Little solution to the little hierarchy problem: A vectorlike generation

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We present a simple solution to the little hierarchy problem in the minimal supersymmetric standard model: a vectorlike fourth generation. With $\mathcal{O}(1)$ Yukawa couplings for the new quarks, the Higgs mass can naturally be above 114 GeV. Unlike a chiral fourth generation, a vectorlike generation can solve the little hierarchy problem while remaining consistent with precision electroweak and direct production constraints, and maintaining the success of the grand unified framework. The new quarks are predicted to lie between ∼300–600 GeV and will thus be discovered or ruled out at the LHC. This scenario suggests exploration of several novel collider signatures.

I. INTRODUCTION

The hierarchy problem has for years been taken as a strong motivation for theories of physics beyond the standard model (SM). The minimal supersymmetric standard model (MSSM) is one of the most attractive ideas for solving this problem as it naturally gives gauge coupling unification and a dark matter candidate. However the MSSM predicts a light Higgs boson, near the Z mass, while LEP placed a lower limit on the Higgs mass of 114 GeV. To satisfy the LEP bound, the top quark quark must be taken to be ∼1 TeV so that radiative corrections from the top quark increase the Higgs mass sufficiently. Thus supersymmetry (SUSY) must be broken above the weak scale, recreating a fine-tuning of ∼1% or worse in the soft SUSY-breaking parameters in order to reproduce the observed value of the weak scale. This is how the little hierarchy problem appears in the context of the MSSM [1–4]. In this paper we point out that a vectorlike fourth generation can solve this problem by adding extra radiative corrections to the Higgs mass.

This solution is straightforward, relying mostly on having new quarks, and is thus predictive. In order to remove the fine-tuning and avoid current experimental constraints there must be at least one new colored particle with mass between roughly 300 GeV and 600 GeV, easily discoverable at the LHC. As we will show, this solves the little hierarchy problem while naturally preserving the success of unification. Alternative solutions to the little hierarchy problem in the MSSM either involve large couplings which spoil unification or require new gauge or global symmetries, a very low messenger scale, or a carefully chosen set of soft SUSY-breaking parameters [4–29]. Other solutions involve extensions of the Higgs sector to create unusual decays of the Higgs in order to avoid LEP bounds [30,31]. Twin and little Higgs theories have also been proposed to solve the little hierarchy problem [32–37] by extending the symmetries of the standard model to a larger structure with

a collective breaking pattern, though these theories only push the cutoff up to ∼10 TeV [38]. A chiral fourth generation has been considered before but does not solve the little hierarchy problem and runs into difficulty with the large Yukawa couplings necessary to avoid experimental constraints leading to low scale Landau poles [39–46]. A vectorlike generation has been proposed [45,47] but its possible use in solving the Little Hierarchy problem was only appreciated in [48]. We discuss in Sec. V why we believe the problem is more fully ameliorated than was claimed in [48].

In Sec. II the model is presented. Section III presents the renormalization group analysis. Section IV presents the physical masses of the new particles. In Sec. V we calculate the experimental constraints on our model from direct collider production and precision electroweak observables. In Sec. VI we evaluate the Higgs mass. In Sec. VII we discuss collider signatures of this model.

II. THE MODEL

We add a full vectorlike generation to the MSSM with the following Yukawa interactions:

$$\mathcal{W} \supset y_4 Q_4 U_4 H_u + z_4 \bar{Q}_4 \bar{D}_4 H_u \qquad (1)$$

and mass terms

$$\begin{aligned} \mathcal{W} \supset{} & \mu_Q Q_4 \bar{Q}_4 + \mu_U U_4 \bar{U}_4 + \mu_D D_4 \bar{D}_4 + \mu_L L_4 \bar{L}_4 \\ & + \mu_E E_4 \bar{E}_4 \end{aligned} \qquad (2)$$

in the superpotential. The subscript 4 denotes the new generation. In Eqs. (1) and (2) and the rest of the paper, we use the familiar notation of the MSSM [49]. The superpotential (1) implicitly assumes a discrete parity under which the new matter is charged. This parity forbids mixing between the new generation and the standard model. This parity does not affect the Higgs mass in this model but has other interesting phenomenological consequences that

are discussed in Sec. VII B. It is also possible to write the model without this parity in which case the first term in Eqn. (1) is extended to a full 4×4 Yukawa matrix allowing mixing between all the generations. These mixings, if present, have to be small from flavor-changing neutral currents limits [46,50] and we will assume this to be the case.

Upon SUSY breaking, the terms in (1) contribute to the Higgs quartic. Including the contribution from the top Yukawa y_3, the Higgs mass m_h in this model is roughly given by

$$m_h^2 \sim M_z^2 \cos^2 2\beta + \left(\frac{3}{2\pi^2}\right) v^2 \sin^4 \beta$$
$$\times \left(y_3^4 \log \frac{m_{\tilde{t}}}{m_t} + y_4^4 \log \frac{m_{\tilde{Q}_4}}{m_{Q_4}} + z_4^4 \log \frac{m_{\tilde{\bar{Q}}_4}}{m_{\bar{Q}_4}}\right) \quad (3)$$

where $v \sim 174$ GeV is the electroweak symmetry breaking vev. The contributions from the new Yukawa couplings add linearly to m_h^2 and so can increase m_h^2 more effectively than the usual logarithmic contribution from raising the top quark masses. As a result, this model can be compatible with the LEP limit on the Higgs mass with smaller soft scalar masses, and is significantly less tuned. We calculate the Higgs mass more precisely in Sec. VI.

For example, with $y_4 \cong z_4 \cong y_3$, the size of the logarithmic corrections in (3) is roughly 3 times that of the top sector alone. In this case, a Higgs mass ~ 114 GeV can be obtained with soft masses ~ 300 GeV (taking $\tan\beta \sim 5$ and the vector masses $\mu_Q, \mu_U, \mu_D \sim 300$ GeV). For similar parameters, in the MSSM, the top quark has to be $\gtrsim$ 1.1 TeV in order that $m_h > 114$ GeV [49]. Since the Higgs vev is quadratically sensitive to the soft scalar masses, we expect the tuning in our model to be alleviated by a factor of $\sim(\frac{1.1 \text{ TeV}}{300 \text{ GeV}})^2 \sim \mathcal{O}(10)$.

We first make some qualitative remarks about the parameter space of the model. The corrections to m_h^2 from the new generation scale as the fourth power of the couplings y_4 and z_4 [see Eq. (3)]. If these couplings are much smaller than the top Yukawa, their effects become quickly subdominant. Moreover, these Yukawas renormalize each other and the top Yukawa and can lead to UV Landau poles. Motivated by gauge coupling unification, we impose the requirement that these Landau poles lie beyond the grand unified theory (GUT) scale. This sets an upper bound on the low energy values of y_4 and z_4. Since y_3 is close to its fixed point, we expect this bound to lie around the fixed point. These two considerations lead us to expect y_4 and z_4 to lie in a technically natural, but narrow, range around y_3.

The superpotential (1) can also contain Yukawa terms between the Higgs sector and the leptonic components of the new generation (e.g. $w_4 \bar{L}_4 \bar{E}_4 H_u$). These terms will also contribute to the Higgs quartic. However, the color factor for these loops is a third of the color factor for the quark loops. Hence, we expect these corrections to be subdominant, unless the couplings are large. But, these leptonic

Yukawas become nonperturbative more easily than the quark Yukawas since their one loop beta functions are unaffected by the strong coupling constant g_3 (see Sec. III). This constrains these Yukawas to be smaller than the corresponding quark Yukawas and hence they do not make significant corrections to the Higgs mass. In this paper, we assume that these Yukawas are small and ignore their effects on the phenomenology.

The contributions to m_h^2 from the new vectorlike generation is a function of SUSY breaking in that sector and is suppressed by $\sim \frac{\tilde{m}_{Q_4}^2}{m_{Q_4}^2}$. Here $\tilde{m}_{Q_4}^2$, the soft mass, is the difference between the scalar and fermion mass squares, respectively. These contributions are unsuppressed when $\tilde{m}_{Q_4}^2 \sim m_{Q_4}^2$. Since $\tilde{m}_{Q_4}^2$ contributes quadratically to the Higgs vev, the tuning in this model is minimized when $\tilde{m}_{Q_4}^2 \sim (200 \text{ GeV})^2$. This leads us to expect the masses of the new generation to lie around ~ 200 GeV—a range easily accessible to the LHC.

III. THE RENORMALIZATION GROUP ANALYSIS

In this section, we study the renormalization group evolution of all the parameters. We identify the regions of the y_4-z_4 parameter space where the theory is free of Landau poles up to the GUT scale. The addition of the new vectorlike generation also affects the evolution of gauge couplings. Since the new particles form complete $SU(5)$ multiplets, gauge coupling unification is preserved in this scenario. However, the extra matter fields do change the running of gaugino and soft scalar masses.

The evolution of the gauge couplings g_i are governed by the equations [49]

$$\frac{d}{dt} g_i = \frac{1}{16\pi^2} b_i g_i^3. \quad (4)$$

With the particle content of this model $(b_1, b_2, b_3) = (\frac{53}{5}, 5, 1)$, and the gauge couplings unify perturbatively at roughly $\sim 10^{16}$ GeV. The running of the Yukawa couplings are governed by

$$\frac{d}{dt} y_i = \frac{1}{16\pi^2} y_i \left(6(y_3 y_3^* + y_4 y_4^*) + 3z_4 z_4^* \right.$$
$$\left. - \left(\frac{16}{3} g_3^2 + 3g_2^2 + \frac{13}{15} g_1^2\right)\right) \quad (5)$$

$$\frac{d}{dt} z_4 = \frac{1}{16\pi^2} z_4 \left(3(y_3 y_3^* + y_4 y_4^*) + 6z_4 z_4^* \right.$$
$$\left. - \left(\frac{16}{3} g_3^2 + 3g_2^2 + \frac{7}{15} g_1^2\right)\right). \quad (6)$$

In writing the above, we have ignored contributions to these expressions from the Yukawa couplings in the down and lepton sector. This is reasonable in the regime of moderate $\tan\beta \lesssim 10$ where the down and lepton Yukawas are small. Similarly, we have also ignored mixing terms between the vectorlike generation and the standard

model, since these Yukawas also have to be small to evade flavor-changing neutral current constraints.

The running of these couplings is governed by the competition between the Yukawa and the gauge couplings. Since the gauge interactions themselves get stronger in the UV, in particular, the strong coupling g_3 [see Eq. (4)], the model is able to accommodate low energy values of y_4, $z_4 \sim 0.9$ without any coupling hitting a Landau pole below the GUT scale, in order to preserve perturbative unification. We plot this parameter space in the y_4-z_4 plane in Fig. 1.

The above perturbativity analysis was performed at the one loop level. Higher loop contributions to the beta functions were included in the analysis of [51]. These additional contributions cause the gauge couplings to become nonperturbative roughly around the unification scale $\sim 10^{16}$ GeV. This suggests that in the presence of a vector-like generation, the physics at the unification scale may be more compatible with orbifold GUT constructions instead of simple four-dimensional (4D) unification scenarios. We note that these orbifold constructions offer several advantages over simple 4D unification scenarios, including natural ways to incorporate doublet-triplet splitting and avoiding dimension five proton decay constraints. It is also possible that the flavor structure of the SM is generated at a scale below the GUT scale and thus the Yukawa couplings need only remain perturbative up to that intermediate scale, which will expand the available parameter
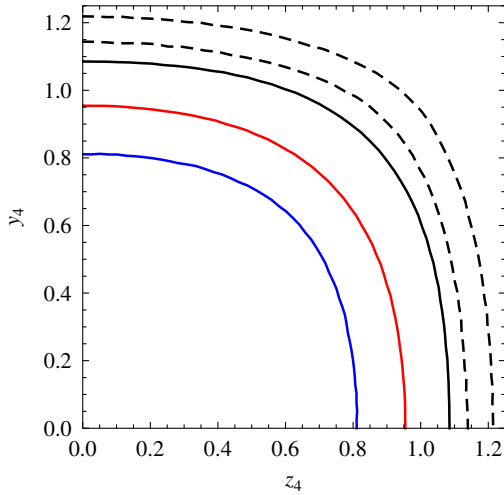

FIG. 1 (color online). The parameter space in the z_4-y_4 plane which solves the little hierarchy problem. The dashed lines show the maximum values of y_4 and z_4 such that all couplings remain perturbative (do not hit a Landau pole) up to the GUT scale (lower line) or 10^9 GeV (upper line). The solid lines show the minimum values of y_4 and z_4 that bring the Higgs mass above 114 GeV. From top to bottom they are $(\tan\beta, A) = (5, 300 \text{ GeV})$ (black), (5, 350 GeV) (red), and (10, 300 GeV) (blue), where A is the unified A term value. Here we have taken a unified vector mass $\mu_4 = \mu_Q = \mu_U = \mu_D = 320$ GeV, a unified soft mass $\tilde{m}^2 = \tilde{m}_Q^2 = \tilde{m}_U^2 = \tilde{m}_D^2 = (350 \text{ GeV})^2$, and the decoupling limit.

space. We show an example where flavor is generated below 10^9 GeV in Fig. 1.

The modified gauge couplings also affect the running of the soft gaugino and scalar masses. The weak scale soft scalar masses were computed (using [52]) after imposing the condition that the low energy gluino and electroweakino masses obey current bounds ($M_3 > 300$ GeV, and M_1, $M_2 > 100$ GeV) [53]. With this constraint on the gaugino masses, we find that the typical size of soft scalar masses in this model is $\sim 400\,\text{GeV} + 50\,\text{GeV}(\log(\frac{M_S}{10^9\,\text{GeV}}))$, where M_S is the scale at which the soft masses (~ 100 GeV) are generated. A primordial SUSY-breaking scale M_S larger than 10^9 GeV will drag the soft scalar masses up and reintroduce tuning. SUSY breaking at scales $\lesssim 10^9$ GeV are natural in many models of SUSY breaking e.g. gauge mediation. These SUSY-breaking models also address many of the other problems that plague the MSSM and they can naturally accommodate our framework. Note that if the soft scalar masses are universal for the various generations at the scale M_S (as one might expect, for example, in gauge mediated SUSY breaking), then the running to the weak scale does not induce large variation between the generations. We will generally assume universal weak scale soft scalar masses for the third and fourth generations when determining the Higgs mass within this model.

The large Yukawa couplings and SUSY-breaking gaugino masses also drive the generation of A terms. These A terms also make small, but significant corrections to the Higgs mass. As with the Yukawa couplings, when computing the Higgs mass we consider only the H_u-type A terms:
$$\mathcal{L} \supset \left(-A_y^{ij}\tilde{Q}_i\tilde{U}_j^*H_u - A_z\tilde{\bar{Q}}_4\tilde{\bar{D}}_4^*H_u\right).$$
The beta functions of A terms consist of terms proportional to themselves and terms proportional to the corresponding Yukawa couplings. With the Yukawa couplings of interest to this paper, A terms $\sim -(300\,\text{GeV} + 30\,\text{GeV}(\log(\frac{M_S}{10^9\,\text{GeV}})))$ are generated for the third and fourth generations even when they are zero at the SUSY-breaking scale. For example, gauge mediation typically gives zero A terms at the SUSY-breaking scale. In the context of such theories, renormalization group evolution can lead to weak scale A terms ~ -300 GeV for the generations that have $O(1)$ Yukawa couplings, with much smaller values for the first two generations. In other SUSY-breaking scenarios such as gravity mediation, the A terms are free parameters and can also easily be ~ -300 GeV. Using this input from the renormalization group flow, in this paper, we generally take A terms ~ -300 GeV for the third and fourth generations while computing the corrections to the Higgs mass.

The vector masses run proportional to themselves, and their running is affected by both the Yukawa couplings (y_4 and z_4) and the gauge couplings g_i. The low energy values of these vector masses depend upon the scale at which they were generated. For the new generation to ameliorate the tuning in the Higgs sector, these vector masses must also be

at the weak scale (see Sec. II). This requirement leads to a "μ problem" in this model. It is conceivable that the physics responsible for creating these masses is tied to the generation of the μ parameter of the MSSM. Since μ is often tied to SUSY breaking, we will assume that these vector masses are also generated at this scale M_S. With vector masses ~ 200 GeV at $M_S \sim 10^9$ GeV, we find that the weak scale masses for the colored particles are ~ 300 GeV. Under the same conditions, the electroweak particles receive masses ~ 200 GeV.

The renormalization group analysis shows that it is theoretically possible for the new generation to have reasonably large Yukawa couplings y_4, $z_4 \subset (0.8, 1.1)$ to the Higgs sector without losing perturbative unification. In this context, with SUSY breaking $M_S \lesssim 10^9$ GeV, this scenario can yield soft scalar masses ~ 400 GeV, while remaining consistent with experimental bounds on gaugino masses. This parameter space also supports vector masses ~ 200 GeV for the colored components of the new generation. In the next section, we examine the mass spectrum of this model after electroweak symmetry breaking.

IV. THE MASS SPECTRUM

Upon electroweak symmetry breaking, the $SU(2)$ doublet Q_4 splits to yield an up-type quark Q_4^u and a down-type quark Q_4^d. In addition to the vector mass, these quarks also receive mass from the Higgs vev. The mass matrices for these quarks can be expressed as

$$
\begin{pmatrix} Q_4^u & \bar{U}_4 \end{pmatrix} \begin{pmatrix} \mu_Q & y_4 v \\ 0 & \mu_U \end{pmatrix} \begin{pmatrix} \bar{Q}_4^u \\ U_4 \end{pmatrix} \quad \text{and}
$$
$$
\begin{pmatrix} Q_4^d & \bar{D}_4 \end{pmatrix} \begin{pmatrix} \mu_Q & 0 \\ z_4 v & \mu_D \end{pmatrix} \begin{pmatrix} \bar{Q}_4^d \\ D_4 \end{pmatrix} \tag{7}
$$

where the superscripts u and d denote the up and down components of the doublet.

The mass eigenstates are obtained by bidiagonalizing the above mass matrices. In the limit $\mu_Q \cong \mu_U \cong \mu_D = \mu_4$ and $\mu_4 \gtrsim y_4 v$, $z_4 v$, the eigenvalues simplify to

$$
M_u \cong \begin{pmatrix} \mu_4 - \frac{y_4 v}{2} + \frac{(y_4 v)^2}{8\mu_4} \\ \mu_4 + \frac{y_4 v}{2} + \frac{(y_4 v)^2}{8\mu_4} \end{pmatrix},
$$
$$
M_d \cong \begin{pmatrix} \mu_4 - \frac{z_4 v}{2} + \frac{(z_4 v)^2}{8\mu_4} \\ \mu_4 + \frac{z_4 v}{2} + \frac{(z_4 v)^2}{8\mu_4} \end{pmatrix}. \tag{8}
$$

Electroweak symmetry breaking splits the spectrum into two mass eigenstates, one with mass $\sim \mu_4 - \frac{v}{2}$ and the other of mass $\sim \mu_4 + \frac{v}{2}$. In the next section, we study the experimental constraints on these new particles from direct collider searches and precision electroweak observables.

V. EXPERIMENTAL CONSTRAINTS

The collider signatures of the new quarks in superpotential (1) depend upon its decay channels to the standard

model. If the new generation mixes with the standard model, these quarks will decay to the standard model through W or Z emission. These decay channels are constrained by the collider detector at Fermilab (CDF), which imposes a lower bound $\gtrsim 256$ GeV on the mass of a new down-type quark [39,54]. The bound on a new up-type quark depends upon its branching fraction for decays to W's. If this branching fraction is 100%, CDF imposes a lower bound $\gtrsim 311$ GeV [55]. However, in this model, this branching fraction can be significantly smaller since it depends upon the unknown mixing angle between the new generation and the standard model. In particular, when both y_4 and z_4 are nonzero, it is possible for the mass of the lightest down-type quark to be smaller than that of the lightest up-type quark. But, their mass difference could nevertheless be smaller than the W mass [see Eq. (7)]. In this case, the up-type quark can dominantly decay to the down-type quark and soft standard model final states as long as it is heavier than the down-type quark by a few GeV. The CDF search does not limit this scenario. Consequently, we will take a lower bound of $\gtrsim 256$ GeV on the mass of the new quarks.

It is also possible that the new generation is endowed with a parity that forbids it from mixing with the standard model. We discuss this phenomenology in Sec. VII B where we show that as a result of the new parity, the new quarks are meta-stable on collider time scales. The lifetimes are somewhat model dependent, but can easily be $\sim 10^6$ s even for a 500 GeV quark. The strongest constraint on this decay mode comes from a CDF search for meta-stable CHAMPS [56]. This study constrains the mass of a meta-stable up-quark to be $\gtrsim 350$–400 GeV. The bound on a meta-stable down quark would be weaker by ~ 30–40 GeV [57]. In this model, it is possible for the meta-stable, lightest colored particle to be a down-type quark [see Eq. (7)]. Consequently, in this scenario, the lower bound on the quark mass would be $\gtrsim 300$–350 GeV.

Irrespective of the mixing between the new generation and the standard model families, this model requires the new generation to couple to the Higgs. This Yukawa interaction contributes to the precision electroweak parameters S, T, and U and will impose a restriction on the vector masses. Using the mass matrices (7), we computed the corrections to S, T, and U in this model using [58]. Taking y_4, $z_4 = 0.9$ and the vector masses $\mu_Q = \mu_U = \mu_D = \mu_4$, we get

$$
\delta T = 0.17 \left(\frac{300 \text{ GeV}}{\mu_4} \right)^2 \qquad \delta S = 0.06 \left(\frac{300 \text{ GeV}}{\mu_4} \right)^2
$$
$$
\delta U = 0.004 \left(\frac{300 \text{ GeV}}{\mu_4} \right)^2. \tag{9}
$$

These contributions are within the 68% confidence limits on these electroweak parameters [39,53]. These corrections decouple as $\sim \mu_4^{-2}$ with the vector mass μ_4 and are quickly suppressed beyond $\mu_4 > 300$ GeV.

The corrections discussed in Eq. (9) were computed for the new fermion sector. Their scalar partners also contribute to these quantities. However, since the scalars are heavier than the fermions, their contributions are more suppressed. Using [59] to estimate these corrections, we find that with vector masses $\mu_4 \gtrsim 300$ GeV and soft scalar masses $\tilde{m}^2 \gtrsim (350 \text{ GeV})^2$, the net electroweak contributions from the new sector are within the 95% confidence limits on the electroweak observables. This constraint was also obtained using $y_4, z_4 = 0.9$. The electroweak corrections are sensitive to the squares of the Yukawas y_4 and z_4. Hence, if either of these Yukawas is a bit small, their contributions are rapidly suppressed and these constraints can be satisfied with even smaller soft scalar masses.

The primary electroweak precision constraint is due to the T parameter, $\delta T \lesssim 0.2$. Note that this seems to be the origin of the difference between our conclusions and those of [48]. While we find that this constraint can be satisfied for $\mu \gtrsim 300$ GeV, they require $\mu \sim$ TeV. They seem to have used a formula for δT that is a factor of 4 larger than ours. We would find their result if we had a symmetric mass matrix instead of the asymmetric matrix in Eq. (7). Further, this factor of roughly 4 between a symmetric and an asymmetric matrix is confirmed by [60]. Taking the vector mass to be a TeV requires also raising the scalar soft masses to around a TeV in order to get a significant contribution to the Higgs quartic, thus recreating the fine-tuning necessary to get the correct Higgs vev.

In order to accommodate these experimental constraints, we will assume that the vector masses $\mu_4 \gtrsim 325$ GeV for the rest of the paper. This forces all new fermions to be heavier than $\gtrsim 256$ GeV [see Eq. (8)], in agreement with the CDF bound on rapidly decaying particles. Note that colored fermions at 256 GeV would still be in conflict with collider searches if these particles are stable on collider timescales. Consequently, $\mu_4 \gtrsim 325$ GeV satisfies all constraints only when the new generation mixes with the standard model. In the absence of this mixing, the collider bound forces the vector mass $\mu_4 \gtrsim 380$ GeV so that the lightest colored particle is heavier than ~ 300 GeV.

The leptonic spectrum is also split by electroweak symmetry breaking [see Eq. (7)]. However, if these leptonic Yukawas are small [see Sec. II], this part of the particle spectrum will be relatively unaffected by electroweak symmetry breaking. Consequently, we expect the new leptons to be roughly around their vector masses μ_L and μ_E. The most stringent constraints on these particles come from LEP, which imposes a lower bound μ_L, $\mu_E >$ 101 GeV [39,53].

VI. THE HIGGS MASS

In this section, we calculate the corrections to the Higgs mass from the new generation. We restrict the parameter space of the model to that allowed by current experimental bounds, namely, $\mu_4 \gtrsim 300$ GeV and the condition that the Yukawa couplings remain perturbative up to the GUT scale.

The correction to the Higgs mass was computed using the one loop effective potential method. The mass matrices in Eq. (7) for the quarks Q_4, $\bar{Q}_4$, U_4, $\bar{U}_4$, D_4, $\bar{D}_4$ were bidiagonalized. For the scalars, diagonal SUSY-breaking soft scalar masses $\tilde{m}^2_{Q_4}$, $\tilde{m}^2_{\bar{Q}_4}$, $\tilde{m}^2_{U_4}$, $\tilde{m}^2_{\bar{U}_4}$, $\tilde{m}^2_{D_4}$, and $\tilde{m}^2_{\bar{D}_4}$ were added to the supersymmetric masses in Eq. (7). We also added A terms ~ -300 GeV to the scalar mass matrix. A terms of this size at the weak scale can be naturally realized from renormalization group effects even if they are negligible at the high scale (see Sec. III). The resultant scalar mass matrix was diagonalized. The mass eigenstates obtained from this procedure were used to calculate the one loop effective potential. This potential was used to obtain the Higgs mass in the decoupling limit.

The above computations were performed numerically and their results are summarized in Figs. 1–3. Figure 1 displays the parameter space in the y_4-z_4 plane which solves the little hierarchy problem while remaining consistent with perturbative unification. This parameter space contains regions where z_4 is negligible and the Higgs mass corrections arise from y_4. This suggests that this framework can solve the little hierarchy problem with just the coupling $y_4 Q_4 U_4 H_u$. This coupling requires the addition of just a vectorlike antisymmetric $SU(5)$ tensor 10. The little hierarchy problem can be solved in this manner, although, the allowed range of y_4 is smaller. This is because the gauge couplings run less without the contributions from the vectorlike $(5_4, \bar{5}_4)$ and hence they compete less against the Yukawa contributions in Eq. (6). This causes the Yukawas to become nonperturbative more rapidly, restricting $y_4 \lesssim 1.05$.

In addition to producing a Higgs mass in excess of the LEP bound, our model must satisfy the experimental constraints on the masses of the top squark and the fourth generation states. In Fig. 2 we plot the allowed region of the vector mass—soft mass parameter space for various values of $\tan\beta$ and a unified A term $A = A_y = A_z$. The vertical dashed line indicates the lower bound on the vector mass discussed in the previous section. LEP searches place a lower bound on the lightest top squark mass of 96 GeV [53]; for given $(\tan\beta, A)$ this places a lower bound on the soft mass parameter $\tilde{m}$. This lower bound appears as the horizontal sections of the two lower curves in Fig. 2. For values of $\tilde{m}$ above this bound the parameter space is constrained by the Higgs mass bound, producing the rising parts of the curves. Given these lower bounds on μ_4 and $\tilde{m}$, the fourth generation squarks are automatically above their experimental bounds [56]. If the soft scalar masses of the first two generations are similar to that of the third, then our assumption of small A terms for the first two generations ensures that those squarks are heavier than the lightest top squark. We see that the Higgs mass in this model is larger than the LEP bound even with soft scalar masses
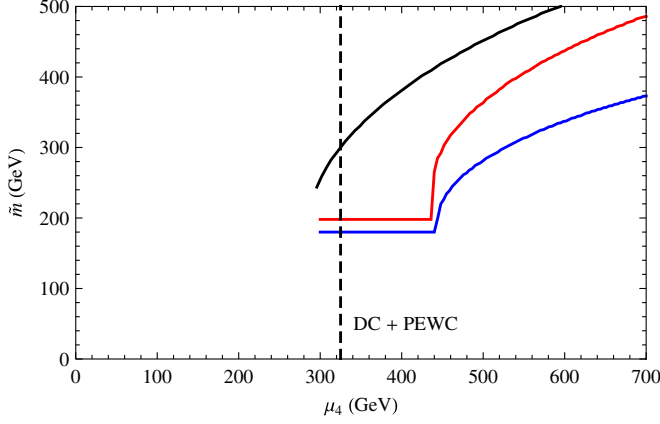
FIG. 2 (color online). The parameter space in the μ_4-$\tilde{m}$ (vector mass—soft mass) plane. The solid curves are the minimal values of $\tilde{m}$ that raise the Higgs above 114 GeV and raise all squarks above the bounds from colliders. From top to bottom they are $(\tan\beta, A) = (5, 300 \text{ GeV})$ (black), $(5, 350 \text{ GeV})$ (red), and $(10, 300 \text{ GeV})$ (blue), where A is the unified A term value. The lower two curves become flat where they are determined by the lower bound on the top squark mass, not the Higgs mass. Here we have taken $y_4 = z_4 = 0.9$ and the decoupling limit. The dashed line is the lower limit on μ_4 from direct collider constraints on the fermions and precision electroweak constraints (DC + PEWC).

$\tilde{m} \sim 300$ GeV. By contrast, in the MSSM, the squarks have to be $\gtrsim 1$ TeV to overcome this bound.

In Fig. 3, we study the Higgs mass as a function of y_4 for $z_4 = 0$, again with various choices of the parameters $\tan\beta$ and A. In this model, Higgs masses up to approximately 120 GeV are possible. This illustrates the possibility of solving the little hierarchy problem adding only the couplings to the vectorlike $(\mathbf{10}, \overline{\mathbf{10}})$ multiplets.
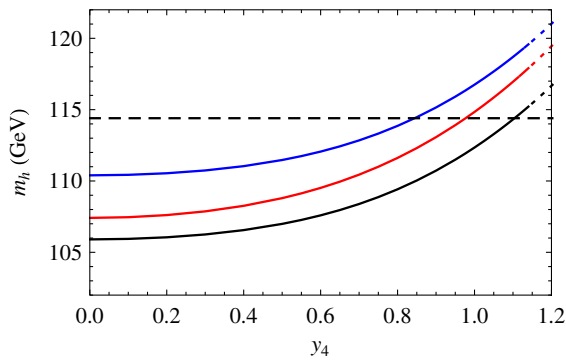

FIG. 3 (color online). The Higgs mass versus y_4 for $z_4 = 0$. From bottom to top they are $(\tan\beta, A) = (5, 300 \text{ GeV})$ (black), $(5, 350 \text{ GeV})$ (red), and $(10, 300 \text{ GeV})$ (blue), where A is the unified A term value. Here we have taken a unified vector mass $\mu_4 = \mu_Q = \mu_U = \mu_D = 320$ GeV, a unified soft mass $\tilde{m}^2 = \tilde{m}_Q^2 = \tilde{m}_U^2 = \tilde{m}_D^2 = (350 \text{ GeV})^2$, and the decoupling limit. Each curve is dotted where it hits the perturbativity limit from Fig. 1. The dashed line is the lower limit on the Higgs mass.

VII. COLLIDER PHENOMENOLOGY

The Yukawa interactions between the new quarks and the Higgs are the biggest source of corrections to the Higgs mass. These quarks have to be light in order to significantly affect the Higgs mass (see Secs. II and VI). For moderate $\tan\beta(\approx 5)$ and soft SUSY-breaking masses $\tilde{m} < 400$ GeV, we expect the vector mass μ_4 of the new quarks to be $\lesssim 550$ GeV (see Fig. 2) so that the Higgs mass is larger than 114 GeV. This vector mass implies the existence of two sets of quarks, one with mass $\lesssim 450$ GeV and the other with mass $\lesssim 650$ GeV [see Eq. (7)]. Of course, the vector mass can be higher than 550 GeV if $\tan\beta$ is larger. For example, with $\tan\beta = 10$, the vector mass can be as large as 700 GeV (giving rise to one ~ 600 GeV quark) and still make the Higgs sufficiently heavy. The model thus predicts an abundance of light colored particles at the LHC: standard model squarks with masses $\lesssim 500$ GeV, two new quarks and their superpartners, with at least one of the quarks being lighter than ~ 600 GeV.

The existence of the leptonic sector of the new generation is not required to explain the Higgs mass (see Sec. II). However, we expect this new sector to exist in order to preserve gauge coupling unification. The model by itself does not place direct bounds on the vector mass of these leptons. But, if the vector mass of the new generation has a common, unified origin, the leptons will be significantly lighter than the quarks due to renormalization (see Sec. III). Hence, we expect leptons with masses $\ll 500$ GeV in this model.

In this section, we discuss the observability of these new particles at the LHC. The phenomenology of this model depends upon the mixing between the new generation and the standard model. In the following, we first discuss the case where the new generation mixes with the standard model and then examine the situation where this mixing is forbidden by a parity. We conclude this section with a discussion of the Higgs phenomenology in this model.

A. Mixing with the standard model

Mixing between the new generation and the standard model leads to rapid (on collider time scales) decays of the new generation to the standard model. The new quarks will decay through the production of W's. The subsequent leptonic decays of the W is a standard, low background way to search for these quarks at the LHC. With 100 fb^{-1} of data, the LHC reach for these new quarks is at least ~ 700 GeV [61,62]. This reach should cover most of the expected mass range of the new quarks.

The phenomenology of the new lepton sector with vector masses is different from that of a new chiral lepton sector. In particular, the neutrinos of this sector will have large ~ 100 GeV masses. This changes the collider signatures of this new lepton sector since the heavy neutrinos can also decay to the standard model. In conventional searches for fourth generation chiral leptons, the fourth

generation neutrinos are assumed to be massless [61], preventing them from decaying to the standard model. On the other hand, a new heavy neutrino can decay to produce lepton-rich signals, discriminating it more from standard model background. The LHC reach for this novel lepton sector needs further study [61,63,64].

B. Discrete parity

The new generation might respect a discrete parity that forbids it from mixing with the standard model [e.g. superpotential (1)]. In this case, the superpotential (1) does not allow the lightest new quark to decay. Naively, this model would appear to be ruled out by stringent constraints on long-lived colored relics. However, it is possible for these quarks to decay to their leptonic counterparts (which are typically lighter than the quarks due to renormalization effects) and the standard model through baryon-number violating operators. A natural source of such operators can be found in supersymmetric GUT theories [65].

For example, if the superpotential (1) is embedded into a $SU(5)$ GUT, the term $y_4 Q_4 U_4 H_u$ can emerge from the $SU(5)$ invariant operator $y_4 10_4 10_4 H_u$. Using the Higgs triplet $H_u^{(3)}$, this GUT operator yields $y_4 U_4 E_4 H_u^{(3)}$. Integrating out the heavy Higgs triplets and using the familiar interactions between the triplets and the standard model, we get the dimension five operator in the superpotential

$$W \supset y_4 y_b \frac{U_4 E_4 U_3 D_3}{M_{\text{GUT}}} \tag{10}$$

which leads to the decay of the quark to its leptonic counterpart and the standard model. The decay lifetime is

$$\tau \sim 3 \times 10^8 \text{ s} \left(\frac{1}{y_4}\right)^2 \left(\frac{2 \times 10^{-2}}{y_b}\right)^2 \left(\frac{M_{\text{GUT}}}{2 \times 10^{16} \text{ GeV}}\right)^2$$
$$\times \left(\frac{200 \text{ GeV}}{\delta M}\right)^3 \tag{11}$$

where y_b is the bottom Yukawa and δM is the phase space (equal to the mass difference between the colored and leptonic components) available for the decay.

Similar decay operators can also be generated from embedding the coupling $z_4 \bar{Q}_4 \bar{D}_4 H_u$ into a GUT. However, these operators will also be suppressed by y_b since the terms in the superpotential (1) connect the new generation to H_u and not H_d. Since a dimension five decay through the Higgs triplet must involve couplings to both H_u and H_d, the standard model enters this operator through its interactions with H_d. Consequently, these operators are at least suppressed by the bottom Yukawa coupling y_b.

The lifetime in Eq. (11) should be regarded as an upper bound for the quarks in a GUT theory, since we expect the Higgs triplet to couple to the light particles in most GUTs. But, it is possible for the quarks to decay more rapidly than (11). For example, new physics at the GUT scale can lead to faster decays with lifetimes ~ 1000 s [65].

The bounds on the lifetime of long-lived colored particles is sensitive to their abundance. This abundance is uncertain due to nonperturbative processes that occur during the QCD phase transition [66,67]. Under the assumptions of [66], decays with lifetimes $\lesssim 10^{14}$ s are unconstrained by cosmology. But, with the caveats in [65,67], the abundance could be larger than the estimates of [66]. In this case, it is possible for these decays to have cosmological implications. For example, these decays may help explain the primordial lithium problems [65,68].

Regardless of their cosmological impact, long-lived colored particles give rise to striking signals in colliders [69,70]. The colored particles, upon production, will travel through the detector where they will lose energy due to electromagnetic and hadronic interactions, giving rise to charged tracks in the detector. A fraction of these particles will stop in the detector and eventually decay. Since these decays are uncorrelated with the beam, they can be distinguished from most backgrounds. The LHC reach for such meta-stable quarks is at least 1 TeV [71].

The discrete parity also leads to interesting phenomenology in the lepton sector, since it stabilizes the lightest leptonic particle. If the lightest particle is the new neutrino, it is a natural candidate for the weakly interacting massive particle dark matter. This neutrino by itself couples too strongly to the Z and is in conflict with bounds from dark matter direct detection experiments. However, this problem can be solved if this neutrino mixes with a standard model singlet S_4 through the term $x_4 L_4 S_4 H_u$ [72]. The singlet S_4 and the Yukawa coupling x_4 could emerge naturally in a $SO(10)$ GUT from the $SO(10)$ invariant Yukawa $16_4 16_4 10_h$.

C. Higgs phenomenology

Gluon fusion is the dominant production channel for the Higgs at the LHC. The new quarks contribute to this channel and can enhance the production cross section. This enhancement is smaller than the case of a chiral fourth generation [39] since the additional contributions are suppressed by the vector mass μ_4. The enhancement should roughly scale as $\sim (y_4^2 + z_4^2)(\frac{v}{\mu_4})^2$ and is $\sim \frac{1}{3}(\frac{300 \text{ GeV}}{\mu_4})^2$ for $y_4, z_4 \sim \mathcal{O}(1)$.

The mass spectrum may also permit the decay of the Higgs to the new leptons. It is possible for this decay channel to be the dominant decay mode of the Higgs since we expect the new generation to have $\mathcal{O}(1)$ Yukawas to the Higgs. The subsequent decay of the new leptons to the standard model might offer a new way to discover the Higgs and the new leptons. This possibility merits further study.

VIII. CONCLUSIONS

A new vectorlike generation with $\mathcal{O}(1)$ Yukawa couplings has interesting phenomenological consequences.

Owing to the vector mass, collider and precision electro-weak constraints are more easily avoided. This is unlike the case of a chiral fourth generation where these constraints rapidly force the theory to become nonperturbative [39]. Low energy nonperturbativity can be avoided in such models at the cost of drastically reduced tree-level contributions to the Higgs mass, further accentuating the little hierarchy problem [39]. A vector mass, on the other hand, easily avoids experimental constraints and simultaneously allows a solution to the little hierarchy problem. Furthermore such a solution is predictive. The new generation can significantly ameliorate the tuning of the Higgs vev if it has a mass between ~300–500 GeV, along with soft scalar masses between ~300–400 GeV. The spectrum contains many light, colored particles which would be well within the LHC reach.

Our model motivates consideration of several novel collider signatures. A vectorlike generation can give rise to unique decay channels for the new leptons and neutrinos. The new generation also changes the collider phenomenology of the Higgs sector. In addition to increasing the Higgs production cross section, the new generation allows for nonstandard Higgs decay channels. It is also possible for this framework to contain long-lived quarks. These quarks give rise to striking signatures at colliders and may help solve the primordial lithium problems. The LHC phenomenology of this new sector deserves further consideration.

In this paper, we discussed the effects of a new vectorlike generation on the Higgs mass. In addition to its effects on Higgs physics, a new vectorlike generation could also have other phenomenological uses. For example, it may help explain the hierarchy in the fermion mass spectrum [73,74]. It could also arise in string constructions where the number of chiral generations emerges as a result of a mismatch between the number of left-handed and right-handed chiral fields. In such a construction, it may also be possible to address the new μ problem raised by this scenario. It is also possible for these new $\mathcal{O}(1)$ Yukawa couplings to modify the electroweak phase transition and stimulate electroweak baryogenesis. In this case, there might be additional signatures of this model, for example, gravitational wave signals that may be observable in upcoming experiments [75–77]. The presence of such a fourth generation would clearly have important implications for UV physics beyond just this model.

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