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Predictions for high-energy pp and $\bar{p}p$ scattering from a finite sum of gluon ladders

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An eikonalized elastic proton-proton and proton-antiproton scattering amplitude $F(s, t)$, calculated from QCD as a finite sum of gluon ladders, is compared with the existing experimental data on the total cross section and the ratio $\rho(s, 0) = \mathrm{Re}F(s, 0)/\mathrm{Im}F(s, 0)$ of the real part to the imaginary part of the forward amplitude. Predictions for the expected LHC energies are given.

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I. INTRODUCTION

According to the common belief, the Pomeron in QCD corresponds to an infinite sum of gluon ladders with Reggeized gluons, resulting [1–3] in the so-called supercritical behavior $\sigma_t \sim s^{\alpha_P(0)-1}$, where $\alpha_P(0) > 1$ is the intercept of the Pomeron trajectory, as discussed in Ref. [4]. In that approach, the main contribution to the inelastic amplitude and to the absorptive part of the elastic amplitude in the forward direction arises from the multi-Regge kinematics in the limit $s \to \infty$ and leading logarithmic approximation. In the next-to-leading logarithmic approximation, corrections require also the contribution from the quasi-multi-Regge kinematics [5]. Hence, the subenergies between neighboring s-channel gluons must be large enough to be in the Regge domain. At finite total energies, this implies that the amplitude is represented by a finite sum of N terms [6], where N increases like $\ln s$, rather than by the solution of the Balitsky-Fadin-Kuraev-Lipatov (BFKL) integral equation [1–3]. The interest in the first few terms of the series is related to the fact that the energies reached by the present accelerators are not high enough to accommodate a large number of s-channel gluons that eventually hadronize and give rise to clusters of secondary particles [7].

The lowest order diagram is that of two-gluon exchange, first considered by Low and Nussinov [8]. The next order, involving an s-channel gluon rung was studied, e.g. in papers [2,9] and generalized in Ref. [1]. The problem of calculating these diagrams is twofold. The first one is connected with the nonperturbative contributions to the scattering amplitude in the ''soft'' region. It may be ignored by ''freezing'' the running coupling constant at some fixed value of the momenta transferred and assuming that the forward amplitude can be cast by a smooth interpolation to $t = 0$. More consistently, one introduces a nonperturbative model [10] of the gluon propagator valid also in the forward direction. The second problem is more technical: as $s \to \infty$ the number of Feynman diagrams that contribute to the leading order rapidly increases and, in each of them, only the leading contribution is usually evaluated. At any order in the coupling, subleading terms coming both from the neglected diagrams and from the calculated ones are present. Although functionally the result is always the sum of increasing powers of logarithms, the numerical values of the coefficients entering the sum is lost unless all diagrams are calculated.

Conversely, one can expand the ''supercritical'' Pomeron $\sim s^{\alpha_P(0)}$ in powers of $\ln(s)$. Such an expansion is legitimate within the range of active accelerators, i.e. near and below the TeV energy region, where fits to total cross sections by power or logarithms are known [11] to be equivalent numerically. Moreover, forward scattering data (total cross sections and the ratio of the real to the imaginary part of the forward scattering amplitude) do not discriminate even between a single and quadratic fit in $\ln(s)$ to the data.

In Ref. [6] a model for the Pomeron at $t = 0$ based on the idea of a finite sum of ladder diagrams in QCD was suggested. According to the idea of that paper, the number of s-channel gluon rungs and correspondingly the powers of logarithms in the forward scattering amplitude depends on the phase space (energy) available, i.e. as energy increases, progressively new prongs with additional gluon rungs in the s-channel open. Explicit expressions for the total cross section involving two and three rungs or, alternatively, three and four prongs (with $\ln^2(s)$ and $\ln^3(s)$ as highest terms, respectively) were fitted to the proton-proton and proton-antiproton total cross section data in the accelerator region.

In a related paper [12] the Pomeron was considered as a finite series of ladder diagrams, including one gluon rung besides the Low-Nussinov ''Born term'' and resulting in a

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constant plus logarithmic term in the total cross section. With a subleading Regge term added, good fits to pp and $p\bar{p}$ total as well as differential cross section were obtained in [12]. There is however a substantial difference between Ref. [6] and that of Ref. [12] or simple decomposition in powers of $\ln(s)$, namely, that we consider the opening channels (in s) as threshold effects, the relevant prongs being separated in rapidity by $\ln s_0$, s_0 being a parameter related to the average subenergy in the ladder. Although such an approach inevitably introduces new parameters, we consider it more adequate in the framework of the finite-ladder approach. We mention these attempts only for the sake of completeness, although we stick to the simplest case of $t = 0$, where there are hopes to have some connection with the QCD calculations.

Within the "finite gluon ladder approach" to the Pomeron (see [6] and references therein), several options are possible. In Ref. [6] a system of interconnected equations was solved with several free parameters, including the value of s_0^i. that determine the opening of each threshold (prong). In that paper finite gluon ladders were calculated from QCD, where the important dynamical information is contained in ρ of Eq. (7) of that paper, including $\ln s$ terms multiplied by the QCD running constant α_s constraining the interconnection between various powers of the logarithms in the total cross section. If one chooses $\alpha_s = 0.5$–0.7, a typical "frozen" value of the QCD coupling constant, the resulting total cross section will rise too fast with respect to the data. Good fits within this option can be achieved only if α_s is an order of magnitude smaller than the above "canonical" value. Whether this is acceptable or not is an open question (see below, Sec. IV and the conclusions in Sec. V of this paper).

In the present paper we include the unitarization procedure: we consider the QCD-inspired amplitude as a Born term, subject to a subsequent unitarization procedure. We use the eikonal formalism and treat the running constant as a free parameter. The resulting eikonalized amplitude, fitted to the data (Sec. IV), gives $\alpha_s \approx 0.2$. This can be considered also as a way of deriving the QCD running coupling in the soft region.

II. TOTAL CROSS SECTIONS FROM A FINITE SUM OF GLUON LADDERS

The Pomeron contribution to the total cross section is represented in the form

$$\sigma_t^{(P)}(s) = \sum_{i=0}^{N} f_i \theta(s - s_0^i)\theta(s_0^{i+1} - s), \tag{1}$$

where

$$f_i = \sum_{j=0}^{i} a_{ij} L^j, \tag{2}$$

s_0 is the prong threshold, $\theta(x)$ is the step function and $L \equiv \ln(s)$. Here, by s and s_0, respectively, $s/(1\ \mathrm{GeV}^2)$ and $s_0/(1\ \mathrm{GeV}^2)$ are implied. The main assumption in Eq. (1) is that the widths of the rapidity gaps $\ln(s_0)$ are the same along the ladder. The functions f_i are polynomials in L of degree i, corresponding to finite gluon ladder diagrams in QCD, where each power of the logarithm collects all the relevant diagrams. When s increases and reaches a new threshold, a new prong opens adding a new power in L. In the energy region between two neighboring thresholds, the corresponding f_i, given in Eq. (1), is supposed to represent adequately the total cross section.

In Eq. (1) the sum over N is a finite one, since N is proportional to $\ln(s)$, where s is the present squared c.m. energy. Hence, this model is quite different from the usual approach where, in the limit $s \to \infty$, the infinite sum of the leading logarithmic contributions gives rise to an integral equation for the amplitude.

To make the idea clearer, we describe the mechanism in the case of three gaps (two rungs). To remedy the effect of the first threshold and get a smooth behavior at low energies, we have included also a Pomeron daughter, going like $\sim 1/s$ in the first two gaps with parameters b_0 and b_1, respectively. Then

$$f_0(s) = a_{00} + b_0/s \quad \text{for } s \le s_0, \tag{3}$$

$$f_1(s) = a_{10} + b_1/s + a_{11}L \quad \text{for } s_0 \le s \le s_0^2, \tag{4}$$

$$f_2(s) = a_{20} + a_{21}L + a_{22}L^2 \quad \text{for } s_0^2 \le s \le s_0^3. \tag{5}$$

By imposing the requirement of continuity (of the cross section and of its first derivative) one constrains the parameters. For example, from the conditions $f_1(s_0) = f_0(s_0)$ and $f_1'(s_0) = f_0'(s_0)$ the relations

$$b_1 = a_{11}s_0 + b_0, \qquad a_{10} = a_{00} - a_{11}\ln(s_0) - a_{11}$$

follow. Furthermore, from $f_2(s_0^2) = f_1(s_0^2)$ and $f_2'(s_0^2) = f_1'(s_0^2)$ one gets

$$a_{20} = a_{22}\ln^2(s_0^2) + a_{10} + b_1(1 + \ln(s_0^2))/s_0^2,$$
$$a_{21} = a_{11} - 2a_{22}\ln(s_0^2) - b_1/s_0^2.$$

The same procedure can be repeated for any number of gaps.

In fitting the model to the data, the authors of Ref. [6] relied mainly on $p\bar{p}$ data that extend to the highest (accelerator) energies, to which the Pomeron is particularly sensitive. To increase the confidence level, pp data were included in the fit as well. To keep the number of the free parameters as small as possible and following the successful phenomenological approach of Donnachie and Landshoff [13], a single "effective" Reggeon trajectory with intercept $\alpha(0)$ will account for nonleading contributions, thus leading to the following form for the total cross section:

$$\sigma_t(s) = \sigma_t^{(P)}(s) + R(s), \tag{6}$$

where $\sigma_t^{(P)}(s)$ is given by Eq. (1) and $R(s) = as^{\alpha(0)-1}$ (note that a is different for $p\bar{p}$ and pp and is considered as an additional free parameter).

Ideally, one would let free the width of the gap s_0 and consequently the number of gluon rungs (highest power of L). Although possible, technically this is very difficult. Therefore we considered only the cases of two and three rungs and, for each of them, we treated s_0 as a free parameter.

Notice that the values of the parameters depend on the energy range of the fitting procedure. For example, the values of the parameters in f_0 if fitted in "its" range, i.e. for $s \leq s_0$, will get modified in f_1 with the higher energy data and correspondingly higher order diagrams included.

As a first attempt, only three rapidity gaps, that correspond to two-gluon rungs in the ladder were considered. Fits to the $p\bar{p}$ and pp data were performed from $\sqrt{s} = 4$ GeV up to the highest energy Tevatron data [14]. Interestingly, the value of s_0 turned out to be very close to 144 GeV^2, i.e. the value for which the energy range considered is covered with equal rapidity gaps uniformly.

Next, the energy span available in the accelerator region by four gaps, resulting in 3 gluon rungs and consequently L^3 as the maximal power were covered [6]. After the matching procedure, ten free parameters remained: first of all s_0, then a_{00}, b_0, a_{11}, a_{22}, a_{32}, a_{33}, each determined in its range, while the two a's and $\alpha(0)$ are fitted in the whole range of the data. The final value for s_0 turned out to be $s_0 \simeq 42.5$ GeV^2 resulting in a sequence of energy intervals ending at $\sqrt{s} = 1800$ GeV. Interestingly, search for the phase space region where the production amplitude in the multicluster configuration has a maximum resulted, with the help of cosmic-ray data, to an average "subenergy" $\langle s_i \rangle \sim 44$ GeV^2 [15], that is very near to the value of s_0 found in the fit.

III. EXPLICIT ITERATIONS OF BFKL

From the theoretical point of view, the phenomenological model of Sec. II corresponds to the explicit evaluation in QCD of gluonic ladders with an increasing number of s-channel gluons. This correspondence is far from literal since each term of the BFKL series takes into account only the dominant logarithm in the limit $s \to \infty$. In the following we give concrete expressions for the forward high-energy scattering amplitudes for hadrons in the form of an expansion in powers of large logarithms in the leading logarithmic approximation.

We start from known results obtained in paper [2] where an explicit expression for the total cross section for hadron-hadron scattering has been obtained. In the high-energy limit, it is convenient to introduce the Mellin transform of the amplitude

$$\mathcal{A}(\omega, t) = \int_0^\infty d\tilde{s}\tilde{s}^{-\omega-1}\frac{\mathrm{Im}_s\mathcal{A}(s,t)}{s}, \qquad \tilde{s} = \frac{s}{m^2}$$

and its inverse

$$\frac{\mathrm{Im}_s\mathcal{A}(s,t)}{s} = \frac{1}{2\pi i}\int_{\delta-i\infty}^{\delta+i\infty} d\omega\tilde{s}^{\omega}\mathcal{A}(\omega,t).$$

The general expression of $A(\omega, t)$ in the leading logarithmic approximation has the form:

$$\mathcal{A}(\omega, t) = \int d^2k\frac{\Phi^a(k,q)F^b_\omega(k,q)}{k^2(q-k)^2},$$

where $\Phi^a(k, q)$ and $\Phi^b(k, q)$ (see next equation) are the impact factors of the colliding hadrons a and b, obeying the gauge conditions $\Phi^j(0, q) = \Phi^j(q, q) = 0$ ($j = a, b$). The quantity $F^b_\omega(k, q)$ obeys the BFKL equation:

$$\omega F^b_\omega(k,q) = \Phi^b(k,q) + \gamma\int\frac{d^2k'}{2\pi} \times\frac{A(k,k',q)F^b_\omega(k',q) - B(k,k',q)F^b_\omega(k,q)}{(k-k')^2},$$

with

$$A(k,k',q) = \frac{-q^2(k-k')^2 + k^2(q-k')^2 + k'^2(q-k)^2}{k'^2(q-k')^2},$$

$$B(k,k',q) = \frac{k^2}{k'^2 + (k'-k)^2} + \frac{(q-k)^2}{(q-k')^2 + (k-k')^2},$$

and

$$\gamma = 3\frac{\alpha_s}{\pi}.$$

The strong coupling α_s is assumed to be frozen at a suitable scale set, for example, by the external particles. The iteration procedure and the inverse Mellin transform give (furthermore, we set $q = 0$):

$$\sigma_t(s) = \frac{\mathrm{Im}_s A(s,0)}{s} = \int d^2k\frac{\Phi^a(k,0)}{(k^2)^2}\bigg[\Phi^b_0(k) + \rho\Phi^b_1(k) + \frac{1}{2!}\rho^2\Phi^b_2 + \ldots\bigg],$$

where

$$\rho = \frac{3\alpha_s}{\pi}\ln\tilde{s} \tag{7}$$

and the subsequent iterations begin from $\Phi^b_0(k) = \Phi^b(k, 0)$. In the previous integral and everywhere in the following, all the momenta are two-dimensional Euclidean vectors, living in the plane transverse to the one formed by the momenta of the colliding particles.

To obtain the cross section of proton-proton scattering, we use the ansatz of Ref. [16] for the impact factor of a hadron in terms of its form factor $F(q^2)$:

$$\Phi^p(k,q) = F^p\left(\frac{q^2}{4}\right) - F^p\left(\left(k - \frac{q}{2}\right)^2\right),$$
$$\Phi^p(0,q) = \Phi(q,q) = 0.$$

Here the two-dimensional Euclidean vector q is related to the four-dimensional transferred momentum Q by the relation $Q^2 = -q^2 < 0$. To get the input value for Φ_0, we use

$$F_0(k) = ak^2 e^{-ck^2}, \tag{8}$$

where a and c are in GeV^{-2}. It is convenient to define

$$\psi_n(k^2) = \frac{F_n(k)}{k^2},$$

then (for $n \geq 1$)

$$\psi_n(k^2) = \int_0^1 \frac{dx}{1-x}(\psi_{n-1}(k^2x) - \psi_{n-1}(k^2)) + \int_1^\infty \frac{dx}{x-1} \times \left(\psi_{n-1}(k^2x) - \frac{1}{x}\psi_{n-1}(k^2)\right)$$

and

$$\sigma_t(s) = \pi \int_0^\infty dk^2 \psi_0(k^2) \sum_n \psi_n(k^2) \frac{\rho^n}{n!}. \tag{9}$$

Integrations can be performed analytically, due to the simple choice of the impact factor in Eq. (8), and the final result is

$$\sigma_t(s) = \frac{\pi a^2}{2c}\left\{1 + 2(\ln 2)\rho + \left[\frac{\pi^2}{12} + 2(\ln 2)^2\right]\rho^2 + \frac{1}{3}\left[\frac{\pi^2}{2}(\ln 2) + 4(\ln 2)^3 - \frac{3}{4}\zeta(3)\right]\rho^3 + \ldots\right\}, \tag{10}$$

where ρ is defined in Eq. (7), and $\zeta(3)$ is the Riemann's Zeta function $\zeta(3) \approx 1.202$.

IV. UNITARIZED FIT

In Sec. II, we quoted the calculated cross sections of the finite-ladder-Pomeron model with the parameters fitted to the existing data, the threshold value (opening prong) of the interconnected ladders playing there an important role. In Sec. III, the parameters were calculated from QCD. With these parameters the resulting cross sections overshoot the data, which is not surprising, since the calculated Born term should be subjected to a unitarization procedure. Below we perform such calculations in the framework of the eikonal formalism and compare the results with the experimental data.

We start from Eq. (10), Sec. III for the pp and $\bar{p}p$ total cross section. Supplying that expressing with an exponential t dependence we get the elastic scattering amplitude

$$F_{\mathrm{Born}}(s,t) = A(-i\tilde{s})^{1+\alpha' t}[a_0 + a_1\gamma\ln(-i\tilde{s}) + a_2\gamma^2\ln^2(-i\tilde{s}) + a_3\gamma^3\ln^3(-i\tilde{s})]e^{Bt}, \tag{11}$$

where α' and B are new fitting parameters, and

$$a_0 = 1 + \frac{\pi^2}{4}\left(\frac{\pi^2}{12} + 2\ln^2 2\right)\gamma^2,$$
$$a_1 = \frac{\pi^2}{4}\left[\frac{\pi^2}{2}\ln 2 + 4\ln^3 2 - \frac{3}{4}\zeta(3)\right]\gamma^2 + 2\ln 2,$$
$$a_2 = \frac{\pi^2}{12} + 2\ln^2 2,$$
$$a_3 = \frac{1}{3}\left[\frac{\pi^2}{2}\ln 2 + 4\ln^3 2 - \frac{3}{4}\zeta(3)\right], \qquad A = -\frac{a^2}{8c},$$

and is the Riemann ζ function, defined above.

Notice that we use a finite Pomeron slope α', and remind that, while in the leading order and without account for the running of the coupling constant, the BFKL Pomeron singularity is a fixed brunch point with vanishing slope, $\alpha' = 0$. The situation changes completely with the running coupling, resulting in an infinite number of Regge poles [3], whose trajectories for small $|t|$ are not calculable from perturbative QCD. The case of moving poles is more relevant to high-energy phenomenology, the subject of the present paper. Consequently, we treat α' in Eq. (11) as a free parameter.

We remind that in Eq. (11) $\tilde{s} = s/m^2$, $m = m_p = 938$ MeV, and, consequently, our approach can be trusted when s is greater than m^2. In fact, our fits shown in Figs. 1 and 2 extend in the range $(5 < \sqrt{s} < 1800)$ GeV.

In the eikonalization procedure we follow Ref. [17], according to which the Pomeron amplitude

$$F_P(s,t) = is\int_0^\infty bdbJ_0(b\sqrt{-t})(1 - e^{i\chi(b,s)}), \tag{12}$$

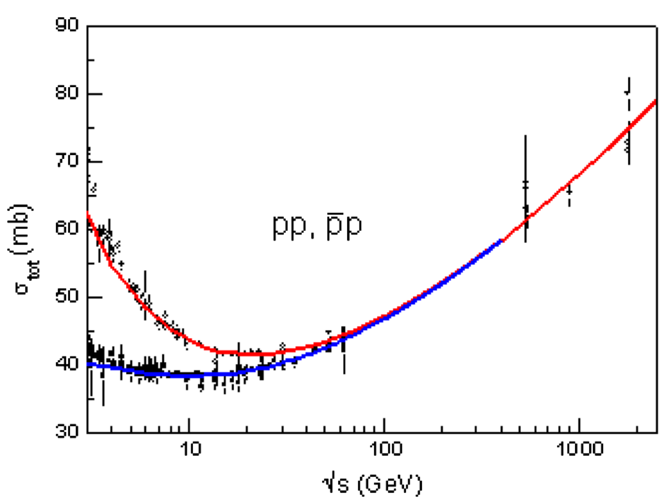

FIG. 1 (color online). Total pp (lower, blue line) and $\bar{p}p$ (upper, red line) cross sections from the uniratized (eikonalized) version of the model.

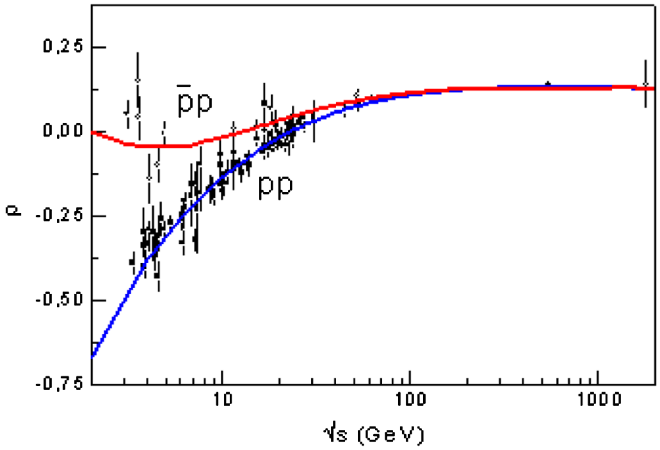

FIG. 2 (color online). The ratio $\rho(s) = \mathrm{Re}A(s,0)/\mathrm{Im}A(s,0)$ from the same model.

where J_0 is the Bessel function of zeroth order and the eikonal χ is

$$\chi(s,b) = \frac{1}{s}\int_0^\infty \sqrt{-t}d\sqrt{-t}I_0(b\sqrt{-t})F_{\mathrm{Born}}(s,t). \qquad (13)$$

Inserting the expression for the Pomeron into Eq. (13) and expanding the exponential in (12), one find for the eikonalized Pomeron amplitude

$$F_P = 2is\xi\sum_{k=1}^{\infty}\frac{1}{kk!}\left(-\frac{\xi}{\mu}\right)^{k-1}e^{\mu t/k}. \qquad (14)$$

Respectively, the forward Pomeron amplitude is

$$F_P(s,t=0) = 2is\mu[C + \ln(\xi/\mu) + E_1(\xi/\mu)], \qquad (15)$$

where

$$\mu = B + \alpha' \ln(-i\tilde{s}); \qquad (16)$$

$\xi = \frac{A}{2m^2}(\xi_0 + \xi_1 + \xi_2 + \xi_3)$, and

$$\xi_0 = a_0, \qquad \xi_1 = a_1\gamma\ln(-i\tilde{s}),$$
$$\xi_2 = a_2\gamma^2\ln^2(-i\tilde{s}), \qquad \xi_3 = a_3\gamma^3\ln^3(-i\tilde{s}),$$

$C = 0.577216$ is the Euler constant and E_1 is the asymptotic form of the first order exponential integral:

$$E_1 = \frac{\exp(\xi/\mu)}{\xi/\mu}\left[1 - \frac{1}{\xi/\mu} + \frac{2}{(\xi/\mu)^2} - \frac{6}{(\xi/\mu)^3} + \dots\right]. \qquad (17)$$

The obtained eikonalized Pomeron term is appended by a contributions from secondary Reggeons, f and ω:

$$F_R^{\pm}(s,t=0) = g_f\tilde{s}^{\alpha_f(0)} \pm ig_\omega\tilde{s}^{\alpha_\omega(0)},$$

where the $+(-)$ sign corresponds to $\bar{p}p(pp)$ scattering, the resulting forward amplitude being

$$F_{pp}^{\bar{p}p}(s,t=0) = F_P(s,t=0) + F_R(s,t=0).$$

TABLE I. Number of used data points and χ^2 per degree of freedom (χ^2/dof) for 5 GeV $\le \sqrt{s} \le$ 1.8 TeV (same as in [18]).

σ_{pp}	104
$\sigma_{\bar{p}p}$	59
ρ_{pp}	64
$\rho_{\bar{p}p}$	11
Total number of points	238
Number of free parameters	8
χ^2/dof	1.11

TABLE II. Values of the fitted parameters.

Parameter	Value	Error
A	0.526	0.198
α_s	0.190	0.033
B	0.116	0.121
α'	0.134	0.004
g_f	−4.56	0.35
$\alpha_f(0)$	0.858	0.086
g_ω	3.73	0.16
$\alpha_\omega(0)$	0.451	0.013

TABLE III. Prediction of the model for the LHC energies.

Energy (TeV)	6	12
$\sigma(pp/\bar{p}p)(mb)$	91.3	101
$\rho(pp/\bar{p}p)$	0.121	0.115

For the total cross section the norm

$$\sigma = \frac{4\pi}{s}\,\mathrm{Im}F_{pp}^{\bar{p}p}(s,t=0)$$

was used and $\rho(s,0) = \mathrm{Re}F_{pp}^{\bar{p}p}(s,t=0)/\mathrm{Im}F_{pp}^{\bar{p}p}(s,t=0)$.

Fits of the total of 8 free parameters to 238 data points on pp and $\bar{p}p$ total cross sections as well as on the ratio ρ (see Table I) were performed in the range 5 GeV $\le \sqrt{s} \le$ 1.8 TeV with the results shown in Figs. 1 and 2 and Table II. Predictions for pp at the expected LHC energies are quoted in Table III.

The quality of the fits (χ^2/dof) is good, comparable to that in, e.g. Ref. [18] (for a recent review on the subject see Ref. [19]).

V. CONCLUSIONS

Our main goal was an adequate picture of the Pomeron exchange at $t = 0$. In our opinion, it is neither an infinite sum of gluon ladders as in the BFKL approach [1–3], nor its power expansion. In fact, the finite series—call it "threshold approach"—considered in Sec. II and in the previous papers [6] realizes a nontrivial dynamical balance between the total reaction energy and the subenergies equally partitioned between the multiperipheral ladders.

The role and the value of the width of the gap, s_0, is an important physical parameter *per se*, independent of the model presented above. We have fitted it and compared successfully with the prediction from cosmic-ray data. However, its value may be estimated, e.g. as the lowest energy where the Pomeron exchange is manifest, although the latter is also a matter of debate.

The case of two terms (logarithmic rise in s) is particularly interesting as it corresponds to a dipole Pomeron with a number of attractive features [20] such as self-reproducibility with respect of unitarity corrections. In the case of a $\ln^2(s)$ rise (three terms), we still should not worry about the Froissart bound, so ultimately the Pomeron as viewed in this paper does not need to be unitarized. This conclusion is an important by-product of our paper. For the dipole Pomeron, relevant calculations for $t \neq 0$ are interesting and important but difficult. In the case of a single gluon rung they were performed in Ref. [12] and, with a nonperturbative gluon propagator, in the last reference of [10]. It is significant that the obtained in this paper value of the α_s as fitted parameter corresponds to one calculated with describing F_2 through of finite sum of gluon ladders [21], which is typical of this kinematical region.

As mentioned in the Introduction, acceptable high-energy asymptotics with the QCD Pomeron of Sec. III, without unitarization, can be achieved only at the expense of substantially lowering the canonical, "frozen" QCD running constant. The unitarization procedure and the subsequent fit to the data, presented in Sec. IV give independently the value $\alpha_s = 0.19$, thus providing one more means for the determination of this fundamental constant of QCD.

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