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Dynamical electroweak symmetry breaking with superheavy quarks and 2 + 1 composite Higgs model

Michio Hashimoto*

Theory Center, Institute of Particle and Nuclear Studies, High Energy Accelerator Research Organization (KEK), 1-1 Oho, Tsukuba, Ibaraki 305-0801, Japan

V. A. Miransky†

Department of Applied Mathematics, University of Western Ontario, London, Ontario N6A 5B7, Canada

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Recently, a new class of models describing the quark mass hierarchy has been introduced. In this class, while the t quark plays a minor role in electroweak symmetry breaking, it is crucial in providing the quark mass hierarchy. In this paper, we analyze the dynamics of a particular model in this class, in which the b' and t' quarks of the fourth family are mostly responsible for dynamical electroweak symmetry breaking. The low energy effective theory in this model is derived. It has a clear signature, a 2 + 1 structure of composite Higgs doublets: two nearly degenerate $\Phi_{b'} \sim \bar{b}'_R(t', b')_L$ and $\Phi_{t'} \sim \bar{t}'_R(t', b')_L$, and a heavier top-Higgs resonance $\Phi_t \sim \bar{t}_R(t, b)_L$. The properties of these composites are described in detail, and it is shown that the model satisfies the electroweak precision data constraints. The signatures of these composites at the Large Hadron Collider are briefly discussed.

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I. INTRODUCTION

The dynamics of electroweak symmetry breaking (EWSB) and fermion (quark and lepton) mass hierarchy are the two central quests in the Large Hadron Collider program. In particular, it is noticeable that the LHC has a potential for discovering the fourth fermion family [1]. The possibility of the existence of the latter has been studied for a long time (for a review, see Ref. [2]). It is noticeable that the fourth family can play an important role in B-CP asymmetries phenomena [3,4].

Since the mass bounds for the fourth family quarks t' and b' are of the order of the EWSB scale [5], the Pagels-Stokar (PS) formula [6] suggests that their contributions to the EWSB should not be small. This leads to an idea of the dynamical EWSB scenario with the fourth family [7,8], which is an alternative version of the top quark condensate model [9–12]. Because the Yukawa couplings of the t' and b' quarks have the Landau pole around several TeV scale, it suggests that the Higgs doublets $\Phi_{t'} \sim \bar{t}'_R(t', b')_L$ and $\Phi_{b'} \sim \bar{b}'_R(t', b')_L$ composed of them could be produced without fine tuning.

Although the top quark mass is obviously near the EWSB scale, it apparently plays no leading role in the EWSB: the PS formula suggests that its contribution to the EWSB is around 10%–20%. On the other hand, the t quark might play an important role in the dynamics responsible for the quark mass hierarchy. Recently, utilizing dynamics considered in Ref. [13] quite time ago, we introduced a new class of models in which the top quark plays just such a role [14]. The main two features of these models are (a) the presence of strong (although subcritical) horizontal diagonal interactions for the t quark, and (b) horizontal flavor-changing neutral interactions between different families. Together with the assumption that the dynamics primarily responsible for the EWSB leads to the mass spectrum of quarks with no (or weak) isospin violation, and with the masses of the order of the observed masses of the down-type quarks, these features allow to reproduce the quark mass hierarchy and essential characteristics of the Cabibbo-Kobayashi-Maskawa (CKM) matrix [14].

This approach can be implemented in the models with different EWSB scenarios. Its signature is the existence of an additional top-Higgs resonance doublet Φ_t composed of the quarks and antiquarks of the 3rd family, $\Phi_t \sim \bar{t}_R(t, b)_L$. In the case of the dynamical EWSB scenario with the fourth family, the top-Higgs Φ_t is heavier than the $\Phi_{t'}$ and $\Phi_{b'}$ composites [14]. For simplicity, in Ref. [14] we considered only the case when the Φ_t mass is ultraheavy and it decouples from TeV dynamics. However, in general, this is not the case, and the Φ_t can be detectable at the LHC. This leads to a model with three Higgs doublets. Actually, because the $\Phi_b \sim \bar{b}_R(t, b)_L$ composite, and those ones connected with the lighter c, s, u, and d quarks, are necessarily ultraheavy and decouple in this scenario [14], and because there is an approximate $SU(2)_{R4}$ symmetry between t'_R and b'_R quarks, it would be appropriate to call it the 2 + 1 composite Higgs model. In this paper, we will study such dynamics.

As for the fourth family leptons, we assume that their masses are around 100 GeV [5], and thus their contributions to the EWSB are smaller than that of the top quark.

*michioh@post.kek.jp
†vmiransk@uwo.ca;
On leave from Bogolyubov Institute for Theoretical Physics, 03680, Kiev, Ukraine.

For the dynamics with very heavy fourth family leptons, and thereby with a lepton condensation, one needs to use, say, a five Higgs model. Also, the Majorana condensation of the right-handed neutrinos should be reanalyzed in that case. This possibility will be considered elsewhere.

The paper is organized as follows. In Sec. II, we describe the model. The qualitative features of its low energy effective theory are discussed in Sec. III. In Sec. IV, the results of the numerical analysis of the renormalization group equations are presented and the properties of the composite Higgs bosons are described. The structure of the CKM matrix and flavor-changing-neutral interactions are discussed in Sec. V. In Sec. VI, we summarize the main results of the paper. In Appendices A, B, and C, useful formulas used in the main text are derived.

II. MODEL

We will utilize a Nambu-Jona-Lasinio (NJL) type model to describe the dynamics with the 2 + 1 Higgs doublets composed of the third and fourth family quarks. Its Lagrangian density has the following form:

$$\mathcal{L} = \mathcal{L}_g + \mathcal{L}_f + \mathcal{L}_{\rm NJL}, \tag{1}$$

where $\mathcal{L}_g$ is the Lagrangian density for the standard model (SM) gauge bosons, the fermion kinetic term is

$$\mathcal{L}_f \equiv \sum_{i=3,4} \bar{\psi}_L^{(i)} i\not{\partial} \psi_L^{(i)} + \sum_{i=3,4} \bar{u}_R^{(i)} i\not{\partial} u_R^{(i)} + \sum_{i=3,4} \bar{d}_R^{(i)} i\not{\partial} d_R^{(i)}, \tag{2}$$

and the NJL interactions are described by

$$\begin{aligned}\mathcal{L}_{\rm NJL} = {} & G_{t'}(\bar{\psi}_L^{(4)} t'_R)(\bar{t}'_R \psi_L^{(4)}) + G_{b'}(\bar{\psi}_L^{(4)} b'_R)(\bar{b}'_R \psi_L^{(4)}) \\ & + G_t(\bar{\psi}_L^{(3)} t_R)(\bar{t}_R \psi_L^{(3)}) + G_{t'b'}(\bar{\psi}_L^{(4)} t'_R) \\ & \times (\bar{b}'^{\,c}_R i\tau_2 (\psi_L^{(4)})^c) + G_{t't}(\bar{\psi}_L^{(4)} t'_R)(\bar{t}_R \psi_L^{(3)}) \\ & + G_{b't}(\bar{\psi}_L^{(3)} t_R)(\bar{b}'^{\,c}_R i\tau_2 (\psi_L^{(4)})^c) + ({\rm H.c.}).\end{aligned} \tag{3}$$

Here $\psi_L^{(i)}$ denotes the weak doublet quarks from the ith family, and $u_R^{(i)}$ and $d_R^{(i)}$ represent the right-handed up- and down-type quarks.

It is useful to rewrite this theory in an equivalent form by introducing auxiliary fields, $\Phi_{t'}^{(0)}$, $\Phi_{b'}^{(0)}$, $\Phi_t^{(0)}$:

$$\mathcal{L} = \mathcal{L}_f + \mathcal{L}_g + \mathcal{L}_{\rm aux}, \tag{4}$$

where

$$\begin{aligned}-\mathcal{L}_{\rm aux} = {} & \bar{\psi}_L^{(4)} t'_R \Phi_{t'}^{(0)} + \bar{\psi}_L^{(4)} b'_R \tilde{\Phi}_{b'}^{(0)} + \bar{\psi}_L^{(3)} t_R \Phi_t^{(0)} + ({\rm H.c.}) \\ & + M^2_{\Phi_{t'}^{(0)}} ((\Phi_{t'}^{(0)})^\dagger \Phi_{t'}^{(0)}) + M^2_{\Phi_{b'}^{(0)}} ((\Phi_{b'}^{(0)})^\dagger \Phi_{b'}^{(0)}) \\ & + M^2_{\Phi_t^{(0)}} ((\Phi_t^{(0)})^\dagger \Phi_t^{(0)}) + M^2_{\Phi_{t'}^{(0)}\Phi_{b'}^{(0)}} ((\Phi_{t'}^{(0)})^\dagger \Phi_{b'}^{(0)}) \\ & + M^2_{\Phi_{t'}^{(0)}\Phi_t^{(0)}} ((\Phi_{t'}^{(0)})^\dagger \Phi_t^{(0)}) + M^2_{\Phi_{b'}^{(0)}\Phi_t^{(0)}} ((\Phi_{b'}^{(0)})^\dagger \Phi_t^{(0)}) \\ & + ({\rm H.c.}),\end{aligned} \tag{5}$$

with

$$\begin{aligned}&\begin{pmatrix} M^2_{\Phi_{t'}^{(0)}} & M^2_{\Phi_{t'}^{(0)}\Phi_{b'}^{(0)}} & M^2_{\Phi_{t'}^{(0)}\Phi_t^{(0)}} \\ M^2_{\Phi_{t'}^{(0)}\Phi_{b'}^{(0)}} & M^2_{\Phi_{b'}^{(0)}} & M^2_{\Phi_{b'}^{(0)}\Phi_t^{(0)}} \\ M^2_{\Phi_{t'}^{(0)}\Phi_t^{(0)}} & M^2_{\Phi_{b'}^{(0)}\Phi_t^{(0)}} & M^2_{\Phi_t^{(0)}} \end{pmatrix} \\ &= \begin{pmatrix} G_{t'} & G_{t'b'} & G_{t't} \\ G_{t'b'} & G_{b'} & G_{b't} \\ G_{t't} & G_{b't} & G_t \end{pmatrix}^{-1},\end{aligned} \tag{6}$$

and $\tilde{\Phi}_{b'}^{(0)} \equiv -i\tau_2 (\Phi_{b'}^{(0)})^*$. The following remark is in order. If we added the Yukawa mixing terms, they could be erased by redefining the composite Higgs fields. For example, for the mixing term $\bar{\psi}_L^{(3)} t_R \Phi_{t'}^{(0)}$, the redefinition would be $\varphi_t^{(0)} = \Phi_t^{(0)} + \Phi_{t'}^{(0)}$, $\varphi_{t'}^{(0)} = \Phi_{t'}^{(0)}$. Such nonunitary (but invertible) transformations are allowed because there are no canonical kinetic terms for the auxiliary fields in $\mathcal{L}$.

As was shown in Ref. [14], the diagonal parts of the NJL interactions, $G_{t'}$, $G_{b'}$ and G_t, can be generated from the top color interactions [15]. In this case, the scales for the dimensionful NJL parameters $G_{t'} \simeq G_{b'}$ and G_t are connected with the coloron masses, $\Lambda^{(4)}$ and $\Lambda^{(3)}$, respectively. The mixing term $G_{t't}$ can be generated by a flavor-changing-neutral (FCN) interaction, t'-t-$\Lambda^{(34)}$ [14]. On the other hand, $G_{t'b'}$ may be connected with top color instantons [15]. In the 2 + 1 composite Higgs model, while the coupling constants $G_{t'}$ and $G_{b'}$ are supercritical and responsible for EWSB, the t quark coupling G_t is subcritical, although also strong [14].

As to the $G_{b't}$ term, the situation is the following. As far as $M^2_{\Phi_{t'}^{(0)}\Phi_{b'}^{(0)}} \neq 0$ and $M^2_{\Phi_{t'}^{(0)}\Phi_t^{(0)}} \neq 0$, there do not appear Nambu-Goldstone (NG) bosons even if the $M^2_{\Phi_{b'}^{(0)}\Phi_t^{(0)}}$ term, which is connected with $G_{b't}$, is ignored. For example, assuming $M^2_{\Phi_{b'}^{(0)}\Phi_t^{(0)}} = 0$, the Peccei-Quinn like $U(1)_A$ symmetry,

$$\begin{aligned}&\psi_L^{(3)} \to e^{-i\theta_A} \psi_L^{(3)}, \qquad \psi_L^{(4)} \to e^{-i\theta_A} \psi_L^{(4)}, \\ &t'_R \to e^{i\theta_A} t'_R, \qquad b'_R \to e^{i\theta_A} b'_R, \qquad t_R \to e^{i\theta_A} t_R,\end{aligned} \tag{7a}$$

$$\begin{aligned}&\Phi_t^{(0)} \to e^{-2i\theta_A} \Phi_t^{(0)}, \qquad \Phi_{t'}^{(0)} \to e^{-2i\theta_A} \Phi_{t'}^{(0)}, \\ &\Phi_{b'}^{(0)} \to e^{2i\theta_A} \Phi_{b'}^{(0)},\end{aligned} \tag{7b}$$

is explicitly broken by the Higgs mass mixing term

$M^2_{\Phi^{(0)}_{t'}\Phi^{(0)}_{b'}} \neq 0$. (Although the mixing term $M^2_{\Phi^{(0)}_{t'}\Phi^{(0)}_{t}}$ does not break this $U(1)_A$ symmetry, it is important: if both $M^2_{\Phi^{(0)}_{t'}\Phi^{(0)}_{t}}$ and $M^2_{\Phi^{(0)}_{b'}\Phi^{(0)}_{t}}$equal zero, a new global $U(1)$ symmetry appears.) Therefore, it is safe to take $M^2_{\Phi^{(0)}_{b'}\Phi^{(0)}_{t}} = 0$. Because of that, although we will keep the $G_{b't}$ and $M^2_{\Phi^{(0)}_{b'}\Phi^{(0)}_{t}}$ terms in a general discussion for a while, they will be ignored in the numerical analysis.

III. DYNAMICS IN THE LOW ENERGY EFFECTIVE MODEL: QUALITATIVE FEATURES

The model introduced in the previous section provides an approximate 2 + 1 structure in the Higgs quartic coupling sector in the low energy effective action. Indeed, in the bubble approximation, while the top-Higgs Φ_t couples only to $\psi^{(3)}_L$ and t_R, the composite $\Phi_{t'(b')}$ couples only to $\psi^{(4)}_L$ and $t'_R(b'_R)$, that leads to such a 2 + 1 structure. When we turn on the electroweak gauge interactions, this structure breaks down. The breaking effects are however suppressed, because the Yukawa couplings are much larger than the electroweak gauge ones.

In this section, we analyze the main characteristics of the 2 + 1 low energy effective Higgs model, in particular, the structure of its vacuum expectation values (VEV). We also discuss the relations between the parameters of the initial NJL model (such as the NJL couplings, etc.) and the observable ones.

In order to illustrate main qualitative features of the effective model, we will employ the bubble approximation in calculating its parameters (such as Yukawa and quartic couplings, etc.). However, the structure of the action will be taken to be more general, based on a numerical analysis of the renormalization group equations (RGE's) with the compositeness conditions [11], which is performed in the next section.

A. Low energy effective model

Since at low energy the composite Higgs fields develop kinetic terms, the Lagrangian density of the low energy effective model is

$$\mathcal{L} = \mathcal{L}_f + \mathcal{L}_g + \mathcal{L}_s + \mathcal{L}_y, \tag{8}$$

with

$$\mathcal{L}_s = |D_\mu \Phi_{b'}|^2 + |D_\mu \Phi_{t'}|^2 + |D_\mu \Phi_t|^2 - V, \tag{9}$$

and

$$-\mathcal{L}_y = y_{b'} \bar{\psi}^{(4)}_L b'_R \tilde{\Phi}_{b'} + y_{t'} \bar{\psi}^{(4)}_L t'_R \Phi_{t'} + y_t \bar{\psi}^{(3)}_L t_R \Phi_t + (\text{H.c.}), \tag{10}$$

where V is the Higgs potential and $\Phi_{t',b',t}$ are the renormalized Higgs fields. Taking into account the renormalization group (RG) improved analysis, which will be presented in the next section, we study the following Higgs potential:

$$V = V_2 + V_4, \tag{11}$$

with

$$V_2 = M^2_{\Phi_{b'}}(\Phi^\dagger_{b'}\Phi_{b'}) + M^2_{\Phi_{t'}}(\Phi^\dagger_{t'}\Phi_{t'}) + M^2_{\Phi_t}(\Phi^\dagger_t\Phi_t) + M^2_{\Phi_{t'}\Phi_{b'}}(\Phi^\dagger_{t'}\Phi_{b'}) + M^2_{\Phi_{b'}\Phi_t}(\Phi^\dagger_{b'}\Phi_t) + M^2_{\Phi_{t'}\Phi_t}(\Phi^\dagger_{t'}\Phi_t) + (\text{H.c.}), \tag{12}$$

$$V_4 = \lambda_1(\Phi^\dagger_{b'}\Phi_{b'})^2 + \lambda_2(\Phi^\dagger_{t'}\Phi_{t'})^2 + \lambda_3(\Phi^\dagger_{b'}\Phi_{b'})(\Phi^\dagger_{t'}\Phi_{t'}) + \lambda_4|\Phi^\dagger_{b'}\Phi_{t'}|^2 + \frac{1}{2}[\lambda_5(\Phi^\dagger_{b'}\Phi_{t'})^2 + (\text{H.c.})] + \lambda_t(\Phi^\dagger_t\Phi_t)^2. \tag{13}$$

While $M^2_{\Phi_{b'}}$ and $M^2_{\Phi_{t'}}$ are negative, the mass square $M^2_{\Phi_t}$ is positive, which reflects a subcritical dynamics of the t quark. The top-Higgs Φ_t acquires a vacuum expectation value only due to its mixing with $\Phi_{t'}$ (as was already indicated above, we assume that its mixing with $\Phi_{b'}$ is negligible).

The bubble approximation yields the following Yukawa couplings

$$y_{q'}(\mu) \equiv y_{t'}(\mu) = y_{b'}(\mu) = \left(\frac{N}{16\pi^2}\ln\frac{(\Lambda^{(4)})^2}{\mu^2}\right)^{-1/2}, \tag{14}$$

$$y_t(\mu) = \left(\frac{N}{16\pi^2}\ln\frac{(\Lambda^{(3)})^2}{\mu^2}\right)^{-1/2}, \tag{15}$$

the Higgs mass terms,

$$M^2_{\Phi_{t'}}(\mu) = y^2_{t'}\left[M^2_{\Phi^{(0)}_{t'}} - \frac{N}{8\pi^2}((\Lambda^{(4)})^2 - \mu^2)\right], \tag{16}$$

$$M^2_{\Phi_{b'}}(\mu) = y^2_{b'}\left[M^2_{\Phi^{(0)}_{b'}} - \frac{N}{8\pi^2}((\Lambda^{(4)})^2 - \mu^2)\right], \tag{17}$$

$$M^2_{\Phi_t}(\mu) = y^2_t\left[M^2_{\Phi^{(0)}_t} - \frac{N}{8\pi^2}((\Lambda^{(3)})^2 - \mu^2)\right], \tag{18}$$

$$M^2_{\Phi_{t'}\Phi_{b'}} = y_{t'}y_{b'}M^2_{\Phi^{(0)}_{t'}\Phi^{(0)}_{b'}}, \tag{19}$$

$$M^2_{\Phi_{t'}\Phi_t} = y_{t'}y_t M^2_{\Phi^{(0)}_{t'}\Phi^{(0)}_t}, \tag{20}$$

$$M^2_{\Phi_{b'}\Phi_t} = y_t y_{b'} M^2_{\Phi^{(0)}_{b'}\Phi^{(0)}_t}, \tag{21}$$

and the Higgs quartic couplings,

$$\lambda_1 = \lambda_2 = \frac{\lambda_3}{2} = -\frac{\lambda_4}{2} = y^2_{q'}, \qquad \lambda_5 = 0, \tag{22}$$

$$\lambda_t = y_t^2, \tag{23}$$

where $N(=3)$ denotes the color number, μ is a renormalization scale, and $\Lambda^{(3),(4)}$ are the composite scales for the top and the fourth family quarks, respectively. For details, see Appendix A.

While the structure of the mass term part V_2 is general for three Higgs doublet models, the V_4 part is presented as the sum of the potential for the two Higgs doublets $\Phi_{t'}$ and $\Phi_{b'}$ and that for the doublet Φ_t, i.e., it reflects the 2 + 1 structure of the present model. For the most general three Higgs potential, see Appendix B.

As far as we ignore the electroweak (EW) gauge interactions, the terms breaking the (2 + 1)-Higgs structure, such as $(\Phi_{t'}^\dagger \Phi_{t'})(\Phi_t^\dagger \Phi_t)$, are not generated by the one-loop diagrams. The (2 + 1)-Higgs approximation should work well even in the numerical analysis: We expect that the errors connected with this approximation is at most around few %, and hence they are less than a 10% level uncertainty of nonperturbative effects, which will be discussed in the next section. Note that while the $1/N$-leading approximation, including the QCD effects, is qualitatively reasonable, it is not good quantitatively, with errors around 30% level.

In passing, because the NJL model is used, eight-Fermi interactions, such as $|\bar{\psi}_L^{(4)} t_R'|^2 |\bar{\psi}_L^{(3)} t_R|^2$, are ignored in the present approach. This point is also important for keeping the (2 + 1)-Higgs structure.

B. The structure of the vacuum expectation values

Let us analyze the VEV structure and the mass spectrum of the fourth family quarks and the Higgs bosons.

We define the components of the Higgs fields by

$$\Phi_X = \begin{pmatrix} \frac{1}{\sqrt{2}}(v_X + h_X - i z_X) \\ -\omega_X^- \end{pmatrix}, \qquad \tilde{\Phi}_X \equiv -i\tau_2 \Phi_X^*, \tag{24}$$

where $X = b', t', t$. Note that the relation

$$v^2 = v_{b'}^2 + v_{t'}^2 + v_t^2, \tag{25}$$

holds, where $v \simeq 246$ GeV. It is convenient to introduce the ratio of VEVs,

$$\tan\beta_4 \equiv \frac{v_{t'}}{v_{b'}}, \qquad \tan\beta_{34} \equiv \frac{v_t}{\sqrt{v_{t'}^2 + v_{b'}^2}}, \tag{26}$$

i.e.,

$$v_{b'} = v \cos\beta_4 \cos\beta_{34}, \tag{27}$$

$$v_{t'} = v \sin\beta_4 \cos\beta_{34}, \tag{28}$$

$$v_t = v \sin\beta_{34}. \tag{29}$$

The notations $s_{\beta_4} \equiv \sin\beta_4$, $s_{\beta_{34}} \equiv \sin\beta_{34}$, etc., will be used. The quark masses are [compare with Eq. (10)]:

$$m_{b'} = \frac{v_{b'}}{\sqrt{2}} y_{b'}(\mu = m_{b'}), \tag{30}$$

$$m_{t'} = \frac{v_{t'}}{\sqrt{2}} y_{t'}(\mu = m_{t'}), \tag{31}$$

$$m_t = \frac{v_t}{\sqrt{2}} y_t(\mu = m_t). \tag{32}$$

Since we expect $\Lambda^{(4)} \sim \Lambda^{(3)}$, the Yukawa couplings are almost the same, $y_{t'}(\mu = m_{t'}) \simeq y_{b'}(\mu = m_{b'}) \sim y_t(\mu = m_t)$. The T-parameter constraint suggests that $m_{t'} \simeq m_{b'}$ is favorable, so that the phenomenological condition $m_{t'} \simeq m_{b'} \gtrsim m_t$ requires $v_{t'} \simeq v_{b'} \gtrsim v_t$, i.e.,

$$\tan\beta_4 \simeq 1, \qquad \tan\beta_{34} \lesssim 1. \tag{33}$$

To obtain $\tan\beta_{34} \lesssim 1$, the subcritical dynamics for the t quark, leading to $M_{\Phi_t}^2 > 0$, is crucial [14].

Let us analyze the VEV structure and how we can obtain the desirable solution. The effective potential expressed through the VEVs is given by

$$\begin{aligned} V_{\text{eff}} = {} & \tfrac{1}{2} M_{\Phi_{b'}}^2 v_{b'}^2 + \tfrac{1}{2} M_{\Phi_{t'}}^2 v_{t'}^2 + \tfrac{1}{2} M_{\Phi_t}^2 v_t^2 + M_{\Phi_{t'}\Phi_{b'}}^2 v_{t'} v_{b'} \\ & + M_{\Phi_{b'}\Phi_t}^2 v_{b'} v_t + M_{\Phi_{t'}\Phi_t}^2 v_{t'} v_t + \tfrac{1}{4}\lambda_1 v_{b'}^4 + \tfrac{1}{4}\lambda_2 v_{t'}^4 \\ & + \tfrac{1}{4}(\lambda_3 + \lambda_4 + \lambda_5) v_{t'}^2 v_{b'}^2 + \tfrac{1}{4}\lambda_t v_t^4, \end{aligned} \tag{34}$$

so that the stationary conditions are

$$\begin{aligned} \frac{\partial V_{\text{eff}}}{\partial v_{b'}} = {} & M_{\Phi_{b'}}^2 v_{b'} + M_{\Phi_{t'}\Phi_{b'}}^2 v_{t'} + M_{\Phi_{b'}\Phi_t}^2 v_t + \lambda_1 v_{b'}^3 \\ & + \frac{1}{2}(\lambda_3 + \lambda_4 + \lambda_5) v_{b'} v_{t'}^2 = 0, \end{aligned} \tag{35}$$

$$\begin{aligned} \frac{\partial V_{\text{eff}}}{\partial v_{t'}} = {} & M_{\Phi_{t'}}^2 v_{t'} + M_{\Phi_{t'}\Phi_{b'}}^2 v_{b'} + M_{\Phi_{t'}\Phi_t}^2 v_t + \lambda_2 v_{t'}^3 \\ & + \frac{1}{2}(\lambda_3 + \lambda_4 + \lambda_5) v_{t'} v_{b'}^2 = 0, \end{aligned} \tag{36}$$

$$\frac{\partial V_{\text{eff}}}{\partial v_t} = M_{\Phi_t}^2 v_t + M_{\Phi_{t'}\Phi_t}^2 v_{t'} + M_{\Phi_{b'}\Phi_t}^2 v_{b'} + \lambda_t v_t^3 = 0. \tag{37}$$

In order to obtain the approximate solution with $v_{t'} \simeq v_{b'} \gtrsim v_t$, we assume

$$|M_{\Phi_{t'}}^2| \approx |M_{\Phi_{b'}}^2| \gtrsim \frac{v_t}{v_{t'}} |M_{\Phi_{t'}\Phi_t}^2|, \qquad \frac{v_t}{v_{b'}} |M_{\Phi_{b'}\Phi_t}^2|. \tag{38}$$

These assumptions are easily satisfied in our dynamical model. If we further impose

$$M_{\Phi_t}^2 \gg \lambda_t v_t^2, \tag{39}$$

and

$$|M_{\Phi_{t'}\Phi_t}^2| v_{t'} \gg \lambda_t v_t^3, \tag{40}$$

the solution is approximately given by

$$[\lambda_1 + \tfrac{1}{2}(\lambda_3 + \lambda_4 + \lambda_5)\tan^2\beta_4]v_{b'}^2 \simeq -M_{\Phi_{b'}}^2 - M_{\Phi_{t'}\Phi_{b'}}^2 \tan\beta_4, \tag{41}$$

$$[\lambda_2 + \tfrac{1}{2}(\lambda_3 + \lambda_4 + \lambda_5)\cot^2\beta_4]v_{t'}^2 \simeq -M_{\Phi_{t'}}^2 - M_{\Phi_{t'}\Phi_{b'}}^2 \cot\beta_4, \tag{42}$$

$$v_t \simeq \frac{-M_{\Phi_{t'}\Phi_t}^2}{M_{\Phi_t}^2} v_{t'} + \frac{-M_{\Phi_{b'}\Phi_t}^2}{M_{\Phi_t}^2} v_{b'}. \tag{43}$$

The last equation essentially determines $\tan\beta_{34}$.

C. Mass spectrum of the Higgs bosons

We now analyze the mass spectrum of the Higgs bosons.

The formulas for the masses of the CP even Higgs bosons are quite complicated, because they are the eigenvalues of the 3×3 matrices. Even for the CP odd and charged Higgs bosons, the mass formulas are still not quite simple (for the analytic formulas, see Appendix C). In order to make the physical meaning of the dynamics more transparent, here we will consider approximate and useful expressions.

The T-parameter constraint suggests $\tan\beta_4 \simeq 1$. At the zeroth approximation, we may take exactly $\tan\beta_4 = 1$. As was pointed out in Sec. II, we may further assume $M_{\Phi_{b'}\Phi_t}^2 \approx 0$. We also find $\lambda_5 = 0$ (see Sec. III A above).

The mass of the charged top-Higgs boson, which mainly couples to the top and bottom, should be constrained by R_b and, therefore, should be rather heavy. We thus conclude that each of the heaviest CP even, CP odd and charged Higgs bosons are mainly provided by the top-Higgs doublet Φ_t.

Then the mass eigenvalues are approximately given by

$$M_{A_1}^2 \simeq -2M_{\Phi_{t'}\Phi_{b'}}^2(1 - \tan^2\beta_{34}), \tag{44}$$

$$M_{A_2}^2 \simeq M_{\Phi_t}^2(1 + 2\tan^2\beta_{34}) + M_{A_1}^2\tan^2\beta_{34}, \tag{45}$$

$$M_{H_1^\pm}^2 \simeq M_{A_1}^2 - \tfrac{1}{2}\lambda_4 v^2 c_{\beta_{34}}^2(1 - \tan^2\beta_{34}), \tag{46}$$

$$M_{H_2^\pm}^2 \simeq M_{A_2}^2 - \tfrac{1}{2}\lambda_4 v^2 s_{\beta_{34}}^2, \tag{47}$$

up to $\mathcal{O}(\tan^2\beta_{34})$. Here for the CP odd Higgs bosons and for the charged Higgs bosons, we defined $M_{A_1} \le M_{A_2}$ and $M_{H_1^\pm} \le M_{H_2^\pm}$, respectively. For the CP even Higgs bosons, we defined $M_{H_1} \le M_{H_2} \le M_{H_3}$. As was indicated above, the heavy Higgs bosons, $H_2^\pm$, A_2, and H_3, consist mainly of the components of the top-Higgs Φ_t.

The stationary condition (43) approximately read

$$\frac{-M_{\Phi_{t'}\Phi_t}^2}{M_{\Phi_t}^2} \approx \sqrt{2}\tan\beta_{34}, \tag{48}$$

where we took $M_{\Phi_{b'}\Phi_t}^2 = 0$. By using Eq. (22), Eqs. (44)–(47), $v^2 c_{34}^2 = v_{t'}^2 + v_{b'}^2$, and $-\lambda_4 v_{t'(b')}^2 = 4m_{t'(b')}^2$ in the bubble approximation, we also find the charged Higgs masses as

$$M_{H_1^\pm}^2 \approx M_{A_1}^2 + 2(m_{t'}^2 + m_{b'}^2)(1 - \tan^2\beta_{34}), \tag{49}$$

$$M_{H_2^\pm}^2 \approx M_{A_2}^2 + 2(m_{t'}^2 + m_{b'}^2)\tan^2\beta_{34}. \tag{50}$$

The upper bound of M_{A_1} for a given value of M_{A_2} is discussed in Appendix C.

There are eight parameters in the initial NJL model: six NJL couplings and two composite scales, $\Lambda^{(3,4)}$. As we discussed above, these parameters are closely connected with physical observables. The values of $\Lambda^{(3,4)}$ determine the Yukawa couplings. Then, by using the experimental value of m_t, we can find v_t. Fixing the value of $\tan\beta_4$, we can determine $v_{t'}$ and $v_{b'}$ through Eq. (25), and thereby can express $m_{t'}$ and $m_{b'}$ through the Yukawa couplings. The masses $M_{\Phi_{t'}\Phi_{b'}}^2$ and $M_{\Phi_t}^2$ are connected with $M_{A_1}^2$ and $M_{A_2}^2$, respectively. The value of $M_{\Phi_{t'}\Phi_t}^2/M_{\Phi_t}^2$ is approximately given by $v_t/v_{t'}$, if we assume $M_{\Phi_{b'}\Phi_t}^2 \approx 0$, as we already did.

In summary, it is convenient to take the following eight parameters instead of the original theoretical ones:

$$v(= 246\ \text{GeV}), \quad m_t(= 171.2\ \text{GeV}), \quad \tan\beta_4(\simeq 1), \quad M_{A_1}, \quad M_{A_2}, \quad \Lambda^{(3)}, \quad \Lambda^{(4)}, \quad M_{\Phi_{b'}\Phi_t}^2(\approx 0). \tag{51}$$

In the next section, we will perform a numerical analysis.

IV. NUMERICAL ANALYSIS

The analysis in the previous section was somewhat schematic. In this section, in order to describe the dynamics in the model more precisely, we will employ the RGE's with the compositeness conditions [11,16]:

$$y_{t'}^2(\mu = \Lambda^{(4)}) = \infty, \quad y_{b'}^2(\mu = \Lambda^{(4)}) = \infty, \quad y_t^2(\mu = \Lambda^{(3)}) = \infty, \tag{52}$$

and

$$\left.\frac{\lambda_1}{y_{b'}^4}\right|_{\mu=\Lambda^{(4)}} = 0, \quad \left.\frac{\lambda_2}{y_{t'}^4}\right|_{\mu=\Lambda^{(4)}} = 0, \quad \left.\frac{\lambda_3}{y_{b'}^2 y_{t'}^2}\right|_{\mu=\Lambda^{(4)}} = 0, \quad \left.\frac{\lambda_4}{y_{b'}^2 y_{t'}^2}\right|_{\mu=\Lambda^{(4)}} = 0, \quad \left.\frac{\lambda_t}{y_t^4}\right|_{\mu=\Lambda^{(3)}} = 0. \tag{53}$$

The RGE's are similar to those for the two Higgs doublet model (THDM) type II [17]. For consistency with the (2 + 1)-Higgs structure, we ignore the one-loop effects of the EW interactions, which should be tiny. On the other

hand, although the Higgs loop effects are of the $1/N$-subleading order, we incorporate them, because they are numerically relevant.

The RGE for the QCD coupling is given by

$$(16\pi^2)\mu\frac{\partial}{\partial\mu}g_3 = -c_3 g_3^3, \qquad c_3 = 11 - \frac{4}{3}N_g, \tag{54}$$

where N_g denotes the number of generations (families). The RGE's for Yukawa couplings are

$$(16\pi^2)\mu\frac{\partial}{\partial\mu}y_{b'} = -8g_3^2 y_{b'} + \frac{9}{2}y_{b'}^3 + \frac{1}{2}y_{t'}^2 y_{b'}, \tag{55a}$$

$$(16\pi^2)\mu\frac{\partial}{\partial\mu}y_{t'} = -8g_3^2 y_{t'} + \frac{9}{2}y_{t'}^3 + \frac{1}{2}y_{b'}^2 y_{t'}, \tag{55b}$$

$$(16\pi^2)\mu\frac{\partial}{\partial\mu}y_{t} = -8g_3^2 y_{t} + \frac{9}{2}y_{t}^3, \tag{55c}$$

where we ignored the bottom Yukawa coupling y_b and the EW loop effects in order to keep the (2 + 1)-Higgs structure. On the other hand, the RGE's for the Higgs quartic self-couplings are

$$(16\pi^2)\mu\frac{\partial}{\partial\mu}\lambda_1 = 24\lambda_1^2 + 2\lambda_3^2 + 2\lambda_3\lambda_4 + \lambda_4^2 + 12\lambda_1 y_{b'}^2 - 6y_{b'}^4, \tag{56}$$

$$(16\pi^2)\mu\frac{\partial}{\partial\mu}\lambda_2 = 24\lambda_2^2 + 2\lambda_3^2 + 2\lambda_3\lambda_4 + \lambda_4^2 + 12\lambda_2 y_{t'}^2 - 6y_{t'}^4, \tag{57}$$

$$(16\pi^2)\mu\frac{\partial}{\partial\mu}\lambda_3 = 2(\lambda_1 + \lambda_2)(6\lambda_3 + 2\lambda_4) + 4\lambda_3^2 + 2\lambda_4^2 + 6\lambda_3(y_{b'}^2 + y_{t'}^2) - 12y_{b'}^2 y_{t'}^2, \tag{58}$$

$$(16\pi^2)\mu\frac{\partial}{\partial\mu}\lambda_4 = 4(\lambda_1 + \lambda_2)\lambda_4 + 4(2\lambda_3 + \lambda_4)\lambda_4 + 6\lambda_4(y_{b'}^2 + y_{t'}^2) + 12y_{b'}^2 y_{t'}^2, \tag{59}$$

$$(16\pi^2)\mu\frac{\partial}{\partial\mu}\lambda_5 = \lambda_5[4(\lambda_1 + \lambda_2) + 8\lambda_3 + 12\lambda_4 + 6(y_{t'}^2 + y_{b'}^2)], \tag{60}$$

$$(16\pi^2)\mu\frac{\partial}{\partial\mu}\lambda_t = 24\lambda_t^2 + 12\lambda_t y_t^2 - 6y_t^4, \tag{61}$$

where we ignored the EW loop effects. Note that the coupling constants λ_1 and λ_2 that we use are twice larger than those in Ref. [17]. In our model, we find $\lambda_5 = 0$.

Since we impose the same compositeness condition for t' and b', and because the RGE's for $y_{t'}$ and $y_{b'}$ are the same, the $SU(2)_{R4}$ symmetry, which is the symmetry between t'_R and b'_R, is exact for both the Yukawa and Higgs quartic couplings, as far as the EW interactions are ignored. The $SU(2)_{R4}$ breaking effects appear only from the Higgs mass mixing terms. This leads to $v_{t'} \neq v_{b'}$ in general, and thereby the mass difference between the t' and b' quarks can arise.

For the numerical calculations, we vary M_{A_1}, M_{A_2}, $\Lambda^{(3)}$, $\Lambda^{(4)}$, $\tan\beta_4$, and, as an input, use $v = 246$ GeV and the $\overline{\text{MS}}$-mass $m_t = 161.8$ GeV. The latter corresponds to the pole mass $M_t = 171.2$ GeV [5]. We also use the QCD coupling constant $\alpha_3(M_Z) = 0.1176$ [5]. As for $M^2_{\Phi_{b'}\Phi_t}$, we fix $M^2_{\Phi_{b'}\Phi_t} = 0$. Numerically, it is consistent with $G_{b't} \approx 0$.

The results are illustrated in Figs. 1–3. The masses of t' and b' are essentially determined by the value of $\Lambda^{(4)}$, where we converted the $\overline{\text{MS}}$-masses $m_{t'}$ and $m_{b'}$ to the on-shell ones, $M_{t'(b')} = m_{t'(b')}[1 + 4\alpha_s/(3\pi)]$. As is seen in Fig. 1, their dependence on $\Lambda^{(3)}/\Lambda^{(4)}(= 1 - 2)$ is mild. When we vary $\tan\beta_4$ in the interval 0.9–1.1, the variations of $M_{t'}$ and $M_{b'}$ are up to 10% (see Fig. 1).

The Higgs masses are sensitive to the value of $\Lambda^{(4)}$ (see Fig. 2), while their sensitivity to $\Lambda^{(3)}/\Lambda^{(4)}(= 1 - 2)$ is low. Note also that the Higgs mass dependence on $\tan\beta_4$ is mild, at most 5% for $\tan\beta_4 = 0.9$–1.1, $\Lambda^{(4)} = 2$–10 TeV, and $\Lambda^{(3)}/\Lambda^{(4)} = 1.5$.

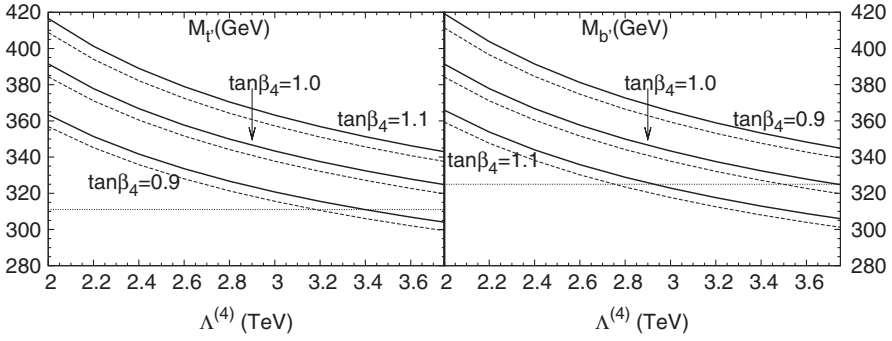

FIG. 1. $M_{t'}$ and $M_{b'}$. The bold and dashed curves are for $\Lambda^{(3)}/\Lambda^{(4)} = 1$, 2, respectively. The dotted lines correspond to the lower bounds for the masses of t' and b' at 95% C.L., $M_{t'} > 311$ GeV and $M_{b'} > 325$ GeV [22].

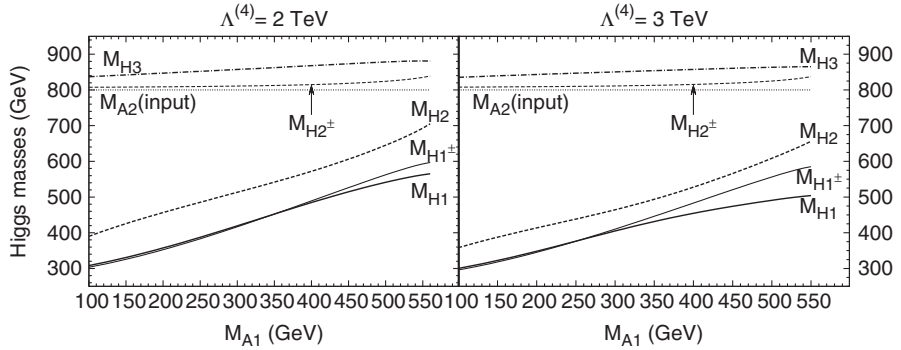

FIG. 2. Mass spectrum of the Higgs bosons for $\Lambda^{(4)} = 2$, 3 TeV. We took $\Lambda^{(3)}/\Lambda^{(4)} = 1.5$ and $\tan\beta_4 = 1$. $M_{A_2} = 800$ GeV is the input. The $2 + 1$ structure in the model is clearly manifested.

It is noticeable that the masses of the $H_2^\pm$ and H_3 Higgs bosons are close and correlate with the mass of the A_2 boson, as shown in Fig. 3. This point agrees with that we identified these heaviest bosons mostly with the top-Higgs doublet Φ_t: it reflects a subcritical dynamics of the t quark. Last but not least, Figs. 2 and 3 clearly illustrate the $2 + 1$ structure of the model.

Since at the compositeness scale the Yukawa couplings go to infinity, there could in principle be uncontrollable nonperturbative effects. In order to estimate them, we studied the RGE's with relaxed compositeness conditions:

$$y_{t'}^2(\mu = \Lambda^{(4)}) = y_\Lambda^2, \qquad y_{b'}^2(\mu = \Lambda^{(4)}) = y_\Lambda^2,$$
$$y_t^2(\mu = \Lambda^{(3)}) = y_\Lambda^2, \qquad y_\Lambda^2 < \infty. \tag{62}$$

For concreteness, we took $y_\Lambda^2 = 25$. It was found that such nonperturbative effects are around $\mathcal{O}(10\%)$, while the loop effects of the EW interactions are expected to be at most $\mathcal{O}(\text{few}\%)$. In fact, the sensitivity of $M_{t'}$ and $M_{b'}$ on y_Λ^2 is 20%–10% for $\Lambda^{(4)} = 2$–10 TeV. On the other hand, the mass M_{H_1} of the H_1 Higgs boson varies about 20% for $\Lambda^{(4)} = 2$–10 TeV, while the sensitivity of the masses of other Higgs bosons is at most 5%. Taking into account these uncertainties, one can safely ignore the EW one-loop corrections.

Since the two charged Higgs bosons couple to t and b quarks, their masses are severely constrained by R_b. Moreover, because in our model $M_{H_{1,2}^\pm}$ are determined by $M_{A_{1,2}}$, it leads to a constraint for $M_{A_{1,2}}$. The 2σ-bound of R_b yields $M_{A_2} \geq 0.70, 0.58, 0.50$ TeV for $\Lambda^{(4)} = 2, 5, 10$ TeV, $\Lambda^{(3)}/\Lambda^{(4)} = 1.5$, and $M_{A_1} > 0.1$ TeV. We note that the above constraint for M_{A_2} is not very sensitive to the values of M_{A_1} and $\Lambda^{(3)}/\Lambda^{(4)}$.

The S and T parameters for a multiple Higgs doublet model are analyzed in Ref. [18]. In our model, the Higgs contributions are $S_h = 0$–0.1 and $T_h = -0.02$–0.2 for $\Lambda^{(4)} = 2$–10 TeV, $\Lambda^{(3)}/\Lambda^{(4)} = 1$–2, $0.1 < M_{A_1} < 0.6$ TeV, and $0.5 < M_{A_2} < 0.8$ TeV. Since the Higgs contribution to the T-parameter is slightly negative, the mass differences of the fermions, depending on the values of T_h, are allowed. For example, following the (S, T) analysis a la LEP EWWG [19], we found that our model is within the 95% C.L. contour of the (S, T) constraint, when the fourth family lepton masses are $M_{\tau'} - M_{\nu'} \sim 150$ GeV.

A noticeable feature of the presence of the fourth family is that because of the extra loop contributions of t' and b', the lightest CP even Higgs boson production via the gluon fusion is considerably enhanced. For example, for $\Lambda^{(4)} = 3$ TeV, $\Lambda^{(3)}/\Lambda^{(4)} = 1.5, \tan\beta_4 = 1, M_{A_1} = 0.50$ TeV, and

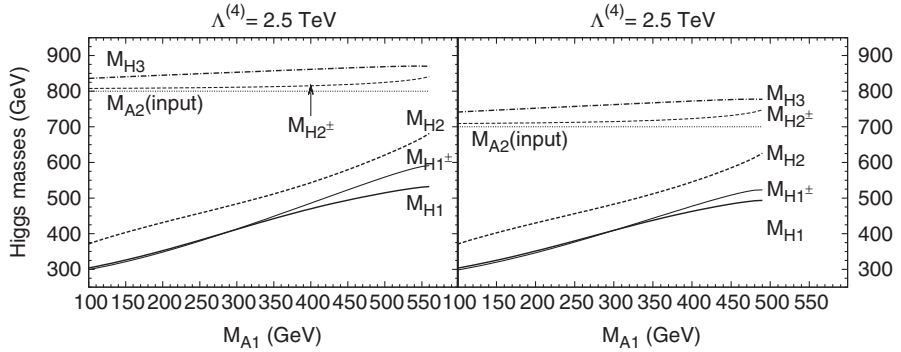

FIG. 3. Mass spectrum of the Higgs bosons for $\Lambda^{(4)} = 2.5$ TeV. We took $\Lambda^{(3)}/\Lambda^{(4)} = 1.5$ and $\tan\beta_4 = 1$. The CP odd Higgs masses $M_{A_2} = 800$, 700 GeV are also the inputs. The $2 + 1$ structure in the model is clearly manifested.

$M_{A_2} = 0.80$ TeV, we obtain $M_{t'} = M_{b'} = 0.33$ TeV and $M_{H_1} = 0.49$ TeV. In this case, the enhancement factor of $\sigma_{gg\to H_1}\mathrm{Br}(H_1 \to ZZ)$ to the SM value is 5.1, where the relative H_1ZZ and $H_1t\bar{t}$ couplings to the SM values are 0.86 and 2.0, respectively. Similarly, the CP odd Higgs production via the gluon fusion should be also enhanced, compared with $gg \to A$ in the two Higgs doublet model.

V. CKM STRUCTURE AND FLAVOR-CHANGING NEUTRAL CURRENT PROCESSES

We use the same approach to constructing the CKM matrix as in Ref. [14]. The Yukawa interactions take the form

$$-\mathcal{L}_Y = \sum_{i,j} \bar{\psi}_L^{(i)} Y_D^{ij} d_R^{(j)} \tilde{\Phi}_{b'} + \sum_{i,j} \bar{\psi}_L^{(i)} Y_U^{ij} u_R^{(j)} \Phi_{t'} + y_t \bar{\psi}_L^{(3)} t_R \Phi_t, \tag{63}$$

where

$$Y_D \equiv \frac{\sqrt{2}}{v_{b'}} M_D, \qquad Y_U \equiv \frac{\sqrt{2}}{v_{t'}} M_U, \tag{64}$$

and

$$M_D = \begin{pmatrix} m_d & \xi_{12} m_d & \xi_{13} m_d & \xi_{14} m_d \\ \xi_{21} m_d & m_s & \xi_{23} m_s & \xi_{24} m_s \\ \xi_{31} m_d & \xi_{32} m_s & m_b & \xi_{34} m_s \\ \xi_{41} m_d & \xi_{42} m_s & \xi_{43} m_s & m_{b'} \end{pmatrix}, \tag{65}$$

$$M_U = \begin{pmatrix} m_u & \eta_{12} m_u & \eta_{13} m_u & \eta_{14} m_u \\ \eta_{21} m_u & m_c & \eta_{23} m_c & \eta_{24} m_c \\ \eta_{31} m_u & \eta_{32} m_c & \eta_{33} m_c & \eta_{34} m_c \\ \eta_{41} m_u & \eta_{42} m_c & \eta_{43} m_c & m_{t'} \end{pmatrix}. \tag{66}$$

In accordance with the essence of the composite (2 + 1)-Higgs model, we assumed that the top-Higgs is responsible for the top mass.

The CKM matrix is approximately given by

$$V_{\mathrm{CKM}}^{4\times 4} \simeq \begin{pmatrix} 1 - \frac{|\xi_{12}|^2}{2}\left(\frac{m_d}{m_s}\right)^2 & \xi_{12}\frac{m_d}{m_s} & \xi_{13}\frac{m_d}{m_b} & -(\eta_{14} - \eta_{12}\eta_{24})\frac{m_u}{m_{t'}} + \xi_{14}\frac{m_d}{m_{b'}} \\ -\xi_{12}^*\frac{m_d}{m_s} & 1 - \frac{|\xi_{12}|^2}{2}\left(\frac{m_d}{m_s}\right)^2 & \xi_{23}\frac{m_s}{m_b} - \eta_{23}\frac{m_c}{m_t} & -\eta_{24}\frac{m_c}{m_{t'}} \\ -\eta_{23}^*\frac{m_c}{m_t}\cdot\xi_{12}^*\frac{m_d}{m_s} - (\xi_{13}^* - \xi_{12}^*\xi_{23}^*)\frac{m_d}{m_b} & -\xi_{23}^*\frac{m_s}{m_b} + \eta_{23}^*\frac{m_c}{m_t} & 1 & -\eta_{34}\frac{m_c}{m_{t'}} \\ -\eta_{24}^*\frac{m_c}{m_{t'}}\cdot\xi_{12}^*\frac{m_d}{m_s} & \eta_{24}^*\frac{m_c}{m_{t'}} & \eta_{34}^*\frac{m_c}{m_{t'}} & 1 \end{pmatrix}. \tag{67}$$

Notice that $m_d/m_s \sim 0.1 = \mathcal{O}(V_{us})$, $m_s/m_b \sim 0.01 = \mathcal{O}(V_{cb})$, $m_c/m_t \sim 0.01$, and $m_d/m_b \sim 0.001 = \mathcal{O}(V_{ub})$. Thus we can reproduce the CKM structure by taking $\xi_{ij} = \mathcal{O}(1)$. Since the mixing between the fourth family and the others is suppressed, $|V_{t'd}| \sim |V_{us}|m_c/m_{t'} \sim \mathcal{O}(10^{-3})$ and $|V_{t's}| \sim |V_{t'b}| \sim m_c/m_{t'} \sim \mathcal{O}(10^{-2})$, the contribution of the flavor-changing neutral current (FCNC) processes with the fourth family quarks in the B-system is negligible: $m_{t'}^2|V_{t'd}^* V_{t'b}|^2 \sim |V_{us}|^2 m_c^4/m_{t'}^2$ for B_d and $m_{t'}^2|V_{t's}^* V_{t'b}|^2 \sim m_c^4/m_{t'}^2$ for B_s. Similarly, $b \to s\gamma$ and $Z \to \bar{b}b$ via the t'-loop are also suppressed (for a related discussion, see Ref. [14]). As to the contribution of a box diagram with t' in the $\Delta S = 2$ processes in the K system, it is very small due to $m_{t'}^2|V_{t'd}^* V_{t's}|^2 \sim m_c^2|V_{us}|^2 m_c^2/m_{t'}^2$. Note also that the contributions of the charged Higgs bosons are negligible, because their masses are relatively heavy and the mixing angles are small.

On the other hand, a new tree FCNC term appears in the up-quark sector, so that the $D^0 - \bar{D}^0$ mixing is potentially dangerous. Let us estimate this effect. In the basis of the fermion mass eigenstates $U_{L,R}$ and $D_{L,R}$, corresponding to the left and right-handed up-type quarks and the down-type ones, respectively, there appear the tree FCNC and flavor-changing charged current (FCCC) terms in the Higgs sector:

$$\begin{aligned} \mathcal{L}_{\mathrm{FCNC/FCCC}} = &-\frac{m_t}{v_{t'}}\bar{U}_L\tilde{M}U_R(h_{t'} - iz_{t'}) \\ &+ \frac{m_t}{v_t}\bar{U}_L\tilde{M}U_R(h_t - iz_t) + (\mathrm{H.c.}) \\ &+ \sqrt{2}\frac{m_t}{v_{t'}}\bar{U}_R\tilde{M}^\dagger V_{\mathrm{CKM}}^{4\times 4} D_L \omega_{t'}^+ \\ &- \sqrt{2}\frac{m_t}{v_t}\bar{U}_R\tilde{M}^\dagger V_{\mathrm{CKM}}^{4\times 4} D_L \omega_t^+ + (\mathrm{H.c.}), \end{aligned} \tag{68}$$

where the fields $h_{t',t}$, $z_{t',t}$, and $\omega_{t',t}^\pm$ are defined in Eq. (24). The matrix $\tilde{M}$ is

$$\tilde{M}_{ij} \equiv (\mathcal{U}_L)_{3i}^*(\mathcal{U}_R)_{3j}, \tag{69}$$

where

$$\mathcal{U}_L \simeq \begin{pmatrix} 1 & \eta_{12}\frac{m_u}{m_c} & \eta_{13}\frac{m_u}{m_t} & \eta_{14}\frac{m_u}{m_{t'}} \\ -\eta_{12}^*\frac{m_u}{m_c} & 1 & \eta_{23}\frac{m_c}{m_t} & \eta_{24}\frac{m_c}{m_{t'}} \\ -(\eta_{13}^* - \eta_{12}^*\eta_{23}^*)\frac{m_u}{m_t} & -\eta_{23}^*\frac{m_c}{m_t} & 1 & \eta_{34}\frac{m_c}{m_{t'}} \\ -(\eta_{14}^* - \eta_{12}^*\eta_{24}^*)\frac{m_u}{m_{t'}} & -\eta_{24}^*\frac{m_c}{m_{t'}} & -\eta_{34}^*\frac{m_c}{m_{t'}} & 1 \end{pmatrix}, \tag{70}$$

and

$$\mathcal{U}_R \simeq \begin{pmatrix} 1 & \eta_{21}^* \frac{m_u}{m_c} & \eta_{31}^* \frac{m_u}{m_t} & \eta_{41}^* \frac{m_u}{m_{t'}} \\ -\eta_{21} \frac{m_u}{m_c} & 1 & \eta_{32}^* \frac{m_c}{m_t} & \eta_{42}^* \frac{m_c}{m_{t'}} \\ -(\eta_{31} - \eta_{21}\eta_{32}) \frac{m_u}{m_t} & -\eta_{32} \frac{m_c}{m_t} & 1 & \eta_{43}^* \frac{m_c}{m_{t'}} \\ -(\eta_{41} - \eta_{21}\eta_{42}) \frac{m_u}{m_{t'}} & -\eta_{42} \frac{m_c}{m_{t'}} & -\eta_{43} \frac{m_c}{m_{t'}} & 1 \end{pmatrix} \tag{71}$$

are the transformation matrices from the weak basis to the mass eigenstates one for up-type quarks. The dangerous contributions to the $D^0 - \bar{D}^0$ mixing come from the u-c-$h_{t',t}$ couplings and thus they are proportional to

$$Y_{u\text{-}c\text{-}h_{t',t}} \simeq \frac{m_t}{v_{t',t}} \frac{m_u}{m_t} \frac{m_c}{m_t}. \tag{72}$$

Therefore, the corresponding contribution to the $D^0 - \bar{D}^0$ mixing parameter $\Delta m_D / m_D$ is of order

$$\frac{Y^2_{u\text{-}c\text{-}h_{t',t}}}{M^2_{H_{2,3}}} f_D^2 B_D \sim \frac{f_D^2 B_D}{M^2_{H_{2,3}}} \times O(10^{-14}), \tag{73}$$

where f_D is the D meson decay constant, B_D denotes the B parameter, and we ignored the mixing between $h_{t',t}$ and $H_{1,2,3}$. Since the experimental value of the $D^0 - \bar{D}^0$ mixing parameter is $\Delta m_D / m_D \sim O(10^{-14})$ [5] and $f_D \sim O(100\ \text{MeV})$, this tree FCNC contribution is negligible for $M_{H_{2,3}}$ of the order of the EWSB scale. Because of the same reasons, the tree FCCC contribution is also suppressed in the first and second families.

VI. CONCLUSION

The 2 + 1 composite Higgs model is an offspring of the top quark condensate one [9–11] but has much richer and more sophisticated dynamics. As a result, this allows to describe rather naturally both the quark mass hierarchy [14] and EWSB. It is quite nontrivial that this model passes the electroweak precision data constraints. Besides, we can naturally evade the constraint of $Z \to b\bar{b}$, because the top-Higgs is sufficiently heavy.

It is also noticeable that the model has a clear signature: the 2 + 1 structure of the composite Higgs bosons. In the heaviest doublet, the top-Higgs $\Phi_t \sim \bar{t}_R(t, b)_L$ component dominates. As is clearly illustrated by Figs. 2 and 3, the masses of the four resonances in this doublet are nearly degenerate that reflects a subcritical dynamics of the t quark.

Other phenomenological manifestations of the model are the following. The gluon-fusion channel with a decay to two Z bosons should be essentially enhanced. For example, for the parameter set with $\Lambda^{(4)} = 3$ TeV, $\Lambda^{(3)}/\Lambda^{(4)} = 1.5$, $\tan\beta_4 = 1$, $M_{A_1} = 0.50$ TeV, and $M_{A_2} = 0.80$ TeV (which yields $M_{t'} = M_{b'} = 0.33$ TeV and $M_{H_1} = 0.49$ TeV), the enhancement factor of $\sigma_{gg\to H_1}\,\mathrm{Br}(H_1 \to ZZ)$ to the SM value is 5.1. Similarly, the CP odd Higgs production via the gluon fusion should be enhanced as well, compared with a three family model. Also, multiple Higgs bosons may be observed as $t\bar{t}$ resonances at the LHC [20]. Detailed analysis of their LHC signatures will be performed elsewhere.

ACKNOWLEDGMENTS

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APPENDIX A: BUBBLE APPROXIMATION

In the bubble approximation, i.e., the $1/N$-leading approximation neglecting the QCD effects, we can easily obtain the low energy effective theory;

$$\mathcal{L} = \mathcal{L}_f + \mathcal{L}_g + \mathcal{L}_s + \mathcal{L}_y, \tag{A1}$$

with

$$\mathcal{L}_s = |D_\mu \Phi_{b'}|^2 + |D_\mu \Phi_{t'}|^2 + |D_\mu \Phi_t|^2 - V, \tag{A2}$$

$$-\mathcal{L}_y = y_{q'} \bar{\psi}_L^{(4)} \mathcal{M}_{\Phi_{t'}\Phi_{b'}} \psi_R^{(4)} + y_t \bar{\psi}_L^{(3)} t_R \Phi_t + (\text{H.c.}), \tag{A3}$$

and

$$V = V_2 + V_4, \tag{A4}$$

$$\begin{aligned} V_2 = {} & M^2_{\Phi_{b'}}(\Phi^\dagger_{b'}\Phi_{b'}) + M^2_{\Phi_{t'}}(\Phi^\dagger_{t'}\Phi_{t'}) + M^2_{\Phi_t}(\Phi^\dagger_t\Phi_t) \\ & + M^2_{\Phi_{t'}\Phi_{b'}}(\Phi^\dagger_{t'}\Phi_{b'}) + M^2_{\Phi_{t'}\Phi_t}(\Phi^\dagger_{t'}\Phi_t) \\ & + M^2_{\Phi_{b'}\Phi_t}(\Phi^\dagger_{b'}\Phi_t) + (\text{H.c.}), \end{aligned} \tag{A5}$$

$$V_4 = \lambda\,\mathrm{tr}(\mathcal{M}^\dagger_{\Phi_{t'}\Phi_{b'}}\mathcal{M}_{\Phi_{t'}\Phi_{b'}})^2 + \tfrac{1}{2}\lambda_t\,\mathrm{tr}(\mathcal{M}^\dagger_{\Phi_t}\mathcal{M}_{\Phi_t})^2, \tag{A6}$$

where we have already renormalized the composite scalar fields and also defined 2×2 Higgs fields,

$$\mathcal{M}_{\Phi_{t'}\Phi_{b'}} \equiv (\Phi_{t'}\tilde{\Phi}_{b'}), \qquad \mathcal{M}_{\Phi_t} \equiv (\Phi_t\tilde{\Phi}_t), \tag{A7}$$

and the right-handed doublet

$$\psi_R^{(4)} \equiv \begin{pmatrix} t'_R \\ b'_R \end{pmatrix}. \tag{A8}$$

Note that

$$\begin{aligned} \mathrm{tr}\,(\mathcal{M}^\dagger_{\Phi_{t'}\Phi_{b'}}\mathcal{M}_{\Phi_{t'}\Phi_{b'}})^2 = {} & (\Phi^\dagger_{b'}\Phi_{b'})^2 + (\Phi^\dagger_{t'}\Phi_{t'})^2 \\ & + 2(\Phi^\dagger_{b'}\Phi_{b'})(\Phi^\dagger_{t'}\Phi_{t'}) \\ & - 2|\Phi^\dagger_{b'}\Phi_{t'}|^2. \end{aligned} \tag{A9}$$

The renormalized quantities are given by

$$y_{q'}(\mu) \equiv y_{t'}(\mu) = y_{b'}(\mu) = \left(\frac{N}{16\pi^2}\ln\frac{(\Lambda^{(4)})^2}{\mu^2}\right)^{-1/2}, \tag{A10}$$

$$y_t(\mu) = \left(\frac{N}{16\pi^2}\ln\frac{(\Lambda^{(3)})^2}{\mu^2}\right)^{-1/2}, \tag{A11}$$

$$M^2_{\Phi_{t'}}(\mu) = y^2_{t'}\left[M^2_{\Phi_{t'0}} - \frac{N}{8\pi^2}((\Lambda^{(4)})^2 - \mu^2)\right], \tag{A12}$$

$$M^2_{\Phi_{b'}}(\mu) = y^2_{b'}\left[M^2_{\Phi_{b'0}} - \frac{N}{8\pi^2}((\Lambda^{(4)})^2 - \mu^2)\right], \tag{A13}$$

$$M^2_{\Phi_t}(\mu) = y^2_t\left[M^2_{\Phi_{t0}} - \frac{N}{8\pi^2}((\Lambda^{(3)})^2 - \mu^2)\right], \tag{A14}$$

$$M^2_{\Phi_{t'}\Phi_{b'}} = y_{t'}y_{b'}M^2_{\Phi_{t'0}\Phi_{b'0}}, \tag{A15}$$

$$M^2_{\Phi_{t'}\Phi_t} = y_{t'}y_t M^2_{\Phi_{t'0}\Phi_{t0}}, \tag{A16}$$

$$M^2_{\Phi_{b'}\Phi_t} = y_t y_{b'}M^2_{\Phi_{b'0}\Phi_{t0}}, \tag{A17}$$

$$\lambda = y^2_{q'}, \tag{A18}$$

$$\lambda_t = y^2_t. \tag{A19}$$

The part of V_4 has the global symmetry,

$$SU(2)_{L4} \times SU(2)_{R4} \times SU(2)_{Lt} \times SU(2)_{Rt} \times U(1)_A, \tag{A20}$$

where the transformation property is

$$\mathcal{M}_{\Phi_{t'}\Phi_{b'}} \to g_{L4}\mathcal{M}_{\Phi_{t'}\Phi_{b'}}g^\dagger_{R4}, \qquad \mathcal{M}_{\Phi_t} \to g_{Lt}\mathcal{M}_{\Phi_t}g^\dagger_{Rt}, \tag{A21}$$

with $g_{L4} \in SU(2)_{L4}$, $g_{R4} \in SU(2)_{R4}$, $g_{Lt} \in SU(2)_{Lt}$ and $g_{Rt} \in SU(2)_{Rt}$. The hypercharge $U(1)_Y$ is included in the $U(1)$ parts of $SU(2)_{R4}$ and $SU(2)_{Rt}$,

$$\begin{aligned}\mathcal{M}_{\Phi_{t'}\Phi_{b'}} &\to \mathcal{M}_{\Phi_{t'}\Phi_{b'}}e^{-i\theta_Y(\tau_3/2)},\\ \mathcal{M}_{\Phi_t} &\to \mathcal{M}_{\Phi_t}e^{-i\theta_Y(\tau_3/2)},\end{aligned} \tag{A22}$$

and the $U(1)_A$ corresponds to

$$\mathcal{M}_{\Phi_{t'}\Phi_{b'}} \to \mathcal{M}_{\Phi_{t'}\Phi_{b'}}e^{-2i\theta_A}, \qquad \mathcal{M}_{\Phi_t} \to \mathcal{M}_{\Phi_t}e^{-2i\theta_A\tau_3}. \tag{A23}$$

Beyond the bubble approximation, another $SU(2)_{R4}$ symmetric coupling, $\tilde{\lambda}[\mathrm{tr}(\mathcal{M}^\dagger_{\Phi_{t'}\Phi_{b'}}\mathcal{M}_{\Phi_{t'}\Phi_{b'}})]^2$, is generated at low energy. This is the reason why we consider more general expressions in Sec. III.

Since there is no bottom Yukawa coupling, the $SU(2)_{Rt}$ symmetry is explicitly broken down in the Yukawa sector. Moreover, the Higgs mass mixing terms V_2 respect only the $SU(2)_L \times U(1)_Y$ gauge symmetry. Thus the Higgs mass spectrum does not have the $SU(2)_{R4}$ and $SU(2)_{Rt}$ symmetries in general.

APPENDIX B: GENERAL THREE HIGGS DOUBLET RENORMALIZABLE MODEL

Let us consider a potential of a general three Higgs doublet renormalizable model with the scalars $\phi_{1,2,3}$.

The most general potential is

$$V = V_2 + V_4, \tag{B1}$$

where the mass terms,

$$\begin{aligned}V_2 \equiv\ & m^2_1\phi^\dagger_1\phi_1 + m^2_2\phi^\dagger_2\phi_2 + m^2_3\phi^\dagger_3\phi_3\\ &+ [m^2_{12}\phi^\dagger_1\phi_2 + (\mathrm{H.c.})] + [m^2_{23}\phi^\dagger_2\phi_3 + (\mathrm{H.c.})]\\ &+ [m^2_{31}\phi^\dagger_3\phi_1 + (\mathrm{H.c.})],\end{aligned} \tag{B2}$$

and the quartic couplings,

$$\begin{aligned}V_4 \equiv\ & \lambda_{1111}(\phi^\dagger_1\phi_1)^2 + \lambda_{2222}(\phi^\dagger_2\phi_2)^2 + \lambda_{3333}(\phi^\dagger_3\phi_3)^2 + \lambda_{1122}(\phi^\dagger_1\phi_1)(\phi^\dagger_2\phi_2) + \lambda_{2233}(\phi^\dagger_2\phi_2)(\phi^\dagger_3\phi_3) + \lambda_{3311}(\phi^\dagger_3\phi_3)(\phi^\dagger_1\phi_1)\\ &+ \lambda_{1221}|\phi^\dagger_1\phi_2|^2 + \lambda_{2332}|\phi^\dagger_2\phi_3|^2 + \lambda_{3113}|\phi^\dagger_3\phi_1|^2 + [\lambda_{1212}(\phi^\dagger_1\phi_2)^2 + (\mathrm{H.c})] + [\lambda_{2323}(\phi^\dagger_2\phi_3)^2 + (\mathrm{H.c})]\\ &+ [\lambda_{3131}(\phi^\dagger_3\phi_1)^2 + (\mathrm{H.c})] + [\lambda_{1112}(\phi^\dagger_1\phi_1)(\phi^\dagger_1\phi_2) + (\mathrm{H.c})] + [\lambda_{1113}(\phi^\dagger_1\phi_1)(\phi^\dagger_1\phi_3) + (\mathrm{H.c})] + [\lambda_{2221}(\phi^\dagger_2\phi_2)(\phi^\dagger_2\phi_1)\\ &+ (\mathrm{H.c})] + [\lambda_{2223}(\phi^\dagger_2\phi_2)(\phi^\dagger_2\phi_3) + (\mathrm{H.c})] + [\lambda_{3331}(\phi^\dagger_3\phi_3)(\phi^\dagger_3\phi_1) + (\mathrm{H.c})] + [\lambda_{3332}(\phi^\dagger_3\phi_3)(\phi^\dagger_3\phi_2) + (\mathrm{H.c})]\\ &+ [\lambda_{1123}(\phi^\dagger_1\phi_1)(\phi^\dagger_2\phi_3) + (\mathrm{H.c})] + [\lambda_{1213}(\phi^\dagger_1\phi_2)(\phi^\dagger_1\phi_3) + (\mathrm{H.c})] + [\lambda_{1231}(\phi^\dagger_1\phi_2)(\phi^\dagger_3\phi_1) + (\mathrm{H.c})]\\ &+ [\lambda_{2213}(\phi^\dagger_2\phi_2)(\phi^\dagger_1\phi_3) + (\mathrm{H.c})] + [\lambda_{2123}(\phi^\dagger_2\phi_1)(\phi^\dagger_2\phi_3) + (\mathrm{H.c})] + [\lambda_{2132}(\phi^\dagger_2\phi_1)(\phi^\dagger_3\phi_2) + (\mathrm{H.c})]\\ &+ [\lambda_{3312}(\phi^\dagger_3\phi_3)(\phi^\dagger_1\phi_2) + (\mathrm{H.c})] + [\lambda_{3132}(\phi^\dagger_3\phi_1)(\phi^\dagger_3\phi_2) + (\mathrm{H.c})] + [\lambda_{3123}(\phi^\dagger_3\phi_1)(\phi^\dagger_2\phi_3) + (\mathrm{H.c})].\end{aligned} \tag{B3}$$

For the mass terms, the number of the real parameters is

$$N_M = N^2_H - (N_H - 1), \tag{B4}$$

where N_H denotes the number of the Higgs doublets and we used the rephasing degrees of freedom of the Higgs fields. For the quartic couplings, the number of the real parameters is

$$N_Q = \tfrac{1}{2}N_H^2(N_H^2+1). \tag{B5}$$

In particular, the formula for the two and three Higgs doublets yields 10 and 45, which agree with expressions (B2) and (B3) above.

The RGE's for the multi-Higgs models are discussed in Ref. [21]. The S and T parameters for the multi-Higgs doublet model are analyzed in Ref. [18].

APPENDIX C: ANALYTIC MASS FORMULAS FOR $M_{A_{1,2}}$ AND $M_{H^{\pm}_{1,2}}$

Let us define the mixing angles of the CP odd and charged Higgs fields,

$$\begin{pmatrix} z_{b'} \\ z_{t'} \\ z_t \end{pmatrix} = O^A \begin{pmatrix} \pi_z \\ A_1 \\ A_2 \end{pmatrix}, \qquad \begin{pmatrix} w^{\pm}_{b'} \\ w^{\pm}_{t'} \\ w^{\pm}_{t} \end{pmatrix} = U^{H^{\pm}} \begin{pmatrix} \pi^{\pm}_w \\ H^{\pm}_1 \\ H^{\pm}_2 \end{pmatrix}, \tag{C1}$$

where π_z and $\pi^{\pm}_w$ denote the would-be NG bosons eaten by the weak bosons. The mass eigenvalues of the corresponding Higgs mass matrices are $M_{A_{1,2}}$ and $M_{H^{\pm}_{1,2}}$.

Eliminating $M^2_{\Phi_{b'}}$, $M^2_{\Phi_{t'}}$, and $M^2_{\Phi_t}$ by using the stationary conditions, we obtain the mass eigenvalues of the CP odd and charged Higgs bosons:

$$M^2_{A_{1,2}} = \frac{1}{2}\Bigg(\frac{-M^2_{\Phi_{t'}\Phi_{b'}}}{s_{\beta_4}c_{\beta_4}} + \frac{-M^2_{\Phi_{t'}\Phi_t}}{s_{\beta_4}s_{\beta_{34}}c_{\beta_{34}}}(s^2_{\beta_{34}} + s^2_{\beta_4}c^2_{\beta_{34}}) + \frac{-M^2_{\Phi_{b'}\Phi_t}}{c_{\beta_4}s_{\beta_{34}}c_{\beta_{34}}}(s^2_{\beta_{34}} + c^2_{\beta_4}c^2_{\beta_{34}}) \mp M^2_{A_2-A_1}\Bigg), \tag{C2}$$

$$(M^2_{A_2-A_1})^2 \equiv \Bigg(\frac{-M^2_{\Phi_{t'}\Phi_{b'}}}{s_{\beta_4}c_{\beta_4}} + \frac{-M^2_{\Phi_{t'}\Phi_t}}{s_{\beta_4}s_{\beta_{34}}c_{\beta_{34}}}(s^2_{\beta_{34}}c^2_{\beta_4} - s^2_{\beta_4}) + \frac{-M^2_{\Phi_{b'}\Phi_t}}{c_{\beta_4}s_{\beta_{34}}c_{\beta_{34}}}(s^2_{\beta_{34}}s^2_{\beta_4} - c^2_{\beta_4})\Bigg)^2 + \frac{4}{c^2_{\beta_{34}}}(M^2_{\Phi_{t'}\Phi_t}c_{\beta_4} - M^2_{\Phi_{b'}\Phi_t}s_{\beta_4})^2, \tag{C3}$$

and

$$M^2_{H^{\pm}_{1,2}} = \frac{1}{2}\Bigg(\frac{-M^2_{\Phi_{t'}\Phi_{b'}}}{s_{\beta_4}c_{\beta_4}} + \frac{-M^2_{\Phi_{t'}\Phi_t}}{s_{\beta_4}s_{\beta_{34}}c_{\beta_{34}}}(s^2_{\beta_{34}} + s^2_{\beta_4}c^2_{\beta_{34}}) + \frac{-M^2_{\Phi_{b'}\Phi_t}}{c_{\beta_4}s_{\beta_{34}}c_{\beta_{34}}}(s^2_{\beta_{34}} + c^2_{\beta_4}c^2_{\beta_{34}}) - \frac{1}{2}\lambda_4 v^2 c^2_{\beta_{34}} \mp M^2_{H^{\pm}_2-H^{\pm}_1}\Bigg), \tag{C4}$$

$$(M^2_{H^{\pm}_2-H^{\pm}_1})^2 \equiv \Bigg(\frac{-M^2_{\Phi_{t'}\Phi_{b'}}}{s_{\beta_4}c_{\beta_4}} + \frac{-M^2_{\Phi_{t'}\Phi_t}}{s_{\beta_4}s_{\beta_{34}}c_{\beta_{34}}}(s^2_{\beta_{34}}c^2_{\beta_4} - s^2_{\beta_4}) + \frac{-M^2_{\Phi_{b'}\Phi_t}}{c_{\beta_4}s_{\beta_{34}}c_{\beta_{34}}}(s^2_{\beta_{34}}s^2_{\beta_4} - c^2_{\beta_4}) - \frac{1}{2}\lambda_4 v^2 c^2_{\beta_{34}}\Bigg)^2 + \frac{4}{c^2_{\beta_{34}}}(M^2_{\Phi_{t'}\Phi_t}c_{\beta_4} - M^2_{\Phi_{b'}\Phi_t}s_{\beta_4})^2. \tag{C5}$$

Although in principle the analytic formulas for the mass eigenvalues of the CP even Higgs can be derived, they are too complicated and, therefore, not very useful.

For $M^2_{\Phi_{b'}\Phi_t} = 0$, the upper bound of M_{A_1} for a given M_{A_2} is obtained as

$$\frac{M^2_{A_1}}{M^2_{A_2}} < 1 + 2\cot^2\beta_4\sin^2\beta_{34} - 2\cot\beta_4\sin\beta_{34}\sqrt{1+\cot^2\beta_4\sin^2\beta_{34}} \le 1, \tag{C6}$$

where the equality on the right-hand side satisfies only when $\cot\beta_4\sin\beta_{34} = 0$ (by definition, $M_{A_1} \le M_{A_2}$).

The mixing matrices are defined by

$$O^A \equiv (\mathbf{n}_v \mathbf{e}^A_1 \mathbf{e}^A_2), \qquad U^{H^{\pm}} \equiv (\mathbf{n}_v \mathbf{e}^{H^{\pm}}_1 \mathbf{e}^{H^{\pm}}_2), \tag{C7}$$

with

$$\mathbf{n}_v \equiv \left(\frac{v_{b'}}{v}\ \frac{v_{t'}}{v}\ \frac{v_t}{v}\right)^T, \tag{C8}$$

$$= \begin{pmatrix} \cos\beta_4\cos\beta_{34} \\ \sin\beta_4\cos\beta_{34} \\ \sin\beta_{34} \end{pmatrix}, \tag{C9}$$

The analytic formulas for the eigenvectors are

$$\mathbf{e}^X_1 = \cos\eta_X \mathbf{e}_1 - \sin\eta_X \mathbf{e}_2, \tag{C10}$$

$$\mathbf{e}^X_2 = \sin\eta_X \mathbf{e}_1 + \cos\eta_X \mathbf{e}_2, \tag{C11}$$

where $X = A, H^{\pm}$,

$$\mathbf{e}_1 \equiv \begin{pmatrix} -\sin\beta_4 \\ \cos\beta_4 \\ 0 \end{pmatrix}, \qquad \mathbf{e}_2 \equiv \begin{pmatrix} -\cos\beta_4\sin\beta_{34} \\ -\sin\beta_4\sin\beta_{34} \\ \cos\beta_{34} \end{pmatrix}, \tag{C12}$$

$$\tan\eta_X \equiv \frac{M^2_{X_1} - \kappa}{\rho}, \tag{C13}$$

and

$$\rho \equiv c^{-1}_{\beta_{34}}(-M^2_{\Phi_{b'}\Phi_t}s_{\beta_4} + M^2_{\Phi_{t'}\Phi_t}c_{\beta_4}), \tag{C14}$$

$$\kappa \equiv c^{-1}_{\beta_{34}}\left(-M^2_{\Phi_{b'}\Phi_t}\frac{c_{\beta_4}}{s_{\beta_{34}}} - M^2_{\Phi_{t'}\Phi_t}\frac{s_{\beta_4}}{s_{\beta_{34}}}\right). \tag{C15}$$

The approximate expressions for the mixing matrices are:

$$O^A \simeq \begin{pmatrix} \frac{1}{\sqrt{2}}(1-\frac{\tan^2\beta_{34}}{2}) & -\frac{1}{\sqrt{2}}(1+\frac{\tan^2\beta_{34}}{2}) & \frac{M^2_{A_1}}{\sqrt{2}M^2_{A_2}}\tan\beta_{34} \\ \frac{1}{\sqrt{2}}(1-\frac{\tan^2\beta_{34}}{2}) & \frac{1}{\sqrt{2}}(1-\frac{3\tan^2\beta_{34}}{2}) & -\sqrt{2}(1+\frac{M^2_{A_1}}{2M^2_{A_2}})\tan\beta_{34} \\ \tan\beta_{34} & (1+\frac{M^2_{A_1}}{M^2_{A_2}})\tan\beta_{34} & 1-\tan^2\beta_{34} \end{pmatrix}, \tag{C16}$$

and

$$U^{H^\pm} \simeq \begin{pmatrix} \frac{1}{\sqrt{2}}(1-\frac{\tan^2\beta_{34}}{2}) & -\frac{1}{\sqrt{2}}(1+\frac{\tan^2\beta_{34}}{2}) & \frac{M^2_{H^\pm_1}}{\sqrt{2}M^2_{H^\pm_2}}\tan\beta_{34} \\ \frac{1}{\sqrt{2}}(1-\frac{\tan^2\beta_{34}}{2}) & \frac{1}{\sqrt{2}}(1-\frac{3\tan^2\beta_{34}}{2}) & -\sqrt{2}(1+\frac{M^2_{H^\pm_1}}{2M^2_{H^\pm_2}})\tan\beta_{34} \\ \tan\beta_{34} & (1+\frac{M^2_{H^\pm_1}}{M^2_{H^\pm_2}})\tan\beta_{34} & 1-\tan^2\beta_{34} \end{pmatrix}, \tag{C17}$$

up to $\mathcal{O}(\tan^2\beta_{34})$, $\mathcal{O}(\tan\beta_{34}M^2_{A_1}/M^2_{A_2})$, and $\mathcal{O}(\tan\beta_{34}M^2_{H^\pm_1}/M^2_{H^\pm_2})$.

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