sinh(\kappa|1+\alpha|\phi_c)^{-3} \times \left[\frac{1-4\alpha+\cosh(2\kappa|1+\alpha|\phi_c)}{4(1-\alpha)} \right]^{1+\alpha} \right\}. \quad (3.8)$$

⁵A similar potential for a different equation of state was suggested in Ref. [23].

⁶Here, we are making use of a slow-roll approximation [24]; i.e. values of the scalar field such that $\epsilon, |\eta| \ll 1$ where ϵ and η are the standard slow-roll parameters [24] [see also Eq. (4.4)].

IV. OBSERVATIONAL CONSTRAINTS

We constrain our model through CMBR/LSS measurements, using the values $P_s(k_c) = (2.45 \pm 0.23) \times 10^{-9}$ and $n_s = 1.0 \pm 0.1$ for the power spectrum and spectral index of the scalar perturbations, respectively, evaluated at the pivot wave number $k_c = 0.05 \text{ Mpc}^{-1}$, that corresponds to CMBR/LSS scales that exited the horizon during inflation [24]. To first order in the slow-roll approximation, the amplitude of the power spectrum for density perturbations is given by [24]

$$P_s(k) \approx \frac{\kappa^6}{12\pi^2} \frac{V^3}{V_\phi^2} \Big|_{k=aH}, \quad (4.1)$$

where $V_\phi \equiv \frac{dV}{d\phi}$. Notice that P_s is proportional to V_0 and therefore the power spectrum is extremely useful to constrain the energy scale of inflation [cf. Eqs. (2.3), (3.2), and (3.3)]. We will follow two different methods to constrain V_0 :

- (i) Method 1: As a first step to constrain our model, we use the bounds on N_c , i.e. the number of e folds since a given mode exits the horizon at ϕ_c until the end of inflation. We consider as a conservative range $47 \leq N_c \leq 62$ [24]. Then for a given N_c , on the allowed range, and for a fixed α , we obtain the corresponding value ϕ_c [cf. Eq. (3.8)]. Finally, using the pair (α, ϕ_c) in Eq. (4.1) with $P_s(k_c) = (2.45 \pm 0.23) \times 10^{-9}$, we deduce the scale of inflation; i.e. V_0 . This is precisely, what is shown on the dashed-dotted and dotted lines of Fig. 2.
- (ii) Method 2: A tighter constraint on V_0 is obtained by evaluating P_s at the pivot scale k_c when the mode exits the horizon; i.e. $k_c = aH$, during the inflationary era. Using Eqs. (2.6), (3.4), and (3.6), this scale reads

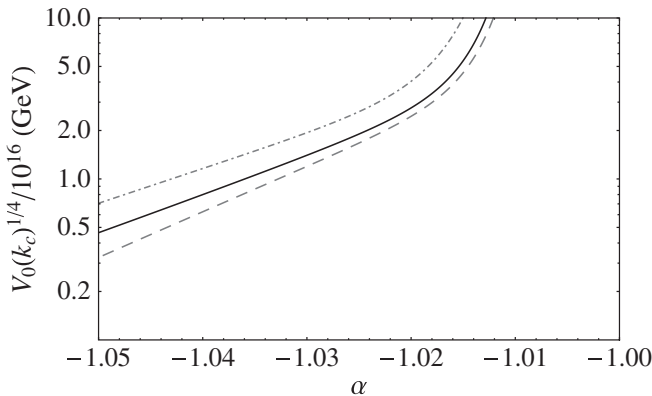


FIG. 2. Variation of the inflationary scale versus α . Using the first method, i.e. the number of e folds and P_s , we obtain the dashed-dotted and dashed lines corresponding to $N_c = 47$ and $N_c = 62$, respectively. Using the second method; i.e. P_s at the pivot scale $k_c = 0.05 \text{ Mpc}^{-1}$, we obtain the solid line.

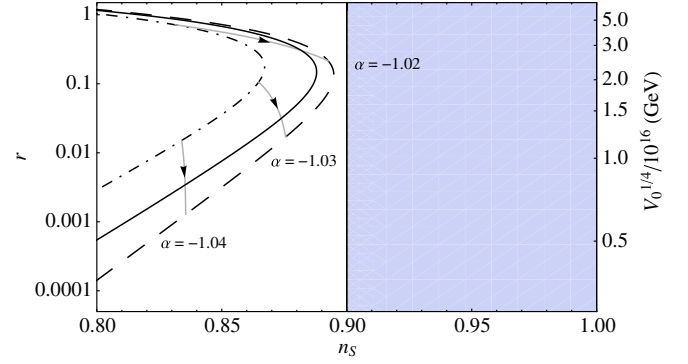


FIG. 3 (color online). Constraints on the $n_s - r$ parameter space. Using the first method; i.e. the number of e folds and P_s , we obtain the dashed-dotted and dashed lines corresponding to $N_c = 47$ and $N_c = 62$, respectively. Using the second method; i.e. P_s at the pivot scale $k_c = 0.05 \text{ Mpc}^{-1}$, we obtain the solid line. The black lines are for a constant α value, parameterized by an increase in the number of e folds.

$$k_c = \frac{2\pi}{\sqrt{3}} \kappa \rho_{r0}^{1/4} V_0^{1/4} a_0 \times (\cosh(\kappa|1 + \alpha|\phi_c)) \times \coth(\kappa|1 + \alpha|\phi_c))^{1/(2(1+\alpha))}. \quad (4.2)$$

Finally, by combining the previous equation and Eq. (4.1), we deduce the corresponding scale V_0 for a given parameter α . The results are shown on the solid line in Fig. 2.

We next consider the tensor to scalar ratio

$$r \equiv \frac{P_t(k)}{P_s(k)} = 16\epsilon, \quad (4.3)$$

where $P_t(k)$ is the tensorial power spectrum [24]. We analyze the variation of this ratio against the spectral index of the scalar perturbations $n_s = 1 - 6\epsilon + 2\eta$. In the previous expressions the slow-roll parameters read

$$\epsilon(\alpha, \phi_c) = \frac{1}{2\kappa^2} \left(\frac{V_\phi}{V} \right)^2; \quad \eta(\alpha, \phi_c) = \frac{1}{\kappa^2} \frac{V_{\phi\phi}}{V}. \quad (4.4)$$

We follow the two approaches enumerated before to evaluate r and n_s . Our results are shown on the $n_s - r$ parameter space in Fig. 3. We see that the spectrum is slightly more red than preferred by the observation. However, Fig. 3 shows that $r < 1$ is in agreement with the observation [24]. Our best fit value corresponds to $\alpha = -1.024$; i.e. an energy scale for inflation about 10^{16} GeV . It is worthy to notice that for a standard Chaplygin gas accommodated to the late-time expansion of the Universe, negative values of α are allowed by Type Ia Supernova [25].

V. GRAVITATIONAL-WAVE SPECTRUM

The stochastic GW spectrum, generated during the expansion of the Universe, can be calculated using the method of continuous Bogoliubov coefficients, using the methods first developed by Parker [12] and Starobinsky

[13], and later applied in [15,26]. Indeed, as the Universe evolves, the annihilation and creation operators change with time.

We can write these operators in terms of time-fixed annihilation and creation operators through a Bogoliubov transformation, involving the Bogoliubov coefficients α and β , which must satisfy the constraint $|\alpha|^2 - |\beta|^2 = 1$ [12,15].

It can be shown that the dimensionless relative logarithmic energy spectrum of the GWs, Ω_{GW} , at the present time η_0 , is related with β [26]:

$$\Omega_{\text{GW}} \equiv \frac{1}{\rho_c} \frac{d\rho_{\text{GW}}}{d\ln\omega} = \frac{\hbar\kappa^2}{3\pi^2 c^5 H^2(\eta_0)} \omega^4 \beta^2, \quad (5.1)$$

where ρ_{GW} is the energy density of GWs and ω the respective angular frequency; ρ_c and H are the critical density of the Universe and Hubble parameter, respectively, evaluated at the present time.

Therefore, the evolution of the Bogoliubov coefficient β , since early times, gives us the power spectrum at the present time. The same coefficient can be written in terms of two continuous functions of time, X and Y , in the form $|\beta|^2 = (X - Y)^2/4$. These new functions are determined by the set of equations [26]

$$X' = -ikY, \quad (5.2)$$

$$Y' = -\frac{i}{k} \left(k^2 - \frac{a''}{a} \right) X. \quad (5.3)$$

The Friedmann equation (2.6) and the conservation of the energy momentum tensor imply $a''/a \propto \rho - 3p$. Therefore, the power spectrum depends on the matter content of the Universe [see, for example, Eq. (5.3)]. Consequently, the GWs power spectrum can provide us with important information about the Universe evolution and its matter content, as long as the latter leaves a significant imprint on the spectrum.

As our model describes a transition between the inflationary period and the radiation-dominated era, the integration must begin at the inflationary period, where we have a constant energy density. Hence, it is necessary to specify, at this inflationary period, the initial conditions for $X(\eta)$ and $Y(\eta)$.

To solve the set of differential Eqs. (5.2) and (5.3), for our model, we must use numerical methods, but, since it admits an exact analytical solution for the case of a dS universe [26], i.e. similar to our model at early time [see Eq. (2.3)], we can specify appropriate initial conditions for $X(\eta)$ and $Y(\eta)$, at an appropriate scale of inflation. In this case, Eqs. (5.2) and (5.3) yield a solution for a dS expansion that takes the form [26]

$$X(\eta_i) = \left(1 + i \frac{a_i H}{k} \right) e^{i(k/(a_i H))}, \quad (5.4)$$

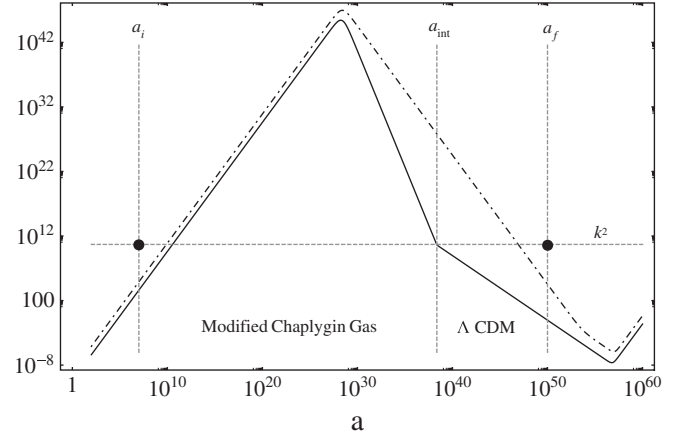


FIG. 4. A scheme of the integration method used in the numerical simulations for a given mode k . The solid line refers to $\frac{a''}{a}$ and the dashed-dotted one to $k_H^2 = (aH)^2$.

$$Y(\eta_i) = \left(1 + i \frac{a_i H}{k} - \frac{a_i^2 H^2}{k^2} \right) e^{i(k/(a_i H))}, \quad (5.5)$$

where a_i is a scale factor during the dS phase. We use these expressions as initial conditions for the numerical integration of Eqs. (5.2) and (5.3).

The other piece of information we need to solve Eqs. (5.2) and (5.3) is the whole evolution of the Universe:

$$\frac{a''}{a} = \begin{cases} \frac{2}{3} \kappa^2 a^2 A \left(A + \frac{B}{a^{4(1+\alpha)}} \right)^{-(\alpha/(1+\alpha))} & \text{early time} \\ \frac{\kappa^2}{6} a^2 \left[\rho_{m0} \left(\frac{a_0}{a} \right)^3 + 4\rho_\Lambda \right] & \text{late time} \end{cases}, \quad (5.6)$$

where the transition between the two epochs is determined by the moment when a''/a (for both periods) is approximately equal at $a = a_{\text{int}}$. We use the values for the energy densities parameters given by WMAP5 [27]: $\Omega_m = 0.291 \pm 0.014$ and $\Omega_\Lambda = 0.709 \pm 0.014$, through the usual definition $\Omega = \frac{8\pi}{3H_0^2} \rho$, where $H_0 = 71.3$ km/s/Mpc.

For a given mode k , the integration is done in terms of the scale factor a . We start at $a \lesssim a_i$, just before the mode exits the horizon; i.e. $k < k_H$ where $k_H = aH$, until $a \gtrsim a_f$, when the mode reenters the horizon; i.e. $k < k_H$ (c.f. Fig. 4). The maximum value of k^2 corresponds to the maximum height of the potential a''/a as the gravitons are produced only below the barrier a''/a (see Fig. 4).

To ensure the accuracy of the numerical results are not a numerical artifact we use the condition on the Bogoliubov coefficients $|\alpha|^2 - |\beta|^2 = 1$, which in terms of the new variables X, Y reads $2XY = 1$, as a constraint equation of the numerical solution.

VI. NUMERICAL SIMULATIONS

The GW spectra resulting from the numerical integrations for the two different methods presented in Sec. IV to constrain the model observationally are shown in Figs. 5

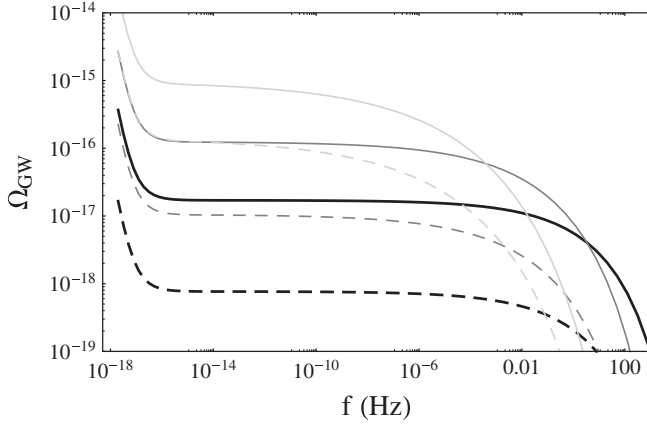


FIG. 5. Gravitational-wave spectra for the values $\alpha = -1.05$ (black), -1.04 (gray), and -1.03 (light gray), along with $N_c = 47$ (solid) and $N_c = 62$ (dashed). The spectrum suffers a downward shift with an increase in $|\alpha|$, and, simultaneously, an increase in the high frequency region.

and 6. In the former method, we use as a constraint equation the number of e folds, N_c , and in the latter the mode function, k_c . Therefore, as one would expect, the variation with the values of α is the same, independently of the method used to calculate the inflationary scale which gives us the constant A of our model [see Eq. (3.3) and Fig. 2]. As we can see in Figs. 5 and 6, the spectrum changes with α , in particular, it suffers simultaneously a vertical displacement and a variation in the high frequency region.

Furthermore, to understand the vertical displacement of the plateau in the intermediate region of the spectrum, it is necessary to consider the inflationary scale as a function of α , plotted in Fig. 2. The same figure shows that, for α constant, an increase in the number of e folds from $N_c = 47$ (dashed-dotted line) to $N_c = 62$ (dotted line) results in a decrease in the inflationary scale V_0 . Therefore, the decrease of the plateau height,⁷ which can be observed in Fig. 5 by the difference between the solid lines ($N_c = 47$) and the dashed lines ($N_c = 62$) for a constant α (a given type of line), is easily interpreted as a decrease in the inflationary scale.

On the other hand, having showed that the vertical displacement of the spectra depends only on the value of the inflationary scale, we immediately conclude that the increase in the high frequency region, when α moves away from -1 , is a function of the parameter α only, as we can see, for example, in Fig. 6.

The results presented above show that, in the gravitational-wave energy spectra for the unified inflationary and radiation eras consistent with CMBR/LSS constraints, and, in particular, the one by means of the mode

⁷An early dS phase results in a flat plateau in the spectra. The height of the plateau is an increasing function of the Hubble rate of the dS expansion; i.e. the energy scale of the dS inflationary era.

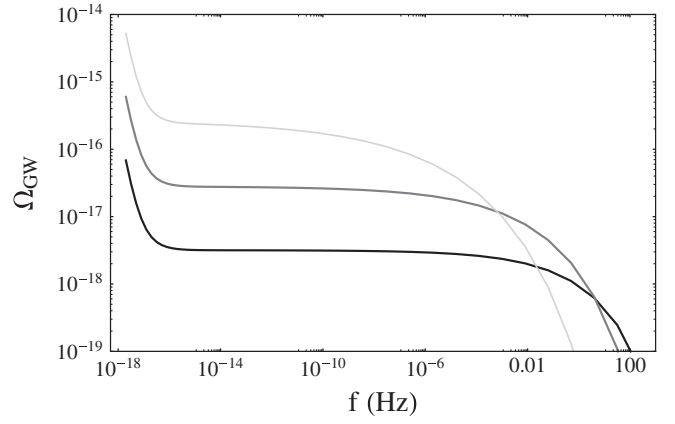


FIG. 6. Gravitational-wave spectra for the values $\alpha = -1.05$ (black), -1.04 (gray), and -1.03 (light gray). The spectrum suffers a downward shift with an increase in $|\alpha|$, and, simultaneously, an increase in the high frequency region.

function in Fig. 6, present a variation in the spectrum that is a combination of the last effects: a vertical shift as a result from the decrease in the inflationary scale; and an increase in the high frequency region that results from a increase in the $|\alpha|$ value.

The last variation is the more important one because it can give us a new insight about the relation between the frequency limits of the GW spectrum and the model behind the inflationary and the radiation eras.

Although the spectra presented on this section are smooth, reflecting the smoothness of the transition between the inflationary and the radiation-dominated periods, characteristic of the present model, this is not always the case. For instances, when the system goes through a complex transition period, with changes in the equation of state, the structure of the spectrum is equally complex, particularly in the hundreds of MHz and GHz regime, allowing us to read a lot of information about the properties of the transition (see Ref. [26]).

What about the reheating of the Universe in our model? such a process can be modeled by coupling the scalar field to a radiation fluid, where the decay rate of the scalar field into the radiation fluid is governed by a phenomenological parameter. In addition, the tail of the potential (3.2) is *substituted* by a potential with a vanishing minimum. For the modified model the trend and the level of the spectra of the GW would not be much different to the one presented on this section. Indeed, it is still the parameter α that characterizes the spectra. In addition, the inclusion of a reheating process is computationally very much time consuming.

VII. CONCLUSIONS

In this work, we investigate the production of GWs in a new modified generalized Chaplygin gas for the early Universe (cf. Secs. II and III). Through recent measure-

ments of CMBR/LSS, the parameters of the model have been constrained.

We have used the method of the continuous Bogoliubov coefficients to calculate the GW energy spectrum for different scales of energy and values of the α parameter of the model.

Besides the fact that the model suffers a small deviation from the preferred values of the spectral index n_s , the obtained spectra reveal a consistent picture corresponding to a smooth transition between the inflationary and the radiation-dominated epochs of the Universe, and constitute a significant imprint of this modified generalized Chaplygin gas.

Finally, the strong variation of the high frequency range, is directly related with the decrease in the maximum of the potential a''/a [see Eq. (5.3)] for $|\alpha|$ approaching 1. This feature implies strong limits to the maximum frequency allowed in our model and which will be within the reach of future gravitational-waves detectors like BBO and DECIGO [24], for the KHz range of frequencies. In fact,

for the most consistent values of the α parameter, the spectrum shows a frequency as low as in the Hz region.

Concluding, these results show that, for this model, the Hz-KHz frequency range comes directly from the transition between the inflationary regime and the radiation era, and addresses important issues about the limits of the GW energy spectrum.

Last but not least, we would like to highlight that one of the merits of the model we have presented for the transition from the inflationary era to the radiation-dominated epoch is its relative simplicity.

ACKNOWLEDGMENTS

The authors are grateful to O. Bertolami and A.Y. Kamenshchik for very useful comments on a previous version of the manuscript. M.B.L. is supported by the Portuguese Agency Fundação para a Ciência e Tecnologia through Grant No. SFRH/BPD/26542/2006. This research was supported by Grant No. FEDER-POCI/P/FIS/57547/2004.

-
- [1] A. R. Liddle and D. H. Lyth, *Cosmological Inflation and Large-Scale Structure* (Cambridge University Press, Cambridge, England, 2000).
 - [2] W. H. Kinney, arXiv:0902.1529.
 - [3] A. H. Guth, Phys. Rev. D **23**, 347 (1981); A. A. Starobinsky, Phys. Lett. **91B**, 99 (1980); A. J. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. **48**, 1220 (1982); A. D. Linde, Phys. Lett. **108B**, 389 (1982).
 - [4] E. Komatsu *et al.* (WMAP Collaboration), Astrophys. J. Suppl. Ser. **180**, 330 (2009).
 - [5] O. Bertolami and V. Duvvuri, Phys. Lett. B **640**, 121 (2006).
 - [6] A. Y. Kamenshchik, U. Moschella, and V. Pasquier, Phys. Lett. B **511**, 265 (2001); N. Bilić, G. B. Tupper, and R. D. Viollier, Phys. Lett. B **535**, 17 (2002); M. C. Bento, O. Bertolami, and A. A. Sen, Phys. Rev. D **66**, 043507 (2002).
 - [7] M. Bouhmadi-López and J. A. Jiménez Madrid, J. Cosmol. Astropart. Phys. 05 (2005) 005; M. Bouhmadi-López and P. Vargas Moniz, Phys. Rev. D **71**, 063521 (2005).
 - [8] V. Gorini, A. Kamenshchik, U. Moschella, V. Pasquier, and A. Starobinsky, Phys. Rev. D **72**, 103518 (2005).
 - [9] H. Sandvik, M. Tegmark, M. Zaldarriaga, and I. Waga, Phys. Rev. D **69**, 123524 (2004); R. Bean and O. Doré, Phys. Rev. D **68**, 023515 (2003); M. C. Bento, O. Bertolami, and A. A. Sen, Phys. Rev. D **70**, 083519 (2004); M. C. Bento, O. Bertolami, N. M. C. Santos, and A. A. Sen, Phys. Rev. D **71**, 063501 (2005); T. Barreiro, O. Bertolami, and P. Torres, Phys. Rev. D **78**, 043530 (2008).
 - [10] A. Kamenshchik, C. Kiefer, and B. Sandhöfer, Phys. Rev. D **76**, 064032 (2007); M. Bouhmadi-López, P. F. González-Díaz, and P. Martín-Moruno, Int. J. Mod. Phys. D **17**, 2269 (2008); Z. Keresztes, L. Á. Gergely, V. Gorini, U. Moschella, and A. Y. Kamenshchik, Phys. Rev. D **79**, 083504 (2009); M. Bouhmadi-López, C. Kiefer, B. Sandhöfer, and P. V. Moniz, Phys. Rev. D **79**, 124035 (2009).
 - [11] J. C. Fabris, S. V. B. Gonçalves, and M. S. Santos, Gen. Relativ. Gravit. **36**, 2559 (2004).
 - [12] L. Parker, Phys. Rev. **183**, 1057 (1969).
 - [13] A. A. Starobinsky, JETP Lett. **30**, 682 (1979).
 - [14] B. Allen, Phys. Rev. D **37**, 2078 (1988).
 - [15] L. E. Mendes, A. B. Henriques, and R. G. Moorhouse, Phys. Rev. D **52**, 2083 (1995); R. G. Moorhouse, A. B. Henriques, and L. E. Mendes, Phys. Rev. D **50**, 2600 (1994).
 - [16] H. B. Benaoum, arXiv:hep-th/0205140.
 - [17] P. F. González-Díaz, Phys. Lett. B **562**, 1 (2003).
 - [18] U. Debnath, A. Banerjee, and S. Chakraborty, Classical Quantum Gravity **21**, 5609 (2004).
 - [19] L. P. Chimento and R. Lazkoz, Classical Quantum Gravity **23**, 3195 (2006).
 - [20] S. Nojiri and S. D. Odintsov, Phys. Rev. D **70**, 103522 (2004); S. Nojiri, S. D. Odintsov, and S. Tsujikawa, Phys. Rev. D **71**, 063004 (2005); S. Nojiri and S. D. Odintsov, arXiv:0903.5231.
 - [21] I. S. Gradshteyn and I. Ryzhik, *Tables of Integrals, Series and Products* (Academic Press, New York, 1994).
 - [22] M. Abramowitz and I. Stegun, *Handbook of Mathematical Functions* (Dover, New York, 1980).
 - [23] A. E. Bernardini and O. Bertolami, Phys. Rev. D **77**, 083506 (2008).
 - [24] T. L. Smith, M. Kamionkowski, and A. Cooray, Phys. Rev.

- D **73**, 023504 (2006).
- [25] O. Bertolami, A. A. Sen, S. Sen, and P. T. Silva, Mon. Not. R. Astron. Soc. **353**, 329 (2004).
- [26] A. B. Henriques, Classical Quantum Gravity **21**, 3057 (2004); Classical Quantum Gravity **24**, 6431 (2007); P. Sá and A. B. Henriques, Phys. Rev. D **77**, 064002 (2008); P. M. Sá and A. B. Henriques, Gen. Relativ. Gravit. **41**, 2345 (2009); A. B. Henriques, R. Potting, and P. M. Sá, Phys. Rev. D **79**, 103522 (2009).
- [27] <http://lambda.gsfc.nasa.gov>.