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Possible observational effects of loop quantum cosmology

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In this paper, we consider realistic model of inflation embedded in the framework of loop quantum cosmology. Phase of inflation is preceded here by the phase of a quantum bounce. We show how parameters of inflation depend on the initial conditions established in the contracting, prebounce phase. Our investigations indicate that phase of the bounce easily sets proper initial conditions for the inflation. Subsequently, we study observational effects that might arise due to the quantum gravitational modifications. We perform preliminary observational constraints for the Barbero-Immirzi parameter γ, critical density ρ_c, and parameter λ. In the next step, we study effects on power spectrum of perturbations. We calculate spectrum of perturbations from the bounce and from the joined bounce + inflation phase. Based on these studies, we indicate a possible way to relate quantum cosmological models with the astronomical observations. Using the Sachs-Wolfe approximation, we calculate the spectrum of the superhorizontal CMB anisotropies. We show that quantum cosmological effects can, in the natural way, explain suppression of the low CMB multipoles. We show that fine-tuning is not required here, and the model is consistent with observations. We also analyze other possible probes of the quantum cosmologies and discuss perspectives of their implementation.

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I. INTRODUCTION

A main obstacle in formulating the quantum theory of gravitational interactions is the lack of any empirical clue. Here, the problem is that quantum gravity effects are expected to be significant at the energies approaching 10^{19} GeV (the Planck scale). With the present generation of accelerators, energies up to 10^3 GeV can be reached, what is 16 orders of magnitude below the Planck scale. Therefore, direct experimental investigation of quantum gravity effects becomes inaccessible. In other words, it is like probing atomic structure with Earth size resolution devices. This suggests that alternative methods of investigation should be taken into account. One, perhaps most promising, possibility of escape from this impasse is *indirect methods*. In this paper, we consider one particular type of indirect probing of quantum gravitational effects, which is based on cosmological observations.

In order to perform any quantitative predictions, the mathematical model of the given process has to be known. The process considered here is the behavior of the Universe in the Planck epoch. In this epoch, evolution of the Universe is determined by the quantum gravitational effects. In our considerations, this phase is described by loop quantum cosmology (LQC) [1]. LQC is based on a nonperturbative approach to quantize gravity called loop quantum gravity (LQG) [2]. The starting point in formulating both LQG and LQC is the parametrization of the phase space of the gravitational field by the $SU(2)$ connection and by its conjugated momenta. These canonical fields are so-called Ashtekar variables ($A = A^i_a \tau_i dx^a$, $E = E^a_i \tau^i \partial_a$) which take value in $\mathfrak{su}(2)$ and $\mathfrak{su}(2)^*$ algebras, respectively, and fulfil the Poisson bracket

$$\{A^i_a(\mathbf{x}), E^b_j(\mathbf{y})\} = \gamma\kappa\delta^b_a\delta^i_j\delta^{(3)}(\mathbf{x}-\mathbf{y}), \tag{1}$$

where $\kappa = 8\pi G$ and γ is the Barbero-Immirzi parameter. Parameter γ is the unknown constant of the theory. However, because γ is related with a black hole entropy, its value can be recovered from comparison with the Hawking-Bekenstein formula $S_{BH} = \frac{k}{4l^2_{\mathrm{Pl}}}A$. In our considerations, we use the value $\gamma = \gamma_{\mathrm{M}} = 0.2375$ calculated by Meissner in Ref. [3].

Loop quantum cosmology can be considered on the two levels: the first is the purely quantum approach and the second is a semiclassical, effective framework. The first approach is more complete with respect to the effective approach. However, the semiclassical approach is more useful in relating quantum cosmological effects with classical physics. Moreover, main features of the complete approach are sufficiently well reproduced by the effective approach. Because our aim here is to relate effects of LQC with subsequent classical evolution, the semiclassical approach will be more adequate. Therefore, in all of the considerations performed in this paper, we base them on semiclassical LQC.

In the cosmological applications, canonical variables (A, E) can be split for the homogeneous and perturbation parts:

$$A^i_a = \bar{A}^i_a + \delta A^i_a \quad \text{and} \quad E^b_j = \bar{E}^b_j + \delta E^b_j. \tag{2}$$

In this paper, the cosmological background is described by

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the flat Friedmann-Robertson-Walker (FRW) metric, then $\bar{A}^i_a = \bar{p}\delta^i_a$ and $\bar{E}^b_j = \gamma\bar{k}\delta^b_j$. Here, $\bar{k}$ and $\bar{p}$ are new canonical variables which fulfill the Poisson bracket $\{\bar{k}, \bar{p}\} = \frac{\kappa}{3V_0}$. The parameter V_0 is the fiducial cell which regularizes integration over the infinite spatial part. The $\bar{p}$ variable can be expressed in terms of the scale factor, $\bar{p} = a^2$ and $\bar{k} = \dot{\bar{p}}/2\bar{p}$. Having canonical variables, one can introduce the Hamiltonian. The Hamiltonian can be also decomposed for the homogeneous and the background parts, $H^{\rm phen}_{\rm G} = \bar{H}^{\rm phen}_{\rm G} + \delta H^{\rm phen}_{\rm G}$. Here, the upper index symbolizes that the classical Hamiltonian contains additional phenomenological quantum corrections. The lower index indicates that the gravitational part is considered. However, in the realistic models, the matter Hamiltonian also contributes, then $H^{\rm phen} = H^{\rm phen}_{\rm G} + H_{\rm m}$. In the considered models, there are no quantum corrections to the matter Hamiltonian, what is indicated by the lack of the upper index. The matter Hamiltonian can also be decomposed for the homogeneous and perturbations parts. We will analyze such a case in this paper.

In this paper, we consider effective LQC with a scalar field. In particular, we concentrate our attention on the model with a massive potential. In this case, it will be possible to investigate realization of the inflationary phase. In this approach, parameters of inflation are dependent on the previous quantum cosmological evolution. We will consider perturbations in the emerging bounce + inflation scenario. This will allow us to relate quantum cosmological effects with classical perturbations. This finally lets us study LQC modifications of the CMB anisotropy.

In this paper, we consider possible observational effects due to the modified background dynamics only. The modifications are introduced by the so-called holonomy corrections. In general, other types of corrections are also expected, as inverse volume corrections. In the present paper, we however take into account holonomy corrections only. The inverse volume corrections in the flat FRW models depend on the fiducial volume, and therefore interpretation of their effects is rather obscure. Moreover, self-interaction of the scalar field can, in principle, also lead to the additional quantum gravitational corrections. Such an effect was discussed in Ref. [4], where effective equations for the loop quantum cosmology were derived. In the present paper, we neglect any corrections due to the self-interacting scalar field. The studies of perturbations are also simplified. This is mainly because the theory of perturbations in loop quantum cosmology is at present incomplete. In particular, equations for the scalar models with holonomy corrections are not derived yet.

Despite the fact that equations of loop quantum cosmology are at present incomplete, the main results obtained in the simplified considerations should survive also in the more detailed models. Therefore, we concentrate on these main features of the loop quantum cosmology. Such a feature is a cosmic bounce predicted in the framework of LQC. In this paper, we concentrate on the possible observational consequences of the quantum bounce. In particular, we study how the standard inflationary scenario is realized in the bouncing universe and how effects of the cosmic bounce can influence a spectrum of the cosmic microwave radiation.

The organization of the text is the following. In Sec. II, we introduce the concept of cosmic bounce in the framework of the effective LQC. Subsequently, in Sec. III we construct a model of inflation in the applied framework. We set initial conditions in the contracting phase and study how parameters of inflation vary with them. We show that the phase of bounce can easily set proper initial conditions for the subsequent phase of inflation. In the next step, in Sec. IV, we discuss perturbations in the considered model. We restrict ourselves to the fluctuations of the scalar field. We calculate the spectrum of the perturbations from the bounce and from the joined bounce + inflation phase. These results can be applied in calculating the spectrum of the CMB anisotropies. In Sec. V, based on the Sachs-Wolfe approximation, we calculate the spectrum of temperature anisotropies in CMB. We show that the phase of bounce can lead to suppression of the low multipoles in the spectrum of CMB anisotropies. Subsequently, in Sec. VI, we discuss other kinds of the observational probes of LQC. Finally, in Sec. VII we summarize our results.

II. BACKGROUND DYNAMICS

In our considerations, the Hamiltonian of the gravitational homogeneous part is given by

$$\bar{H}^{\rm phen}_G = -\frac{3V_0}{\kappa}\sqrt{\bar{p}}\left(\frac{\sin\bar{\mu}\gamma\bar{k}}{\bar{\mu}\gamma}\right)^2. \quad (3)$$

The crucial element of this Hamiltonian is the factor $\bar{\mu}$. This parameter contains details of the quantum modifications, and when $\bar{\mu} \to 0$, the classical limit is recovered. The main ambiguity in LQC comes from the choice of $\bar{\mu}$. The mostly used form of $\bar{\mu}$ is that given by $\bar{\mu} = \sqrt{\frac{\Delta}{\bar{p}}}$, where $\Delta = 2\sqrt{3}\pi\gamma l^2_{\rm Pl}$. In our investigations, we use this particular expression for $\bar{\mu}$. The choice of Δ is motivated by the existence of the gap in the spectrum of area operator in LQG. However, Δ is the result of the kinematic sector of LQG, and extrapolation of this result to LQC is an assumption. This issue is discussed in Refs. [5,6]. The problem here is that the relation between LQC and LQG is not fully understood. In particular, it should be possible to derive LQC directly from LQG, and then the problem of ambiguities in LQC should be overcome. Recently, one promising step has been taken towards deriving LQC from LQG. In their work, Rovelli and Vidotto show how LQC can be derived in the spin networks formalism. Another possibility is that presented in [5,7], where $\bar{\mu} = \lambda/\sqrt{\bar{p}}$, and λ is some unknown constant unrelated with Δ. From this point

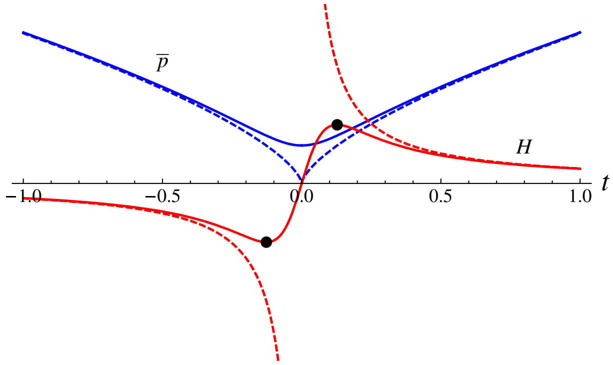

FIG. 1 (color online). Evolution of variable $\bar{p}$ and Hubble factor H in the bouncing universe (thick lines) with a free scalar field. Dashed lines represent classical evolution. Dots represent the points $(t_\pm, \mp H_{\max}) = (\pm\frac{1}{\sqrt{3\kappa\rho_c}}, \mp\sqrt{\frac{\kappa}{12}\rho_c})$.

of view, λ is some phenomenological parameter which should be determined from the observations rather from the theory.

Taking the Hamilton equation $\dot{\bar{p}} = \{\bar{p}, \bar{H}_m + \bar{H}_G^{\mathrm{phen}}\}$ together with the scalar constraint $\bar{H}_m + \bar{H}_G^{\mathrm{phen}} = 0$, one can derive the modified Friedmann equation

$$H^2 := \left(\frac{1}{2\bar{p}}\frac{d\bar{p}}{dt}\right)^2 = \frac{\kappa}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right), \tag{4}$$

where

$$\rho_c = \frac{\sqrt{3}}{16\pi^2\gamma^3 l_{\mathrm{Pl}}^4} = 0.82 m_{\mathrm{Pl}}^4 \tag{5}$$

is the critical energy density. This value was calculated with the fixed area gap parameter Δ. As one can find from Eq. (4), the physical solutions are allowed only for $\rho \leq \rho_c$. The $\rho = \rho_c$ is a turning point, commonly called a bounce. Moreover, while $\rho \to 0$, the classical dynamics is recovered. One can also find that the maximal value of the Hubble factor defined in Eq. (4) is reached for $\rho_{\max} = \frac{\rho_c}{2}$. The maximal value of the Hubble factor is $H_{\max}^2 = \frac{\kappa}{12}\rho_c$. In Fig. 1, we show a typical symmetric bounce obtained for the model with a free scalar field.

III. QUANTUM BOUNCE AND INFLATION

In this section, we are going to construct a realistic model of inflation embedded in the framework of effective loop quantum cosmology. Qualitative studies of inflation in LQC have been already performed in Ref. [8]. The issue of inflation in LQC has been raised also in Refs. [9,10]. In our studies, we model the phase of inflation with the massive scalar field. Since we consider the flat FRW model, the equation for the homogeneous component of the field φ holds the classical form

$$\frac{d^2\bar{\varphi}}{dt^2} + 3H\frac{d\bar{\varphi}}{dt} + \frac{dV}{d\bar{\varphi}} = 0, \tag{6}$$

where the massive potential is given by

$$V(\varphi) = \frac{m^2}{2}\varphi^2. \tag{7}$$

The energy density of the considered homogeneous scalar field is

$$\rho = \frac{1}{2}\left(\frac{d\bar{\varphi}}{dt}\right)^2 + V(\bar{\varphi}). \tag{8}$$

Dynamics of the model is governed by the set of equations

$$\frac{dH}{dt} = \kappa\frac{P^2}{2}\left[\frac{2}{\rho_c}\left(\frac{P^2}{2} + V(\bar{\varphi})\right) - 1\right], \tag{9}$$

$$\frac{d\bar{p}}{dt} = 2H\bar{p}, \tag{10}$$

$$\frac{d\bar{\varphi}}{dt} = P, \tag{11}$$

$$\frac{dP}{dt} = -3HP - \frac{dV(\bar{\varphi})}{d\bar{\varphi}}. \tag{12}$$

It should be clear that the parameter P introduced here is not a canonical momentum. It can be however related with the canonical momentum by $\bar{\pi} = P\bar{p}^{3/2}$. Phase space of this system has been studied in Ref. [8]. Analogous dynamics for the closed FRW model has been studied recently in Ref. [11].

In our consideration, we are going to restrict ourselves to the subset of initial conditions. We consider the initial condition in the prebounce stage at the time t_0. We make a very general assumption that at this arbitrary time, the field is placed in the bottom of the potential, therefore $\bar{\varphi}(t_0) = 0$. Since $\rho \leq \rho_c$, we obtain a restriction for P at t_0, namely $|P(t_0)| \leq \sqrt{2\rho_c}$. Taking a particular value of $P(t_0)$, we can compute the value of the Hubble factor

$$H(t_0) = -\sqrt{\frac{\kappa}{3}\frac{P^2(t_0)}{2}\left(1 - \frac{P^2(t_0)}{2\rho_c}\right)}. \tag{13}$$

In the dynamical system defined by Eqs. (9)–(12), one can distinguish the subsystem $(H, \bar{\varphi}, P)$ whose evolution does not depend on $\bar{p}$. In this subsystem, initial conditions are specified by the value of $P(t_0)$, because $\bar{\varphi}(t_0) = 0$ and $H(t_0)$ is given by Eq. (13). The initial value of $\bar{p}$ can be set arbitrarily, since only changes of $\bar{p}$ have physical meaning.

In the subsequent part of this section, we will show how parameters of inflation depend on the choice of $P(t_0)$ and m.

A. Analytical approximations

Dynamics of the considered model is nonlinear and cannot be traced analytically. However, we can distinguish two regions where approximated analytical solutions can be found. The first is the phase of contraction, and the second is the phase of slow-roll inflation. In this first phase, the field oscillates in the bottom of the potential well. Therefore, the value of $\bar{\varphi} = 0$ is reached many times, what motivates our choice of the initial condition $\bar{\varphi}(t_0) = 0$. The oscillations are amplified when the moment of the bounce is approached. In this regime, evolution of the field is approximated by

$$\bar{\varphi}(t) = C\frac{\cos[m(t-t_0)]}{\bar{p}^{3/4}}. \tag{14}$$

In the subsequent phase of the slow-roll inflation, evolution of the scalar field is approximated by

$$\bar{\varphi}(t) = \bar{\varphi}_{\max} - \frac{m}{\sqrt{12\pi G}}t. \tag{15}$$

In Fig. 2, we show exemplary evolution of the scalar field for the considered model. We show also how approximated solutions fit to the solution obtained numerically. In the contracting phase, the scalar field follows the approximated solution given by Eq. (14). Therefore, energy density behaves like $\rho \simeq \frac{C^2 m^2}{2\bar{p}^{3/2}}$. This is equivalent with the case of the Universe filled by the dust matter. Corresponding evolution of the parameter $\bar{p}$ is in this case given by

$$\bar{p}(t) = \left(-\frac{\sqrt{3\kappa}}{2}Cmt + \bar{p}_i^{3/4}\right)^{4/3}. \tag{16}$$

In the subsequent phase of the slow-roll inflation, the approximated solution is given by

$$\bar{p}(t) = \bar{p}_i \exp\left[\sqrt{\frac{16\pi G}{3}}m\left(\bar{\varphi}_{\max}t - \frac{m}{\sqrt{48\pi G}}t^2\right)\right]. \tag{17}$$

In Fig. 3, we show exemplary evolution of the canonical variable $\bar{p}$ for the considered model. We show also how

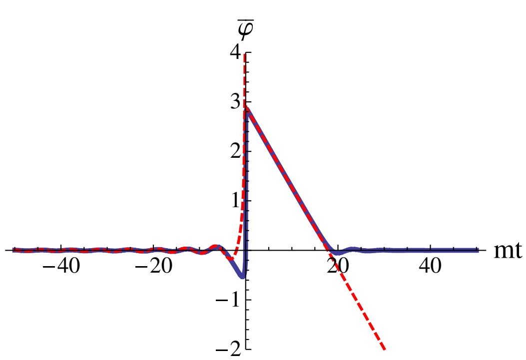

FIG. 2 (color online). Evolution of the field $\bar{\varphi}$. Dashed lines represent analytical approximations. Here, $m = 10^{-4}m_{\mathrm{Pl}}$ and $\bar{\varphi}_{\max} = 2.9m_{\mathrm{Pl}}$.

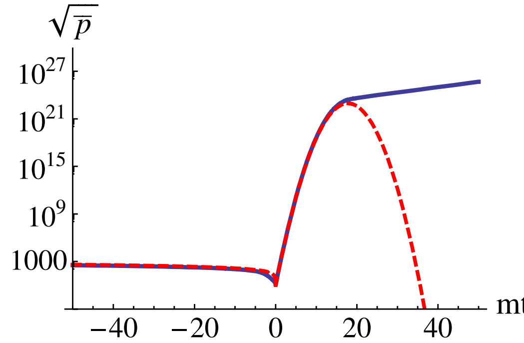

FIG. 3 (color online). Evolution of the variable $\bar{p}$. Dashed lines represent analytical approximations. Here, $m = 10^{-4}m_{\mathrm{Pl}}$ and $\bar{\varphi}_{\max} = 2.9m_{\mathrm{Pl}}$.

approximated solutions fit to the solution obtained numerically.

B. Conditions for inflation

When the field $\bar{\varphi}$ reaches a point of maximal displacement $\bar{\varphi}_{\max}$, then it turns back and consequently, $P(\bar{\varphi}_{\max}) = 0$. At this point, energy of the field is given only by the potential part. Because the total energy density is restricted by $\rho \leq \rho_c$, we obtain the following constraint:

$$|\bar{\varphi}_{\max}| \leq \frac{\sqrt{2\rho_c}}{m} = 1.3\frac{m_{\mathrm{Pl}}^2}{m}. \tag{18}$$

The parameter $\bar{\varphi}_{\max}$ is important, since it gives good characterization of the inflation. Moreover, based on its value, one can express the e-folding number as follows:

$$N \simeq 2\pi\frac{\bar{\varphi}_{\max}^2}{m_{\mathrm{Pl}}^2}. \tag{19}$$

Based on this expression and Eq. (18), we obtain another bound

$$Nm^2 \leq \frac{4\pi\rho_c}{m_{\mathrm{Pl}}^2} = 10.3m_{\mathrm{Pl}}^2. \tag{20}$$

In the bounds given by Eqs. (18) and (20), we have assumed the value of ρ_c given by Eq. (5). However, these bounds can be seen also in the different way. Namely, having parameters of inflation, one can restrict the value of ρ_c.

In the considered setup, the value of $\bar{\varphi}_{\max}$ depends only on $P(t_0)$ and m. It is worth studying how the value of $\bar{\varphi}_{\max}$ is sensitive on them. The results of our investigation are shown in Table I.

In this table, we collected values of $\bar{\varphi}_{\max}$ obtained for the different values of initial parameters. The main conclusion coming from this data is that despite the substantial change of the parameters, the value of $\bar{\varphi}_{\max}$ does not change considerably. Therefore, no fine-tuning is required to obtainthe proper phase of inflation. Moreover, it is worth stressing that the phase of bounce indeed leads to the

TABLE I. Values of $\bar{\varphi}_{\max}$ for the different $P(t_0)$ and m (all parameters in Planck units).

$P(t_0)/m$	1	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}
1	0.5	0.8	1.1	1.4	1.8	2.1	2.2
10^{-1}	0.9	1.1	1.5	1.8	2.2	2.4	2.7
10^{-2}	0.7	1.6	1.8	2.2	2.5	2.8	3.0
10^{-3}		1.3	2.2	2.5	2.9	3.2	3.4
10^{-4}			2.0	3.0	3.2	3.6	4.0
10^{-5}				2.7	3.7	3.9	4.2
10^{-6}					3.4	4.4	4.5
10^{-7}						4.1	5.0

proper inflationary scenario. In the classical considerations, the high value of $\bar{\varphi}_{\max}$ has to be assumed, while in the LQC inspired model, this value can be obtained naturally.

C. Constraining ρ_c, γ, and λ

Now let us assume that parameters of inflation are given by $m = 10^{-6} m_{\mathrm{Pl}}$ and $\bar{\varphi}_{\max} = 3.4 m_{\mathrm{Pl}}$, which gives $N \simeq 73$. These values comes partially from the CMB observations and partially from the requirement of solving the horizon problem. Based on these values and from Eq. (18), we obtain the following constraint:

$$\rho_c \geq 6 \cdot 10^{-12} m_{\mathrm{Pl}}^4. \tag{21}$$

This constraint is not very useful, because it is very weak. However, the point is just to show the possibility of constraining and indicate the presently available cosmological bounds. Lets us also examine the restriction of the Barbero-Immirzi parameter. Taking Eq. (5) together with the constraint Eq. (21), we obtain

$$\gamma \leq 1222. \tag{22}$$

Now one can also constrain the value of parameter λ from the formulation presented in Ref. [7], where $\bar{\mu} = \lambda/\sqrt{\bar{p}}$. Here, λ is the phenomenological scale of the loop quantization (polymerization). Now $\rho_c = \frac{3}{8\pi}\frac{m_{\mathrm{Pl}}^2}{\gamma^2\lambda^2}$ and assuming $\gamma = \gamma_M = 0.2375$, we obtain

$$\lambda \leq 7 \cdot 10^4 l_{\mathrm{Pl}}. \tag{23}$$

IV. PERTURBATIONS

Cosmological perturbations are essential elements in searching for the quantum gravitational signatures. It is because the superhorizontal modes of perturbations can carry information from the inflationary and even preinflationary epoch. Another important issue is that generation of perturbations can have quantum gravitational origin.

In LQC, as we already mentioned in the introduction, the perturbations are introduced according to Eq. (2). Perturbations (δA, δE) can be split for the:

(i) *Scalar modes* (coupled with a scalar field)
This type of perturbation with LQG corrections was studied in Refs. [12–14]. However, until now, only inverse volume corrections have been introduced systematically. Consistent introduction of the holonomy corrections to the scalar modes is still awaiting.

(ii) *Vector modes*
This kind of perturbation is, in general, of the secondary importance in cosmology. It is because they are decaying modes and cannot affect the CMB substantially. However, in the contracting phase, the vector modes are amplified and therefore can be of potential importance. This type of perturbations was studied in the context of LQC in Ref. [15].

(iii) *Tensor modes* (gravitational waves)
Tensor modes in LQC were studied in numerous papers. In contrast to the other two types of perturbations, in this case phenomenological implications were also studied. The effect of inverse volume corrections were studied in Refs. [16–19], while effects of the holonomy corrections were investigated in Refs. [16,20–23]. In these papers, creation of gravitational waves was considered either during the phase of a bounce or during the phase of inflation. The next natural step here is to consider creation of the gravitational waves at the joined bounce + inflation phase considered in Sec. III.

In this section, we will consider a simplified model of perturbations. Namely, we will consider perturbations of the matter field only. The gravitational field is set to be homogeneous. This is only idealization, however, many results from these studies can be extrapolated to the case of scalar and tensor perturbations.

A. Scalar field perturbations

The Hamiltonian of the scalar field is given by

$$H_\varphi = \int_{V_0} d^3\mathbf{x}\left(\frac{1}{2}\frac{\pi_\varphi^2}{\sqrt{|\det E|}} + \frac{1}{2}\frac{E_i^a E_i^b \partial_a\varphi\partial_b\varphi}{\sqrt{|\det E|}} + \sqrt{|\det E|}V(\varphi)\right). \tag{24}$$

Similarly like in the case of gravitational field, the scalar field can be decomposed for homogeneous and perturbation parts

$$\varphi = \bar{\varphi} + \delta\varphi\pi_\varphi = \bar{\pi}_\varphi + \delta\pi_\varphi. \tag{25}$$

Here, homogeneous parts are defined as follows:

$$\bar{\varphi}(t) = \frac{1}{V_0}\int_{V_0} d^3\mathbf{x}\,\varphi(\mathbf{x}, t), \tag{26}$$

$$\bar{\pi}_\varphi(t) = \frac{1}{V_0}\int_{V_0} d^3\mathbf{x}\,\pi_\varphi(\mathbf{x}, t). \tag{27}$$

Equations of motion for the background and perturbation parts are given by

$$\dot{\bar{\varphi}} = \{\bar{\varphi}, H_\varphi\} = \bar{p}^{-3/2}\bar{\pi}_\varphi, \tag{28}$$

$$\dot{\bar{\pi}}_\varphi = \{\bar{\pi}_\varphi, H_\varphi\} = -\bar{p}^{3/2}\frac{dV(\bar{\varphi})}{d\bar{\varphi}}, \tag{29}$$

$$\delta\dot{\varphi} = \{\delta\varphi, H_\varphi\} = \bar{p}^{-3/2}\delta\pi_\varphi, \tag{30}$$

$$\delta\dot{\pi}_\varphi = \{\delta\pi_\varphi, H_\varphi\} = \left(\sqrt{\bar{p}}\nabla^2\delta\varphi - \bar{p}^{3/2}\frac{d^2V(\bar{\varphi})}{d\bar{\varphi}^2}\delta\varphi\right). \tag{31}$$

Combining Eqs. (28) and (29), we obtain Eq. (6). Variable $\delta\varphi$ can be decomposed for the Fourier modes

$$\delta\varphi(\eta, \mathbf{x}) = \int \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{u(\eta, \mathbf{k})}{\sqrt{\bar{p}}}e^{i\mathbf{k}\cdot\mathbf{x}}. \tag{32}$$

Based on this decomposition and on Eqs. (30) and (31), we obtain the equation

$$\frac{d^2}{d\eta^2}u(\eta, \mathbf{k}) + [k^2 + m_{\text{eff}}^2]u(\eta, \mathbf{k}) = 0, \tag{33}$$

where $k^2 = \mathbf{k}\cdot\mathbf{k}$ and

$$m_{\text{eff}}^2 = \bar{p}\frac{d^2V(\bar{\varphi})}{d\bar{\varphi}^2} - \frac{1}{\sqrt{\bar{p}}}\frac{d^2\sqrt{\bar{p}}}{d\eta^2}. \tag{34}$$

In order to describe properties of the perturbations, it is useful to introduce a quantity called the power spectrum which is defined as follows:

$$\mathcal{P}_u = \frac{k^3}{2\pi^2}|u|^2. \tag{35}$$

In the following two subsections, we compute this quantity for two quantum cosmological models. The first will be the model of the symmetric bounce with the free scalar field. The second will be the model with the joined bounce + inflation phase.

B. Symmetric bounce

As in the first case, we consider scalar field perturbations at the symmetric bounce. In the considered case the field is free, $V = 0$. We set the initial conditions to be the Minkowski vacuum

$$u_{\text{in}} = \frac{e^{-ik\eta}}{\sqrt{2k}}. \tag{36}$$

This approximation works, however, only for the subhorizontal modes. With these initial conditions, we solve Eq. (33) numerically. Based on these computations, we obtain the power spectrum of the field u in the post-bounce phase. We show the results in Fig. 4. In this figure, the black straight line represents the analytical approximation of the spectrum. In order to derive this approximation, we assume

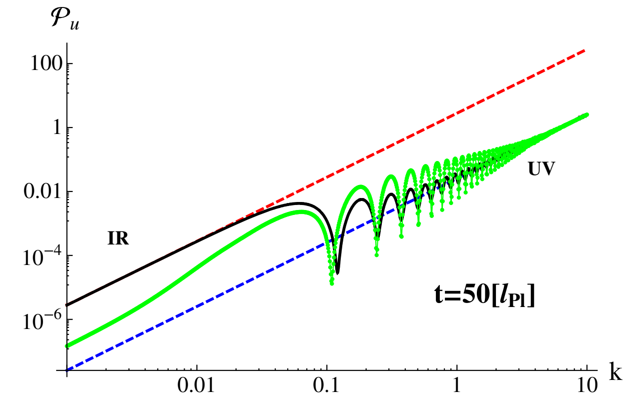

FIG. 4 (color online). Numerical power spectrum of the field u with $\bar{\pi}_\varphi = 0.1m_{\text{Pl}}^2$ (green points). The black line represents the analytical spectrum given by Eq. (43) with $U_0 = 2m_{\text{Pl}}^2$ and $\eta_0 = 0.1l_{\text{Pl}}$. Dashed lines represent UV and IR limits given by Eqs. (44) and (45).

$$u_{\text{out}} = \frac{\alpha_k}{\sqrt{2k}}e^{-ik\eta} + \frac{\beta_k}{\sqrt{2k}}e^{ik\eta}. \tag{37}$$

Here, the relation $|\alpha_k|^2 - |\beta_k|^2 = 1$ holds, as a consequence of the normalization condition. Now, we base on the integral representation

$$u(\eta, \mathbf{k}) = u_{\text{in}}(\eta, \mathbf{k}) + \frac{1}{k}\int_{-\infty}^{\eta} d\eta' U(\eta') \times \sin k(\eta - \eta')u(\eta, \mathbf{k}) \tag{38}$$

of Eq. (33), where $U(\eta) = -m_{\text{eff}}^2(\eta)$. Solving this equation in the first order of perturbative expansion, we compute values of α_k and β_k. We approximate the $U(\eta)$ function by the step function $U(\eta) = U_0\Theta(\eta + \eta_0)\Theta(\eta_0 - \eta)$ of the width $2\eta_0$. The values of parameters U_0 and η_0 can be fixed from the numerical computation of the full model. However, we expect that

$$U_0 \sim -m_{\text{eff}}^2(t = 0) = \frac{\kappa}{3}(2\bar{\pi}_\varphi\rho_c)^{2/3}, \tag{39}$$

where the equality comes from the analytical expression for the m_{eff}^2 function (see Ref. [20]). Moreover, it was shown in Ref. [20] that η_0 can be related with the value of conformal time at H_{max}, then

$$\eta_0 = \eta(t_0) = \frac{2F_1[\frac{1}{2}, \frac{1}{6}, \frac{3}{2}; -1]}{\sqrt{3\kappa}\rho_c^{1/3}(\bar{\pi}_\varphi^2/2)^{1/6}}. \tag{40}$$

The value of parameter η_0 does not have to be however precisely equal to $\eta(t_0)$. We expect rather $\eta_0 \sim \eta(t_0)$. Namely, its value can not be much bigger or much lower than $\eta(t_0)$. In Fig. 4, we showed the case $\bar{\pi}_\varphi = 0.1m_{\text{Pl}}^2$ which was approximated by the model above with $U_0 = 2m_{\text{Pl}}^2$ and $\eta_0 = 0.1l_{\text{Pl}}$. Based on the values of $\eta(t_0)$ and

$-m^2_{\rm eff}(t=0)$, we obtain $\eta_0 = 0.3 l_{\rm Pl}$ and $U_0 = 2.5 m^2_{\rm Pl}$. The better fit to numerical data is obtained when the step function is more narrow than $2\eta(t_0)$ and a bit lower than $-m^2_{\rm eff}(t=0)$.

Based on the performed step function approximation, we find

$$\alpha_k \approx 1 + \frac{i}{2k}\int_{-\infty}^{\infty} d\eta U(\eta) = 1 + i\frac{U_0\eta_0}{k}, \quad (41)$$

$$\beta_k \approx -\frac{i}{2k}\int_{-\infty}^{\infty} d\eta U(\eta) e^{-2ik\eta} = -i\frac{U_0}{2k^2}\sin(2k\eta_0). \quad (42)$$

Now, with use of the definition of the power spectrum, we obtain the analytical expression

$$\mathcal{P}_u = \left(\frac{k}{2\pi}\right)^2 + \frac{U_0(\sin[2k\eta_0]^2 U_0 + 4k^2 U_0 \eta_0^2 + 4k\sin[2k\eta_0](k\sin[2\eta k] - \cos[2\eta k]U_0\eta_0))}{16\pi k^2}, \quad (43)$$

which was shown in Fig. 4. In the UV and IR limits, the power spectrum given by Eq. (43) behaves like

$$\mathcal{P}_u^{\rm UV} \to \left(\frac{k}{2\pi}\right)^2, \quad (44)$$

$$\mathcal{P}_u^{\rm IR} \to \left(\frac{k}{2\pi}\right)^2(1 + 4U_0\eta_0\eta + 4U_0^2\eta_0^2\eta^2). \quad (45)$$

The term in the second bracket in Eq. (45) is the difference between the UV and IR slopes in Fig. 4.

The analytical model correctly reproduces the structure of oscillations obtained from the numerical simulations. However, the relative amplitudes of the modes are not exactly recovered. In particular, the analytical model predicts more power for the low values of k.

The spectrum obtained in this subsection is similar to this obtained in the case of gravitational waves in Ref. [20]. The difference is that in the case of the scalar field, the effective mass $m^2_{\rm eff}$, is negative in the vicinity of the bounce, while in the case of the gravitational waves with holonomy corrections, the effective mass is a positive function during the whole evolution.

C. Bounce + Inflation model

The phase of symmetric bounce is a very idealized situation. More physically relevant is the joined bounce + inflation phase. In this subsection, we show a simple analytical model of perturbations in this phase. It will be a model of perturbations created in the scenario described in Sec. III. In the contracting phase, the subhorizontal modes are given by Eq. (36). The subsequent phase of inflation is approximated by a de Sitter phase where evolution of the scale factor is given by $a = -\frac{1}{H_0\eta}$. In this phase, modes of fluctuations are given by the superposition of Bunch-Davies modes

$$u_{\rm out} = \frac{\alpha_k}{\sqrt{2k}} e^{-ik\eta}\left(1 - \frac{i}{k\eta}\right) + \frac{\beta_k}{\sqrt{2k}} e^{ik\eta}\left(1 + \frac{i}{k\eta}\right), \quad (46)$$

where relation $|\alpha_k|^2 - |\beta_k|^2 = 1$ holds. Performing matching conditions $u_{\rm in}(\eta_i) = u_{\rm out}(\eta_i)$ and $u'_{\rm in}(\eta_i) = u'_{\rm out}(\eta_i)$, we determinate coefficients

$$\alpha_k = -\frac{1 - 2ik\eta_i - 2k^2\eta_i^2}{2k^2\eta_i^2}, \quad (47)$$

$$\beta_k = -\frac{e^{-2ik\eta_i}}{2k^2\eta_i^2}. \quad (48)$$

Based on this, we can derive the power spectrum. At the time $\eta \to 0^-$, it takes a form

$$\mathcal{P}_{\delta\varphi} = \left(\frac{H_0}{2\pi}\right)^2 + \left(\frac{H_0}{2\pi}\right)^2 \frac{k_*^4}{2k^4}\left[1 + \cos\left(\frac{2k}{k_*}\right)\left(-1 + \frac{2k^2}{k_*^2}\right) - \frac{2k}{k_*}\sin\left(\frac{2k}{k_*}\right)\right], \quad (49)$$

where we have defined $k_* = -1/\eta_i$. We show plot of this spectrum in Fig. 5.

The obtained spectrum is characterized by the suppression for the low values of k. For the large k, the spectrum holds the inflationary form. Another important feature is oscillations which are the residue of the bouncing phase. We see that damping begins when $-\eta_i k \sim 1$. This corresponds to the k on the horizon scale at time η_i. At the time η_i, a value of the scale factor is given by $a_i = -\frac{1}{H_0\eta_i}$,

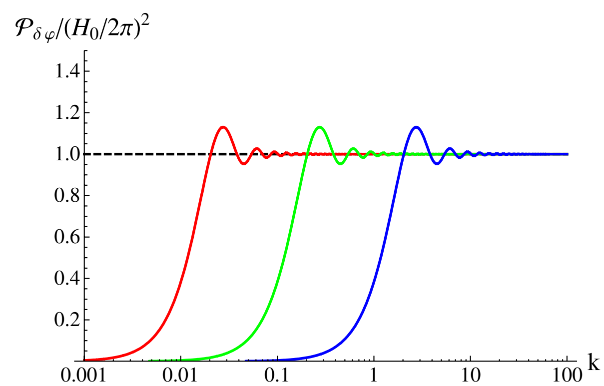

FIG. 5 (color online). Power spectrum of the field $\delta\varphi$. Here, $\eta_i = -1, -10, -100$ (from right to left).

therefore $k_* = a_i H_0$. Defining the length scale $\lambda_* = a_i/k_*$, we obtain $\lambda_* = \frac{1}{H_0}$. Today, the value of λ_* is given by $\lambda_* a_0/a_i$, where a_0 is the present value of a scale factor.

The similar power spectrum to this obtained here was derived also in Refs. [24,25].

V. CMB ANISOTROPY

As we have shown, there are two effects of the bounce phase on the primordial power spectrum: damping of the low energy modes and oscillations. This first effect is dominant, and in this section we are going to investigate its impact of the CMB anisotropy.

A. Sachs-Wolfe approximation

Because we expect that effects of the bounce can affect superhorizontal modes, the Sachs-Wolfe approximation can be used to study the resulting spectrum of CMB. In this approximation, subhorizontal evolution of the primordial plasma is neglected since it does not affect the considered modes. Expression for the CMB multipoles is given by

$$C_l = \frac{4\pi}{25}\int_0^\infty \frac{dk}{k}\mathcal{P}_{\mathcal{R}}(k) j_l^2(kD_\star), \tag{50}$$

where $D_\star = 1.4 \cdot 10^4$ Mpc is distance to last scattering shell. Moreover, $\mathcal{P}_{\mathcal{R}}(k) \equiv \frac{k^3}{2\pi^2}|\mathcal{R}_{\mathbf{k}}|^2$, where $\mathcal{R} = -\frac{v}{z}$ and v is the Mukhanov variable which fulfils the equation

$$v'' + \left[k^2 - \frac{z''}{z}\right]v = 0, \qquad \text{and} \quad \text{where } z = \frac{\sqrt{\bar{p}}\dot{\bar{\varphi}}}{H}.$$

We see that while the approximation $z''/z \approx a''/a$ holds, then the evolution of v and u variables are the same. The validity of this approximation was indicated in Refs. [24,25]. The assumption we made here is the lack of the quantum modification to the equation for the v variable. However, we do not have a right equation for the scalar modes in presence of holonomy corrections. In case of the tensor modes, it was shown in Ref. [20] that corrections to the mode equation do not change the spectrum significantly. It was shown that the shape of the spectrum is determined mainly by the background evolution. Therefore, we assume here that the spectrum of the scalar perturbations is not affected significantly by the holonomy corrections to the mode equation. Then, the spectrum from the joined bounce and inflation phase should have the generic form derived in Sec. IV. In order to build the analytic model, we can average the spectrum over the secondary oscillations. Then, the power spectrum from the joined bounce + inflation is approximated by

$$\mathcal{P}_{\mathcal{R}}(k) = \mathcal{A}_{\mathcal{R}}^2\Theta(k-k_*) + \mathcal{A}_{\mathcal{R}}^2\Theta(k_*-k)\left(\frac{k}{k_*}\right)^2. \tag{51}$$

Based on this spectrum, one can derive the analytical formula for the spectrum of the low multipoles of the CMB anisotropy. Instead of the variable C_l, it is convenient to consider the variable

$$\mathcal{C}_l \equiv \frac{l(l+1)}{2\pi}C_l. \tag{52}$$

It is motivated by the fact that for the scale invariant Harrison-Zeldovich power spectrum, this quantity holds constant value. Performing integral (50) with the spectrum (51), we obtain

$$\mathcal{C}_l = \mathcal{C}_l^{\text{inflation}} + \mathcal{C}_l^{\text{bounce}}, \tag{53}$$

where

$$\mathcal{C}_l^{\text{inflation}} = \frac{\mathcal{A}_{\mathcal{R}}^2}{25}, \tag{54}$$

and

$$\mathcal{C}_l^{\text{bounce}} = \frac{\mathcal{A}_{\mathcal{R}}^2}{25}\frac{x_*^2}{4^{1+l}\Gamma^2(l+3/2)}[l\,{}_2F_3(1+l,1+l;l+3/2,2+l,2l+2;-x_*^2) - (1+l)\,{}_1F_2(l;l+3/2,2l+2;-x_*^2)]. \tag{55}$$

Here, we have introduced the parameter $x_* = k_* D_\star$. In Fig. 6, we show the $\mathcal{C}_l$ spectrum with the parameter $\mathcal{A}_{\mathcal{R}}^2 = 2.6 \cdot 10^{-9}$ set to fit the CMB data and $T_{\text{CMB}} = 2.726$ K. We see that effects of the bounce lead to suppression of the low multipoles in CMB, what is favored observationally. This possibility was indicated earlier in Refs. [21,25].

The value of parameter $\mathcal{A}_{\mathcal{R}}^2$ can be calculated from the slow-roll inflation model

$$\mathcal{A}_{\mathcal{R}}^2 = \frac{1}{2m_{\text{Pl}}^2\epsilon}\left(\frac{H}{2\pi}\right)^2, \tag{56}$$

where ϵ is the slow-roll parameter

$$\epsilon := \frac{1}{2\kappa}\left(\frac{1}{V}\frac{dV}{d\bar{\varphi}}\right)^2 = \frac{1}{4\pi}\frac{m_{\text{Pl}}^2}{\bar{\varphi}^2}. \tag{57}$$

Based on this, we can calculate the value of the Hubble factor at the beginning of inflation

$$H_0 = \frac{\sqrt{2\pi\mathcal{A}_{\mathcal{R}}^2 m_{\text{Pl}}^2}}{\bar{\varphi}_{\max}} = 3.8 \cdot 10^{-5} m_{\text{Pl}}, \tag{58}$$

where in the last equality we assumed $\bar{\varphi}_{\max} = 3.4 m_{\text{Pl}}$.

An important limitation of the method based on the low multipoles in CMB comes from the so-called *cosmic variance*. It is a purely statistical effect which is significant at

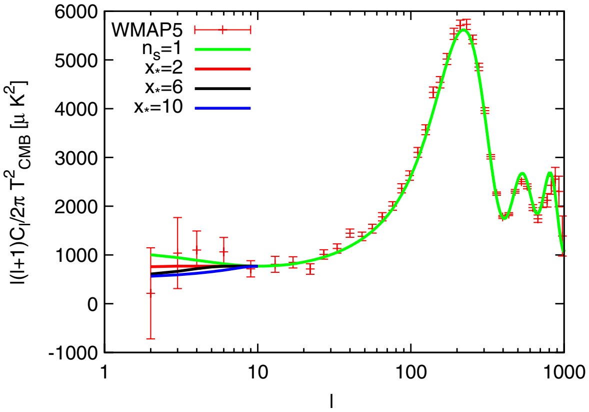

FIG. 6 (color online). Spectrum of CMB anisotropy.

the horizontal scales. In case of CMB, this corresponds to the low multipoles regime. Relative uncertainty coming from the cosmic variance is given by

$$\frac{\Delta C_l}{C_l} = \sqrt{\frac{2}{2l+1}}. \tag{59}$$

Taking for example $l = 2$, we obtain a relative uncertainty equal 0.63. Therefore, the outcome of the measurement is comparable with its uncertainty. This effect cannot be removed and imposes a substantial limitation on our approach to constraint quantum cosmological models.

B. Discussion

In the previous subsection, we showed that bounce can lead to observed suppression of the low CMB multipoles. Now, we would like to discuss whether this scenario is realistic and does non require fine-tuning. At the beginning, we would like to however mention one important adjustment. Namely, the observed scale of cutoff in the CMB spectrum overlaps with the present size of the cosmic horizon. This property was indicated in Ref. [24]. Therefore, there is an intriguing possibility that the observed cutoff is due to unknown evolution of the superhorizontal modes. Therefore, it is not related to the preinflationary dynamics. Such an explanation seems to be more likely. However, the model of this superhorizontal damping does not exist yet.

It is indicated by the observations that the present scale of cutoff $\lambda_*(t_0)$ is comparable with $D_\star$, $\lambda_*(t_0) \approx D_\star$. Therefore,

$$D_\star \approx \lambda_*(t_i)\frac{a_0}{a_i} = \lambda_*(t_i)e^N \frac{T_{\rm GUT}}{T_{\rm dec}}(1+z_{\rm dec}). \tag{60}$$

Because $\lambda_*(t_i) = 1/H_0$, taking Eqs. (58) and (19) we obtain

$$2Ne^{2N} = \xi, \tag{61}$$

where

$$\xi = \frac{2D_\star^2(2\pi)^2 m_{\rm Pl}^2 \mathcal{A}_{\mathcal{R}}^2}{(1+z_{\rm dec})^2}\left(\frac{T_{\rm dec}}{T_{\rm GUT}}\right)^2. \tag{62}$$

It is worth noting that Eq. (61) has a form of the Lambert equation $W(z)e^{W(z)} = z$ which defines the Lambert W function, therefore $N = \frac{1}{2}W(\xi)$. In order to determinate the parameter ξ, we take $z_{\rm dec} \simeq 1070$, $T_{\rm dec} \simeq 0.2$ eV, $T_{\rm GUT} \simeq 10^{14}$ GeV, and $\mathcal{A}_{\mathcal{R}}^2 = 2.6 \cdot 10^{-9}$. Based on this, we obtain $\xi = 5.1 \cdot 10^{62}$ and subsequently $N = 69.7$. Now, one can determinate the second independent parameter of inflation e.g. m. We can easily derive the equation

$$m \simeq \sqrt{\frac{3}{2}\mathcal{A}_{\mathcal{R}}^2}\,\frac{2\pi}{N}m_{\rm Pl}, \tag{63}$$

which gives $m = 5.6 \cdot 10^{-6} m_{\rm Pl}$. As we see, the obtained parameters of inflation are fully consistent with these usually considered. The model is therefore in full agreement with the present observational facts. Moreover, a fine-tuning of inflationary parameters is not required to explain suppression of the low multipoles by the quantum cosmological effects. This however can be seen as a coincidence that the scale of horizon at the beginning of inflation coincides with the present size of the horizon. It remains therefore to check whether this effect is generic and does not depend on the details of inflation, or it is realized only for the particular inflation type.

VI. OTHER OBSERVATIONAL PROBES OF LQC

Besides the effect of suppression of the low multipoles, other potential probes of quantum cosmologies are also available. In this section, we review four possible approaches.

A. Polarization of CMB

In Sec. V, we have shown how the quantum cosmological effects can be related with the spectrum of CMB anisotropy. This method gives us one possible approach to constraint quantum cosmologies. However, observations of CMB bring us much more information than only anisotropies of temperature. Another important measured quantity is the polarization of CMB radiation. This polarization can be described by the spectrum, which depends on the primordial perturbations. While E-type polarization depends on both the scalar and tensor components of perturbations, the B-type polarization depends only on the tensor component. Spectrum of the E type is already observed and can be used e.g. to constrain the cutoff of the power spectrum. Recently, such a study was performed in Ref. [26]. Based on the joined anisotropy and polarization data, it was shown that the scale of cutoff in the power spectrum is $C = \frac{k_c}{10^{-4}\ {\rm Mpc}^{-1}} < 4.2$ at 95% confidence level, while the constraint based only on the polarization gives $C < 5.2$. This result shows that polarization is a good tool to constrain the cutoff in the power spectrum. In particular, while the considerations based only on the CMB anisotropies indicate a cutoff, the joined anisotropy and polarization data indicate a limit on the cutoff. This is crucial while constraining quantum cosmologies based on cutoffs resulting from them.

The second, B-type polarization still remains unreachable observationally. However, there is presently a huge effort to detect it. In particular, a mission like PLANCK aims to detect this type of polarization. If the B-type polarization will be measured, then the amplitude of the tensor power spectrum can be determined. In the case of slow-roll inflation, this amplitude is given by

$$\mathcal{A}_T^2 = \frac{16}{\pi}\left(\frac{H}{m_{\rm Pl}}\right)^2. \quad (64)$$

Because $H^2 \simeq \frac{\kappa}{3}\rho$, the measurements of the B-type polarization enable us to determine energy scale of inflation. Therefore, absolute values of parameters N and m can be found. This also gives us restriction on the prebounce initial condition, in particular, on $P(t_0)$.

B. Nongaussianity

When different modes of perturbations interact with one another, the field becomes non-Gaussian. This interaction can be produced by the potential of the scalar field, higher order corrections in the perturbative expansion or by the quantum gravitational effects.

In case of the Gaussian field, all of its statistical properties are fully described by the two point function $\langle \varphi_{\mathbf{k}_1}\varphi^*_{\mathbf{k}_2}\rangle = \delta^{(3)}(\mathbf{k_1}-\mathbf{k_2})\frac{2\pi^2}{k^3}\mathcal{P}_\varphi(k)$, where $\mathcal{P}_\varphi(k)$ is the power spectrum given by Eq. (35). In order to describe non-Gaussian effects, it is necessary to consider higher order correlation functions. The first contribution of nonlinearity comes in a tree-point function

$$\langle \varphi_{\mathbf{k}_1}\varphi_{\mathbf{k}_2}\varphi_{\mathbf{k}_3}\rangle = \delta^{(3)}(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3})P_3(\mathbf{k_1},\mathbf{k_2},\mathbf{k_3}), \quad (65)$$

where $P_3(\mathbf{k_1},\mathbf{k_2},\mathbf{k_3})$ is called the bispectrum. In case of the Gaussian field, this spectrum is equal zero.

The primordial non-Gaussianity, if present, could affect also the spectrum of CMB anisotropies leading to its non-Gaussianity. Present observations indicate however that the CMB spectrum is nearly Gaussian. This gives us constraint on the cosmological models with a huge amount of nonlinearity. In particular, based on these observations, some quantum cosmological models can be constrained or even excluded. For example, preliminary studies on non-Gaussianity in LQC were performed in Ref. [27]. In these studies, non-Gaussianity is produced by the specific scalar field potential, not by the quantum gravity effects itself. However, this model gives an example of non-Gaussianity production in the bouncing universe. It was shown that in this model, non-Gaussianity is produced in the vicinity of the points t_+ and t_-, where the Hubble factor reaches its maximal value. Another example of non-Gaussianity production in the bouncing cosmology can be found in Ref. [28]. In this paper, production of the non-Gaussianity at the matter bounce is considered and indicates that this form of non-Gaussianity can be potentially distinguished from this produced during the inflationary phase.

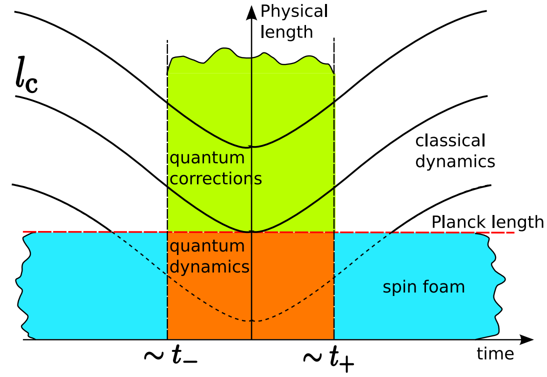

FIG. 7 (color online). Evolution of the different length scales in the bouncing universe.

C. Trans-Planckian modes

When the length of the mode of perturbation approaches the Planck scale, then semiclassical approximation fails. The notion of a continuous wave is missed, and fully quantum gravitational considerations have to be applied. Therefore, these so-called trans-Planckian modes cannot be studied with use of quantum field theory on curved spaces as it was done in the present paper. More adequate would be the application of the quantum field theory on quantum spaces. Some preliminary studies of such a theory were performed in Ref. [29]. However, only the case of a quantum isotropic background is discussed there. The complete approach should take into account also the inhomogeneous backgrounds.

In Fig. 7, we show the evolution of the different length scales in the bouncing universe. We see that modes with $\lambda > l_c$ can be described by the semiclassical approximation. It is exactly the case considered in this paper. However, when $\lambda < l_c$, then new formulation has to be applied. This would lead to potentially new effects. However, these trans-Planckian modes can decay before crossing the horizon during the inflation. Then, any signature of the quantum gravity effects can be lost. However, investigations as those performed in Ref. [30] suggest that the effects of trans-Planckian modes can lead to potentially observational effects.

D. Large scale structures

If the quantum cosmologies can give imprints on CMB, then some of these effects could be seen also in the subsequent structures. The region of the low multipoles corresponds now to the largest visible distances in the

Universe. Gravitational structures on these scales are called large scale structures. Therefore, observations of the large scale structures are complementary to the observations of the low multipoles in CMB. Both of these methods were already applied to investigate the effects of the bouncing cosmological scenario in Ref. [31]. In this paper, the authors predict oscillations in the power spectrum on the horizontal scales due to the bounce. However, available observational data from e.g. Sloan Digital Sky Survey are still not sufficient to probe these effects.

VII. SUMMARY

In this paper, we have investigated realization of the inflationary phase in the framework of loop quantum cosmology. We have shown that the phase of a quantum bounce sets proper initial conditions for inflation. Moreover, we found that parameters of inflation depend only logarithmically on the prebounce initial conditions. Subsequently, we considered the model of perturbations at the bounce and at the joined bounce + inflation phase. We showed that this second model can explain the suppression of the low multipoles in CMB. Moreover, we have indicated that the model is fully consistent and the scale of the cutoff agrees with the present size of horizon. This indicates that fine-tuning is not required to produce suppression on the horizontal scales.

The results of the paper indicate the possible way to relate quantum cosmological models with astronomical observations. The presented method can be used to constrain models of the Universe in the Planck epoch. In particular, we constrained the Barbero-Immirzi parameter γ, critical density ρ_c, and parameter λ. The obtained constraints are however still very weak and ambiguous. The part of this ambiguity can be removed by fixing the parameters of inflation (N and m). At present, we can put a lower limit on N from the so-called tensor-to-scalar ratio, which gives $N > 36$. The value of N can be estimated by the requirement of solving the horizon problem. One can also try to determine N based on the observed value of the spectral index n_s. Namely, one can find that $N = 2/(1 - n_s)$, which for e.g. $n_s = 0.97$ gives $N = 67$. However, the value of the e-folding number obtained in this way can be treated, only as the lower limit on N. There can be more e folding before the seeds of the observed structures were created (Then $\bar{\varphi}_{\max} \geq \bar{\varphi} = 3.4 m_{\mathrm{Pl}}$). With the upper limit on N, the situation is even worse. In the classical cosmology, the total value of N can be arbitrarily high. However, in the framework of loop quantum cosmology, the total value of e folding is bounded. Namely, inserting the upper bound (18) into expression (19), we find $N_{\max} \approx 11 \frac{m_{\mathrm{Pl}}^2}{m^2}$. Therefore, by knowing the vale of m, one can put the upper constraint on the e-folding number. In particular, for the realistic vale $m = 10^{-6} m_{\mathrm{Pl}}$, we find the upper limit on the e-folding number, $N_{\max} \approx 10^{13}$. This limitation is however very weak. Therefore, with the present cosmological observations, only the lower limit on N can be estimated.

Having N, the value of m can be determined from the amplitude of the CMB anisotropies. However, due to the problems discussed above, N remains unknown. Therefore, at present, one can only fix the relation between N an m but not their absolute values. In order to fix one of these parameters, another observable has to be measured. The most promising is an amplitude of tensor perturbations. These perturbations produce B-type polarization in CMB. Therefore, if this polarization would be measured, then parameters N and m can be fixed. This will enable us to perform a less ambiguous constraint on the LQC parameters, in particular, on critical energy density.

Besides the cosmological approach to constrain quantum gravity effects, other indirect methods are also available, in particular, astrophysical measurements of the Lorentz symmetry violation. It is in principle possible to derive quantitative predictions about this effect directly from LQG. However, the process of derivation requires construction of the semiclassical states, where unknown phenomenological parameters appear. Since their values are unknown, the predictive power of this approach is marginal. If these difficulties would be removed, then this method can be complementary with the cosmological approach. Also, the quantum effects on the gravitational collapses give a possible way to put constraint on the LQG. Here, the theoretical predictions are less ambiguous; however, it is harder to relate them with any astrophysical data.

As we see, the difficulties lie here on both the theoretical and empirical sides. Without knowledge about the relation between the LQC and LQG, we can treat parameters of this first rather as phenomenological. If these difficulties would not be overcome, it will be hard to perform complementary constraints of the same parameters, with the use of the different methods, e.g. observations of CMB and gamma ray bursts.

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