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Friedmann equations from entropic force

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In this paper, by use of the holographic principle together with the equipartition law of energy and the Unruh temperature, we derive the Friedmann equations of a Friedmann-Robertson-Walker universe.

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It is a long-held idea that gravity is not regarded as a fundamental interaction in Nature. The earliest idea on this was proposed by Sakharov in 1967 [1]. In this so-called induced gravity, spacetime background emerges as a mean field approximation of underlying microscopic degrees of freedom, similar to hydrodynamics or the continuum elasticity theory from molecular physics [2]. This idea has been further developed since the discovery of the thermodynamic properties of black holes in the 1970s. Black hole thermodynamics tells us that a black hole has an entropy proportional to its horizon area and a temperature proportional to its surface gravity at the black hole horizon, and the entropy and temperature together with the mass of the black hole satisfy the first law of thermodynamics [3–5].

The geometric feature of thermodynamic quantities of a black hole leads Jacobson to ask an interesting question about whether it is possible to derive Einstein's equations of gravitational fields from a point of view of thermodynamics [6]. It turns out that it is indeed possible. Jacobson derived Einstein's equations by employing the fundamental Clausius relation $\delta Q = TdS$ together with the equivalence principle. Here the key idea is to demand that this relation hold for all the local Rindler causal horizons through each spacetime point, with δQ and T interpreted as the energy flux and Unruh temperature seen by an accelerated observer just inside the horizon. In this way, Einstein's equation is nothing but an equation of state of spacetime.

Further, assuming the apparent horizon of a Friedmann-Robertson-Walker (FRW) universe has temperature T and entropy S satisfying $T = 1/2\pi\tilde{r}_A$ and $S = A/4G$, where $\tilde{r}_A$ is the radius of the apparent horizon and A is the area of the apparent horizon, one is able to derive Friedmann equations of the FRW universe with any spatial curvature by applying the Clausius relation to apparent horizon [7]. This works not only in Einstein's gravitational theory, but also in Gauss-Bonnet and Lovelock gravity theories. Here a key ingredient is to replace the entropy area formula in Einstein's theory by using entropy expressions of a black hole horizon in those higher order curvature theories. Recently the Hawking temperature associated with the apparent horizon of a FRW universe has been shown [8]. There exist a lot of papers investigating the relation between the first law of thermodynamics and the Friedmann equations of FRW universe in various gravity theories. For more references see, for example, [9,10] and references therein.

Another hint appears on the relation between thermodynamics and gravitational dynamics by investigating the relation between the first law of thermodynamics and gravitational field equation in the setup of black hole spacetime. Padmanabhan [11] first noticed that the gravitational field equation in a static, spherically symmetric spacetime can be rewritten as a form of the ordinary first law of thermodynamics at a black hole horizon. This indicates that Einstein's equation is nothing but a thermodynamic identity. This observation was then extended to the cases of stationary axisymmetric horizons and evolving spherically symmetric horizons in Einstein's gravity [12], static spherically symmetric horizons [13], and dynamical apparent horizons [14] in Lovelock gravity, and three dimensional Banados-Teitelboim-Zanelli black hole horizons [15]. Very recently it has been shown that it also holds in Horava-Lifshitz gravity [16]. For a recent review on this topic and some relevant issues, see [17].

In a very recent paper by Verlinde [18], the viewpoint of gravity being not a fundamental interaction has been further advocated. Gravity is explained as an entropic force caused by changes in the information associated with the positions of material bodies. Among various interesting observations made by Verlinde, here we mention two of them. One is that with the assumption of the entropic force together with the Unruh temperature [19], Verlinde is able to derive the second law of Newton. The other is that the assumption of the entropic force together with the holographic principle and the equipartition law of energy leads to Newton's law of gravitation. Similar observations are also made by Padmanabhan [20]. He observed that the equipartition law of energy for the horizon degrees of freedom combing with the thermodynamic relation $S = E/2T$, also leads to Newton's law of gravity, here S and T are thermodynamic entropy and temperature associated

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with the horizon and E is the active gravitational mass producing the gravitational acceleration in the spacetime [21]. Finally, we mention that there exist some earlier attempts to build microscopic models of spacetime, for example, see [22–24].

In this short paper we are going to derive the Friedmann equations governing the dynamical evolution of the FRW universe from the viewpoint of entropic force together with the equipartition law of energy and the Unruh temperature by generalizing some arguments of Verlinde to dynamical spacetimes.

Consider the FRW universe with metric

$$ds^2 = -dt^2 + a^2(t)(dr^2 + r^2 d\Omega^2), \tag{1}$$

where $a(t)$ is the scale factor of the universe. Following [18], consider a compact spatial region $\mathcal{V}$ with a compact boundary $\partial\mathcal{V}$, which is a sphere with physical radius $\tilde{r} = ar$. The compact boundary $\partial\mathcal{V}$ acts as the holographic screen. The number of bits on the screen is assumed as

$$N = \frac{Ac^3}{G\hbar}, \tag{2}$$

where A is the area of the screen (note that there is a factor difference $1/4$ from the Bekenstein-Hawking area entropy formula of black hole). Assuming the temperature on the screen is T, and then according to the equipartition law of energy, the total energy on the screen is

$$E = \tfrac{1}{2}Nk_BT. \tag{3}$$

Further, just as in [18], we need the relation

$$E = Mc^2, \tag{4}$$

where M represents the mass that would emerge in the compact spatial region $\mathcal{V}$ enclosed by the boundary screen $\partial\mathcal{V}$.

Suppose the matter source in the FRW universe is a perfect fluid with stress-energy tensor

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu}. \tag{5}$$

Because of the pressure, the total mass $M = \rho V$ in the region enclosed by the boundary $\partial\mathcal{V}$ is no longer conserved, the change in the total mass is equal to the work made by the pressure $dM = -pdV$, which leads to the well-known continuity equation

$$\dot{\rho} + 3H(\rho + p) = 0, \tag{6}$$

where $H = \dot{a}/a$ is the Hubble parameter.

The total mass in the spatial region $\mathcal{V}$ can be expressed as

$$M = \int_{\mathcal{V}} dV(T_{\mu\nu}u^\mu u^\nu), \tag{7}$$

where $T_{\mu\nu}u^\mu u^\nu$ is the energy density measured by a comoving observer. On the other hand, the acceleration for a radial comoving observer at r, namely, at the place of the screen, is

$$a_r = -d^2\tilde{r}/dt^2 = -\ddot{a}r, \tag{8}$$

where the negative sign arises because we consider the acceleration is caused by the matter in the spatial region enclosed by the boundary $\partial\mathcal{V}$. Note that the proper acceleration vanishes for a comoving observer. However, the acceleration (8) is crucial in the following discussions. According to the Unruh formula, we assume that the acceleration corresponds to a temperature

$$T = \frac{1}{2\pi k_B c}\hbar a_r. \tag{9}$$

Now it is straightforward to derive the following equation from Eqs. (2)–(4), (7), and (9):

$$\ddot{a} = -\frac{4\pi G}{3}\rho a. \tag{10}$$

This is nothing but the dynamical equation for Newtonian cosmology (page 10 in [25]). Note that Ref. [25] derives (10) from the Newtonian gravity law, while we obtain (10) by using the holographic principle and the equipartition law of energy in statistical physics. To produce the Friedmann equations of the FRW universe in general relativity, let us notice that producing the acceleration is the so-called active gravitational mass $\mathcal{M}$ [21], rather than the total mass M in the spatial region $\mathcal{V}$. The active gravitational mass is also called Tolman-Komar mass, defined as

$$\mathcal{M} = 2\int_{\mathcal{V}} dV\Big(T_{\mu\nu} - \frac{1}{2}Tg_{\mu\nu}\Big)u^\mu u^\nu. \tag{11}$$

Replacing M by $\mathcal{M}$, we have in this case

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p). \tag{12}$$

This is just the acceleration equation for the dynamical evolution of the FRW universe. Multiplying $\dot{a}a$ on both sides of Eq. (12), and using the continuity equation (6), we integrate the resulting equation and obtain

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho. \tag{13}$$

Note that k appears in (13) as an integration constant, but it is clear that the constant k has the interpretation as the spatial curvature of the region $\mathcal{V}$ in the Einstein theory of gravity. $k = 1$, 0 and -1 correspond to a closed, flat, and open FRW universe, respectively.

The above discussion can be extended to any spacetime dimension $d \geq 4$. In that case, the number of bits on the screen is changed to [18]

$$N = \frac{1}{2}\frac{d-2}{d-3}\frac{Ac^3}{G\hbar}, \tag{14}$$

the continuity equation becomes $\dot{\rho} + (d-1)H(\rho + p) = 0$, and the active mass $\mathcal{M}$ is defined as

$$\mathcal{M} = \frac{d-2}{d-3}\int_{\mathcal{V}} dV\Big(T_{\mu\nu} - \frac{1}{d-2}Tg_{\mu\nu}\Big)u^\mu u^\nu. \quad (15)$$

The acceleration equation (12) is changed to

$$\frac{\ddot{a}}{a} = -\frac{8\pi G}{(d-1)(d-2)}((d-3)\rho + (d-1)p). \quad (16)$$

Integrating (16) we then have

$$H^2 + \frac{k}{a^2} = \frac{16\pi G}{(d-1)(d-2)}\rho. \quad (17)$$

This is just the Friedmann equation of the FRW universe in d dimensions.

Thus we have derived the Friedmann equations of a FRW universe starting from the holographic principle and the equipartition law of energy by using Verlinde's argument that gravity appears as an entropic force. Before we close this paper, however, some remarks are in order. First, it is claimed that Verlinde's arguments are applicable to any spacetime, but Verlinde mainly discusses the cases of static and/or stationary spacetimes. In particular, when he derives Einstein's equation, a timelike Killing vector is employed. The timelike Killing vector exists for static or stationary spacetimes, and it does not for a dynamical spacetime. Here we have applied his arguments to the FRW universe, a special dynamical spacetime, and obtained the dynamical equations governing the evolution of the FRW universe. Second, in deriving Newton's law of gravity, Verlinde considers a spherical surface with a fixed radius as the holographic screen, and does not take into account the evolution of the background spacetime itself. This is right since in Newton's gravity, the background spacetime is a fixed one. In our case, the holographic screen is a dynamical one, in some sense, so it can be viewed as a surface of the spherical symmetric dust matter [25]. The surface evolves due to the self-gravity. Thus, an observer (or a test particle) on the screen will feel a force which leads to an acceleration (8). The final comment is concerned with the assumed relation (9). According to Unruh, the acceleration could correspond to a local Unruh temperature on the screen (9). Note that the acceleration (8) is not a proper acceleration; the proper acceleration vanishes for a comoving observer in the FRW universe. In fact, a_r is just the acceleration of geodesic deviation vector [26]. Let us recall that Verlinde arrives at the second law of Newton starting from entropic force together with the Unruh relation (Eq. (3.8) in [18]), which relates the temperature on the screen to an acceleration. Note that the second law of Newton is a nonrelativistic form, where the acceleration has a form $\ddot{x}$. The situation is the same as the case of the discussions in the present paper. Indeed Eq. (10) has a nonrelativistic origin. It is argued by Verlinde that here the Unruh relation should be read as a formula for the temperature on the screen that is required to cause the acceleration, not as usual, as the temperature caused by an acceleration. Therefore, the relation (9) may be regarded as a working ansatz here. Thus it is a very interesting issue to see whether there exists such a relation between the Unruh temperature and the acceleration. To this aim, some useful references are already available [27]: It is known that Hawking temperature in de Sitter space and in some black hole spacetimes can be viewed as an Unruh temperature for a Rindler observer in higher-dimensional flat spacetimes in which the de Sitter spacetime and black holes spacetime can be embedded.

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